Exercises for Chapter 2

Exercise 1: Electromagnetic Waves in Conducting Materials

Explain how the parameters of an electromagnetic wave change (compared to a non-conducting material) when it is traveling inside a conducting material with relative permittivity ϵ_r, permeability μ, and conductivity σ.

Visualize a linearly-polarized, monochromatic electromagnetic wave traveling within such a material for arbitrary (realistic[1]) choices of dielectric properties.

Try to answer the following questions:

- How does the wave-equation change if we consider an electromagnetic wave traveling inside a conducting, linear material?
- Which physical meaning can be assigned to the complex part of the wave-number?
- How is the penetration-depth δ_A defined, and how is it related to σ and ϵ_r?

Exercise 2: Polarization

Explain and visualize the different possibilities for polarization of an electromagnetic wave (linear-, circular-, elliptic-polarization) as presented in Chapter 2.3

Try to answer the following questions:

- Why is it meaningful to specify the polarization of an electromagnetic wave?
  - Why is this not meaningful when talking about sound-waves?
- Which parameters can be used to characterize the state of polarization of a wave?
- Is the state of polarization important when talking about interference-effects between two waves?

Exercise 3: Interference Patterns

Visualize the interference-pattern of two identical (isotropic) sources that are separated a distance Δx from each other dependent on the choice of the wavelength of the emitted radiation.

Try to answer the following questions:

- How can the appearance of the interference-pattern be explained in terms of an approximation formula?
- In which region is the approximate formula applicable? How are the boundaries of this region defined?
- What does the generated interference-pattern tell us if we replace the sources by detectors that are both equally sensitive to radiation incoming from all directions?

[1] When searching for possible dielectric properties of materials, the conductivity is usually expressed in form of the 'loss-tangent' $\tan(\delta)$ which is related to the real-and complex part of the relative permeability via:
$\tan(\delta) = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\epsilon_0 \epsilon_r \omega} \qquad \Rightarrow \qquad \tilde{\epsilon}_r = \epsilon' + i\epsilon'' = \epsilon' \left[1 + i \ \tan(\delta)\right]$

Exercises for Chapter 3

Exercise 1: Blackbody radiation

Visualize and explain the behavior of the brightness of a blackbody as a function of frequency and temperature.

Your visualizations should include:

- two different units: $\left[\mathrm{Wm^{-2}sr^{-2}m^{-1}}\right]$ and $\left[\mathrm{Wm^{-2}sr^{-2}Hz^{-1}}\right]$
- two different scales: linear and logarithmic

Try to answer the following questions:

- Which approximations of Planck's law exist and what are their validity conditions? Demonstrate the validity using your visualization.

Exercise 2: Antenna Directivity

Visualize a polar-plot of the far-field power-pattern of an antenna-array consisting of N omnidirectional antennas positioned on a straight line. Employ a method that allows you to control the directivity of your antenna-array. (Check also lessons learned from Section 2.7!) Explain how such a (passive) microwave sensing system can determine the direction of incoming signals.

Try to answer the following questions:

- How can the design of a microwave antenna control its directivity?
- What are the analogies to the concept for hearing audio waves?
- What are the fundamental differences between microwave-/audio-systems and optical systems?

Exercise 3: Signal mixing

Visualize and explain the functionality of a coherent measurement system. You should be able to visualize all input-, intermediate-, and output signal components (LO, RF, IF, I- and Q-channel). You can use a unit amplitude for simplicity.

Try to answer the following questions:

- How does the mixed signal change in response to the frequency difference of the input signals?
- How can you synthesize the filter in your code using a simple mathematical operation (given the knowledge of the amplitude and the frequency of the signals)?

Hint for evaluating the real- and complex part of the wavenumber:

In the following it is shown how one can calculate the real- and complex part of a quantity x defined via $x^2 = a + ib$ if the following restrictions apply:

- a and b are real, **positive** numbers
  - This applies to the wavenumber k since μ, ϵ, σ and ω are positive numbers.
- the complex part of x is a **positive** number
  - From physical observations we know that the amplitude of electromagnetic waves gets attenuated when entering a conducting material (and not enhanced), therefore the complex part of k must be a positive number.

$$x = \sqrt{a + ib} = x' + ix'' \qquad \Rightarrow \qquad a + ib = x'^2 + i2x'x'' - x''^2$$

$$\Rightarrow \quad a = x'^2 - x''^2 \qquad\qquad b = 2x'x''$$

$$\Rightarrow \quad x' = \frac{b}{2x''} \qquad \Rightarrow \qquad x''^4 + a\,x''^2 - \left(\frac{b}{2}\right)^2 = 0$$

$$\left(x''_{1,2}\right)^2 = -\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$x''_{1,2,3,4} = \pm\sqrt{-\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$$

Since we only seek for real-solutions of x' and x'' (and a and b are real, positive numbers), only the + sign inside the root is meaningful. Furthermore in order to get a positive complex part, only the + sign in front of the root is meaningful. Thus, we find for the complex part:

$$x'' = \sqrt{-\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}} = \sqrt{\frac{a}{2}}\sqrt{\sqrt{1 + \left(\frac{b}{a}\right)^2} - 1}$$

...and inserting this in the above formula for the real-part x' we find:

$$x' = \frac{b}{2x''} = \ldots = \sqrt{\frac{a}{2}}\sqrt{\sqrt{1 + \left(\frac{b}{a}\right)^2} + 1}$$