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Master-Thesis

Path Planning for Dynamic Maneuvers with Micro Aerial Vehicles

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Declaration of Originality

I hereby declare that the written work I have submitted entitled

Path Planning for Dynamic Maneuvers with Micro Aerial Vehicles

is original work which I alone have authored and which is written in my own words.[1]

Author(s)

First name Last name

Supervising lecturer

First name Last name

With the signature I declare that I have been informed regarding normal academic citation rules and that I have read and understood the information on 'Citation etiquette' (`http://www.ethz.ch/students/exams/plagiarism_s_en.pdf`). The citation conventions usual to the discipline in question here have been respected.

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Contents

Abstract

The goal of this Master-Thesis is to develop a numerical robust trajectory-planning algorithm for aggressive multi-copter flights in dense environments. The trajectory generated by this algorithm is represented by polynomials which are jointly optimized. The cost function of the optimization consists of the total trajectory-time as well as the total quadratic snap (second derivation of the acceleration). Including the snap into the cost function guaranties a trajectory without abrupt or expensive control inputs.

Furthermore the process of exploring the state space using the Rapidly-Exploring Random Tree (RRT) algorithm is embedded into the numerical robust algorithm. The sampling points oft the RRT (or RRT*) algorithm are then used as the vertices in the polynomial optimization.

Symbols

Symbols

ϕ, θ, ψ	roll, pitch and yaw angle

Indices

x	x axis
y	y axis

Acronyms and Abbreviations

ETH	Eidgenössische Technische Hochschule
UAV	Unmanned Aerial Vehicle
RRT	Rapidly-Exploring Random Tree
QP	Quadratic Programming

Chapter 1

Introduction

1.1 State of the Art

A lot of research has been done in the field of Unmanned Aerial Vehicles (UAV) in the last years leading to a strong improvement in planning [**?**] as well as in control [[**?**], [**?**]]. Another research field is machine learning [**?**] which is suitable to enhance the performance of aerobatic maneuvers but seams to have a downside regarding motion planning and trajectory generation in dense environments.

Speaking of trajectory planning, there are two different strategies which are pursued. On the one hand, the geometric and the temporal planning are decoupled [**?**] on the other hand, geometric and temporal information are coupled and the trajectory is the result of a minimization problem. For the couplet problem one can make use of the differential flatness of a quadrocopter to derive constraint on the trajectory. Then formulate a cost-function which could be the trajectory-time [**?**] or the total snap [**?**] (second derivation of acceleration).

Another aspect of planning is exploring the state space in the first place. A strong tool to do so are incremental search techniques as for instance the A* [**?**] or the RRT* algorithm [**?**]. The sampling points of the solution of the incremental search can then be used as the vertices for the polynomial optimization.

1.2 Quadratic Programming

1.2.1 Constrained Quadratic Programming

Quadratic Programming (QP) is a special case of optimization problem in which a quadratic function is optimized with respect to its optimizations variables (which are represented with the vector x in Equation **??**)

$$f(x) = \frac{1}{2} \cdot x^T Q x + c^T x \tag{1.1}$$

The optimization is performed under linear constraints on the optimizations variables. Whereas a distinction between equality($E\mathbf{x} = \mathbf{d}$) and inequality constraints ($A\mathbf{x} \leq \mathbf{b}$) has to be made.

Quadratic programming is particularly simple when there are only equality constraints; specifically, the problem is linear. By using Lagrange multipliers and seeking the extremum of the Lagrangian, it may be readily shown that the solution to the equality constrained problem is given by the linear system:

$$\begin{bmatrix} Q & E^T \\ E & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} -\mathbf{c} \\ \mathbf{d} \end{bmatrix} \tag{1.2}$$

where λ is a set of Lagrange multipliers which come out of the solution alongside x.

1.2.2 Unconstrained Quadratic Programming

1.3 Polynomial Trajectory

Regarding the differentiability of polynomials, they are a profound choice to represent a trajectory. Especially for the use in a differentially flat representation of the UAV dynamics. (Flatness in the proper sense of system theory means that all the states and inputs can be expressed in terms of the flat output and a finite number of its derivative).
Furthermore, the differentiability of polynomials enables the possibility to check the derivatives of the trajectory for bounding violations to avoid input saturation. This saturation-check can be perform during trajectory optimization and therefore guarantees the feasibility of the resulting trajectory.

Chapter 2

Einige wichtige Hinweise zum Arbeiten mit LATEX

Nachfolgend wird die Codierung einiger oft verwendeten Elemente kurz beschrieben. Das Einbinden von Bildern ist in LATEX nicht ganz unproblematisch und hï¿½ngt auch stark vom verwendeten Compiler ab. Typisches Format fï¿½r Bilder in LATEX ist EPS[1].

2.1 Gliederungen

Ein Text kann mit den Befehlen `\chapter{.}`, `\section{.}`, `\subsection{.}` und `\subsubsection{.}` gegliedert werden.

2.2 Referenzen und Verweise

Literaturreferenzen werden mit dem Befehl `\cite{.}` erzeugt. Ein Beispiel: [**?**].
Zur Erzeugung von Fussnoten wird der Befehl `\footnote{.}` verwendet. Auch hier ein Beispiel[2].
Querverweise im Text werden mit `\label{.}` verankert und mit `\ref{.}` erzeugt. Beispiel einer Referenz auf das zweite Kapitel: Kapitel **??**.

2.3 Aufzï¿½hlungen

Folgendes Beispiel einer Aufzï¿½hlung ohne Numerierung,

- Punkt 1
- Punkt 2

wurde erzeugt mit:

```
\begin{itemize}
  \item Punkt 1
  \item Punkt 2
\end{itemize}
```

Folgendes Beispiel einer Aufzï¿½hlung mit Numerierung,

[1] Encapsulated Postscript
[2] Bla bla.

1. Punkt 1
2. Punkt 2

wurde erzeugt mit:

```
\begin{enumerate}
  \item Punkt 1
  \item Punkt 2
\end{enumerate}
```

Folgendes Beispiel einer Auflistung,

P1 Punkt 1

P2 Punkt 2

wurde erzeugt mit:

```
\begin{description}
  \item[P1] Punkt 1
  \item[P2] Punkt 2
\end{description}
```

2.4 Erstellen einer Tabelle

Ein Beispiel einer Tabelle:

Table 2.1: Daten der Fahrzyklen ECE, EUDC, NEFZ.

Kennzahl	Einheit	ECE	EUDC	NEFZ
Dauer	s	780	400	1180
Distanz	km	4.052	6.955	11.007
Durchschnittsgeschwindigkeit	km/h	18.7	62.6	33.6
Leerlaufanteil	%	36	10	27

Die Tabelle wurde erzeugt mit:

```
\begin{table}[h]
\begin{center}
 \caption{Daten der Fahrzyklen ECE, EUDC, NEFZ.}\vspace{1ex}
 \label{tab:tabnefz}
 \begin{tabular}{ll|ccc}
 \hline
 Kennzahl & Einheit & ECE & EUDC & NEFZ \\ \hline \hline
 Dauer & s & 780 & 400 & 1180 \\
 Distanz & km & 4.052 & 6.955 & 11.007 \\
 Durchschnittsgeschwindigkeit & km/h & 18.7 &  62.6 & 33.6 \\
 Leerlaufanteil & \% & 36 & 10 & 27 \\
 \hline
 \end{tabular}
\end{center}
\end{table}
```

2.5 Einbinden einer EPS-Graphik

Das Einbinden von Graphiken kann wie folgt bewerkstelligt werden:

```
\begin{figure}[h]
   \centering
   \includegraphics[width=0.75\textwidth]{pics/k_surf.eps}
   \caption{Ein Bild.}
   \label{pics:k_surf}
\end{figure}
```

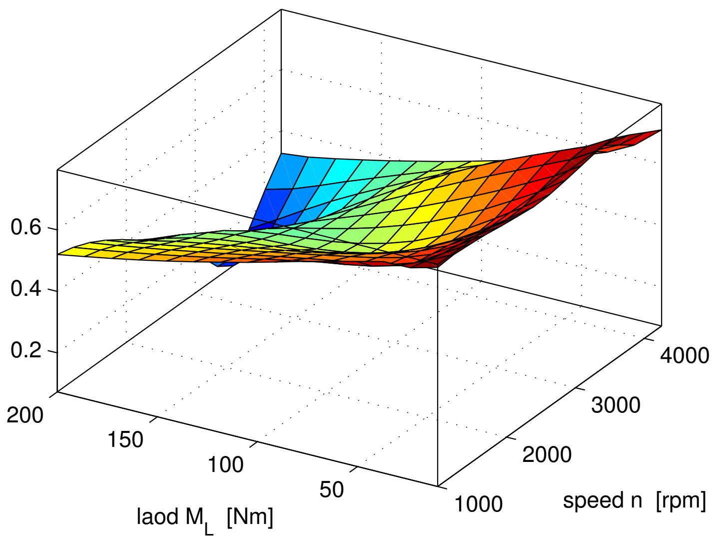

Figure 2.1: Ein Bild.

oder bei zwei Bildern nebeneinander mit:

```
\begin{figure}[h]
  \begin{minipage}[t]{0.48\textwidth}
    \includegraphics[width = \textwidth]{pics/cycle_we.eps}
  \end{minipage}
  \hfill
  \begin{minipage}[t]{0.48\textwidth}
    \includegraphics[width = \textwidth]{pics/cycle_ml.eps}
  \end{minipage}
  \caption{Zwei Bilder nebeneinander.}
  \label{pics:cycle}
\end{figure}
```

Bemerkung: Ersetzt man den Positionierungsparameter `h` durch `H`, so wird das Gleiten der Abbildung verhindert.

2.6 Mathematische Formeln

Einfache mathematische Formeln werden mit der equation-Umgebung erzeugt:

$$p_{me0f}(T_e, \omega_e) \;=\; k_1(T_e) \cdot (k_2 + k_3 S^2 \omega_e^2) \cdot \Pi_{max} \cdot \sqrt{\frac{k_4}{B}}\,. \tag{2.1}$$

Der Code dazu lautet:

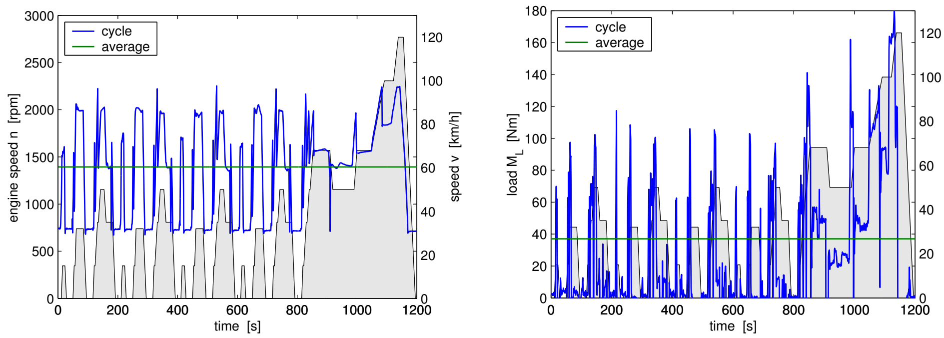

Figure 2.2: Zwei Bilder nebeneinander.

```
\begin{equation}
 p_{me0f}(T_e,\omega_e) \ = \ k_1(T_e) \cdot (k_2+k_3 S^2
 \omega_e^2) \cdot \Pi_{max} \cdot \sqrt{\frac{k_4}{B}} \, .
\end{equation}
```

Mathematische Ausdrï¿½cke im Text werden mit \$formel\$ erzeugt (zB: $a^2+b^2 = c^2$).

2.7 Weitere nï¿½tzliche Befehle

Hervorhebungen im Text sehen so aus: *hervorgehoben*. Erzeugt werden sie mit dem `\epmh{.}` Befehl.

Appendix A

Irgendwas

Bla bla . . .

Appendix B

Nochmals irgendwas

Bla bla ...

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