Exercises for Chapter 6

Exercise 1: Speckle

Visualize the concept of the *Central Limit Theorem* by using self-generated samples of random variables drawn from one or more arbitrary probability distributions (e.g. Exponential, Gumbel, Rayleigh, Uniform, Poisson, ...).

Try to answer the following questions:

- What is the main finding of the *Central Limit Theorem*?
- Why is it important for the interpretation of speckle?
- What constraints must the considered variables obey if their behavior is described within the concept of the *Central Limit Theorem*?
- How does the validity of the *Central Limit Theorem* depend on the sample size?

Exercise 2: Backscattering Coefficient

Visualize the **backscattering coefficient (and its individual contributions) of a bare- and vegetated soil-surface (in dB)** for various soil and vegetation conditions within the angular range of $\theta \in [20 - 75°]$ using an observation frequency of approx. 5 GHz.
The vegetative coverage should be modelled using the Cloud-Model as given in Eq. 6.64 of the lecture notes. (Use reasonable values for ω and τ !).
For the bare-soil backscattering coefficient, a simple bare-soil model as proposed by Champion[1] can be used which assumes that the bare-soil backscattering coefficient is linearly related to soil-moisture in the microwave-domain, and it's angular dependency can be described by a simple cosine-model as given below:

$$\sigma^0_{soil}(\theta)[dB] = C(\theta) + D\, m_v \qquad \text{with} \qquad C(\theta) = C_1 + C_2\, \cos(\theta)^{C_3}$$

The values for the parameters C1, C2, C3 from Champion[1] can be found in Fig.1. An estimate on the parameter D which is usually referred to as the soil moisture sensitivity can be obtained from the findings of Ulaby et.al[2] shown in Fig. 2. Use the values where the measurement specifications fit best.

Try to answer the following questions:

- Is the consideration of vegetation effects necessary for measurements in the microwave domain?
- How is a vegetation coverage treated within the Cloud-Model? (i.e. what simplifications are applied?)
- What are the main effects of a vegetation coverage on the backscattering coefficient under the Cloud-Model approximation?

Hint: Note that within the above empirical bare-soil model σ^0_{soil} soil is already given in dB! In order to use this result within the Cloud-Model, one first has to convert it back to linear units and then convert the total result (including the vegetation contribution) back to dB! This is due to the fact that for the logarithmic quantities we have:

$$\log(A + B) \neq \log(A) + \log(B)$$

[1] Champion L.: Simple modelling of radar backscattering coefficient over a bare soil: variation with incidence-angle, frequency and polarization, Int. J. Remote Sensing (1996), Vol.17, No.4, 783-800

[2] F.T.Ulaby, P.P. Batlivala, M,C, Dobson: Microwave Backscatter Dependence on Surface Roughness, Soil-Moisture, and Soil-Texture: Part I - Bare Soil, IEEE Transactions on Geoscience Electronics, Vol.GE-16, No.4, Oct. 1978

Table 2. Values of cosine model parameters C_1, C_2 and C_3 fitted on the radar measurements. The residual error is indicated as R_2.

	C_1	C_2	C_3	R_2
5·3 GHz, *HH*	−29·2	27·2	2·8	1·9
5·3 GHz, *VV*	−26·0	24·0	2·7	1·9
5·3 GHz, *HV*	−33·0	16·0	3·2	1·6
9·0 GHz, *HH*	−20·9	17·4	3·9	2·1
9·0 GHz, *VV*	−21·5	17·1	3·1	2·2
9·0 GHz, *HV*	−27·0	12·9	4·2	2·4

Figure 1: Table taken from Champion (1996)[1]

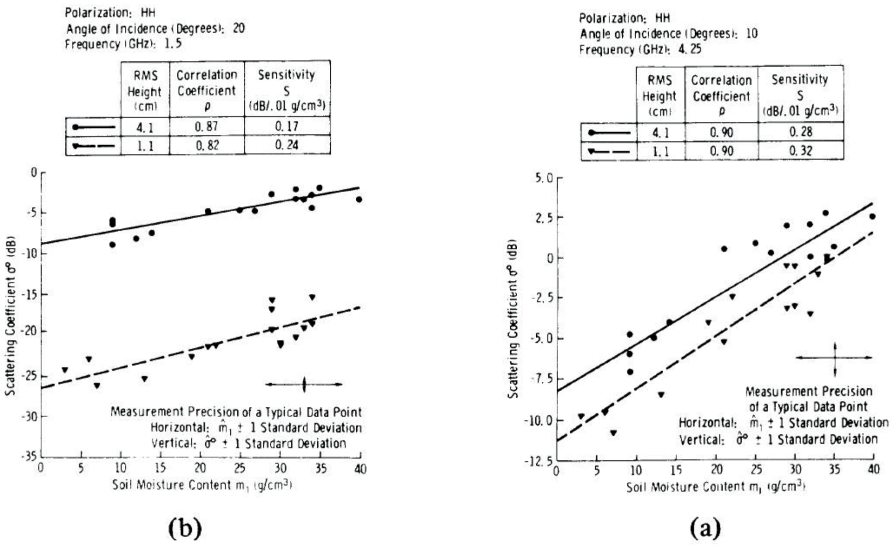


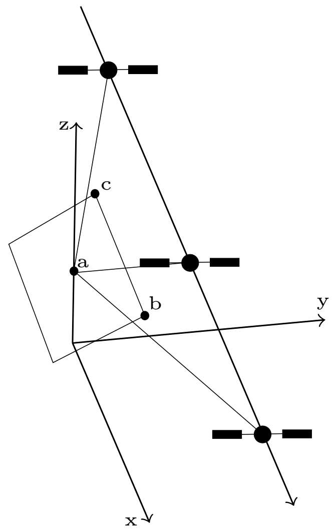
Fig. 9. Comparison of the backscatter response to m_1 of a smooth surface and a rough surface of (a) θ = 10°, f = 4.25 GHz and (b) θ = 20°, f = 1.5 GHz. (Note the difference in the vertical scale.)

Figure 2: Figure taken from Ulaby et.al. (1978)[2]

Exercise 3: Incidence Angles

A terrain facet is given by three points a (center point), b and c:

$$a = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} \quad b = \begin{pmatrix} 5 \\ 5 \\ 15 \end{pmatrix} \quad c = \begin{pmatrix} -5 \\ 5 \\ 15 \end{pmatrix}$$



A radar satellite passes by this facet and measures a radar cross section σ equal to $1.256\,\mathrm{m}^2$ with an across-track spatial resolution (dR) of 5 m and an along-track spatial resolution (dA_{az}) of 10 m. In this example it is assumed that the surface is a perfect scatterer and all measurements only differ by the incidence angle. The measurements start at x_s and continue as the satellite reaches the position x_e (orbit direction follows x-axis):

$$x_s = \begin{pmatrix} -2000 \\ 0 \\ 1000 \end{pmatrix} \quad x_e = \begin{pmatrix} 2000 \\ 0 \\ 1000 \end{pmatrix}$$

First, transform σ into a radar brightness value β^0. Then compute the incidence angle for each measurement, which takes place every 10 m along the orbit path. You can determine the incidence angles by computing the angles between the surface normal and the surface-satellite vector. Finally, transform the radar brightness into sigma nought (σ^0) and gamma nought (γ^0) for each incidence angle and plot the result.

Try to answer the following questions:

- What is the general relation between backscatter and incidence angles?
- Explain the terms radar brightness, sigma nought and gamma nought.
- Are there advantages of using a specific backscattering coefficient?

Exercises for Chapter 8

Exercise 1: InSAR Measurement Simulation

This exercise simulates the actual measurement of the phases based on a given Digital Elevation Model (DEM). First, download or use the Python file:
"~/shared/mrs22w/scripts/mrs_ue_utils.py"
at the JupyterHub server and put it into the same directory as your Jupyter Notebook script. Import the `generate_dem()` function at the beginning of your script and generate 3D coordinates with it:

```
from mrs_ue_utils import generate_dem
x, y, z = generate_dem()
```

Two C-band SAR sensors observe the region of interest (defined by the generated DEM) at 5 cm wavelength. The master sensor position is located at p_m and is separated by a baseline b into the direction $\vec{b}_e$ from the slave sensor position p_s (all units in m):

$$p_m = \begin{pmatrix} 0 \\ -12000 \\ 10000 \end{pmatrix} \quad \vec{b}_e = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \quad b = 100$$

Use the generated DEM and the sensor positions to create a master and a slave phase image.

Hint: derive relative phase values ϕ ($\phi \in [0, 2\pi[$) from the computed distances between the sensors and terrain points.

Try to answer the following questions:

- What kind of baselines exist?
- What does the term decorrelation mean in terms of InSAR?
- Name some terms of decorrelation.

Exercise 2: Phase Difference

Take the master (*"master_img.npy"*) and slave (*"slave_img.npy"*) phase image located on the JupyterHub server under:
"~/shared/mrs22w/data/"
and generate a flat-earth corrected phase interferogram. Use the same input parameters as given in **Exercise 1**. Note: You can save/pickle *NumPy* arrays to disk via `np.save(filepath, np_ar)` and load them with `np_ar = np.load(filepath)`

Hint: One possible solution to generate a flat-earth interferogram would be to use a flat-earth model:

```
x, y, z = generate_dem(z_min=0., z_max=0.)
```

Try to answer the following questions:

- What is a flat-earth correction and why is it necessary?
- What do the fringes show in the interferogram?
- What parameters influence the fringe pattern?

Exercise 3: Phase Unwrapping

For this exercise you need three phase interferograms as input: *"interferogram_1.npy"*, *"interferogram_2.npy"* and *"interferogram_3.npy"*, which you can find on the JupyterHub server under: *"~/shared/mrs22w/data/"*
First, load and visualise them. Next, try to write your own unwrapping algorithm and unwrap the 1-D phase array for line number 400 and the whole column in all images.

Hint: You have to implement a cumulative summation/subtraction, where the sign changes according to phase jumps larger than π.

There are already ready-to-use phase unwrapping algorithms available in Python. For the 2-D images, unwrap the phases with the following function:

```
from skimage.restoration import unwrap_phase
unwrapped_phases = unwrap_phase(np_ar)
```

Apply this function on all three input interferograms. Plot the input data and the result.

Try to answer the following questions:

- Which interferogram(s) relate(s) to the given input DEM?
- Do all results look valid? If not, what could be the reason?
- What problems can occur during phase unwrapping?
- To what input parameter do the fringes in the interferogram relate to, i.e. what parameter(s) need(s) to be considered to compute the relative heights?