How to do reasonably well in SCI 238

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1 Quick reference

1.1 Values

$$
\begin{aligned}
\text{earth radius} &= 6371 \text{ km} \\
\text{earth mass} &= 5.97 \times 10^{24} \text{ kg} \\
\text{sun radius} &= 6.95 \times 10^{5} \text{ km} \\
\text{sun mass} &= 1.99 \times 10^{30} \text{ kg} \\
\text{milky center dist} &= 2.6 \times 10^{4} \text{ ly} \\
\text{milky radius} &= 5.3 \times 10^{4} \text{ ly} \\
\text{andromeda dist} &= 2.5 \times 10^{6} \text{ ly} \\
\text{solar day} &= 24\text{h} \\
\text{sidereal day} &= 23\text{h}56\text{m}4.09\text{s} = 23.93447\text{h} \\
\text{synodic month} &= 29.53 \text{ solar days} \\
\text{sidereal month} &= 27.32 \text{ solar days} \\
\text{tropical year} &= 365.242 \text{ solar days} \\
\text{sidereal year} &= 365.256 \text{ solar days} \\
v_{\text{escape earth}} &= 11.2 \text{ km/s} \\
\text{sun luminosity} &= 3.828 \times 10^{26} \text{ W} \\
\text{white dwarf radius} &\approx 7000 \text{ km} \\
\text{neutron star radius} &\approx 10 \text{ km} \\
\text{neutron star density} &\approx 4 \times 10^{17} \text{ kg/m}^3
\end{aligned}
$$

1.2 Constants

$$
\begin{aligned}
G &= 6.674 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2) \\
c &= 2.997 \times 10^{8} \text{ m/s} \\
h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s}
\end{aligned}
$$

1.3 Units

$$
\begin{aligned}
1 \text{ eV} &= 1.602 \times 10^{-19} \text{ J} \\
1 \text{ AU} &= 1.496 \times 10^{8} \text{ km} \\
1 \text{ ly} &= 9.461 \times 10^{12} \text{ km} \\
1 \text{ pc} &= 3.086 \times 10^{13} \text{ km}
\end{aligned}
$$

1.4 Formulas

$$F_g = G\frac{m_1 m_2}{d^2} \qquad \text{(Gravity)}$$

$$L = mvr \qquad \text{(Angular momentum)}$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2} \qquad \text{(Ellipse eccentricity)}$$

1.5 Mathematical insights

Page numbers are for The Cosmic Perspective 6th.

- 92: Math S1.1. Orbital and synodic period
- 99: Math S1.2. Sidereal time, hour angle
- 131: Math 4.3. Law of periods

$$T^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

- 133: Math 4.4. v_{escape}
- 150: Math 5.1. $c = \lambda f$, $E = hf$
- 161: Math 5.2. Stefan-Boltzmann law, Wien's law.

$$j^* = \sigma T^4$$
$$\lambda_{\max} = b/T$$

where

$$\sigma = 5.670 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$$
$$b = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

- 165: Math 5.3. Doppler shift

$$\frac{v}{c} = \frac{\lambda_{\text{shift}}}{\lambda_{\text{rest}}} - 1$$

- 175: Math 6.1. Angular separation:

$$\theta = \frac{d}{r} \cdot \frac{360^\circ}{2\pi}$$

- 180: Math 6.2. Diffraction limit of telescope

$$\theta = 1.220 \frac{\lambda}{D} \cdot 360^\circ$$

- 249: Math 9.1. Sphere surface area / volume
- 382: Math 13.1. Exoplanet orbital distance (via law of periods)
- 383: Math 13.2. Exoplanet mass (via conservation of momentum)
- 384: Math 13.3. Exoplanet size (via light fraction blocked)
- 416: Math S2.1. Time dilation

$$t_{\text{moving}} = t_{\text{rest}}\sqrt{1-(v/c)^2}$$

- 417: Math S2.2. Special relativity

$$\ell_{\text{moving}} = \ell_{\text{rest}}\sqrt{1-(v/c)^2}$$
$$m_{\text{moving}} = \frac{m_{\text{rest}}}{\sqrt{1-(v/c)^2}}$$
$$\text{speed addition} = \frac{v_1+v_2}{1+(v_1v_2/c^2)}$$

- 420: Math S2.3. Deriving $E = mc^2$
- 478: Math 14.1. $E = mc^2$ with hydrogen fusion
- 479: Math 14.2. Ideal gas law: $P = nkT$
- 494: Math 15.1. Apparent brightness

$$\text{Apparent brightness} = \frac{L}{4\pi d^2}$$

- 496: Math 15.2. Parallax
- 497: Math 15.3. Magnitude
- 503: Math 15.4. Binary star mass (via law of periods)
- 506: Math 15.5. Star radius (via thermal radiation)
- 523: Math 16.1. Gravity vs. pressure (Jean's mass)

$$M_{\text{balance}} = 18M_{\text{Sun}}\sqrt{T^3/n}$$

- 570: Math 18.1. Schwarzschild radius

$$R_S = \frac{2GM}{c^2}$$

- 586: Math 19.1. Interior mass via velocity

$$M_r = \frac{rv^2}{G}$$

- 612: Math 20.1. Standard candles
- 617: Math 20.2. Redshift

$$z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{rest}}} - 1$$

- 618: Math 20.3. Hubble's law
- 621: Math 20.4. Age from Hubble's constant
- 622. Math 20.5. Redshift and light stretching
- 642. Math 21.1. Feeding a black hole (via $E = mc^2$)
- 644. Math 21.2. Supermassive black hole (via interior mass)
- 653. Math 22.1. Mass-to-light ratio
- 655. Math 22.2. Cluster mass (via interior mass)
- 656. Math 22.3. Cluster mass (via gas temperature)

$$v_H = (140\text{m/s}) \times \sqrt{T}$$

- 685. Math 23.1. Background radiation temperature/wavelength (via Wien's law, redshift)

1.6 Figures and tables

- 148: Figure 5.7. Electromagnetic spectrum
- 163: Figure 5.21. Interpreting a spectrum
- 185: Figure 6.22. Electromagnetic penetration
- 200: The solar system and the planet sequence
- 212: Table 7.1. Solar system planet data
- 501: Table 15.1. The spectral sequence
- 504: Figure 15.10. H-R diagram
- 680: Figure 23.5. The early universe
- 703: Figure 24.4. Geological time scale