Boundary Types

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The internal boundary kind is given by:

$$\frac{\cdot}{\text{int} \mathrel{:} \cdot \text{Unit}} \qquad \frac{\cdot}{\text{bool} \mathrel{:} \cdot \text{Unit}} \qquad \frac{A \mathrel{:} \cdot \kappa_A \quad B \mathrel{:} \cdot \kappa_B}{A \times B \mathrel{:} \cdot \kappa_A \times \kappa_B} \qquad \frac{A \mathrel{:} \cdot \kappa_A \quad B \mathrel{:} \cdot \kappa_B}{A + B \mathrel{:} \cdot \kappa_A \times \kappa_B}$$

$$\frac{A \mathrel{:} \cdot \kappa_A \quad B \mathrel{:} \cdot \kappa_B}{A \to B \mathrel{:} \cdot \kappa_A \to \text{Type} \times \kappa_\text{B}}$$

The boundary type is given by:

$$\frac{\cdot}{\Gamma \vdash i : \text{int} \rhd [\text{unit}, \_.()]} \qquad \frac{\cdot}{\Gamma \vdash b : \text{bool} \rhd [\text{unit}, \_.()]}$$

$$\frac{\Gamma \vdash e : \text{int} \rhd [\tau, \_.()]}{\Gamma \vdash \texttt{hold}\ e : \bigcirc\text{int} \rhd [\tau \times \text{int}, \_.()]} \qquad \frac{x : A \ \rhd\ \sigma \in \Gamma}{\Gamma \vdash x : A \rhd [\text{unit}, \_.\sigma]}$$

$$\frac{\Gamma \vdash e_1 : A \rhd [\tau_1, l_1.\sigma_1] \quad \Gamma \vdash e_2 : B \rhd [\tau_2, l_2.\sigma_2]}{\Gamma \vdash (e_1, e_2) : A \times B \rhd [\tau_1 \times \tau_2, (l_1, l_2).(\sigma_1, \sigma_2)]}$$

$$\frac{\Gamma \vdash e : A \times B \rhd [\tau, l.\sigma]}{\Gamma \vdash \texttt{pi1}\ e : A \rhd [\tau, l.(\pi_1 \sigma)]} \qquad \frac{\Gamma \vdash e : A \times B \rhd [\tau, l.\sigma]}{\Gamma \vdash \texttt{pi2}\ e : B \rhd [\tau, l.(\pi_2 \sigma)]}$$

$$\frac{A \mathrel{:} \cdot \kappa \quad \Gamma, x : A \ \rhd\ \alpha \vdash e : B \rhd [\tau, l.\sigma]}{\Gamma \vdash \lambda x{:}A.e : A \to B \rhd [\text{unit}, \_.(\lambda \alpha : \kappa.(\tau, \sigma))]}$$

$$\frac{\Gamma \vdash e_1 : A \to B \rhd [\tau_1, l_1.\sigma_1] \quad \Gamma \vdash e_2 : A \rhd [\tau_2, l_2.\sigma_2]}{\Gamma \vdash e_1\ e_2 : B \rhd [\tau_1 \times \tau_2 \times \pi_1(\sigma_1\ \sigma_2), (l_1, l_2, l).(\pi_2(\sigma_1\ \sigma_2))]}$$

$$\frac{\Gamma \vdash e_1 : \text{bool} \rhd [\tau_1, \_.()] \quad \Gamma \vdash e_2 : A \rhd [\tau_2, l_2.\sigma_2] \quad \Gamma \vdash e_3 : A \rhd [\tau_3, l_3.\sigma_3]}{\Gamma \vdash \begin{pmatrix} \texttt{if}\ e_1 \\ \texttt{then}\ e_2 \\ \texttt{else}\ e_3 \end{pmatrix} : A \rhd [\tau_1 \times (\tau_2 + \tau_3), (l_1, l).(\texttt{case}\ l\ \texttt{of}\ l_2.\sigma_2 \mid l_3.\sigma_3)]}$$