Introduction: A fitness center has a high demand for bikes from 6:00-7:30 am. Since we want to model the arrival of the clients, to simplify the problem, we assume they only arrive at discrete time intervals. For example, no client would arrive 30 seconds past the 5th minute; they only arrive at 5th or 6th minute. The assumptions are:

1- First come first serve basis
2- Clients wait in the line when there are at most 3 people ahead of them
3- Clients who are in the line already, will not leave until they get a bike
4- Clients use the bikes for exactly 8 minutes each

In order to make the model even simpler, we assume that all clients use bikes for exactly 8 minutes. Otherwise, we would have a complicated availability for each bike. The rates of arrivals are as follows: 6:00-6:30: 30 people, 6:31-7:00 45 people, 7:01-7:30 20 people. People who arrive when all bikes have 3 people in the line, will leave the gym unsatisfied. If a client becomes dissatisfied for more than 4 times in a month, there is 50% chance that they drop their membership. A complete process of this model is shown in the flowchart in FlowchartUnit3.docx.

Methods: Since we are using excel, we had to come up with a way to create a chain of events which are dependent on each other. The events on nth minute depend on the last n-1 steps, thus we start with a column and number it 1 to 90. For minutes 0-30 (6-6:30), we have the rate of 1person/minute, for 31-60 (6:31-7:00) we have rate of 1.5 person/minute and for 61-90 (7:00-7:30) we have rate of 0.67 person/minute. Since we want to find the number of arrivals in one-minute intervals, for the rate of arrival, we determine the number of clients arriving at random, based on Poisson probability distribution tables. First, we ask Excel to generate a random number between 0 and 1, then we match the probabilities to the number of arrivals based on Table 1.

- Now for the number of initial bikes: Model for the 1st minute: We start by the total number of bikes for the first step. Next, we determine the number of final bikes as follows: =IF((E3-D3)<=0,0,(E3-D3)) means if #arrivals-#available bikes ≥ 0 , put in the value otherwise, put zero. Then the initial queue value for the first step is 0 and final queue value is : =IF(D3-E3>0,IF(D3-E3<=4,D3-E3,4),0). This is asking excel that if #arrivals-#available bikes > 0 , and if #arrivals-#available bikes ≤ 4, then print the value of arrivals without bikes and if #arrivals-#available bikes > 4, which is capacity of the queue, then print the maximum value, 4. Also, it states that if #arrivals-#available ≤ 0 , there is nobody in the queue, so just print zero.
- Number of dissatisfied customers follows a general formula for <u>all the minutes</u> which is : =IF(H3=4,IF(D3>E3,D3-E3,0),0). This is asking Excel that if the queue is full and there are clients left without a bike, then count those and print the number as dissatisfied. Otherwise, if they are enough bikes or more space in the queue, just print zero as dissatisfied. Dissatisfied clients will leave the gym.

Minutes 2-8

- Number of Initial bikes for minutes 2-8: =IF(F3<=0,0,F3) This is stating that number of initial bikes are just the final bikes from the last step. If the last step has 0 bikes, then Excel prints zero.
- Number of final bikes for minutes 2-8: =IF(E4-H3>=0,IF(E4-H3-D4>=0,E4-H3-D4,0),0) This is simply asking Excel that if there are enough bikes left so that all the people waiting in the queue can get a bike and there

are bikes left over, then take in some arrivals, and print the number of left over bikes, if on any step there are no more bikes, print zero.

- ❖ Value of initial queue for minutes 2-8: This is number of final queue from the previous step.

Minutes 9-90

- ❖ Number of Initial bikes for minutes 9-90:=IF(F10=0,0,F10)+IF(H10=0,E3-F3+IF(H2-G3>0,H2-G3,0),IF(E3-F3+IF(H2-G3>0,H2-G3,0)-H10>0,E3-F3+IF(H2-G3>0,H2-G3,0)-H10,0)) This is the longest formula, thus, color coding it would help. The yellow part is just asking to add the final bikes from the previous step. The next part is a long loop. The condition is if previous final queue is zero, then add the number of used bikes from 8 minutes ago (no longer being used) which equals to a number of bikes used by the arrivals+ number of bikes used by people waiting in the queue. Otherwise, if the previous final qeueu is not zero, we have to subtract the value from the addition of total bikes used from 8 minutes ago (green part). Then at the end, we add the value of the yellow portion to it and print the value as the initial bike.
- ❖ Number of final bikes for minutes 9-90: =IF((E11-D11)<=0,0,(E11-D11)) This is asking Excel to print #initial bikes-#arrivals if it is positive, otherwise print zero.
- ❖ Value of initial queue for minutes 9-90: =IF(H10-(E3-F3)-IF(H2-G3>0,H2-G3,0)<=0,0,H10-(E3-F3)-IF(H2-G3>0,H2-G3,0)).
- ❖ Value of finish queue for minutes 2-90: =IF(G4+IF(D4-E4>0,IF(D4-E4<=4,D4-E4,4),0)<4,G4+IF(D4-E4>0,IF(D4-E4<=4,D4-E4,4),0),4).This rather looks very confusing. If we break the formula by parts, it is not too bad. The loop starts to ask Excel that if arrivals are more than initial bikes, add the extra clients to the initial queue value and print. Since there is a limit to the size of the queue, for every loop we state that if the number of the final qeueu equals more than 4, just print 4.
- ❖ Finally, the results are simulated for 5 to 12 bikes.We run the loop for 1000 trials.Tab **Table 4.19** is made based on these simulations. The formula 4.10 is used to estimate the probability of cancellation. Net Reveneu=#visitors x $30-#bikes x$100- #visotors x probability of cancellation x $30

Results: The results in Table 2 match the data from table 4.19 in the textbook. The data in Table 2 comes from averaging the results produced by running the simulation explained in the methods section for 1000 trials. For this simulation, we assumed there is a 50% chance of cancellation after four unsuccessful visits. Also, we assume that the same clients visit every day. In this case, there are around 94 clients per day. Overall, the optimal number of bikes would be 8. Although the cancellation count is not zero, the net profit is the highest when we add 3 extra bikes. The cost of new bikes plus the cost of cancellations are relatively low comparing to membership fees revenue, thus the difference would make the highest profit. For question 4, as we see in Table 3 and Table 4, as the probability of cancellation increases, we need more bikes to get the best profit. This is very intuitive as more bikes mean fewer cancellation expenses. It is important to note the number of bikes which gives the best profit depends on the probability of cancellation. Overall, for the range of 30%-70% probability of cancellation per unsatisfied person, we need 7 to 8 bikes to get the highest profit.

Conclusion: Based on the Excel model and analysis of data, as we add more bikes to the gym, the probability of unsatisfied members decreases which leads to fewer membership cancellations. There is an optimal number of bikes which maximizes the net revenue. When there are 8 bikes, the fitness center still loses a couple of members per month, but the net profit is the highest. If the chance of cancellation stays 50% after four unsuccessful visits, the recommendation is to add 3 more bikes to maximize the profit.

Figures

Look up table	6:30-7:00
Cutoffs	Arrivals
0	0
0.22313016	1
0.5578254	2
0.808846831	3
0.934357546	4
0.981424064	5
0.995544019	6
0.999074008	7
0.999830434	8
0.999972264	9
0.999995902	10
0.999999448	11
0.999999932	12
0.999999992	13
0.999999999	14
1	15

Table 1: Example of lookup table to determine the number of arrivals

# Bikes	% disappointment	Probability of cancellation	Cost of cancellation	cost of new bike	Net Revenue
5	38.79%	0.29351484	837.9085551	0	2016.831445
6	30%	0.173320437	495.0291675	100	2261.120833
7	22.28%	0.082172987	233.8503521	200	2411.979648
8	16.37%	0.032934073	93.85815468	300	2456.021845
9	11.50%	0.010364044	29.59349047	400	2425.80651
10	7%	0.002211191	6.296189712	500	2341.12381
11	4.92%	0.000482905	1.380901347	600	2258.189099
12	3.21%	9.59334E-05	0.273726635	700	2153.026273

Table 2:4.19 simulation

# Bikes	% Disappointment	Probability of cancellation	Cost of cancellation	cost of new bike	Net Revenue
5	38.85%	0.176523213	502.9322876	0	2346.167712
6	30%	0.105563613	301.3250009	100	2453.114999
7	22.21%	0.048866073	139.4368964	200	2514.013104
8	16.27%	0.019384645	55.22762915	300	2493.812371
9	11.29%	0.005843401	16.56884614	400	2418.911154
10	7%	0.001363724	3.882400426	500	2343.0276
11	4.87%	0.00027995	0.79774901	600	2248.812251
12	3.06%	4.78135E-05	0.136048893	700	2145.273951

Table 3: 30% chance of cancellations

# Bikes	% disappointment	Probability of cancellation	Cost of cancellation	cost of new bike	Net Revenue
5	38.85%	0.411887498	1173.508671	0	1675.591329
6	30%	0.246315098	703.0916688	100	2051.348331
7	22.21%	0.114020837	325.3527582	200	2328.097242
8	16.27%	0.045230838	128.864468	300	2420.175532
9	11.29%	0.013634602	38.66064098	400	2396.819359
10	7%	0.003182023	9.058934328	500	2337.851066
11	4.87%	0.000653217	1.861414357	600	2247.748586
12	3.06%	0.000111565	0.317447417	700	2145.092553

Table 4:70% chance of cancellation