Mockingbird Semantics

Nicolas Feltman

June 26, 2013

1 MbFilter

1.1 Grammar

$$
\begin{aligned}
\langle \mathit{breakMethodG} \rangle &= \texttt{1G} \mid \texttt{2\_GP} \\
\langle \mathit{breakMethodS} \rangle &= \texttt{1S} \mid \texttt{16}^2\texttt{\_SP} \\
\langle \mathit{breakMethodH} \rangle &= \texttt{1H}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathit{expr} \rangle = &\ \langle \mathit{expr} \rangle \texttt{>>} \langle \mathit{expr} \rangle \\
\mid &\ \texttt{fix}\ x.\langle \mathit{expr} \rangle \\
\mid &\ x \\
\mid &\ \texttt{test}\{\langle \mathit{expr} \rangle\} \\
\mid &\ \texttt{hit} \\
\mid &\ \texttt{Shade} \\
\mid &\ \texttt{if}\ |g| > n\ \texttt{then}\ \langle \mathit{expr} \rangle\ \texttt{else}\ \langle \mathit{expr} \rangle \\
\mid &\ \texttt{BreakMapReduceG}(\langle \mathit{breakMethodG} \rangle)\{\langle \mathit{expr} \rangle\} \\
\mid &\ \texttt{BreakMapReduceS}(\langle \mathit{breakMethodS} \rangle)\{\langle \mathit{expr} \rangle\} \\
\mid &\ \texttt{BreakMapReduceH}(\langle \mathit{breakMethodH} \rangle)\{\langle \mathit{expr} \rangle\}
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathit{sizeRange} \rangle &= [a, b] \mid [a+] \\
\langle \mathit{entity\text{-}type} \rangle &= \texttt{GeoSamp}\ \langle \mathit{sizeRange} \rangle\ \langle \mathit{sizeRange} \rangle \\
&\mid \texttt{Hit}\ \langle \mathit{sizeRange} \rangle \\
&\mid \texttt{Frag}\ \langle \mathit{sizeRange} \rangle \\
\langle \mathit{transition\text{-}type} \rangle &= \langle \mathit{entity\text{-}type} \rangle \to \langle \mathit{entity\text{-}type} \rangle \\
\langle \mathit{breakG\text{-}type} \rangle &= (\langle \mathit{sizeRange} \rangle \to \langle \mathit{sizeRange} \rangle)\ \langle \mathit{sizeRange} \rangle \\
\langle \mathit{breakS\text{-}type} \rangle &= \langle \mathit{sizeRange} \rangle\ (\langle \mathit{sizeRange} \rangle \to \langle \mathit{sizeRange} \rangle) \\
\langle \mathit{breakH\text{-}type} \rangle &= \langle \mathit{sizeRange} \rangle \to \langle \mathit{sizeRange} \rangle
\end{aligned}
$$

1.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau' \to \tau''}{\Gamma \vdash e_1 >> e_2 : \tau \to \tau''} \tag{1}$$

$$\frac{\Gamma, x : \phi \vdash e : \phi}{\Gamma \vdash \texttt{fix}\ x.e : \phi} \tag{2}$$

$$\frac{\Gamma(x) = \phi}{\Gamma \vdash \texttt{call}\ x : \phi} \tag{3}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp}\ \sigma_1\ \sigma_2 \to \tau}{\Gamma \vdash \texttt{test}\{e\} : \texttt{GeoSamp}\ \sigma_1\ \sigma_2 \to \tau} \tag{4}$$

$$\frac{.}{\Gamma \vdash \texttt{hit} : \texttt{GeoSamp}\ [1,1]\ [1,1] \to \texttt{Hit}} \tag{5}$$

$$\frac{.}{\Gamma \vdash \texttt{Shade} : \texttt{Hit}\ [1,1] \to \texttt{Frag}} \tag{6}$$

$$\frac{\Gamma \vdash e_1 : \texttt{GeoSamp}\ [n+1,b]\ \sigma \to \tau \quad \Gamma \vdash e_2 : \texttt{GeoSamp}\ [a,n]\ \sigma \to \tau}{\Gamma \vdash \texttt{if}\ |g| > n\ \texttt{then}\ e_1\ \texttt{else}\ e_2 : \texttt{GeoSamp}\ [a,b]\ \sigma \to \tau} \tag{7}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp} \to \tau}{\Gamma \vdash \texttt{BreakMapReduceG}(d)\{e\} : \texttt{GeoSamp} \to \tau} \tag{8}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp} \to \tau}{\Gamma \vdash \texttt{BreakMapReduceS}(d)\{e\} : \texttt{GeoSamp} \to \tau} \tag{9}$$

$$\frac{\Gamma \vdash e : \texttt{Hit} \to \tau}{\Gamma \vdash \texttt{BreakMapReduceH}(d)\{e\} : \texttt{Hit} \to \tau} \tag{10}$$

1.3 Denotational Semantics

$$|e_1 \texttt{>>} e_2| = |e_2| \circ |e_1| \tag{11}$$

$$|\texttt{fix } x.e| = |[(\texttt{fix } x.e)/(\texttt{call } x)]e| \tag{12}$$

$$|\texttt{test}\{e\}| = |e| \circ id_{\emptyset} \cup (\lambda(G,S).\{(\bot,k) : (s,k) \in S\}) \circ id_{\not\emptyset} \tag{13}$$

$$|\texttt{hit}| = \{((\{g\},\{(s,k)\}),\{(h,k)\}) : h = isect(g,s)\} \tag{14}$$

$$|\texttt{Shade}| = \{(\{(s,k)\},\{(f,k)\}) : f = shade(h)\} \tag{15}$$

$$|\texttt{if } |g| > n \texttt{ then } e_1 \texttt{ else } e_2| = |e_1| \circ id_{|g|>n} \cup |e_2| \circ id_{|g|\leq n} \tag{16}$$

$$|\texttt{BreakMapReduceG}(d)\{e\}| = \begin{array}{l} \{((G,S),\{F_1 \oplus \cdots \oplus F_n\}) \\ : ((G_i,S),f_i) \in |e|, (G,\{G_1,...,G_n\}) \in |d|\} \end{array} \tag{17}$$

$$|\texttt{BreakMapReduceS}(d)\{e\}| = \begin{array}{l} \{((G,S),\{F_1 \cup \cdots \cup F_n\}) \\ : ((G,S_i),f_i) \in |e|, (S,\{S_1,...,S_n\}) \in |d|\} \end{array} \tag{18}$$

$$|\texttt{BreakMapReduceH}(d)\{e\}| = \begin{array}{l} \{(H,\{F_1 \cup \cdots \cup F_n\}) \\ : (H_i,F_i) \in |e|, (H,\{H_1,...,H_n\}) \in |d|\} \end{array} \tag{19}$$

1.4 Theorems

1. For any expression e, if $((G,S),F) \in |e|$, then $keys(S) = keys(F)$.
2. For any expression $e : \texttt{GeoSamp} \to \texttt{Frag}$, if $((G,S),F) \in |e|$, then for all $(s,k) \in S$, $(\bigoplus_{g \in G} shade(isect(g,s)),k) \in F$.

2 MbOrder

2.1 Grammar

$$
\begin{aligned}
e = \; & e \mathbin{>>} e \\
\mid \; & \texttt{fix}\ x.\ e \\
\mid \; & x \\
\mid \; & \texttt{hit} \\
\mid \; & \texttt{test}\{e\} \\
\mid \; & \texttt{filt}\{e\} \\
\mid \; & \texttt{ifsizeG}(> n)\ e\ \texttt{else}\ e \\
\mid \; & \texttt{buildG}(d_G) \\
\mid \; & \texttt{buildS}(d_S) \\
\mid \; & \texttt{unboundG} \\
\mid \; & \texttt{unboundS} \\
\mid \; & \texttt{mmrG}\{e\} \\
\mid \; & \texttt{mmrS}\{e\}
\end{aligned}
$$

$$
\begin{aligned}
d_\alpha = \; & \texttt{id} \\
\mid \; & p_\alpha \mathbin{>=>} d_\alpha \\
\mid \; & \texttt{fix}\ x.\ d_\alpha \\
\mid \; & x \\
\mid \; & \mathit{bound}(d_\alpha) \\
\mid \; & \texttt{ifsize}\alpha(> n)\ d_\alpha\ \texttt{else}\ d_\alpha
\end{aligned}
$$

$$
\begin{aligned}
p_G &= \texttt{1G} \mid \texttt{2\_GP} \\
p_S &= \texttt{1S} \mid \texttt{16}^2\texttt{\_SP}
\end{aligned}
$$

$$
\begin{aligned}
\tau = \; & \sigma \to \sigma \\
\sigma = \; & \texttt{GeoSamp}\ \delta\ \delta \\
\mid \; & \texttt{Hit}\ \delta \\
\delta = \; & * \\
\mid \; & [\delta] \\
\mid \; & \#\delta \\
\mid \; & \delta + \delta \\
\mid \; & \texttt{fix}\ x.\ \delta \\
\mid \; & x
\end{aligned}
$$

2.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \sigma \to \sigma' \quad \Gamma \vdash e_2 : \sigma' \to \sigma''}{\Gamma \vdash e_1 >> e_2 : \sigma \to \sigma''} \tag{20}$$

$$\frac{\Gamma, x : \tau \vdash e : \tau}{\Gamma \vdash \texttt{fix}\ x.e : \tau} \tag{21}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \tag{22}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp}\ \#\delta_1\ \#\delta_2 \to \sigma}{\Gamma \vdash \texttt{test}\{e\} : \texttt{GeoSamp}\ \#\delta_1\ \#\delta_2 \to \sigma} \tag{23}$$

$$\frac{.}{\Gamma \vdash \texttt{hit} : \texttt{GeoSamp}\ *\ * \to \texttt{Hit}\ *} \tag{24}$$

$$\frac{\Gamma \vdash e : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \texttt{Hit}\ *}{\Gamma \vdash \texttt{mmrG}\{e\} : \texttt{GeoSamp}\ [\delta_1]\ \delta_2 \to \texttt{Hit}\ *} \tag{25}$$

$$\frac{\Gamma \vdash e_1 : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \sigma \quad \Gamma \vdash e_2 : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \sigma}{\texttt{ifsizeG}(> n)\ e_1\ \texttt{else}\ e_2 : \texttt{GeoSamp}\ \delta_1\ \delta_2 \to \sigma} \tag{26}$$

$$\frac{.}{\texttt{unboundG} : \texttt{GeoSamp}\ \#\delta_1\ \delta_2 \to \texttt{GeoSamp}\ \delta_1\ \delta_2} \tag{27}$$

$$\frac{.}{\texttt{unboundS} : \texttt{GeoSamp}\ \delta_1\ \#\delta_2 \to \texttt{GeoSamp}\ \delta_1\ \delta_2} \tag{28}$$

2.3 Big Step Semantics

$$\frac{e_1\ v \Downarrow v' \quad e_2\ v' \Downarrow v''}{(e_1 >> e_2)\ v \Downarrow v''} \tag{29}$$

$$\frac{d\ g \Downarrow_G g'}{\texttt{buildG}(d)\ (g, s) \Downarrow_G (g', s)} \tag{30}$$

$$\frac{d\ g \Downarrow_G g'}{\mathit{bound}(d)\ g \Downarrow_G (\mathit{box}(g'), v)} \tag{31}$$

$$\frac{p(g) = [g_1, g_2, ..., g_n] \quad d\ g_i \Downarrow_G g'_i}{\mathit{bound}(p >=> d)\ g \Downarrow_G [g'_1, g'_2, ..., g'_n]} \tag{32}$$

$$\frac{\mathit{intersects}(b_1, b_2) \quad e\ ((b_1, g), (b_2, s)) \Downarrow v}{\texttt{test}\{e\}\ ((b_1, g), (b_2, s)) \Downarrow v} \tag{33}$$