Mockingbird Semantics

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1 MbFilter

1.1 Grammar

$$
\begin{aligned}
\langle \mathit{breakMethodG} \rangle &= \texttt{1G} \mid \texttt{2\_GP} \\
\langle \mathit{breakMethodS} \rangle &= \texttt{1S} \mid \texttt{16}^2\texttt{\_SP} \\
\langle \mathit{breakMethodH} \rangle &= \texttt{1H}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathit{expr} \rangle = {} & \langle \mathit{expr} \rangle \texttt{>>} \langle \mathit{expr} \rangle \\
\mid {} & \texttt{fix } x.\langle \mathit{expr} \rangle \\
\mid {} & x \\
\mid {} & \texttt{test}\{\langle \mathit{expr} \rangle\} \\
\mid {} & \texttt{hit} \\
\mid {} & \texttt{Shade} \\
\mid {} & \texttt{if } |g| > n \texttt{ then } \langle \mathit{expr} \rangle \texttt{ else } \langle \mathit{expr} \rangle \\
\mid {} & \texttt{BreakMapReduceG}(\langle \mathit{breakMethodG} \rangle)\{\langle \mathit{expr} \rangle\} \\
\mid {} & \texttt{BreakMapReduceS}(\langle \mathit{breakMethodS} \rangle)\{\langle \mathit{expr} \rangle\} \\
\mid {} & \texttt{BreakMapReduceH}(\langle \mathit{breakMethodH} \rangle)\{\langle \mathit{expr} \rangle\}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathit{sizeRange} \rangle &= [a, b] \mid [a+] \\
\langle \mathit{entity\text{-}type} \rangle &= \texttt{GeoSamp } \langle \mathit{sizeRange} \rangle \ \langle \mathit{sizeRange} \rangle \\
&\mid \texttt{Hit } \langle \mathit{sizeRange} \rangle \\
&\mid \texttt{Frag } \langle \mathit{sizeRange} \rangle \\
\langle \mathit{transition\text{-}type} \rangle &= \langle \mathit{entity\text{-}type} \rangle \to \langle \mathit{entity\text{-}type} \rangle \\
\langle \mathit{breakG\text{-}type} \rangle &= (\langle \mathit{sizeRange} \rangle \to \langle \mathit{sizeRange} \rangle) \ \langle \mathit{sizeRange} \rangle \\
\langle \mathit{breakS\text{-}type} \rangle &= \langle \mathit{sizeRange} \rangle \ (\langle \mathit{sizeRange} \rangle \to \langle \mathit{sizeRange} \rangle) \\
\langle \mathit{breakH\text{-}type} \rangle &= \langle \mathit{sizeRange} \rangle \to \langle \mathit{sizeRange} \rangle
\end{aligned}
$$

1.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau' \to \tau''}{\Gamma \vdash e_1 \texttt{>>} e_2 : \tau \to \tau''} \tag{1}$$

$$\frac{\Gamma, x : \phi \vdash e : \phi}{\Gamma \vdash \texttt{fix}\ x.e : \phi} \tag{2}$$

$$\frac{\Gamma(x) = \phi}{\Gamma \vdash \texttt{call}\ x : \phi} \tag{3}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp}\ \sigma_1\ \sigma_2 \to \tau}{\Gamma \vdash \texttt{test}\{e\} : \texttt{GeoSamp}\ \sigma_1\ \sigma_2 \to \tau} \tag{4}$$

$$\frac{.}{\Gamma \vdash \texttt{hit} : \texttt{GeoSamp}\ [1,1]\ [1,1] \to \texttt{Hit}} \tag{5}$$

$$\frac{.}{\Gamma \vdash \texttt{Shade} : \texttt{Hit}\ [1,1] \to \texttt{Frag}} \tag{6}$$

$$\frac{\Gamma \vdash e_1 : \texttt{GeoSamp}\ [n+1,b]\ \sigma \to \tau \quad \Gamma \vdash e_2 : \texttt{GeoSamp}\ [a,n]\ \sigma \to \tau}{\Gamma \vdash \texttt{if}\ |g| > n\ \texttt{then}\ e_1\ \texttt{else}\ e_2 : \texttt{GeoSamp}\ [a,b]\ \sigma \to \tau} \tag{7}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp} \to \tau}{\Gamma \vdash \texttt{BreakMapReduceG}(d)\{e\} : \texttt{GeoSamp} \to \tau} \tag{8}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp} \to \tau}{\Gamma \vdash \texttt{BreakMapReduceS}(d)\{e\} : \texttt{GeoSamp} \to \tau} \tag{9}$$

$$\frac{\Gamma \vdash e : \texttt{Hit} \to \tau}{\Gamma \vdash \texttt{BreakMapReduceH}(d)\{e\} : \texttt{Hit} \to \tau} \tag{10}$$

1.3 Denotational Semantics

$$|e_1 \texttt{>>} e_2| = |e_2| \circ |e_1| \tag{11}$$

$$|\texttt{fix}\ x.e| = |[(\texttt{fix}\ x.e)/(\texttt{call}\ x)]e| \tag{12}$$

$$|\texttt{test}\{e\}| = |e| \circ id_{\emptyset} \cup (\lambda(G,S).\{(\perp,k) : (s,k) \in S\}) \circ id_{\not\emptyset} \tag{13}$$

$$|\texttt{hit}| = \{(((\{g\},\{(s,k)\}),\{(h,k)\}) : h = isect(g,s)\} \tag{14}$$

$$|\texttt{Shade}| = \{(\{(s,k)\},\{(f,k)\}) : f = shade(h)\} \tag{15}$$

$$|\texttt{if}\ |g| > n\ \texttt{then}\ e_1\ \texttt{else}\ e_2| = |e_1| \circ id_{|g|>n} \cup |e_2| \circ id_{|g|\leq n} \tag{16}$$

$$|\texttt{BreakMapReduceG}(d)\{e\}| = \begin{array}{l} \{((G,S),\{F_1 \oplus \cdots \oplus F_n\}) \\ : ((G_i,S),f_i) \in |e|, (G,\{G_1,...,G_n\}) \in |d|\} \end{array} \tag{17}$$

$$|\texttt{BreakMapReduceS}(d)\{e\}| = \begin{array}{l} \{((G,S),\{F_1 \cup \cdots \cup F_n\}) \\ : ((G,S_i),f_i) \in |e|, (S,\{S_1,...,S_n\}) \in |d|\} \end{array} \tag{18}$$

$$|\texttt{BreakMapReduceH}(d)\{e\}| = \begin{array}{l} \{(H,\{F_1 \cup \cdots \cup F_n\}) \\ : (H_i,F_i) \in |e|, (H,\{H_1,...,H_n\}) \in |d|\} \end{array} \tag{19}$$

1.4 Theorems

1. For any expression e, if $((G,S),F) \in |e|$, then $keys(S) = keys(F)$.
2. For any expression $e : \texttt{GeoSamp} \to \texttt{Frag}$, if $((G,S),F) \in |e|$, then for all $(s,k) \in S$, $(\bigoplus_{g \in G} shade(isect(g,s)),k) \in F$.

2 MbOrder

2.1 Grammar

$$
\begin{aligned}
e = {} & e \texttt{ >> } e \\
& \mid \texttt{fix}\ x.\ e \\
& \mid x \\
& \mid \texttt{hit} \\
& \mid \texttt{test}\{e\} \\
& \mid \texttt{filt}\{e\} \\
& \mid \texttt{ifsizeG}(> n)\ e\ \texttt{else}\ e \\
& \mid \texttt{buildG}(d_G) \\
& \mid \texttt{buildS}(d_S) \\
& \mid \texttt{unboundG} \\
& \mid \texttt{unboundS} \\
& \mid \texttt{mmrG}\{e\} \\
& \mid \texttt{mmrS}\{e\}
\end{aligned}
$$

$$
\begin{aligned}
d_\alpha = {} & \texttt{id} \\
& \mid p_\alpha \texttt{ >=> } d_\alpha \\
& \mid \texttt{fix}\ x.\ d_\alpha \\
& \mid x \\
& \mid \texttt{bound}(d_\alpha) \\
& \mid \texttt{ifsize}\alpha(> n)\ d_\alpha\ \texttt{else}\ d_\alpha
\end{aligned}
$$

$$
\begin{aligned}
p_G &= \texttt{1G} \mid \texttt{2\_GP} \\
p_S &= \texttt{1S} \mid \texttt{16}^2\texttt{\_SP}
\end{aligned}
$$

$$
\begin{aligned}
\tau = {} & \sigma \to \sigma \\
\sigma = {} & \texttt{GeoSamp}\ \delta\ \delta \\
& \mid \texttt{Hit}\ \delta \\
\delta = {} & * \\
& \mid [\delta] \\
& \mid \#\delta \\
& \mid \delta + \delta \\
& \mid \mu\ \alpha.\ \delta \\
& \mid \alpha
\end{aligned}
$$

2.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \sigma \to \sigma' \quad \Gamma \vdash e_2 : \sigma' \to \sigma''}{\Gamma \vdash e_1 >> e_2 : \sigma \to \sigma''} \tag{20}$$

$$\frac{\Gamma, x : \tau \vdash e : \tau}{\Gamma \vdash \texttt{fix}\ x.e : \tau} \tag{21}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \tag{22}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp}\ \#\delta_1\ \#\delta_2 \to \sigma}{\Gamma \vdash \texttt{test}\{e\} : \texttt{GeoSamp}\ \#\delta_1\ \#\delta_2 \to \sigma} \tag{23}$$

$$\frac{.}{\Gamma \vdash \texttt{hit} : \texttt{GeoSamp}\ *\ * \to \texttt{Hit}\ *} \tag{24}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \texttt{Hit}\ *}{\Gamma \vdash \texttt{mmrG}\{e\} : \texttt{GeoSamp}\ [\delta_1]\ \delta_2 \to \texttt{Hit}\ *} \tag{25}$$

$$\frac{\Gamma \vdash e_1 : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \sigma \quad \Gamma \vdash e_2 : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \sigma}{\texttt{ifsizeG}(> n)\ e_1\ \texttt{else}\ e_2 : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \sigma} \tag{26}$$

$$\frac{.}{\texttt{unboundG} : \texttt{GeoSamp}\ \#\delta_1\ \delta_2 \to \texttt{GeoSamp}\ \delta_1\ \delta_2} \tag{27}$$

$$\frac{.}{\texttt{unboundS} : \texttt{GeoSamp}\ \delta_1\ \#\delta_2 \to \texttt{GeoSamp}\ \delta_1\ \delta_2} \tag{28}$$

$$\frac{d : \delta_1}{\texttt{buildG}(d) : \texttt{GeoSamp}\ *\ \delta_2 \to \texttt{GeoSamp}\ \delta_1\ \delta_2} \tag{29}$$

$$\frac{d : \delta_2}{\texttt{buildS}(d) : \texttt{GeoSamp}\ \delta_1\ * \to \texttt{GeoSamp}\ \delta_1\ \delta_2} \tag{30}$$

2.3 Big Step Semantics

$$
\begin{aligned}
g &= [t, \ldots, t] \\
&\mid\ [g, \ldots, g] \\
&\mid\ (bound, g) \\
&\mid\ inL(g) \\
&\mid\ inR(g) \\
s &= [r, \ldots, r] \\
&\mid\ [s, \ldots, s] \\
&\mid\ (bound, s) \\
&\mid\ inL(s) \\
&\mid\ inR(s) \\
v &= (g, s)
\end{aligned}
$$

$$\frac{e_1\ v \Downarrow v' \quad e_2\ v' \Downarrow v''}{(e_1 \texttt{>>} e_2)\ v \Downarrow v''} \tag{31}$$

$$\frac{h = intersect(g, s)}{\texttt{hit}\ ([g], [s]) \Downarrow [h]} \tag{32}$$

$$\frac{d\ g \Downarrow_G g'}{\texttt{buildG}(d)\ (g, s) \Downarrow (g', s)} \tag{33}$$

$$\frac{d\ g \Downarrow_G g'}{\texttt{bound}(d)\ g \Downarrow_G (box(g'), v)} \tag{34}$$

$$\frac{p(g) = [g_1, g_2, ..., g_n] \quad d\ g_i \Downarrow_G g_i'}{(p \texttt{>=>} d)\ g \Downarrow_G [g_1', g_2', ..., g_n']} \tag{35}$$

$$\frac{intersects(b_1, b_2) \quad e\ ((b_1, g), (b_2, s)) \Downarrow v}{\texttt{test}\{e\}\ ((b_1, g), (b_2, s)) \Downarrow v} \tag{36}$$

$$\frac{e\ (g_i, s) \Downarrow v_i \quad v = v_1 \oplus \cdots \oplus v_n}{\texttt{mmrG}\{e\}\ ([g_1, \ldots, g_n], s) \Downarrow v} \tag{37}$$

$$\frac{e\ (g, s_i) \Downarrow v_i \quad v = v_1 \cup \cdots \cup v_n}{\texttt{mmrS}\{e\}\ (g, [s_1, \ldots, s_n]) \Downarrow v} \tag{38}$$

$$\frac{e_1\ (g, s) \Downarrow v}{\texttt{caseG}\{e_1 | e_2\}\ (inL(g), s) \Downarrow v} \tag{39}$$

$$\frac{e_2\ (g, s) \Downarrow v}{\texttt{caseG}\{e_1 | e_2\}\ (inR(g), s) \Downarrow v} \tag{40}$$

3 Examples

We start with the source for the repeat-work raytracer.

```
buildS(1S >=> id) >>
mmrS{
    fix x.
        buildS(bound(id)) >>
        buildG(bound(id)) >>
        test{
            unboundS >>
            unboundG >>
            ifsizeG(> 1) buildG(2_GP >=> id) >> mmrG{x}
            else hit
        }
}
```

We now precompute the 2GP split.

```
buildS(1S >=> id) >>
mmrS{
    fix x.
        buildS(bound(id)) >>
        buildG(bound(id)) >>
        test{
            unboundS >>
            unboundG >>
            buildG(ifsizeG(> 1) (2_GP >=> id) else id) >>
            caseG mmrG{x} | hit
        }
}
```

Reorder the bounding and unbounding to make the next step less daunting.

```
buildS(1S >=> id) >>
mmrS{
    fix x.
        buildS(bound(id)) >>
        buildG(bound(id)) >>
        test{
            unboundG >>
            buildG(ifsizeG(> 1) (2_GP >=> id) else id) >>
            unboundS >>
            caseG mmrG{x} | hit
        }
}
```

Pull through the bound-test-unbound (I have to show this all at once, since the intermediate stages are inexpressible in this language).

```
buildS(1S >=> id) >>
mmrS{
    fix x.
        buildS(bound(id)) >>
        buildG(bound(ifsizeG(> 1) (2_GP >=> id) else id)) >>
        test{
            unboundG >>
            unboundS >>
            caseG mmrG{x} | hit
        }
}
```

Push the unboundS through the case and into the mmrG.

```
buildS(1S >=> id) >>
mmrS{
    fix x.
        buildS(bound(id)) >>
        buildG(bound(ifsizeG(> 1) (2_GP >=> id) else id)) >>
        test{
            unboundG >>
            caseG mmrG{unboundS >> x} | (unboundS >> hit)
        }
}
```

Lift out the buildS.

```
buildS(1S >=> id) >>
mmrS{
    buildS(bound(id)) >>
    fix x.
        buildG(bound(ifsizeG(> 1) (2_GP >=> id) else id)) >>
        test{
            unboundG >>
            caseG mmrG{x} | (unboundS >> hit)
        }
}
```

Lift the buildG through the fix.

```
buildS(1S >=> id) >>
mmrS{
    buildS(bound(id)) >>
    buildG(fix z. bound(ifsizeG(> 1) (2_GP >=> z) else id)) >>
    fix x.
        test{
            unboundG >>
            caseG mmrG{x} | (unboundS >> hit)
        }
}
```

Finally list the bvh build through mmrS and we have the bvh-accelerated raytracer.

```
buildS(1S >=> id) >>
buildG(fix z. bound(ifsizeG(> 1) (2_GP >=> z) else id)) >>
mmrS{
    buildS(bound(id)) >>
    fix x.
        test{
            unboundG >>
            caseG mmrG{x} | (unboundS >> hit)
        }
}
```

Now here's the dumb raytracer expressed in another language:

$$
\begin{array}{l}
\texttt{fix}\ y. \\
\quad \texttt{map}^{(G\times)} >> push_S^{(G\times)} >> \\
\quad \texttt{fix}\ x. \\
\quad\quad \%G(boundBox) >> \\
\quad\quad \%S(boundRay) >> \\
\quad\quad \texttt{if}\ \%GS(intersect)\ \texttt{then} \\
\quad\quad\quad \%G(unbound) >> \\
\quad\quad\quad \%S(unbound) >> \\
\quad\quad\quad \texttt{if}\ \%G(sizeGT1)\ \texttt{then} \\
\quad\quad\quad\quad \%G(2GP) >> \texttt{semimap}_G^2(x) >> \%H(closer) \\
\quad\quad\quad \texttt{else} \\
\quad\quad\quad\quad \%GS(hit) \\
\quad\quad \texttt{else} \\
\quad\quad\quad \%S(allMiss)
\end{array}
$$

Now here's the dumb raytracer expressed in yet another language:

$$
\begin{array}{l}
\texttt{fix}\ y. \\
\quad \%G. \\
\quad \texttt{fix}\ x. \\
\quad\quad \%G(boundBox) >> \\
\quad\quad \%S(boundRay) >> \\
\quad\quad \texttt{if}\ \%GS(intersect)\ \texttt{then} \\
\quad\quad\quad \%G(unbound) >> \\
\quad\quad\quad \%S(unbound) >> \\
\quad\quad\quad \texttt{if}\ \%G(sizeGT1)\ \texttt{then} \\
\quad\quad\quad\quad \%G(2GP) >> \texttt{semimap}_G^2(x) >> \%H(closer) \\
\quad\quad\quad \texttt{else} \\
\quad\quad\quad\quad \%GS(hit) \\
\quad\quad \texttt{else} \\
\quad\quad\quad \%S(allMiss)
\end{array}
$$

And the bvh-accelerated raytracer:

$$
\begin{aligned}
&\%S(\texttt{1S}) >> \\
&\%G(\texttt{fix}\ z.\ \mathit{boundBox} >> \texttt{map}^{\times\overline{B}}(\\
&\qquad\qquad \texttt{if}\ (\mathit{sizeGT}1)\ \texttt{then} \\
&\qquad\qquad\quad 2GP >> \texttt{map}^2(x) >> \iota_1 \\
&\qquad\qquad \texttt{else}\ \iota_2) >> \\
&*\mathit{mapOverS}*\{ \\
&\quad \%S(\mathit{boundRay}) >> \\
&\quad \texttt{fix}\ x. \\
&\qquad \texttt{if}\ \%GS(\mathit{intersect})\ \texttt{then} \\
&\qquad\quad \%G(\mathit{unbound}) >> \\
&\qquad\quad \texttt{case} \\
&\qquad\quad\ (\texttt{semimap}^2_G(x) >> \%H(\mathit{closer})) \\
&\qquad\quad \mid (\%S(\mathit{unbound}) >> \%GS(\texttt{hit})) \\
&\qquad \texttt{else} \\
&\qquad\quad \%S(\mathit{allMiss}) \\
&\}
\end{aligned}
$$

4 Splitting

$$\frac{\cdot}{e \rightsquigarrow [e, x.x]} \tag{41}$$

$$\frac{e_1 \rightsquigarrow [p_1, x_1.r_1] \quad e_2 \rightsquigarrow [p_2, x_2.r_2]}{(e_1, e_2) \rightsquigarrow [(p_1, p_2), x.\texttt{let}\ (x_1, x_2) = x\ \texttt{in}\ (r_1, r_2)]} \tag{42}$$

$$\frac{e \rightsquigarrow [p, x.r]}{f\ e \rightsquigarrow [p, x.f\ r]} \tag{43}$$

$$\frac{e_2 \rightsquigarrow [p_2, x_2.r_2] \quad e_3 \rightsquigarrow [p_3, x_3.r_3]}{\texttt{case}\ e_1\ \texttt{of}\ v_2 => e_2 \mid v_3 => e_3 \rightsquigarrow [(\texttt{case}\ e_1\ \texttt{of}\ v_2 => p_2 \mid v_3 => p_3), x.(\texttt{case}\ x\ \texttt{of}\ x_2 => r_2 \mid x_3 => r_3)]} \tag{44}$$

$$\frac{e \rightsquigarrow [p, x.r]}{\texttt{eval}\ f\ x = e\ \texttt{with}\ e_2 \rightsquigarrow [\texttt{eval}\ f_2\ x = e\ \texttt{with}\ e_2, x.]} \tag{45}$$