History

$u(v)dv = c_1 v^3 e^{-c_2 v/T} dv$ — Wien
$u(v)dv = \frac{8\pi}{c^3} kTv^2 dv$ — Rayleigh-Jeans
$u(v)dv = \frac{c_1}{e^{c_2 v T}-1} v^3 dv$ — Planck
$p = \frac{E}{c} = \frac{h}{\lambda}$ — Broglie
$E = hv$, $E_k(\max) = hv - \Phi$, $E_t = nhv$
$L = mvr = n/\hbar$ — Angular momentum of electron
$v = R(n_f^{-2} - n_i^{-2})$ — Balmer

Physics

SE: $\hat{H}\Psi = i\hbar\frac{\partial}{\partial t}\Psi$ — TISE: $\hat{H}\psi = E\psi$

Operators

$\hat{p} = -i\hbar\frac{d}{dx}$ $\quad \hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V$ $\quad \hat{T} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$
$\langle T\rangle = \langle p^2\rangle /2m$
$\hat{\ell}_z = -i\hbar\frac{d}{d\phi}$ — Angular momentum of rigid rotator
$\hat{H} = \frac{\ell_z^2}{2I}$ — Energy of rigid rotator

Ehrenfest

$\langle p\rangle = m\frac{d\langle x\rangle}{dt}$ $\quad \frac{d\langle p\rangle}{dt} = \left\langle -\frac{\partial V}{\partial x}\right\rangle$

Momentum space

$\Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}}\int e^{-ipx/\hbar}\Psi(x,t)dx$
$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\int e^{ipx/\hbar}\Phi(p,t)dp$

Generalized uncertainty (p. 110)

$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2$

General solution

$\Psi(x,t) = \sum c_n\psi_n(x)e^{-iE_n t/\hbar}$
$c_n = \int \psi_n^*(x)\Psi(x)dx$

Infinite square well (p. 30)

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & \text{else} \end{cases}$$

$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$, n=1,...
$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

Harmonic oscillator (p. 40)

$V(x) = \frac{1}{2}m\omega^2x^2$

Ladder operators

$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$
$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$ $\quad \hat{p} = i\sqrt{\frac{2m\omega}{\hbar}}(a_+ - a_-)$
$\hat{H} = \hbar\omega\left(a_\pm a_\mp \pm \frac{1}{2}\right)$
$a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$ $\quad a_-\psi_n = \sqrt{n}\,\psi_{n-1}$

Solution

With $\alpha = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$, $\xi = \sqrt{\frac{m\omega}{\hbar}}x$:
$\psi_0 = \alpha e^{-\xi^2/2}$
$\psi_1 = \sqrt{2}\alpha\xi e^{-\xi^2/2}$
$\psi_2 = \frac{\alpha}{\sqrt{2}}\left(2\xi^2 - 1\right)e^{-\xi^2/2}$ $\quad \psi_n = \frac{1}{\sqrt{n!}}a_+^n\psi_0$
$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

Free particle (p. 59)

$\Psi_k(x,t) = Ae^{-i(kx-\omega t)}$, $\omega = \frac{\hbar k^2}{2m}$, k continuous
$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int \phi(k)e^{i(kx-\omega t)}dk$
$\phi(k) = \frac{1}{\sqrt{2\pi}}\int \Psi(x,0)e^{-ikx}dx$

Delta function (p. 68)

$V(x) = -\alpha\delta(x)$
$\Delta\left(\frac{d\Psi}{dx}\right) = -\frac{2m\alpha}{\hbar^2}\Psi$
$\Psi(x) = \frac{\sqrt{m\alpha}}{\hbar}e^{-m\alpha|x|/\hbar^2}$, $E = -\frac{m\alpha^2}{2\hbar^2}$ — Bound

Finite square well (p. 78)

$$V(x) = \begin{cases} -V_0 & -a \leq x \leq a \\ 0 & \text{else} \end{cases}$$

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < -a \\ C\sin(\ell x) + D\cos(\ell x) & -a < x < a \\ Fe^{ikx} & x > a \end{cases}$$

$k = \sqrt{2mE}/\hbar$, $\quad \ell = \sqrt{2m(E+V_0)}/\hbar$
Resonance at $n\lambda = 2a, \kappa a = n\pi$ $(\kappa = 2\pi/\lambda)$