Unit 8

a. **Analytical**: Let the decision variable be:

Number of Residential Renovation = x1

Number of Residential Construction = x2

Number of Commercial Construction = x3

Objective function = max (12000*x1+35000*x2+205000*x3)

Now, to formulate the problem, we need to add all the needed crew for electricians, plumbers, drywallers and carpenters. It is important to remember there is a constraint for each of the crew groups:

Constraints Table					
Variables	x1	x2	x3		
Carpenters	2	3	50	<=	65
Drywallers	2	4	20	<=	35
Plumbers	1	1	12	<=	17
Electricians	2	1	30	<=	38

Which gives us the following inequalities:

A. 2*x1+3*x2+50*x3 <= 65
B. 2*x1+4*x2+20*x3 <= 35
C. x1+x2+12*x3 <= 17
D. 2*x1+x2+30*x3 <= 38

So, now, to find the maximum subject to constraints above, we need to find "corners" or intersections of these equations. This will help us to map out the feasible region. Now, there are 12 x,y,z intercepts which we need to find.
A: (32.50,0,0) , (0,21.67,0),(0,0,1.3)
B: (17.50,0,0), (0,8.75,0), (0,0,1.75)
C: (17,0,0),(0,17,0),(0,0,1.42)
D: (19,0,0), (0,38,0),(0,0,1.27)

Now since all the inequalities are "less than", without graphing this we already know that smallest values of each intercept will be corners of our feasible region. These are (17,0,0), (0,8.75,0), (0,0,1.27) and (0,0,0). Also, we need to find other possible bounds of the feasible region. Since there are 3 unknowns, we need at least 3 planes to find an intersection, otherwise we will get a line. Since there are 4 equations, we must check any possible 3x3 systems of equations. There are four cases, and all four produces positive intersections:
(1.25,2.68, 1.89), (1.31,2.56,1.09),(1.38, 2.63,1.09),(0.75,2.75,1.13)

Now, since the projects go up by whole numbers, we can see that all the four cases are closest to (1,3,1).

We can see that (17,0,0), (0,8.75,0), (0,0,1.27) and (0,0,0) are marked on Figure1.There are two more points, the one which is marked by the star, it is closest to (1,3,1), and the other point is (0,3,1.1) which is marked more precisely on Figure 2.

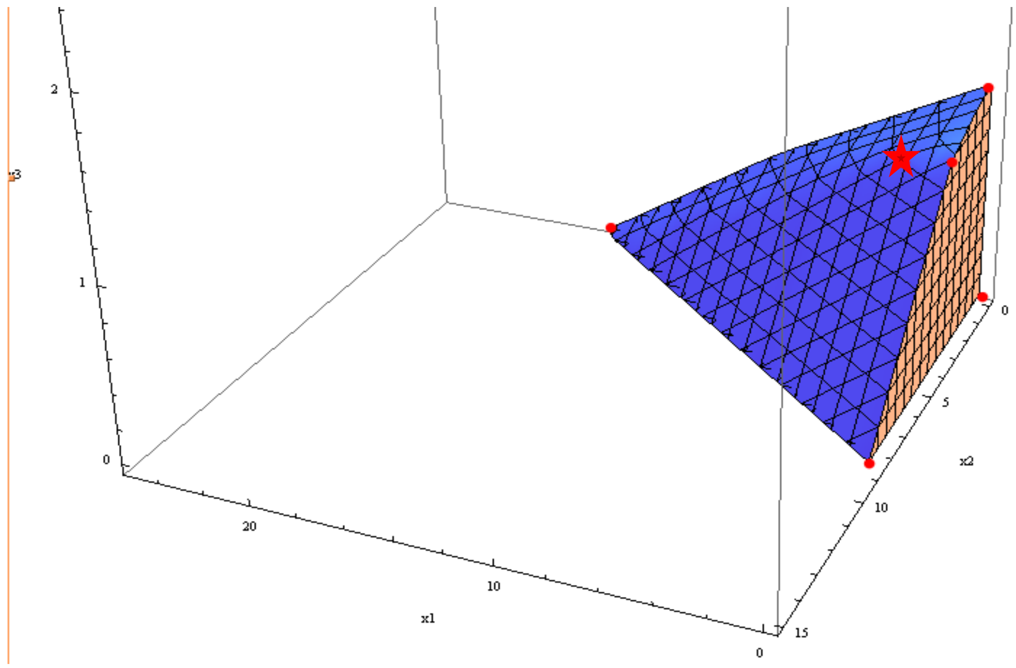

Figure 1

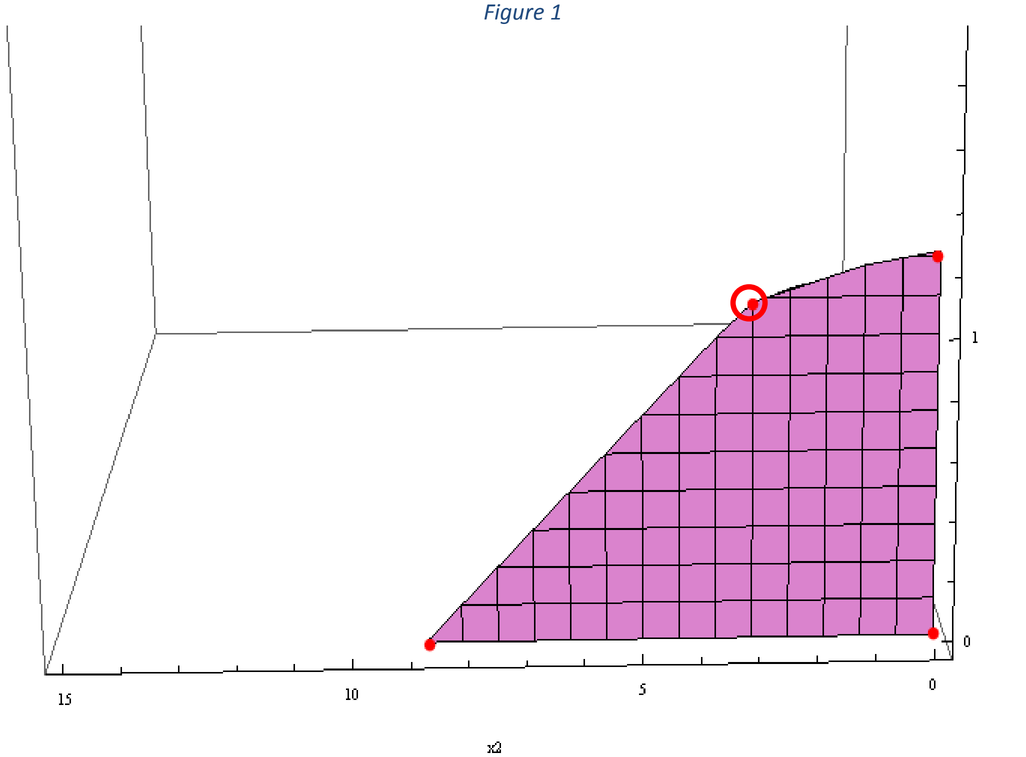

Figure 2

Also, we can see on Figure 3, that x1x2 plane has the similar points as what we found on Figure 1 and 2.

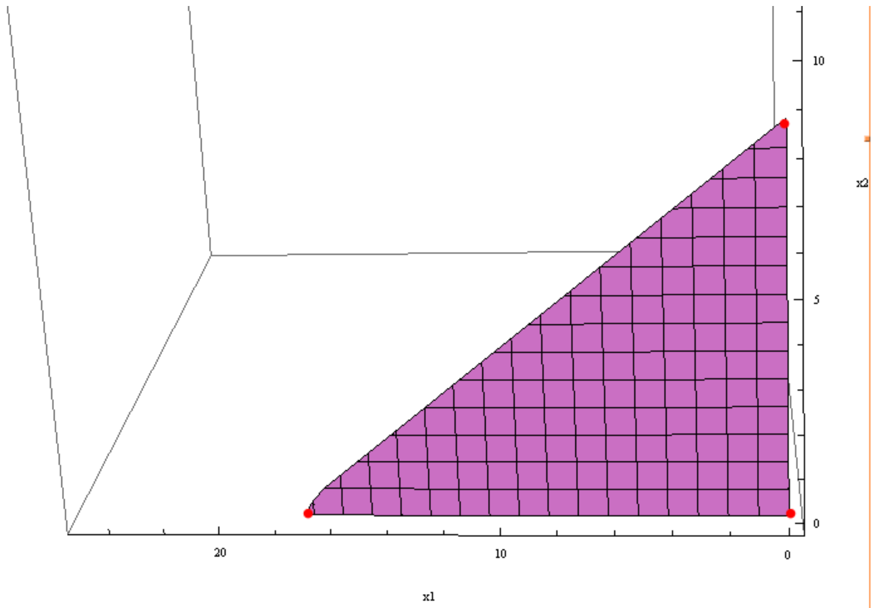

Figure 3

Now we round the points we found to the nearest whole number, because we cannot do half a project and so on. After doing so, we put the values back in to the objective function to find the maximum profit. Below is a chart which gives us the profit for the points we found on Figures 1,2 and 3.

	Case 1	Case 2	Case 3	Case 4	Case5	Case 7
Number of Residential Renovation	0	0	17	0	0	1
Number of Residential Construction	0	8	0	0	3	3
Number of Commercial Construction	1	0	0	0	1	1
Profit	205000	280000	204000	0	310000	322000

Table 1

Modeling: Microsoft excel has the Solver add-on which uses Simplex method to solve linear programming problems.On figure 4, we can see how I have modelled the problem on solver. First, we chose our objective function to be maximized by changing the number of projects (purple dots) then we tell the solver that our actual usage must be smaller or equal to our constrants. (red dots) Next, we have to tell Solver that we are looking for non-negative, whole number results only.(blue dots) When we press solve, we get the highlighted numbers. Using Sumproduct function, we multiply the matching arrays to find how many workers we used and how much maximum profit will be (product strategyxcoeffiecients). (green dots) (File Unit8-Excel-a)Since our analytical result matches the Excel answer, we can confirm 1renovation, 3residental construction and 1 commercial construction will maximize the profit to 322000.

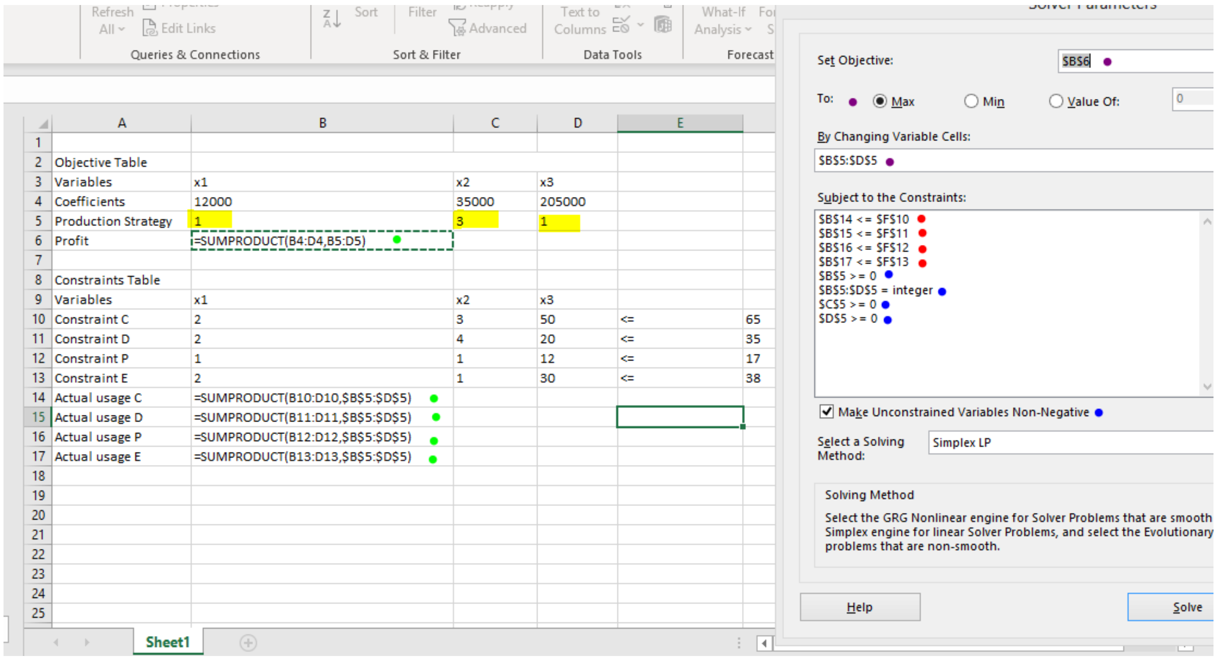

Figure 4

b. **Analytical:** To find a shadow price, we must use the Duality Theorem.

primal problem: Maximize $zP = c^T x$ subject to $x \geq 0, s \geq 0, Ax + s = b$

Objective function = maximize (12000*x1+35000*x2+205000*x3) subject to:

A. 2*x1+3*x2+50*x3 <= 65
B. 2*x1+4*x2+20*x3 <= 35
C. x1+x2+12*x3 <= 17
D. 2*x1+x2+30*x3 <= 38

Dual problem: Minimize $zD = b^T y$ subject to $y \geq 0, t \geq 0$ $A^T y - t = c$

Dual problem=minimize(65*y1+35*y2+17*y3+38*y4) subject to:

A. 2*y1+2*y2+y3+2*y4>=12000
B. 3*y1+4*y2+y3+y4>=35000
C. 50*y1+20*y2+12*y3+30*y4 >=205000

Unlike the objective function, the dual problem shows the pricings. More precisely, it shows the amount of profit per additional unit of the resources. Solving above problem with excel by the simplex method (Figure 5), we get $8108. Thus, to make sure our profit stays the same, we are allowed to pay additional drywallers at most $8108.

Modelling: Using Excel, we can ask for a sensitivity analysis. If we delete B5:D5=integer from part a Solver criteria, we will get the sensitivity analysis report, which points out the shadow price for constraint b as $8108. (Figure 6)

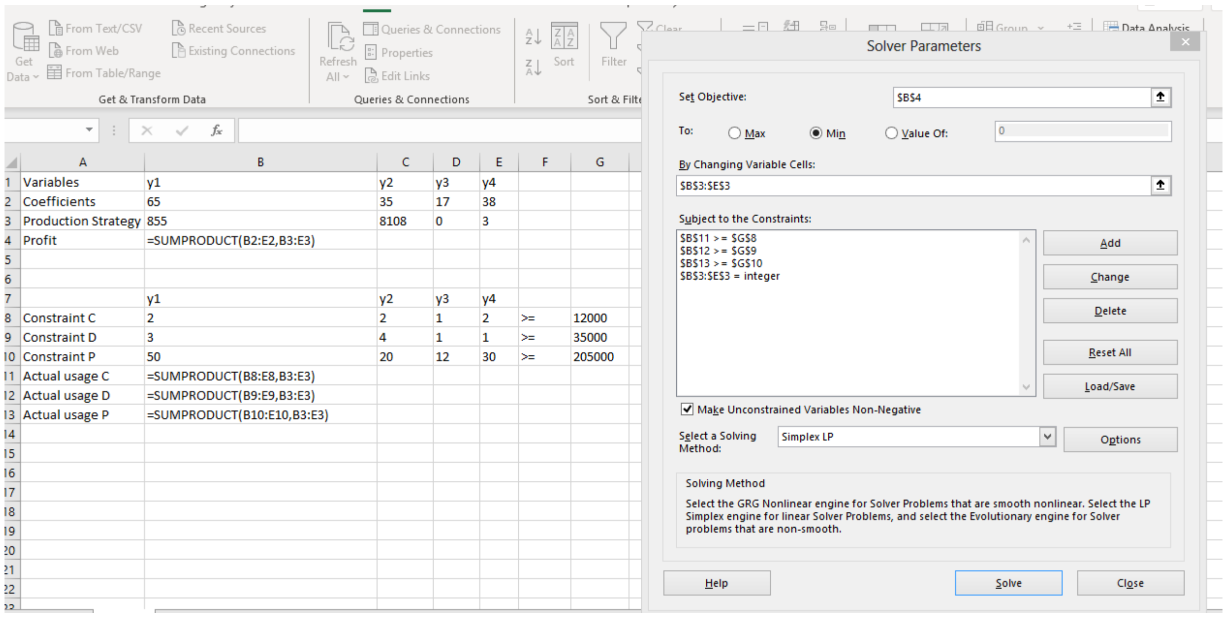

Figure 5

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$5	Production Strategy x1	0	-5928.571429	12000	5928.571429	1E+30
\$C\$5	Production Strategy x2	3.214285714	0	35000	6000	13833.33333
\$D\$5	Production Strategy x3	1.107142857	0	205000	378333.3333	30000

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$14	Actual usage C x1	65	857.1428571	65	2.2	38.75
\$B\$15	Actual usage D x1	35	8107.142857	35	5	5.5
\$B\$16	Actual usage P x1	16.5	0	17	1E+30	0.5
\$B\$17	Actual usage E x1	36.42857143	0	38	1E+30	1.571428571

Figure 6

c. **Analytical:** Using Simplex method on Excel, we change the dual problem objective function coefficient for y2 and find the solution. It is originally 35 (drywallers). We must add 1 unit per time to check if our profit for each additional drywaller stays the same. As soon as \$8108 starts to decrease, we know that we have reached the threshold. For this case, our profit does not change to maximum of additional 5 drywallers. Thus, adding 5 additional drywallers gives us the same profit/drywaller as the shadow price.
Modeling: Using the solver for sensitivity analysis (we must delete integer criteria), we can see on Figure 6 that allowable increase for constraint b is 5, which matches with our analytical method.

All graphs are generated by Wolfram Mathematica

Example code:

```
fr = RegionPlot3D[2 x1 + 3 x2 + 50 x3 ≤ 65 && 2 x1 + 4 x2 + 20 x3 ≤ 36 && x1 + x2 + 12 x3 ≤ 17 && 2 x1 + x2 + 30 x3 ≤ 38
  && x1 ≥ 0 && x2 ≥ 0 && x3 ≥ 0, {x1, 0, 25}, {x2, 0, 15}, {x3, 0, 3}, Mesh → All, PlotPoints → 25, AxesLabel → Automatic]
```

Systems of equations were solved mostly by Wolfram or MathPortal.