

Lecture 12: Convolutional neural networks

Harbour.Space University
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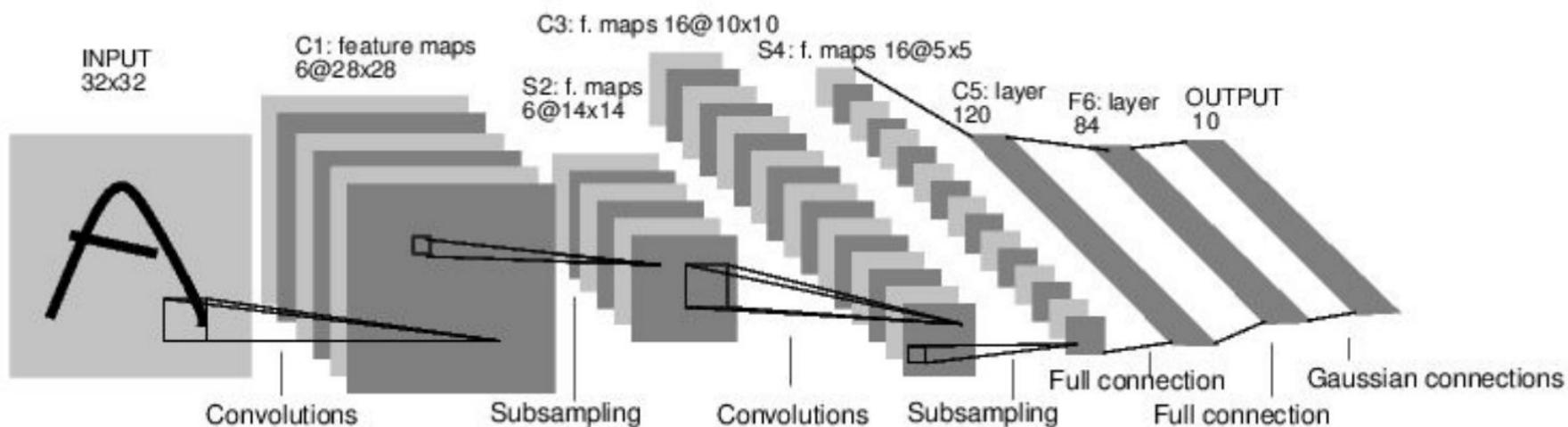
Outline

1. Previous lecture recap.
2. Convolutional layer structure.
3. Pooling layers.
4. Top architectures overview.
5. Q & A.

This lecture is deeply based on [Stanford CS231n Lecture 7](#) materials by Fei-Fei Li & Andrej Karpathy & Justin Johnson

- Layers
 - a. Dense layer (*done*)
 - b. Convolutional layer (*this lecture*)
 - c. Pooling layer (*this lecture*)
 - d. Dropout layer (*done*)
 - e. Batchnorm layer (batch normalization) (*done*)
 - f. Embeddings (aka word2vec, GloVe) (*next lecture*)
 - g. Recurrent layers (*done*)

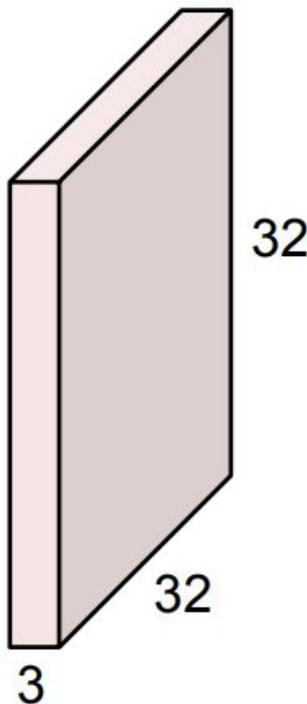
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 - a. Dense layer (*done*)
 - b. Convolutional layer (*this lecture*)
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 - d. Dropout layer (*done*)
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 - f. Embeddings (aka word2vec, GloVe) (*next lecture*)
 - g. Recurrent layers (*done*)



[LeNet-5, LeCun 1998]

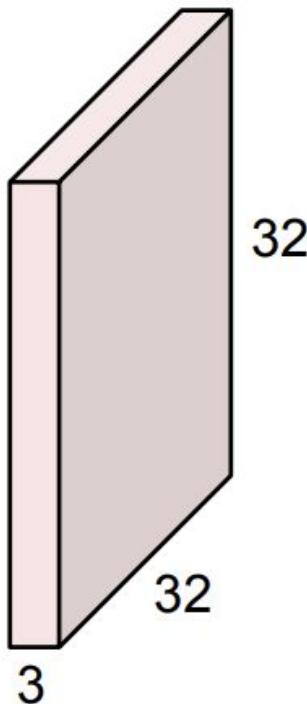
Convolutional layer

32x32x3 image



Convolutional layer

32x32x3 image



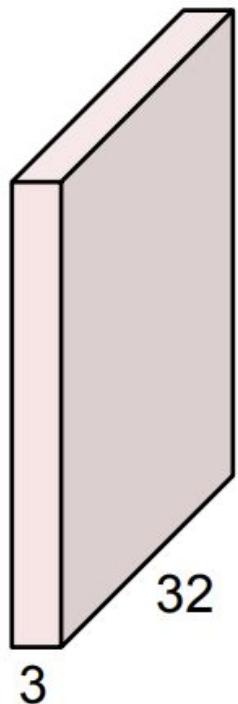
5x5x3 filter



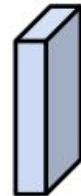
Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolutional layer

32x32x3 image



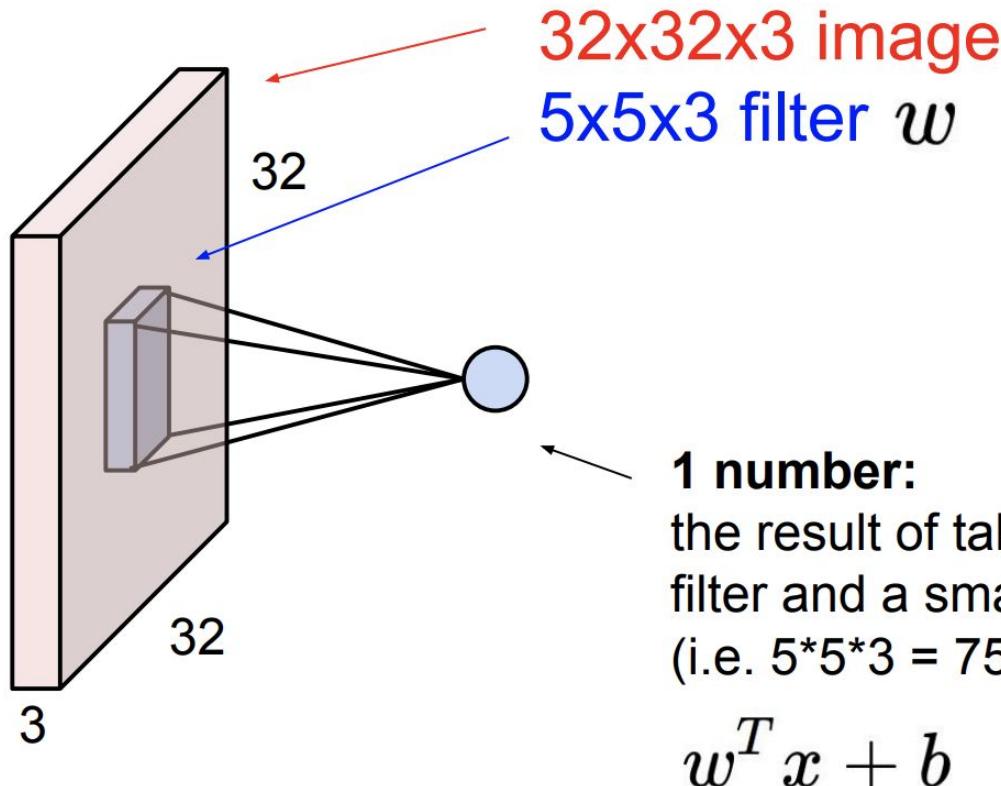
5x5x3 filter



Filters extend the *depth* of the original image

Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolutional layer

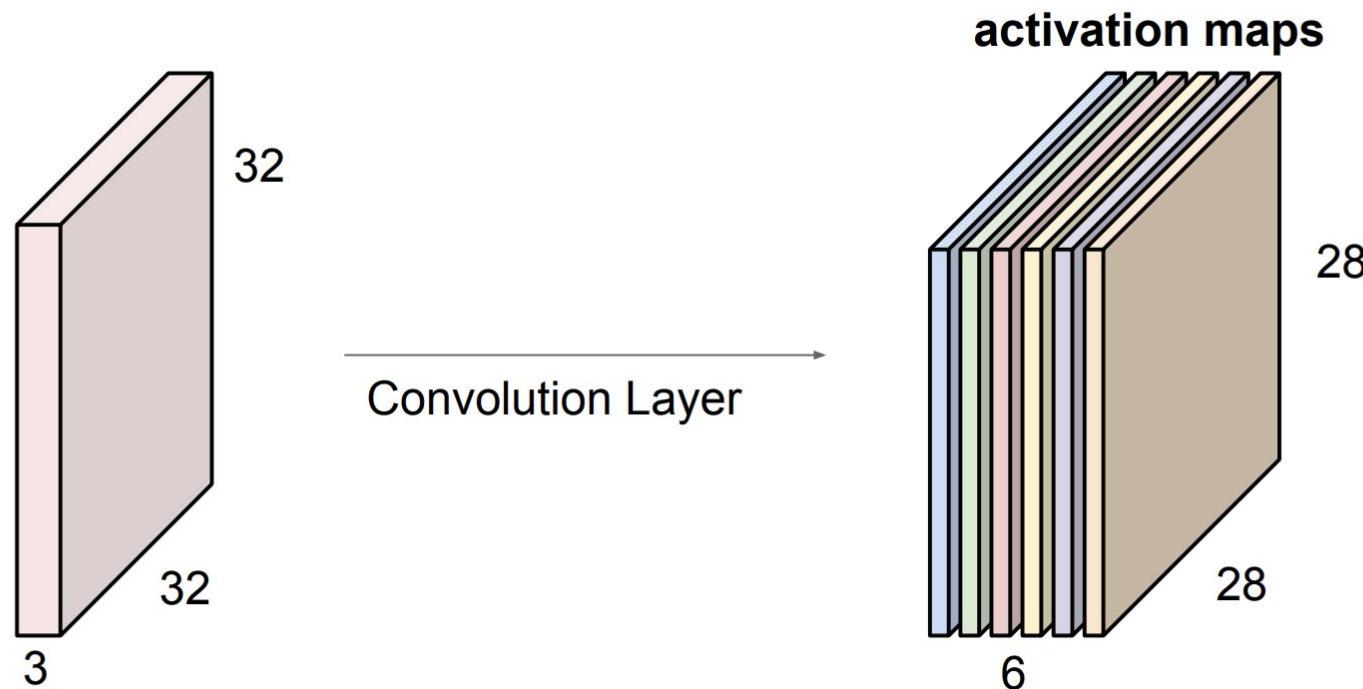


Convolutional layer



Convolutional layer

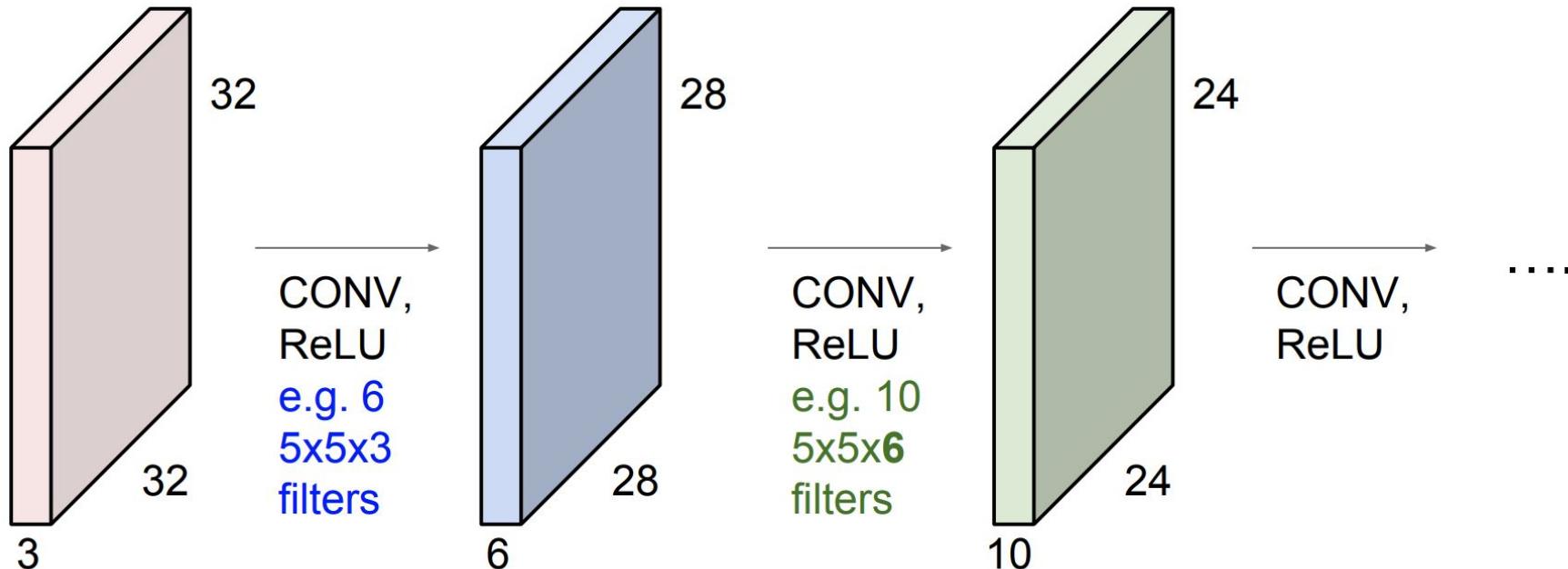
For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



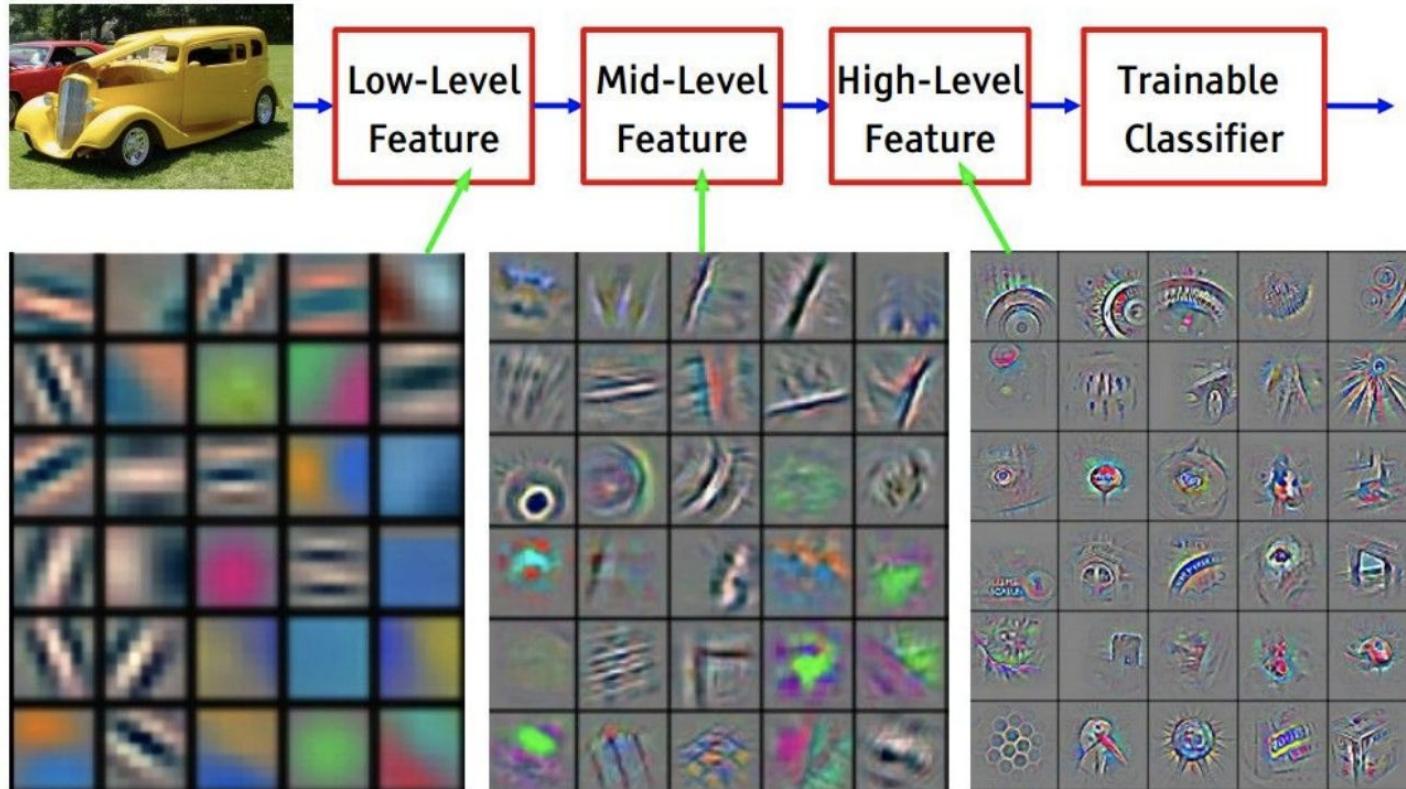
We stack these up to get a “new image” of size 28x28x6!

Convolutional layer

Preview: ConvNet is a sequence of Convolutional Layers, interspersed with activation functions

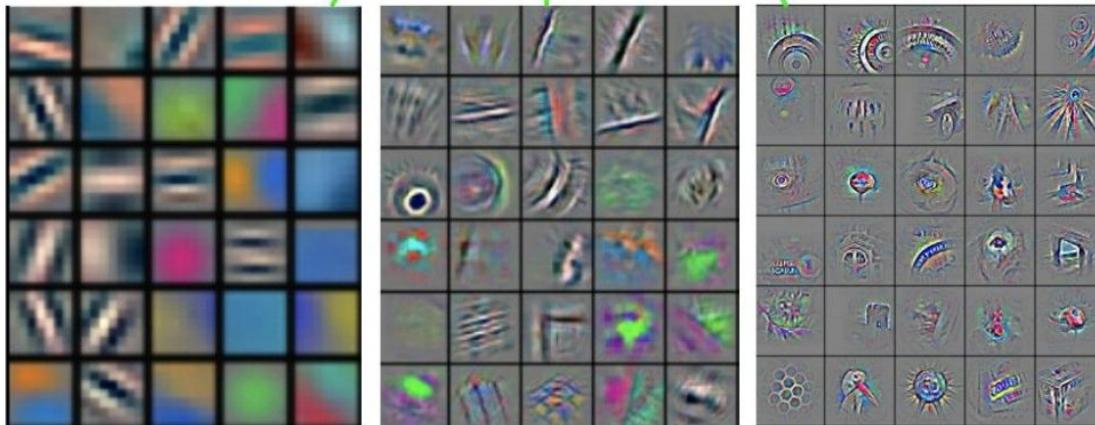


Convolutional layer

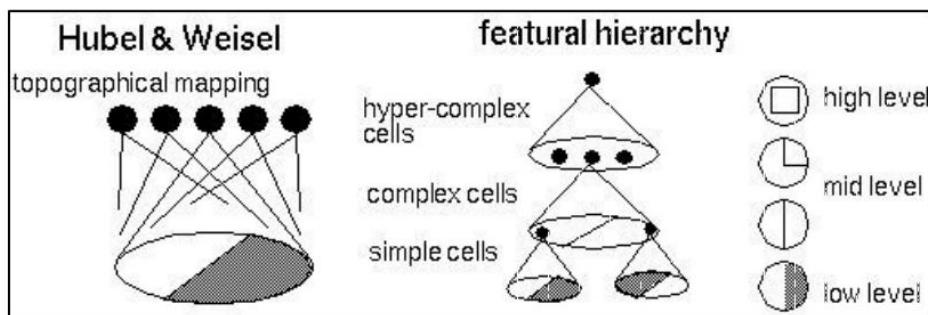


Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

[From Yann LeCun slides]



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]



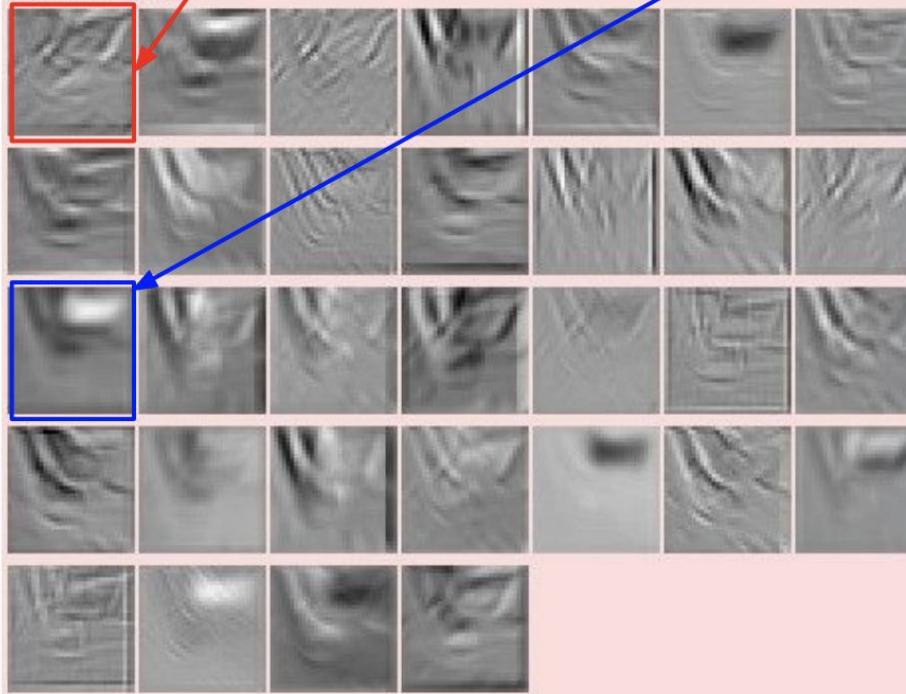
Convolutional layer and visual cortex

[From Yann LeCun slides]



one filter =>
one activation map

Activations:



example 5x5 filters
(32 total)

We call the layer convolutional
because it is related to convolution
of two signals:

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1, n_2] \cdot g[x - n_1, y - n_2]$$



elementwise multiplication and sum of
a filter and the signal (image)

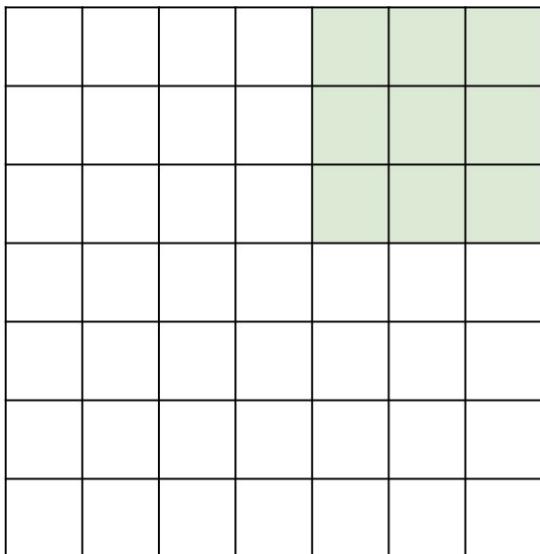
CIFAR-10 online demo:

<https://cs.stanford.edu/people/karpathy/convnetjs/demo/cifar10.html>



A closer look at spatial dimensions:

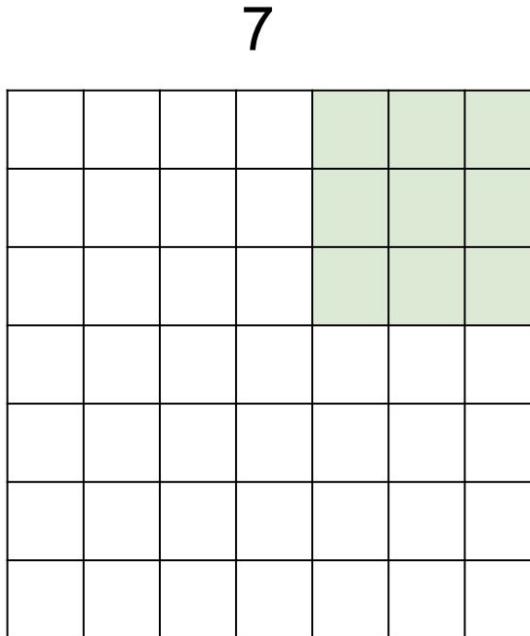
7



7x7 input (spatially)
assume 3x3 filter

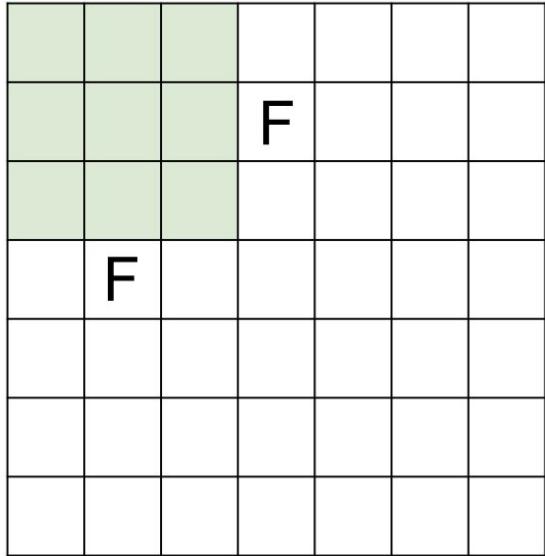
=> 5x5 output

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

N



N

Output size:
(N - F) / stride + 1

e.g. $N = 7, F = 3$:

$$\text{stride } 1 \Rightarrow (7 - 3)/1 + 1 = 5$$

$$\text{stride } 2 \Rightarrow (7 - 3)/2 + 1 = 3$$

$$\text{stride } 3 \Rightarrow (7 - 3)/3 + 1 = 2.33 : \backslash$$

In practice: Common to zero pad the border

0	0	0	0	0	0			
0								
0								
0								
0								

e.g. input 7x7

3x3 filter, applied with stride 1

pad with 1 pixel border => what is the output?

7x7 output!

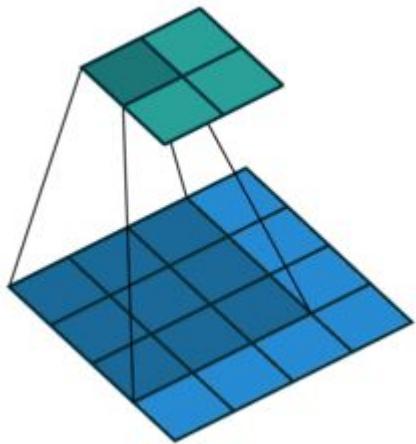
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with $(F-1)/2$. (will preserve size spatially)

e.g. $F = 3 \Rightarrow$ zero pad with 1

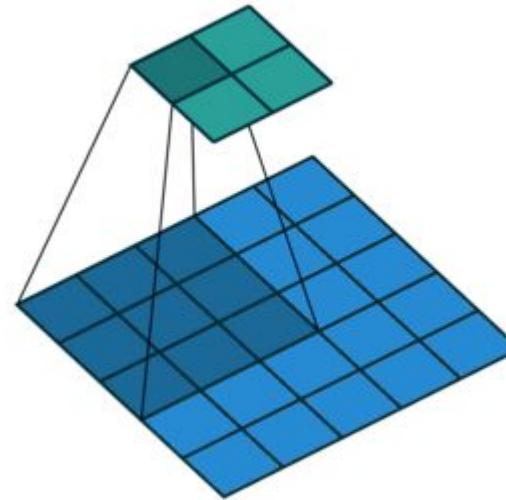
$F = 5 \Rightarrow$ zero pad with 2

$F = 7 \Rightarrow$ zero pad with 3

Strides, padding in convolutional layer

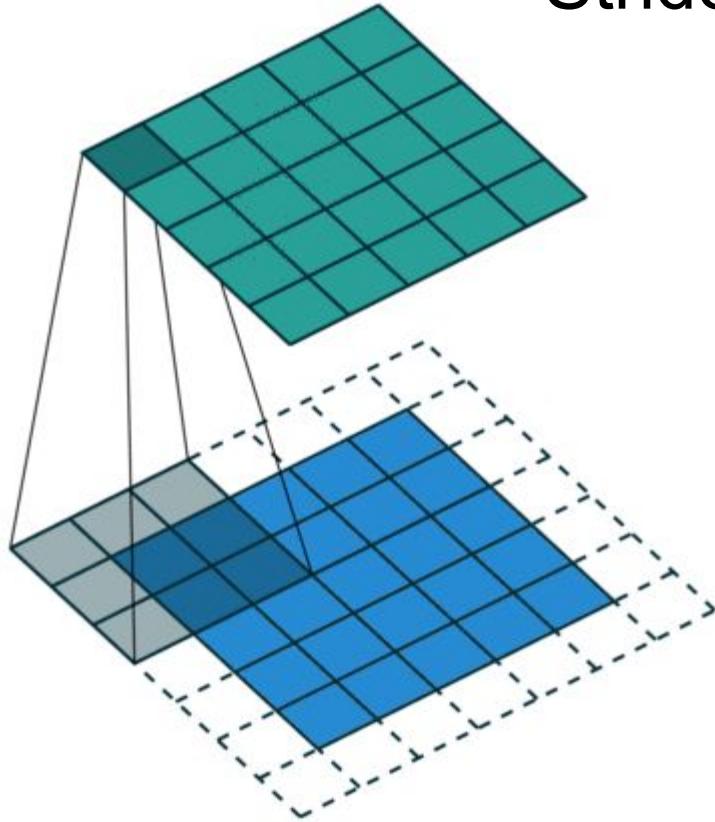


No padding,
no strides

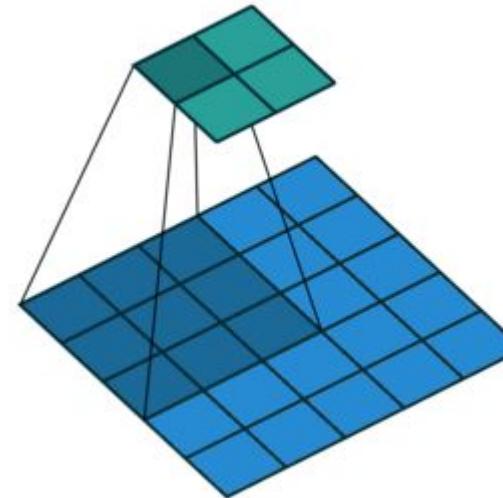


No padding,
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Strides, padding in convolutional layer

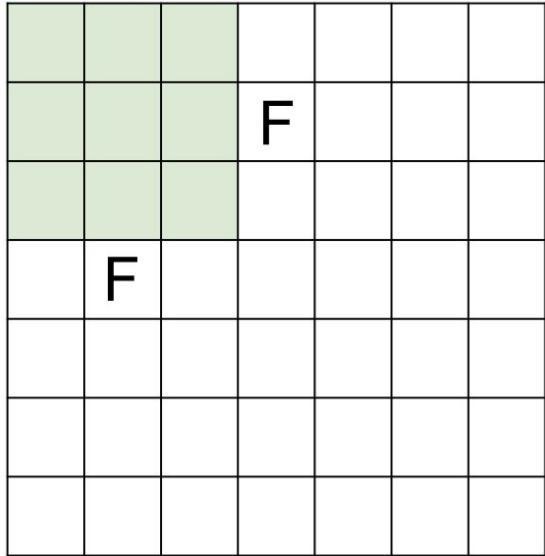


With padding,
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No padding,
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N



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Output size:
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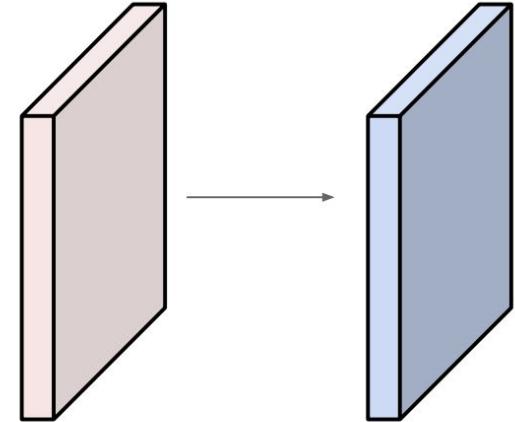
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Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

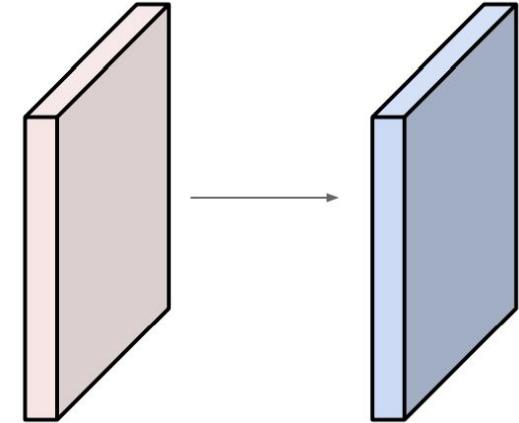


Output volume size: ?

Examples time:

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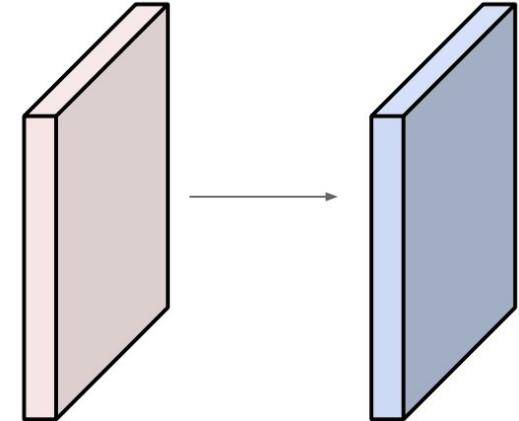
Output volume size:

$(32+2*2-5)/1+1 = 32$ spatially, so
32x32x10

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

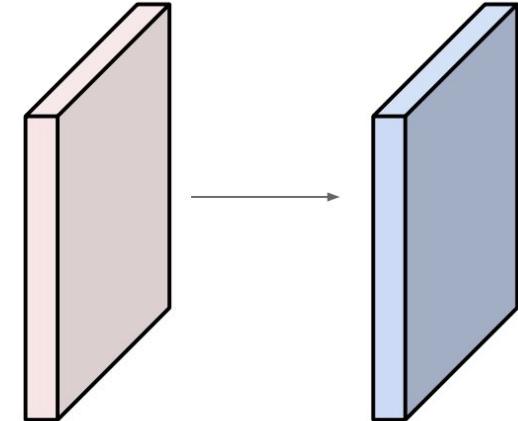


Number of parameters in this layer?

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?

each filter has $5*5*3 + 1 = 76$ params (+1 for bias)
 $\Rightarrow 76*10 = 760$

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
 - Number of filters K ,
 - their spatial extent F ,
 - the stride S ,
 - the amount of zero padding P .
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
 - $W_2 = (W_1 - F + 2P)/S + 1$
 - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
 - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d -th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d -th filter over the input volume with a stride of S , and then offset by d -th bias.

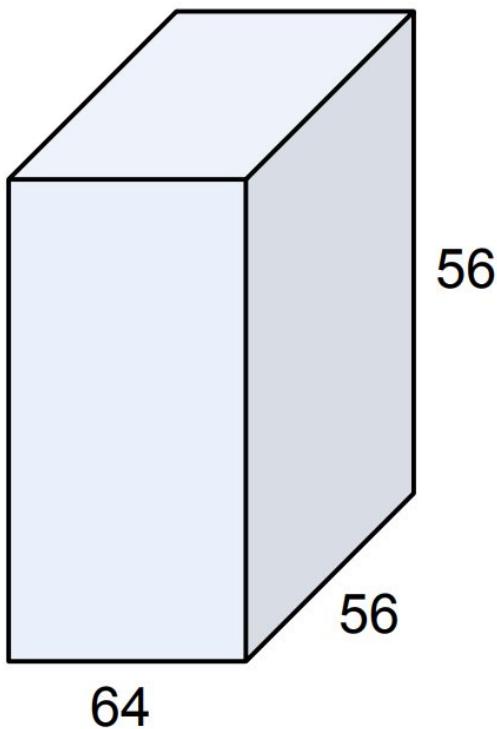
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Common settings:

- $K = (\text{powers of 2, e.g. } 32, 64, 128, 512)$
- $F = 3, S = 1, P = 1$
- $F = 5, S = 1, P = 2$
- $F = 5, S = 2, P = ?$ (whatever fits)
- $F = 1, S = 1, P = 0$

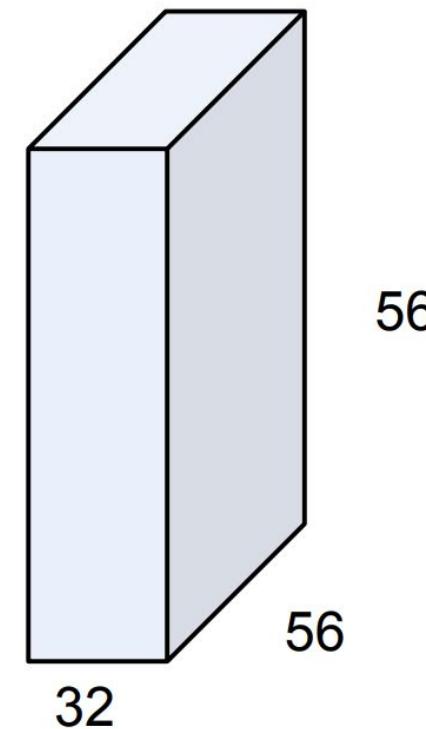
1x1 convolutions



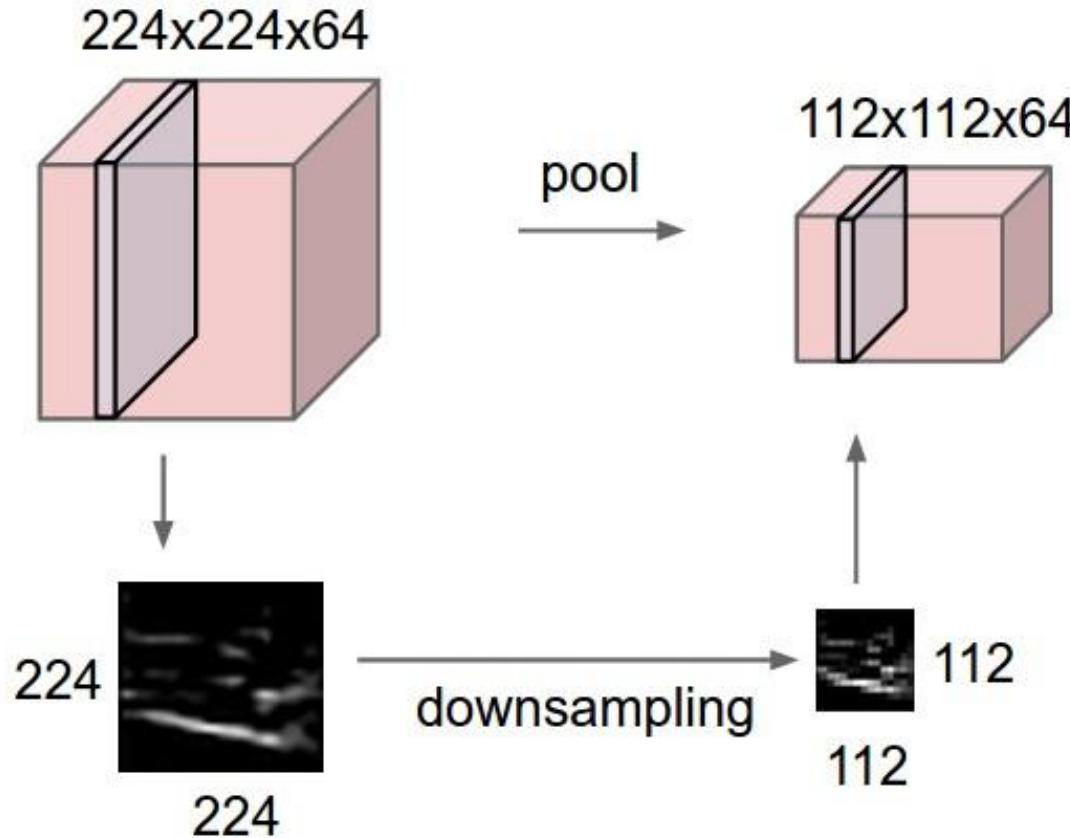
1x1 CONV
with 32 filters

→

(each filter has size
1x1x64, and performs a
64-dimensional dot
product)



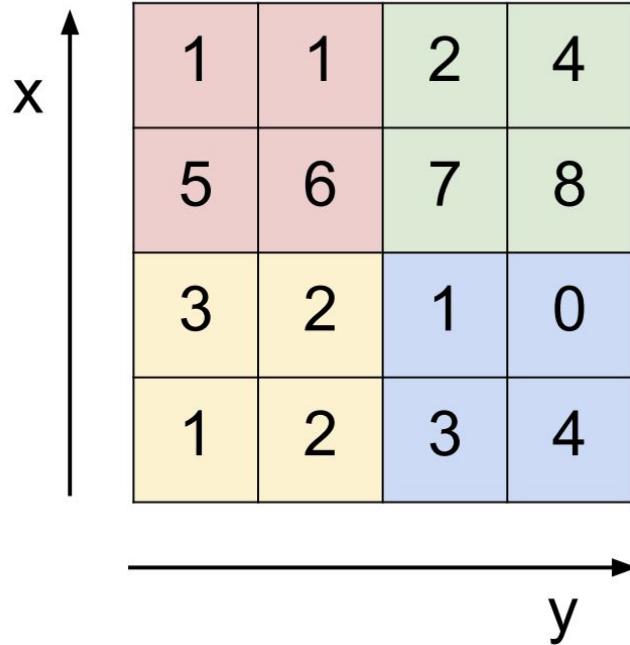
Pooling layer



- Makes the representations smaller and more manageable
- Operates over each activation map independently

Max pooling

Single depth slice



max pool with 2x2 filters
and stride 2



6	8
3	4

Pooling layer: sum up

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
 - their spatial extent F ,
 - the stride S ,
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
 - $W_2 = (W_1 - F)/S + 1$
 - $H_2 = (H_1 - F)/S + 1$
 - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- For Pooling layers, it is not common to pad the input using zero-padding.

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Common settings:

- $F = 2, S = 2$
- $F = 3, S = 2$

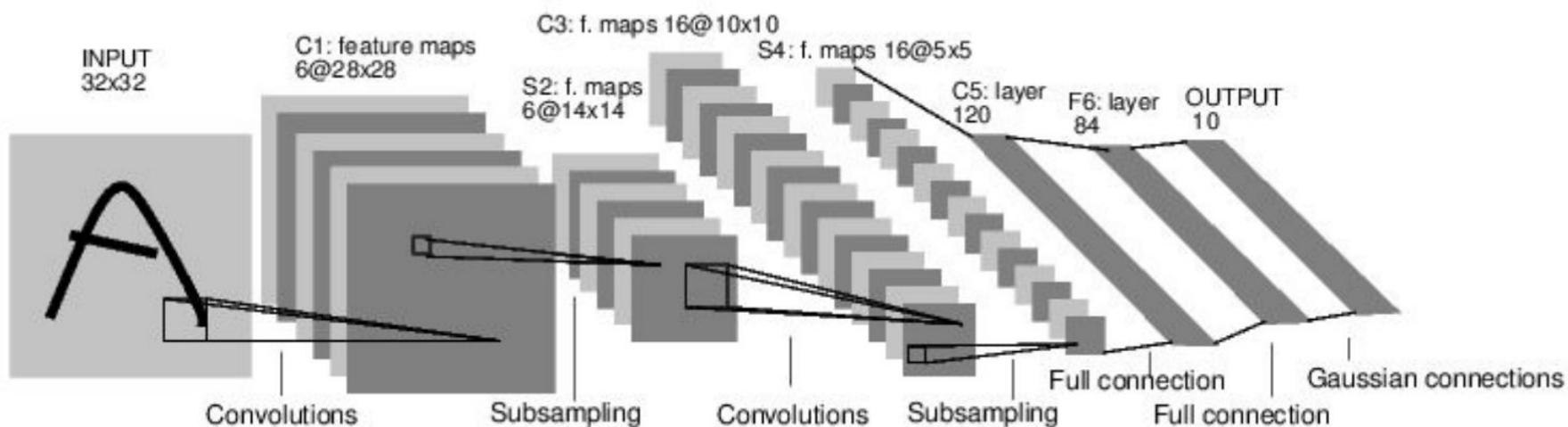
Example time

Great in-browser CNN (using js) demo by Andrej Karpathy:

<https://cs.stanford.edu/people/karpathy/convnetjs/demo/cifar10.html>

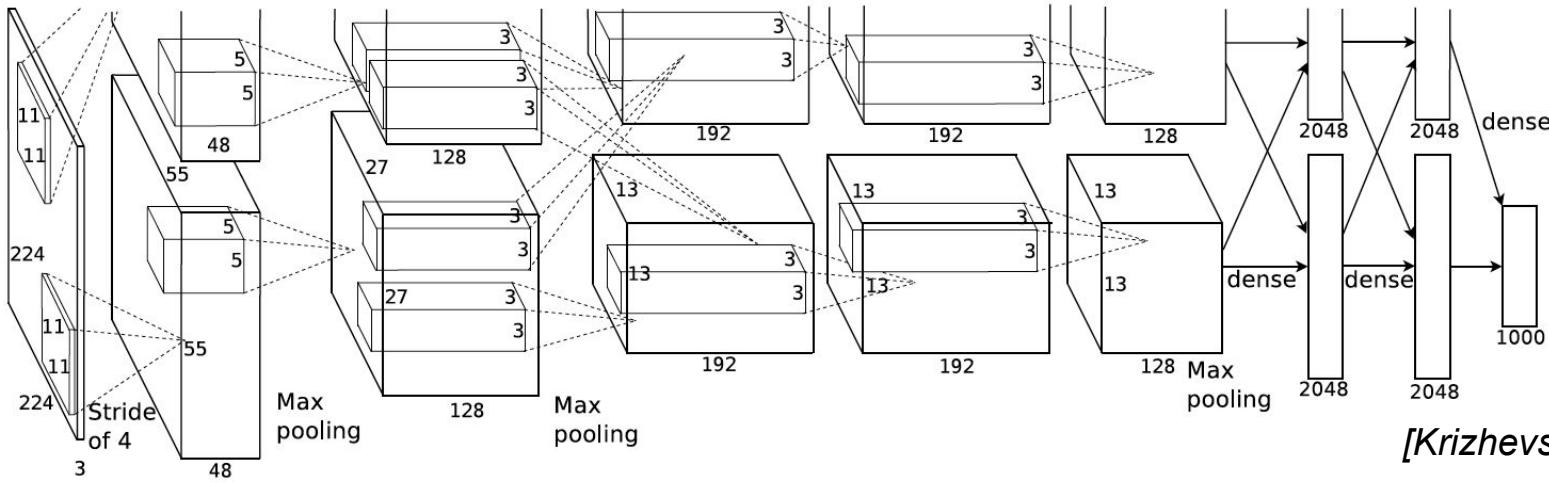
Architectures overview

LeNet-5

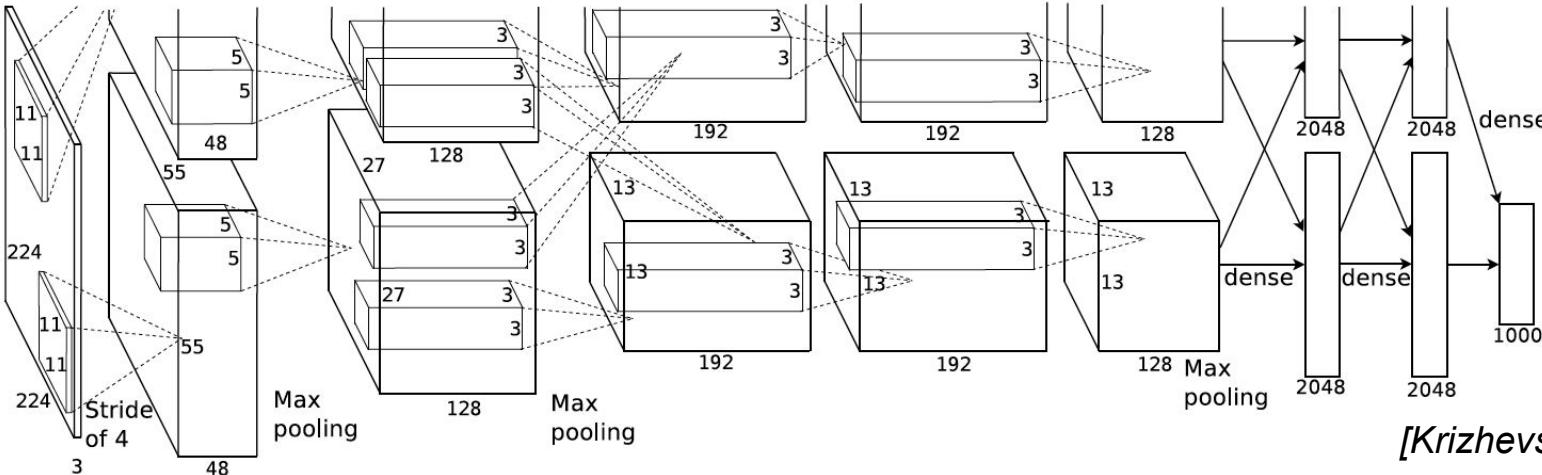


[LeNet-5, LeCun 1998]

AlexNet



[Krizhevsky et al. 2012]

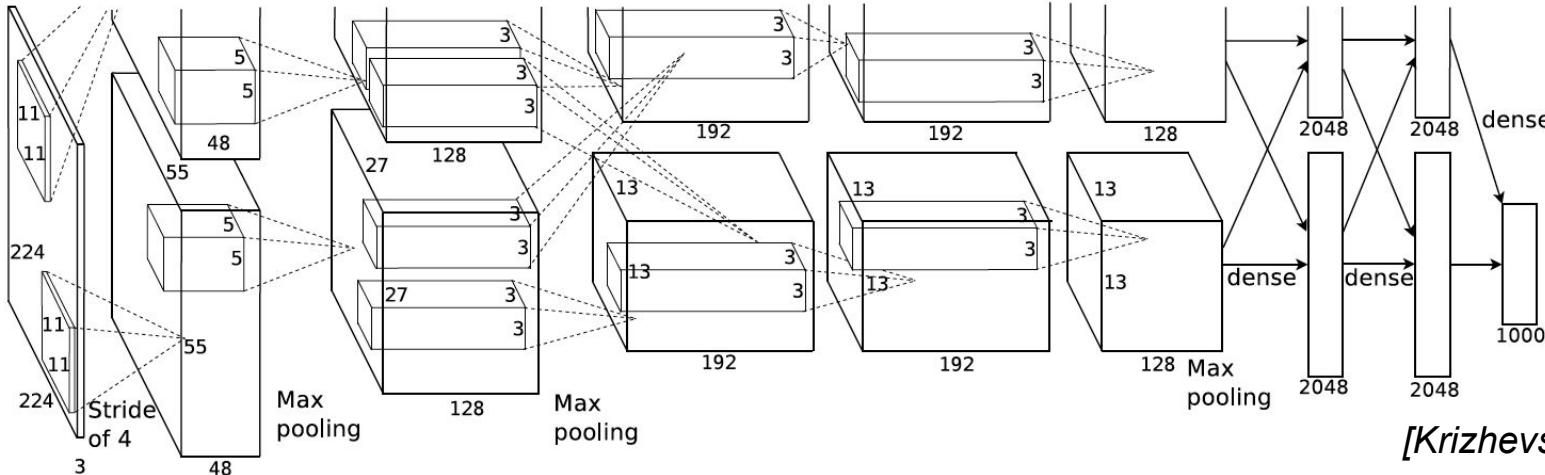


[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture:

[227x227x3] INPUT
 [55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
 [27x27x96] MAX POOL1: 3x3 filters at stride 2
 [27x27x96] NORM1: Normalization layer
 [27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
 [13x13x256] MAX POOL2: 3x3 filters at stride 2
 [13x13x256] NORM2: Normalization layer
 [13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
 [13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
 [13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
 [6x6x256] MAX POOL3: 3x3 filters at stride 2
 [4096] FC6: 4096 neurons
 [4096] FC7: 4096 neurons
 [1000] FC8: 1000 neurons (class scores)

source: http://cs231n.stanford.edu/slides/2016/winter1516_lecture7.pdf



[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture:

[227x227x3] INPUT

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[27x27x96] NORM1: Normalization layer

[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2

[13x13x256] MAX POOL2: 3x3 filters at stride 2

[13x13x256] NORM2: Normalization layer

[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1

[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1

[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1

[6x6x256] MAX POOL3: 3x3 filters at stride 2

[4096] FC6: 4096 neurons

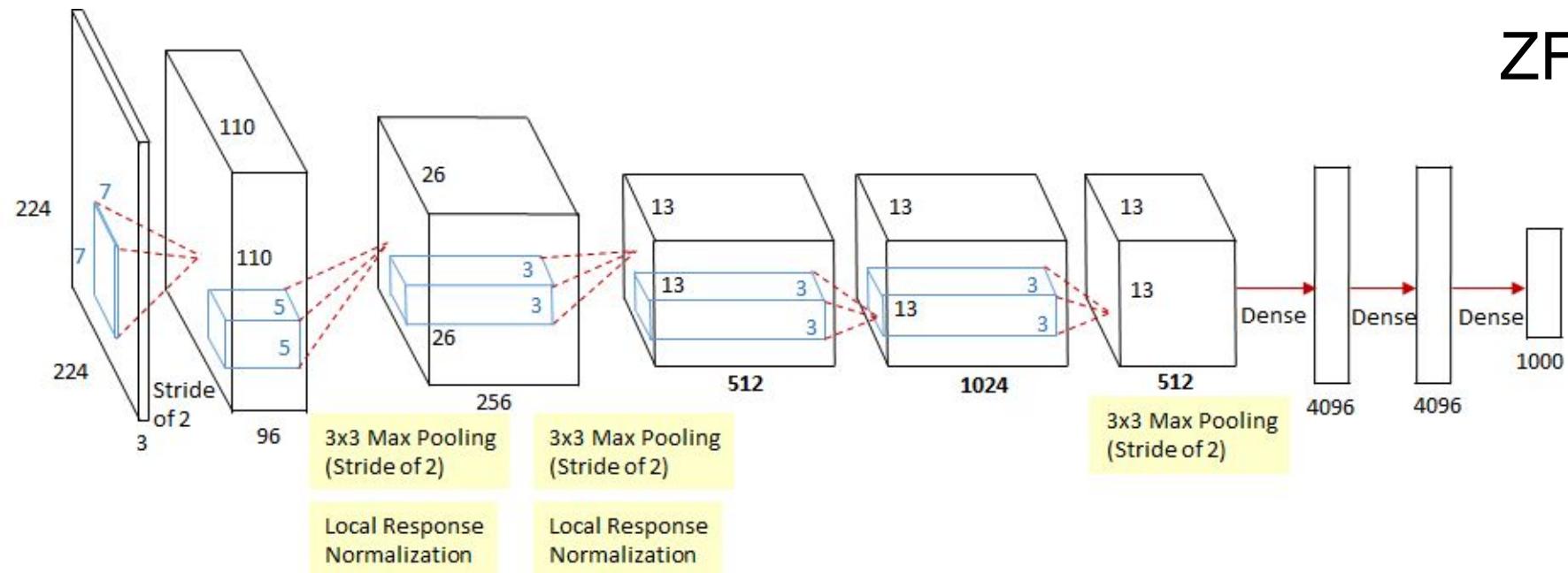
[4096] FC7: 4096 neurons

[1000] FC8: 1000 neurons (class scores)

Details/Retrospectives:

- first use of ReLU
- used Norm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%

ZFNet



[Zeiler and Fergus, 2013]

AlexNet but:

CONV1: change from (11x11 stride 4) to (7x7 stride 2)

CONV3,4,5: instead of 384, 384, 256 filters use 512, 1024, 512

ImageNet top 5 error: 15.4% -> 14.8%

VGGNet

INPUT: [224x224x3] memory: $224 \times 224 \times 3 = 150K$ params: 0 (not counting biases)

CONV3-64: [224x224x64] memory: $224 \times 224 \times 64 = 3.2M$ params: $(3 \times 3 \times 3) \times 64 = 1,728$

CONV3-64: [224x224x64] memory: $224 \times 224 \times 64 = 3.2M$ params: $(3 \times 3 \times 64) \times 64 = 36,864$

POOL2: [112x112x64] memory: $112 \times 112 \times 64 = 800K$ params: 0

CONV3-128: [112x112x128] memory: $112 \times 112 \times 128 = 1.6M$ params: $(3 \times 3 \times 64) \times 128 = 73,728$

CONV3-128: [112x112x128] memory: $112 \times 112 \times 128 = 1.6M$ params: $(3 \times 3 \times 128) \times 128 = 147,456$

POOL2: [56x56x128] memory: $56 \times 56 \times 128 = 400K$ params: 0

CONV3-256: [56x56x256] memory: $56 \times 56 \times 256 = 800K$ params: $(3 \times 3 \times 128) \times 256 = 294,912$

CONV3-256: [56x56x256] memory: $56 \times 56 \times 256 = 800K$ params: $(3 \times 3 \times 256) \times 256 = 589,824$

CONV3-256: [56x56x256] memory: $56 \times 56 \times 256 = 800K$ params: $(3 \times 3 \times 256) \times 256 = 589,824$

POOL2: [28x28x256] memory: $28 \times 28 \times 256 = 200K$ params: 0

CONV3-512: [28x28x512] memory: $28 \times 28 \times 512 = 400K$ params: $(3 \times 3 \times 256) \times 512 = 1,179,648$

CONV3-512: [28x28x512] memory: $28 \times 28 \times 512 = 400K$ params: $(3 \times 3 \times 512) \times 512 = 2,359,296$

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POOL2: [14x14x512] memory: $14 \times 14 \times 512 = 100K$ params: 0

CONV3-512: [14x14x512] memory: $14 \times 14 \times 512 = 100K$ params: $(3 \times 3 \times 512) \times 512 = 2,359,296$

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POOL2: [7x7x512] memory: $7 \times 7 \times 512 = 25K$ params: 0

FC: [1x1x4096] memory: 4096 params: $7 \times 7 \times 512 \times 4096 = 102,760,448$

FC: [1x1x4096] memory: 4096 params: $4096 \times 4096 = 16,777,216$

FC: [1x1x1000] memory: 1000 params: $4096 \times 1000 = 4,096,000$

TOTAL memory: $24M * 4 \text{ bytes} \approx 93\text{MB} / \text{image}$ (only forward! ~ 2 for bwd)

TOTAL params: 138M parameters

ConvNet Configuration			
B	C	D	E
13 weight layers	16 weight layers	16 weight layers	19 weight layers
put (224 x 224 RGB image)			
conv3-64	conv3-64	conv3-64	conv3-64
conv3-64	conv3-64	conv3-64	conv3-64
		maxpool	
conv3-128	conv3-128	conv3-128	conv3-128
conv3-128	conv3-128	conv3-128	conv3-128
		maxpool	
conv3-256	conv3-256	conv3-256	conv3-256
conv3-256	conv3-256	conv3-256	conv3-256
		conv1-256	conv3-256
			conv3-256
			conv3-256
		maxpool	
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
		conv1-512	conv3-512
			conv3-512
			conv3-512
			conv3-512
		maxpool	
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
		conv1-512	conv3-512
			conv3-512
			conv3-512
			conv3-512
		maxpool	
FC-4096			
FC-4096			
FC-1000			
soft-max			

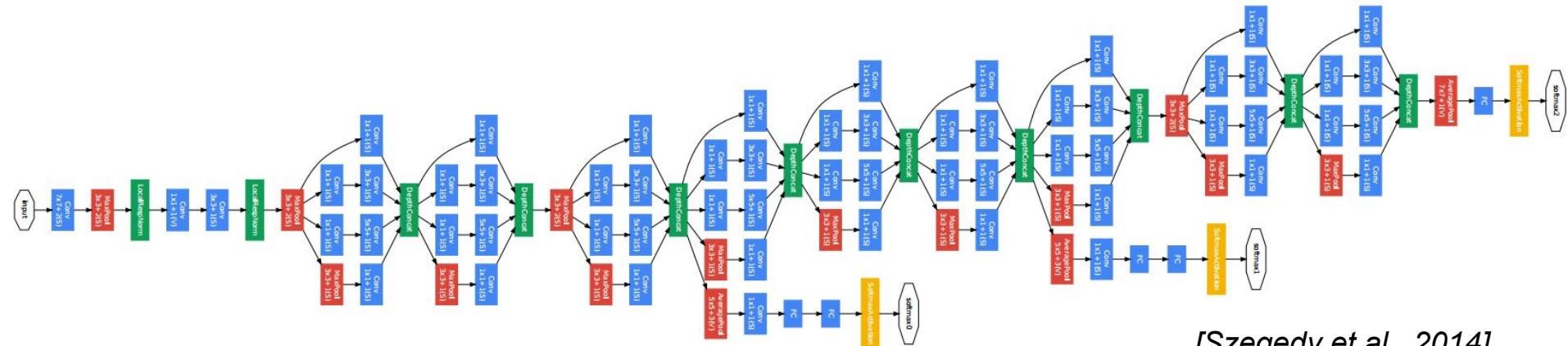
VGGNet

7.3% top 5 error

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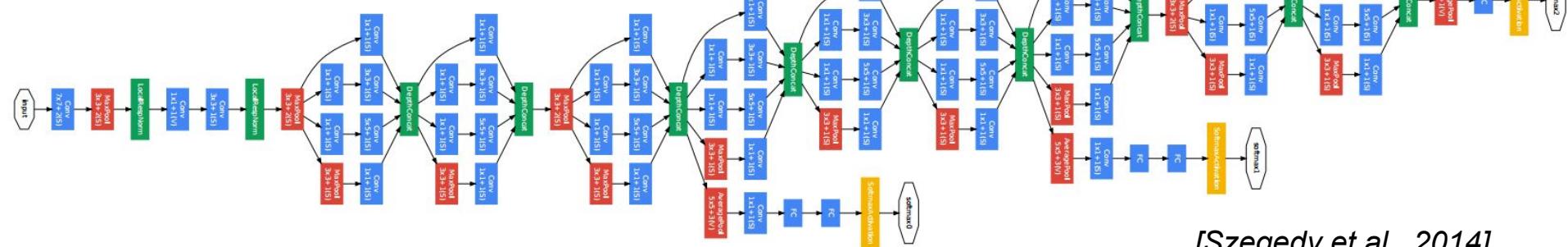
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B	C	D	E
13 weight layers	16 weight layers	16 weight layers	19 weight layers
put (224 × 224 RGB image)			
conv3-64	conv3-64	conv3-64	conv3-64
conv3-64	conv3-64	conv3-64	conv3-64
maxpool			
conv3-128	conv3-128	conv3-128	conv3-128
conv3-128	conv3-128	conv3-128	conv3-128
maxpool			
conv3-256	conv3-256	conv3-256	conv3-256
conv3-256	conv3-256	conv3-256	conv3-256
	conv1-256	conv3-256	conv3-256
maxpool			
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
	conv1-512	conv3-512	conv3-512
maxpool			
conv3-512	conv3-512	conv3-512	conv3-512
conv3-512	conv3-512	conv3-512	conv3-512
	conv1-512	conv3-512	conv3-512
maxpool			
FC-4096			
FC-4096			
FC-1000			
soft-max			

GoogLeNet



[Szegedy et al., 2014]

GoogLeNet

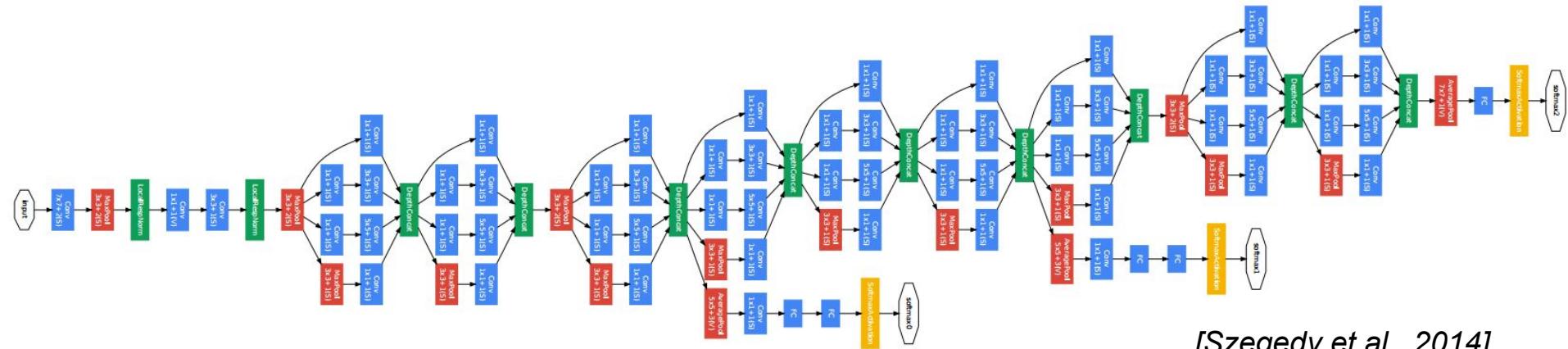


[Szegedy et al., 2014]

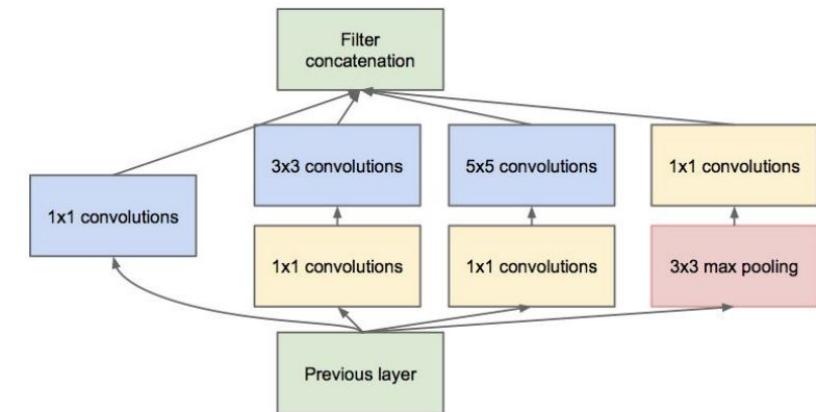
Inception module



GoogLeNet

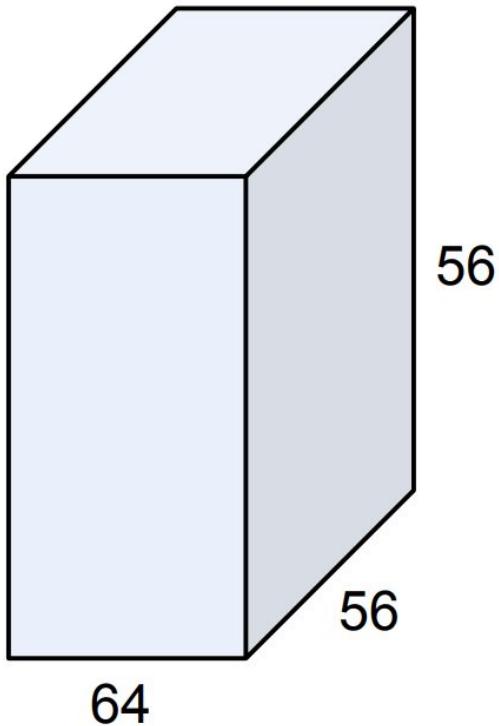


[Szegedy et al., 2014]



Inception module

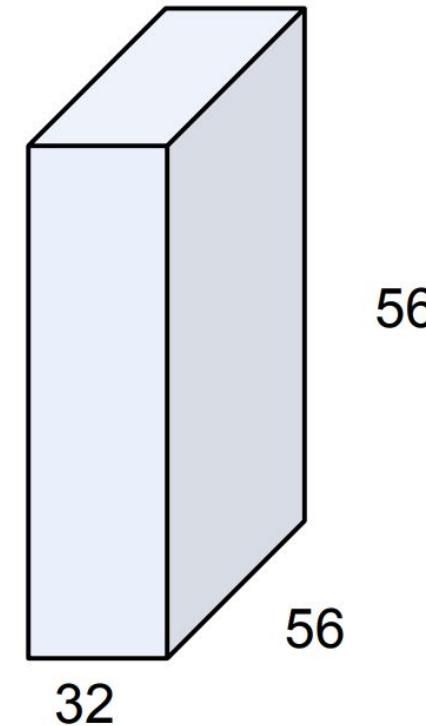
Once again: 1x1 convolutions



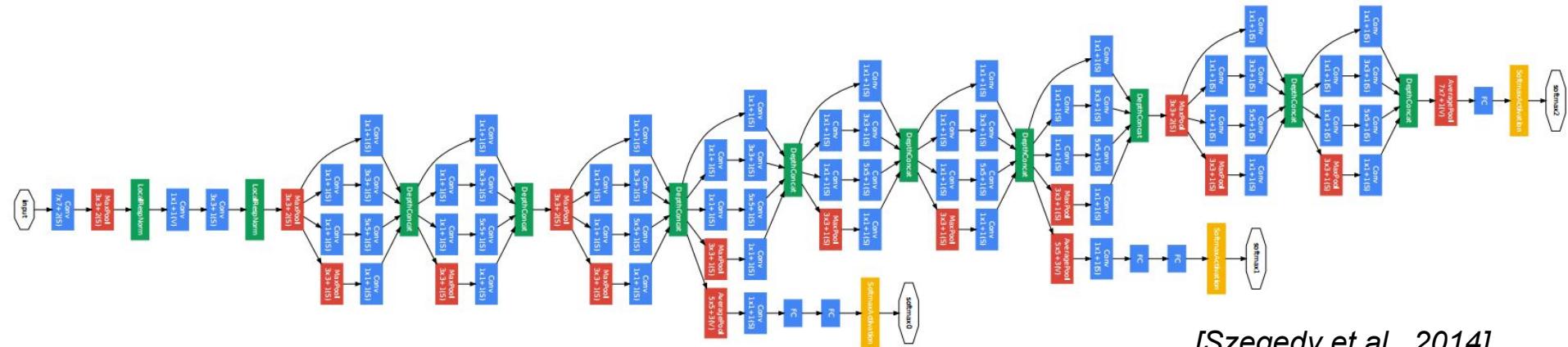
1x1 CONV
with 32 filters

→

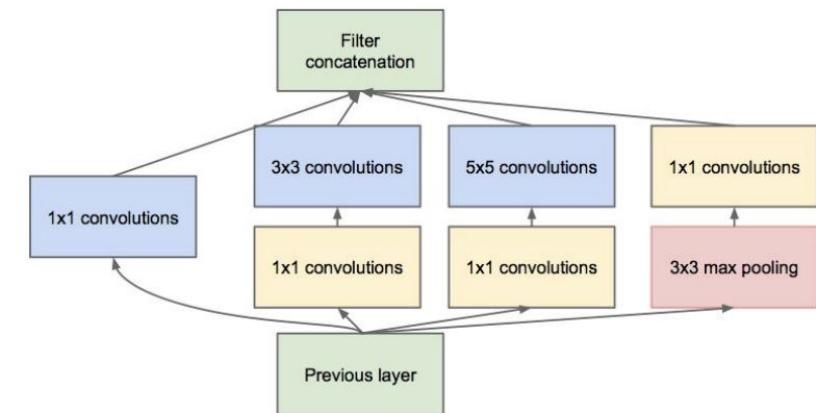
(each filter has size
1x1x64, and performs a
64-dimensional dot
product)



GoogLeNet



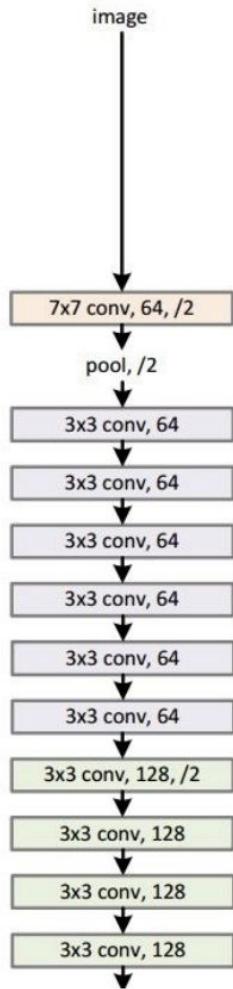
[Szegedy et al., 2014]



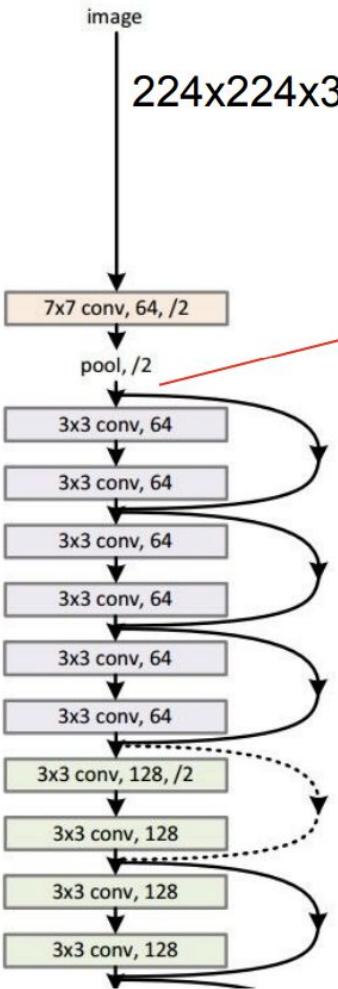
Inception module

ILSVRC 2014 winner (6.7% top 5 error)

34-layer plain



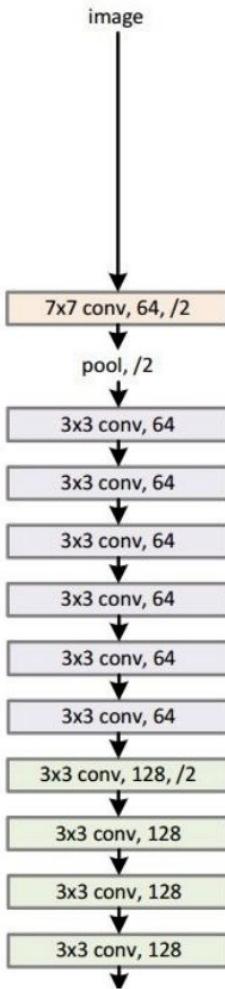
34-layer residual



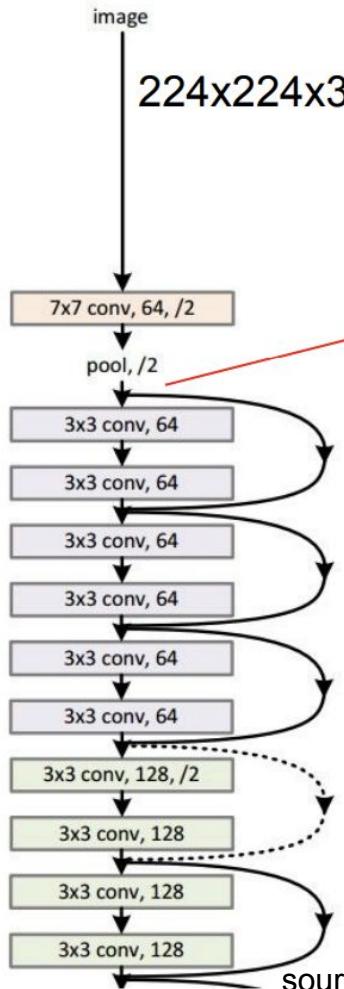
spatial dimension
only 56x56!

ILSVRC 2015 winner (3.6% top 5 error)

34-layer plain



34-layer residual



[He et al., 2015]

spatial dimension
only 56x56!

- Batch Normalization after every CONV layer
- Xavier/2 initialization from He et al.
- SGD + Momentum (0.9)
- Learning rate: 0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of 1e-5
- No dropout used

ILSVRC 2015 winner (3.6% top 5 error)

source: http://cs231n.stanford.edu/slides/2016/winter1516_lecture7.pdf

MSRA @ ILSVRC & COCO 2015 Competitions

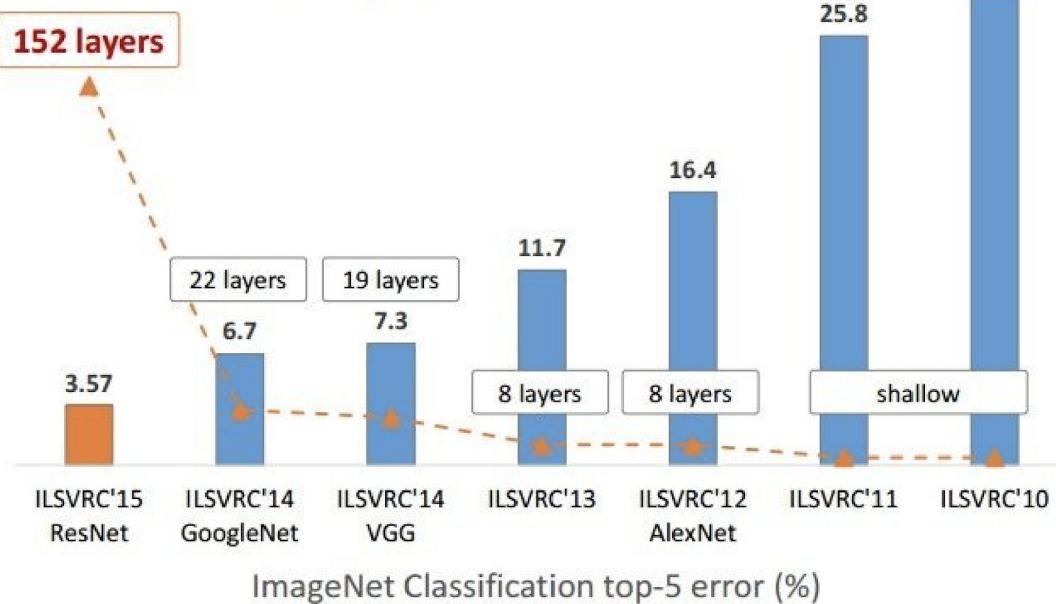
- **1st places in all five main tracks**

- ImageNet Classification: “*Ultra-deep*” (quote Yann) **152-layer nets**
- ImageNet Detection: **16%** better than 2nd
- ImageNet Localization: **27%** better than 2nd
- COCO Detection: **11%** better than 2nd
- COCO Segmentation: **12%** better than 2nd

*improvements are relative numbers

Kaiming He, Xiangyu Zhang, Shaoqing Ren, & Jian Sun. “Deep Residual Learning for Image Recognition”. arXiv 2015.

Revolution of Depth



Batch normalization

It's usually a good idea to normalize linear model inputs.

(c) Every machine learning lecturer, ever

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\dots m}\}$;

Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma, \beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \quad // \text{mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2 \quad // \text{mini-batch variance}$$

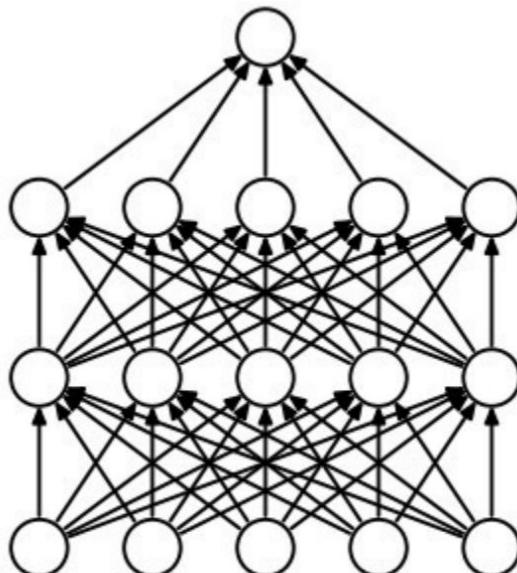
$$\hat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad // \text{normalize}$$

$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma, \beta}(x_i) \quad // \text{scale and shift}$$

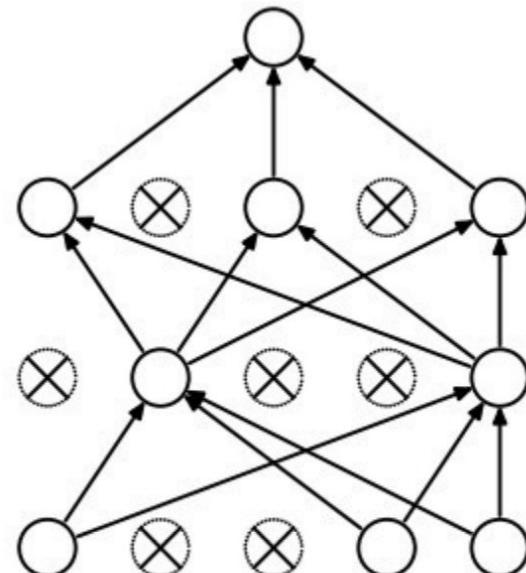
Regularization: Dropout

Some neurons are “dropped” during training.

Prevents overfitting.



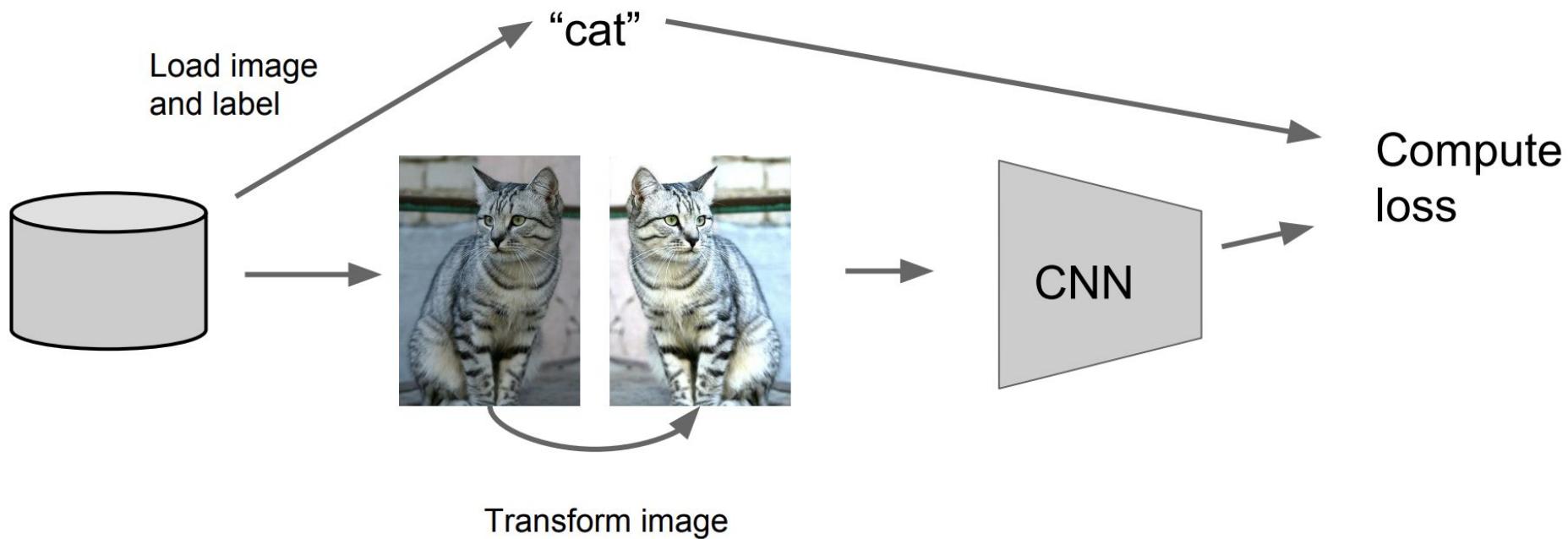
(a) Standard Neural Net



(b) After applying dropout.

Actually, on test case output should be normalized. See sources for more info.

Regularization: data augmentation

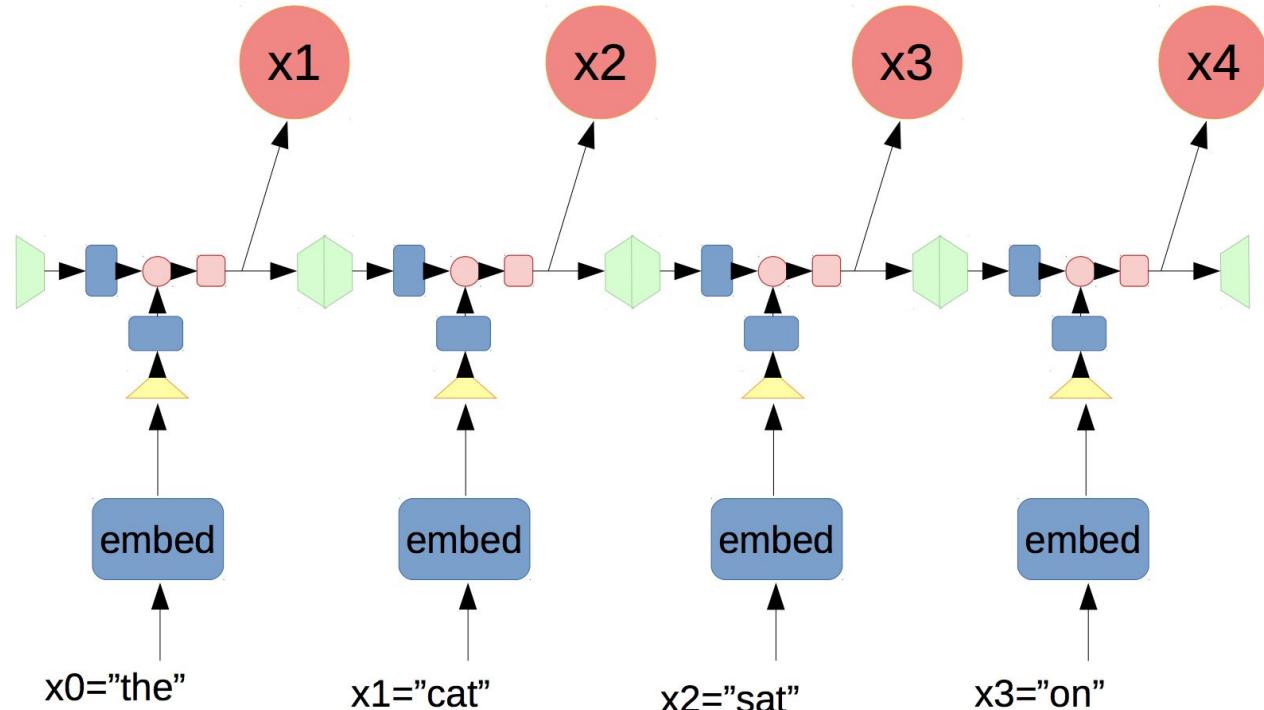


- ConvNets stack convolutional, pooling and dense layers
- Trend towards smaller filters and deeper architectures
- 1x1 convolutions are meaningful
- Humanity is already beaten on ImageNet.

That's all. Feel free to ask any questions.

Backup

Recap: RNN

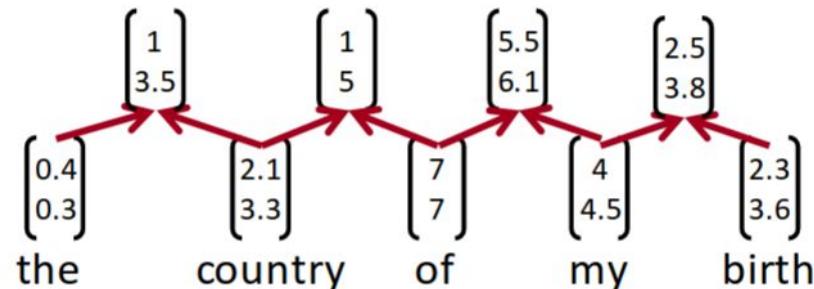


From RNN to CNN

- RNN: Get compositional vectors for grammatical phrases only
- CNN: What if we compute vectors for every possible phrase?
 - Example: “*the country of my birth*” computes vectors for:
 - *the country, country of, of my, my birth, the country of, country of my, of my birth, the country of my, country of my birth*
- Regardless of whether it is grammatical
- Wouldn’t need parser
- Not very linguistically or cognitively plausible

From RNN to CNN

- Imagine using only bigrams



- Same operation as in RNN, but for every pair

$$p = \tanh \left(W \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + b \right)$$

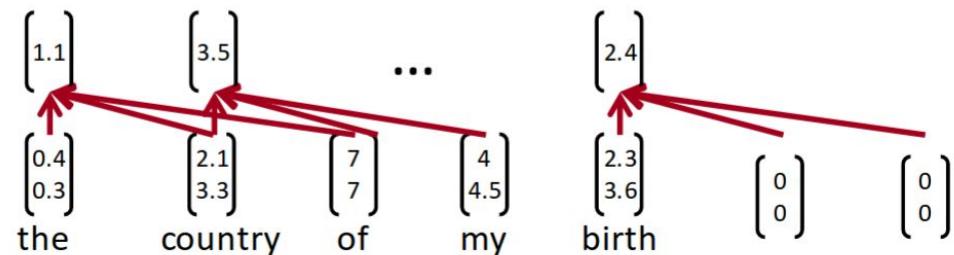
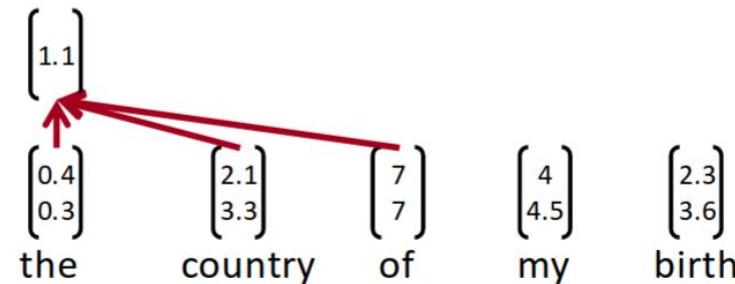
- Can be interpreted as convolution over the word vectors

From RNN to CNN

- Simple convolution + pooling
- Window size may be different (2 or more)
- The feature map based on bigrams:

$$\mathbf{c} = [c_1, c_2, \dots, c_{n-h+1}] \in \mathbb{R}^{n-h+1}$$

$$c_i = f(\mathbf{w}^T \mathbf{x}_{i:i+h-1} + b)$$

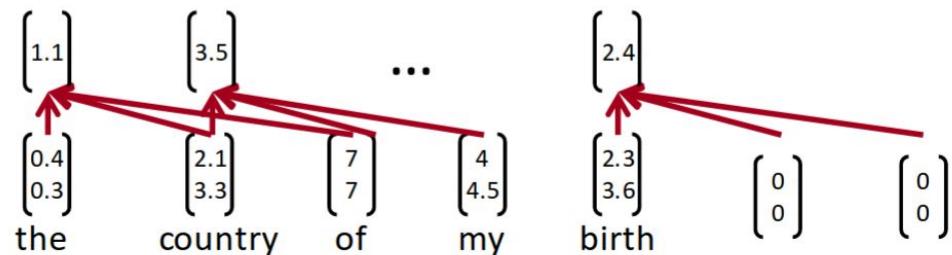
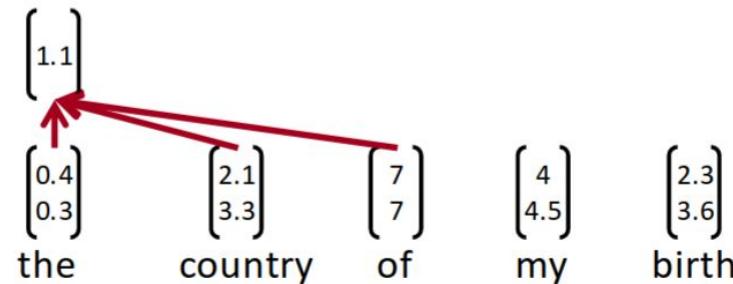


One layer CNN

- Simple convolution + pooling
- Window size may be different (2 or more)
- The feature map based on bigrams:

$$\mathbf{c} = [c_1, c_2, \dots, c_{n-h+1}] \in \mathbb{R}^{n-h+1}$$

$$c_i = f(\mathbf{w}^T \mathbf{x}_{i:i+h-1} + b)$$



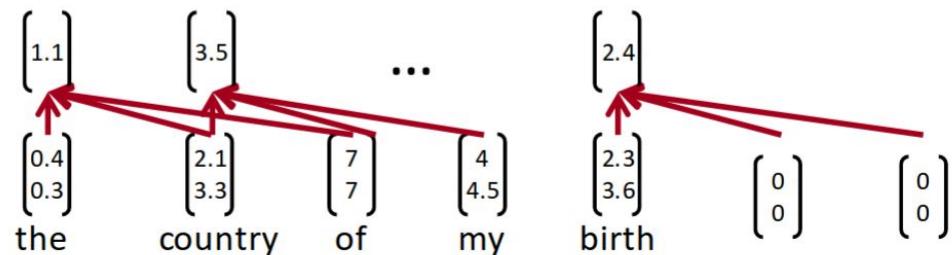
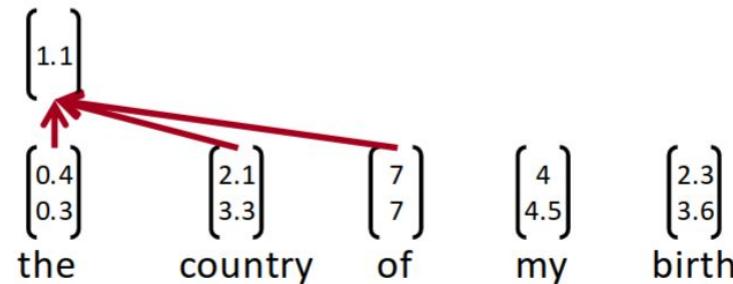
One layer CNN

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What's next?

$$c_i = f(\mathbf{w}^T \mathbf{x}_{i:i+h-1} + b)$$



One layer CNN

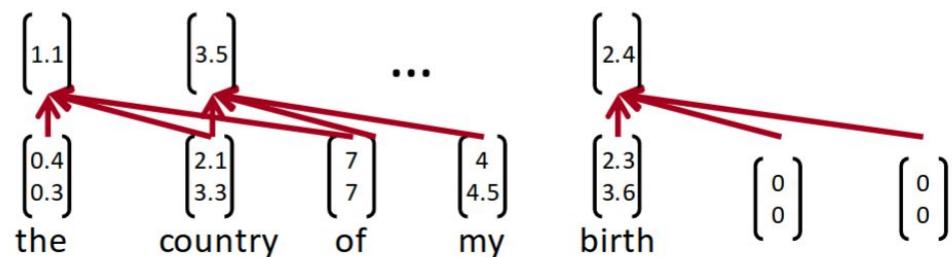
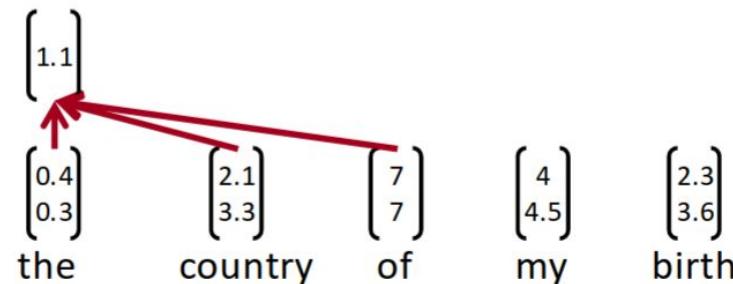
- Simple convolution + pooling
- Window size may be different (2 or more)
- The feature map based on bigrams:

$$\mathbf{c} = [c_1, c_2, \dots, c_{n-h+1}] \in \mathbb{R}^{n-h+1}$$

What's next?

We need more features!

$$c_i = f(\mathbf{w}^T \mathbf{x}_{i:i+h-1} + b)$$



- Feature representation is based on some applied filter:

$$\mathbf{c} = [c_1, c_2, \dots, c_{n-h+1}] \in \mathbb{R}^{n-h+1}$$

- Let's use pooling over the time axis: $\hat{c} = \max\{\mathbf{c}\}$
- Now the length of \mathbf{c} is irrelevant!
- So we can use filters based on unigrams, bigrams, tri-grams, 4-grams, etc.

Another example from Kim (2014) paper

