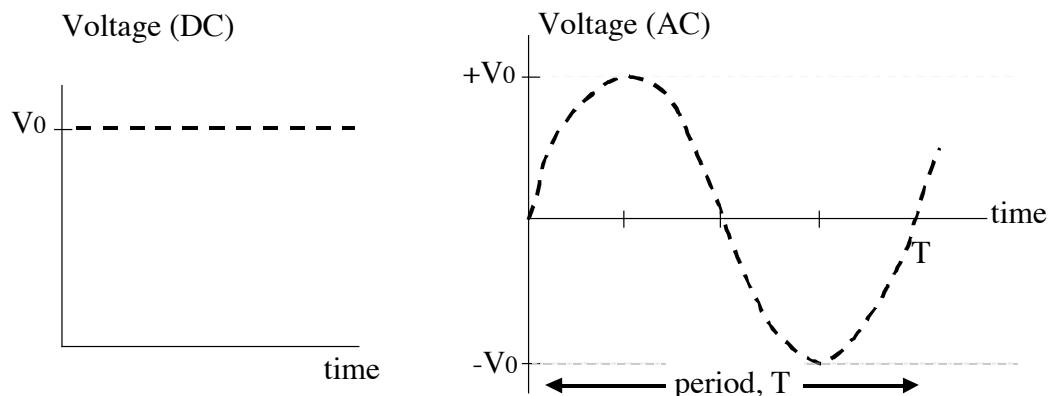


## AC Voltage and Current:

Batteries produce a steady, fixed voltage, called DC, or direct current. (We should probably call them DV, direct voltage, but never mind)

The power company produces a time-varying voltage, AC, or *alternating current*. Here's a sketch of voltage vs. time:



The mathematical formula for AC voltage is

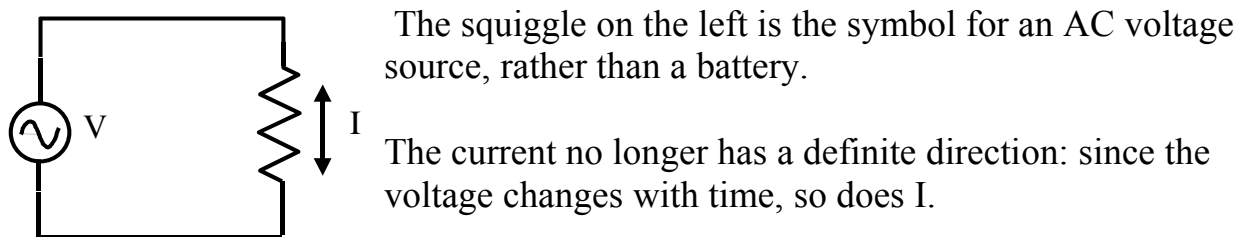
$$V(t) = V_0 \sin(2\pi f t). \text{ (Your calculator MUST be in radian mode!)}$$

$V_0$  is called “peak” or “maximum” voltage.

In the USA the period  $T = 1/60$  s, so frequency  $f = 1/T = 60$  Hz.

(In Europe, it's closer to 50 Hz).

A simple circuit diagram for a light bulb (which is basically a resistor) plugged into the wall might look like this:



In fact, we can figure out  $I(t)$  easily from Ohm's law:

$$I(t) = V(t)/R = \frac{V_0}{R} \sin(2\pi f t) = I_0 \sin(2\pi f t).$$

Here, we have found the maximum current  $I_0 = V_0/R$ .

Clearly, current  $I$  alternates right along with  $V$  (hence the name AC)

Ohm's law continues to hold in AC circuits, and  $V_0 = I_0 R$ ...

The voltage  $V(t)$  is symmetric, it's + as often as -, it averages to 0.

We say  $V(\text{ave}) = 0$  (or  $\bar{V} = 0$ )

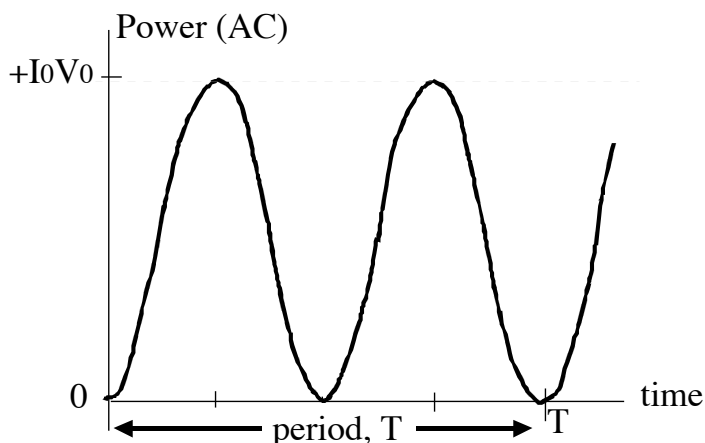
Similarly,  $I(t)$  is also oscillating about 0,  $I(\text{ave}) = 0$ .

Recall that  $P = IV$ , so what is the average power,  $\bar{P}$ ?

(You *might* guess zero, but think about light bulbs: average power used by real bulbs surely can't be zero, otherwise they'd be free)

At any moment in time,  $P(t) = I(t) \cdot V(t) = I_0 V_0 \sin^2(2\pi f t)$ .

Let's graph this, because the "sin squared" changes things a bit:



$\sin^2(\text{anything})$  is always positive.

$\sin^2(\text{anything})$  runs from 0 up to 1 and back again.

On average, it is  $1/2$ .

That means  $\bar{P} = (1/2) I_0 V_0$ . (It is *not* zero.)

Here's an odd question: "what's the average of the *square* of voltage?" (It may not seem obvious why I'd care, but then remember  $P = V^2/R$ , so  $V^2$  *does* appear in formulas... We really will care about this.)

Remember,  $V(\text{ave})=0$ . So you might think the average of  $V^2$  is zero - but no! Just like the power example above:  $V^2(t) = V_0^2 \sin^2(2\pi f t)$ , and the average value of  $\sin^2$  is  $1/2$ , not 0:  $V^2(\text{average}) = V_0^2 / 2$ .

The "average of the square" is NOT the square of the average (which was zero)!

Now we have another way to find average power: find the average of  $P=V^2/R$ , which (we just showed) is  $P(\text{ave}) = (1/2)V_0^2/R$ .

(It's really the SAME result as the previous page, namely

$P(\text{ave}) = (1/2) I_0 V_0$ , because remember  $I_0 = V_0/R$ .)

People have even given a name to  $\text{Sqrt}[V^2(\text{average})]$ .

They call this *Vrms*, the “root mean square” voltage.

$$V_{\text{RMS}} \equiv \sqrt{(V^2)_{\text{Average}}}$$

From its definition (just square both sides):  $V_{\text{rms}}^2 = V^2(\text{ave})$ .

Now, plugging in my result above for  $V^2(\text{ave}) = (1/2) V_0^2$  gives

$$V_{\text{rms}} = V_0 / \text{Sqrt}[2].$$

The rms voltage is *not* the average voltage (which is 0), but it’s kind of a “representative” voltage. After all, voltage runs from  $-V_0$  to  $+V_0$ , it’s (almost) always less than  $V_0$ , so  $V_0/\text{Sqrt}[2]$  is kind of a more “typical” voltage...

Similarly,  $I_{\text{rms}} = I_0 / \text{Sqrt}[2]$  gives the typical current.

Now, remember, we had old (DC) formulas that said

$$P = IV = I^2 R = V^2/R.$$

The new (AC, but averaged) formulas we just derived say

$$P(\text{ave}) = (1/2) I_0 V_0 = (1/2) I_0^2 R = (1/2) V_0^2/R.$$

That recurring factor of  $(1/2)$  is annoying, and perhaps confusing.

It’s there because we’re writing average power in terms of *maximum*  $I$  and  $V$ .

If instead we rewrote  $P(\text{ave})$  in terms of rms values, we’d get a nicer result:

$$P(\text{ave}) = I_{\text{rms}} V_{\text{rms}}. \quad (\text{Check that - convince yourself it’s right.})$$

The AC average formula LOOKS like the old DC formula, exactly:

no factors of 2 at all, if you just use rms values instead of “peak” values.

$$P(\text{ave}) = I_{\text{rms}} V_{\text{rms}} = I_{\text{rms}}^2 R = V_{\text{rms}}^2/R$$

(Convince yourself that they’re *all* right)

*Example:* US Wall sockets really have  $V_{rms} = 120 \text{ V}$ . What is the *peak* voltage,  $V_0$ ?

*Answer:*  $V_0 = \text{Sqrt}[2] * V_{rms} = 170 \text{ V}$ .

The US wall voltage is NOT running from  $-120 \text{ V}$  to  $+120 \text{ V}$ .

It's really running from  $-170 \text{ V}$  to  $+170 \text{ V}$ .

On average it is 0, but it has a “typical” value of  $120 \text{ V}$ .

In particular, when computing POWER, you can just pretend it's  $120 \text{ V DC}$ , and just use the old familiar power formulas.

That's why people say “the wall is  $120 \text{ V}$ ”; they really MEAN  $V_{rms}$ .

In Europe,  $V_{rms} = 240 \text{ V}$ . This causes serious problems if you try to plug a US appliance into a European socket, or vice versa.

E.g., consider a  $100 \text{ W}$  bulb purchased in the US. Plug it into the wall in Europe.

The resistor,  $R$ , is the same of course, but  $V$  is different.

Since  $P(\text{ave}) = V_{rms}^2 / R$ , and  $V_{rms}$  is about 2 times bigger there, squaring gives 4 times more power. It becomes a  $400 \text{ W}$  bulb, but it's not designed to dissipate all that heat - it'll burn out immediately.

(If you go the other way, and plug a European  $100 \text{ W}$  bulb into the wall here, what will happen?)

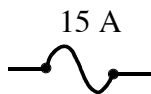
*Example:* Earlier, we computed  $R$  for a short piece of Cu wire, and got  $0.01 \Omega$ .

What's the average power dissipated in this wire, if a current of  $I_{rms} = 25 \text{ A}$  runs through the wire into a big appliance?

*Answer:*  $P(\text{ave}) = I_{rms}^2 * R = (25 \text{ A})^2 * (0.01 \Omega) = 6.3 \text{ W}$ .

That's quite a lot of power. (A toaster might dissipate  $500 \text{ W}$  or so) The wire will certainly heat up. Even though  $R$  is small, it's not 0. If you let appliances consume too much current, the wires in your house could easily start a fire. That's why you have *fuses* (typically rated at  $15\text{-}20 \text{ A}$ ). A fuse is a little device that shuts off all the current in a circuit if it ever exceeds the rated value.

The circuit symbol for a  $15 \text{ A}$  fuse is



**Transformers:**

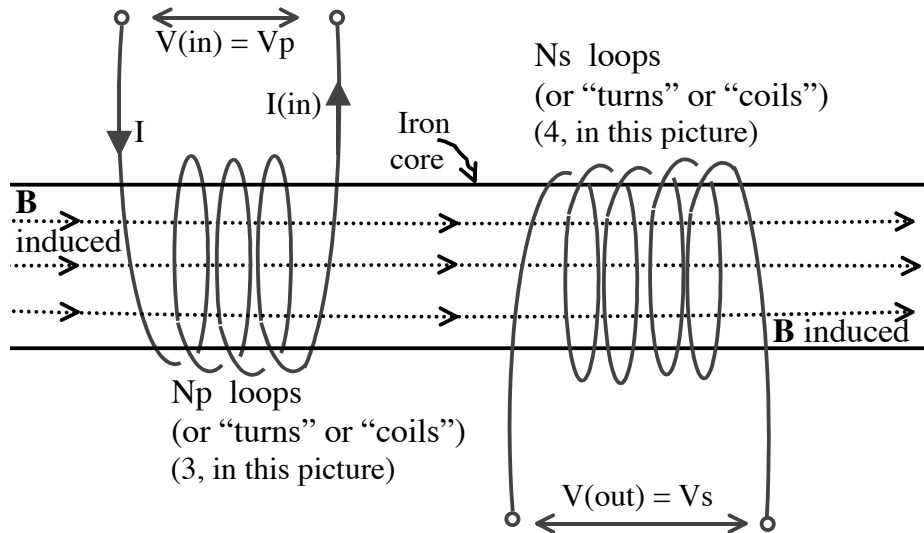
These are clever (and simple) devices to transform AC voltages.

You need two coils:

the “**input wire**” or “*primary winding*”,

and the “**output wire**” or “*secondary winding*”.

The wires are usually wrapped around iron. There are no moving mechanical parts needed.



If  $V(\text{in})$  is steady ( $I(\text{in})$  is steady) then  $\mathbf{B}(\text{induced})$  through the coils is

steady. The flux  $\Phi$  through the secondary coil is therefore steady, and Faraday's law says  $V(\text{out})=0$ : *no output current, or voltage*.

Moral: transformers do *nothing* if the input is steady, or DC.

But, if  $V(\text{in})$  is AC, then the induced  $\mathbf{B}$  keeps flipping direction, which means  $\Phi$  is changing, and Faraday says we *will* induce an EMF in the secondary coil. That means there is a  $V(\text{out})$ .

Faraday says, specifically,

$$V(\text{in}) = N_p \Delta\Phi/\Delta t,$$

$$V(\text{out}) = N_s \Delta\Phi/\Delta t.$$

The flux  $\Phi$  is the *same* through both coils, because an iron core will guide all the  $\mathbf{B}$  field lines through the secondary. ( $\mathbf{B}$  and Area are thus the same for both coils...)

Dividing those last two equations gives

$$\boxed{V(\text{in}) / V(\text{out}) = N_p / N_s} \quad (\text{But, only if } V_{\text{in}} \text{ is AC!})$$

or, of you prefer,  $\boxed{V(\text{out}) = V(\text{in}) * (N_s/N_p)}$  .

A **“step up” transformer** has more secondary windings,  $N_s > N_p$ , which means  $V(\text{out}) > V(\text{in})$ .

You get more voltage out than you had to start with.

A **“step down” transformer** has  $N_s < N_p$ , so  $V(\text{out}) < V(\text{in})$

A step up transformer *almost* looks like something for nothing:  $V(\text{in})$  could be small, and you could get a huge Voltage out (!?) Well....yes,  $V(\text{out})$  can indeed be greater than  $V(\text{in})$ , but

**Power(out) = Power(in).**

or if you prefer

$$V(\text{out}) * I(\text{out}) = V(\text{in}) * I(\text{in})$$

This is conservation of energy. A good transformer is very efficient, but energy is still conserved!

$$\boxed{I(\text{out}) = [V(\text{in}) / V(\text{out})] * I(\text{in}) = (N_p / N_s) * I(\text{in})} \quad .$$

Step up transformers do increase the voltage, but at the cost of decreasing the current you get out. In reality, you always lose a little power to heating, eddy currents in the iron, etc.

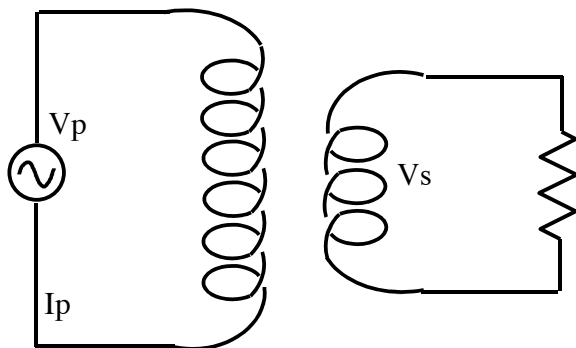
A good transformer might give you  $P(\text{out}) = 99\% P(\text{in})$ ...

Many household devices use step *down* transformers, e.g. chargers, which don't need much voltage (certainly not 120V for a little 1.5 V battery), but they want lots more current than the 15-30 A your wall sockets are limited to.

Other household devices use step *up* transformers, e.g. your TV set needs much higher voltages than 120V, but not much current is needed.

Transformers are a wonderful invention, and allow for much of our electrical technology and distribution system.

Example: A 100:1 step down transformer:



This is the generic symbol used for a transformer.

R “100:1 step down” means  $N_p/N_s = 100:1$

Suppose in this circuit that  $V_p = 120$  V (it’s plugged into a regular wall socket), and  $I_p = 15$  A (a typical value you might get out of normal wall sockets before the circuit breaker blows.)  
 Then,  $V_s = (1/100) V_p = 1.2$  V.  
 $I_s = (100) * I_p = 1,500$  A.

You could never get 1500 A directly out of the wall!  
 This transformer puts lots of currents out, just at a lower voltage.  
 Notice that  $\text{Power}(\text{in}) = 120\text{V} * 15\text{ A} = 1800\text{ W}$   
 $\text{Power}(\text{out}) = 1.2\text{ V} * (1500\text{ A}) = 1800\text{ W}$ . (Energy is conserved)

Example: If you wanted to melt a 12 cm long nail by running current through it, could you do it by just wiring it straight into your wall plug?  
 Using resistivity for iron, I’d estimate the nail’s resistance to be  
 $R = \rho * L / A = (10^{-7}\ \Omega\text{ m}) * (0.12\text{ m} / (10^{-5}\text{ m}^2)) = 1.2\text{E-3}\ \Omega$ .  
 (I got the area by  $\pi r^2$ , with  $r$  about 2 mm, kind of a fat nail?)  
 If you plugged this into the wall (like I did the pickle), you’d expect  
 $I = V/R = (120\text{ V}) / (1.2\text{ E-3}\ \Omega) = 100,000\text{ Amps}$ . Yikes! You’d blow the fuse, nothing would happen. But if this was the “R” in the figure above, then  $I = (1.2\text{ V} / 1.2\text{E-3}\ \Omega) = 1000\text{ A}$ , no problem.  
 (You’d be drawing 10 amps from the wall, do you see why?)  
 But it’s still putting out  $1000\text{ A} * 1.2\text{ V} = 1200\text{ W}$  of power, which almost surely would melt a nail!

The power company (Excel) sends currents over long distances. They do lose power in the lines due to resistance,  $P(\text{loss}) = I^2 R$ .

They'd prefer to transmit a small current, to avoid this power loss (because the resistance of the lines is something constant) But you and I demand a certain average power  $= V \cdot I$ . They want to supply this to us. They want to give us all the  $V \cdot I$  we ask for, but send less current through the cables, so they use VERY HIGH voltages! (Large  $V$  \* small  $I$  gives the power we ask for, while small  $I$  through the lines means less loss.)

They have step up transformers at the power plants, which can take the voltage up to a million volts on some long distance power lines. At the edge of cities, they have step down transformers (at transformer stations, there's one not far from my house) which convert it down to lower voltage, perhaps 2000 V. Then, right at your house, there's one last step-down transformer to take it down to 120 V (or actually 220V).

**Example:** Say my house uses  $10^4$  W of power. (A little high, but not an unusual number for a big household, during the daytime) Suppose the transmission lines have a total resistance, all the way from power station to my house, of  $1 \Omega$ . (That's pretty small)

If they just transmitted at 120 V all the way, I'd use

$$I = P/V = (10^4 \text{ W}/120 \text{ V}) = 83 \text{ A}.$$

This current has to go through their lines, losing power of

$$I^2 R = (83 \text{ A})^2 (1 \Omega) = 7,000 \text{ W}.$$

Yikes! 70% of the power I need is wasted in heating their lines!

But if they used 120,000 V instead (using transformers), then

$$I = P/V = (10^4 \text{ W}/120,000 \text{ V}) = 0.083 \text{ A}.$$

This current goes through their lines, losing power of

$$I^2 R = (.083 \text{ A})^2 (1 \Omega) = .007 \text{ W}. \text{ Nothin'!}$$