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# Epidemiology Visualized: The Prosecutor's Fallacy

**Daniel Westreich* and Noah Iliinsky**

* Correspondence to Dr. Daniel Westreich, Department of Epidemiology, CB 7435 McGavran-Greenberg Hall, University of North Carolina Chapel Hill, Chapel Hill, NC 27599 (e-mail: djw@unc.edu).

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The "prosecutor's fallacy" (the assumption that Pr [probability] ($A|B$) = Pr ($B|A$)) arises often in epidemiology but is often unrecognized as such, in part because investigators do not have strong intuitions about what the fallacy means. Here, we help inform such intuitions and remind investigators of this fallacy by using visualizations. In figures, we demonstrate the prosecutor's fallacy, as well as show conditions under which Pr ($A|B$) can be assumed to be equal to Pr ($B|A$). Visualizations can help build intuition around statistical concepts such as the prosecutor's fallacy and should be more widely considered as teaching tools.

Bayes' rule; Bayes' theorem; logical fallacies; prevention paradox; probability; prosecutor's fallacy; statistics

Abbreviations: FAS, fetal alcohol syndrome; MI, myocardial infarction; Pr, probability.

The "prosecutor's fallacy" (1) is a well-known statistical fallacy (2) arising from both a misunderstanding of conditional probabilities and issues of multiple testing. Here, we focus on the conditional probability aspect of the prosecutor's fallacy, in which the probability of $A$ given $B$ is assumed to be the same as the probability of $B$ given $A$. The classic example of the prosecutor's fallacy arises when a prosecutor (whence its name) argues that 1) if the accused were guilty, the probability of the evidence at hand (for example, a DNA match) would be high; therefore, 2) given the evidence at hand, the probability of the accused's guilt must be high. The fallacy arises in the application of epidemiologic evidence, and statistics generally, in the courts (3).

Such mistakes of reasoning are common in epidemiology and public health. For example, the risk of Down syndrome rises with maternal age, with risks 17 times higher among children born to women aged 40 years and older compared with children born to women under 30 years of age. It is thus easy to assume that most women who give birth to a child with Down syndrome are older; but in fact Alberman and Berry (4) (cited by Rose (5)) note that 51% of total Down syndrome cases occur in children born to women under 30 years of age, chiefly because women under 30 experience far more births. Likewise, although high cholesterol may predispose individuals to coronary artery disease, many more cases emerge from individuals with less-than-high cholesterol (6) (cited by Rose (5)).

The 2 examples above are illustrations of the "prevention paradox" (5, 7), as well as the prosecutor's fallacy; the fallacy also arises in the interpretation of analytical results in epidemiology. For example, Slate.com published a statement that "One in 7 children with fetal alcohol syndrome [FAS] had a mother who drank one to eight drinks per week in the first trimester" (8); that is, that the probability that a mother drank 1–8 drinks is 1/7, given that she gave birth to a child with FAS, or Pr (mother drank | FAS) = 1/7. But this quantity tells us little about the risk of FAS given a specific level of maternal drinking, or Pr (FAS | mother drank). The fallacy also arises in the epidemiologic literature in the confusion of sensitivity (probability of testing positive given true disease status) and positive predictive value (probability of true disease status given a positive test).

Although the prevention paradox is well known in epidemiology and public health, the prosecutor's fallacy is not as widely recognized; a search of PubMed (http://www.ncbi.nlm.nih.gov) for "prosecutor's fallacy" on January 6, 2014, yielded a mere 4 hits, 1 in the field of systems biology (9) and 3 in law (10–12). Fundamentally, the prosecutor's fallacy results from a misunderstanding of Bayes' theorem (reviewed in the Appendix). However, in our experience, many scientists who know

Bayes' theorem still make this mistake, and students—especially less quantitatively experienced students—sometimes have trouble with Bayesian logic, finding it difficult or counterintuitive. Here, we offer a visual depiction of the prosecutor's fallacy as a tool for teaching and building intuition around the concepts.

## VISUALIZATION

Figure 1 shows a population of interest of 100 men aged 50 years who are not being treated for hypertension and who have total cholesterol of 235 mg/dL, high-density lipoprotein cholesterol of 40 mg/dL, and systolic blood pressure of 120 mm Hg. Nine percent of these men (represented by 9 striped squares) will be expected to have a myocardial infarction (MI) in 10 years (13). One-quarter of the men are smokers (represented by 25 gray squares); approximately 16% of the smokers are expected to have an MI in 10 years (13); hence, 4/25 smokers will have an MI (both striped and gray squares).

The number of smokers who have an MI is clearly the same as the number of people who have MIs who are smokers. But is the proportion of smokers who have MIs the same as the proportion of MI victims who are smokers? From Figure 1, the answer is clearly no; the probability of being a smoker given that you had an MI—that is, Pr (smoker | MI)—is not the same as Pr (MI | smoker). In particular, Pr (MI | smoker) = Pr (striped | gray) = 4/25 = 0.16, and Pr (smoker | MI) = Pr (gray | striped) = 4/9 = 0.44. These are not at all the same number, and to assume they are the same is to commit the prosecutor's fallacy. Visually, we note that the proportion of gray squares that overlap with striped squares is not the same as the proportion of striped squares that overlap with gray squares.

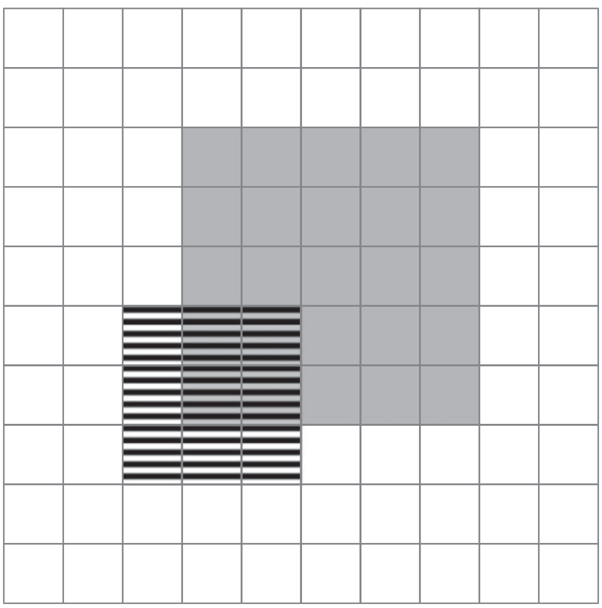

**Figure 1.** A population represented by 100 squares, with 9 striped squares, 25 gray squares, and 4 striped and gray squares. Pr (striped | gray) = 4/25; Pr (gray | striped) = 4/9. Assuming Pr (striped | gray) = Pr (gray | striped) is the prosecutor's fallacy.

Figure 2 shows exceptions to the typical prosecutor's fallacy presented in Figure 1. Now there are 16 striped squares, 16 gray squares, and 4 dotted (and gray) squares. Here, note that because the overall prevalence of striped and gray squares is the same, Pr (striped | gray) = Pr (gray | striped). Also, Pr (striped | dotted) = Pr (dotted | striped), because both probabilities are 0. Both of these (similarly sized populations and populations with no overlap) are narrow exceptions to the fallacy.

On the other hand, the dotted squares in Figure 2 are completely contained by the gray squares; thus, Pr (gray | dotted) = 1, whereas Pr (dotted | gray) = 0.25. The prosecutor's fallacy holds when 1 group is a proper subset of the other.

## DISCUSSION

The prosecutor's fallacy is a common misperception in statistics and epidemiology, as well as in law (10–12), and is committed even by those who are well trained in these disciplines. We would argue that at least part of the reason this fallacy is committed, even by those of us who should know better, is that we sometimes lack strong intuitions about statistics in general and Bayes' theorem in particular. Hopefully, this visual presentation of the fallacy will build intuitions around this important issue.

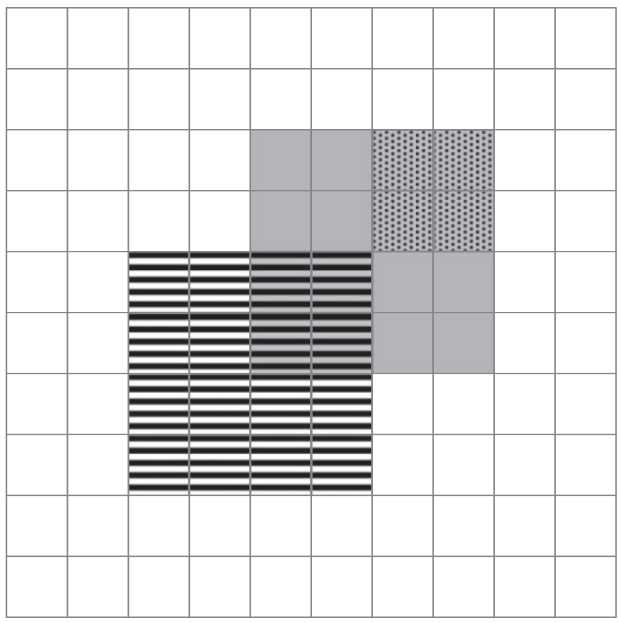

**Figure 2.** A population represented by 100 squares, with 16 striped squares, 16 gray squares, and 4 dotted squares. Because the overall prevalence of striped and gray squares is the same, Pr (striped | gray) = Pr (gray | striped). Also, Pr (striped | dotted) = Pr (dotted | striped), because both probabilities are 0. Both of these (similarly sized populations and populations with no overlap) are narrow exceptions to the fallacy. The dotted squares are completely contained by the gray squares; thus, Pr (gray | dotted) = 1, whereas Pr (dotted | gray) = 0.25. The prosecutor's fallacy holds when 1 group is a proper subset of the other.

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## APPENDIX: BAYES' THEOREM

For 2 events or dichotomous conditions $A$ and $B$, Bayes' theorem is usually stated as

$$\Pr\,(B|A) = \frac{\Pr\,(A|B) \times \Pr\,(B)}{\Pr\,(A)},$$

where Pr (•) indicates the probability of •, and | indicates the "given" operator. So Bayes' theorem can be stated as "the probability of $B$ given $A$ is equal to the probability of $A$ given $B$ times the probability of $B$, all divided by the probability of $A$." As a starting point for intuition building, rewrite the equation as

$$\Pr\,(B|A) = \Pr\,(A|B) \times \frac{\Pr\,(B)}{\Pr\,(A)}$$

and note that it is obvious that Pr ($B$|$A$) and Pr ($A$|$B$) will be equal only if the unconditional probability of $A$ is equal to the unconditional probability of $B$. Assuming that Pr ($B$|$A$) = Pr ($A$|$B$) when either Pr ($A$) != Pr ($B$) or the equality of Pr ($A$) and Pr ($B$) has not been established is precisely the prosecutor's fallacy. However, again, symbolic representations of this issue may not be intuitive for some students, especially those new to statistical thinking. Visualizations in main text may help build intuition.