A Language for Staging

Nicolas Feltman

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1 Language Def

1.1 Grammar

$$
\begin{aligned}
\langle stage \rangle &= \mathbb{1} \mid \mathbb{2} \\
\langle expr \rangle &= f\ \langle expr \rangle \\
&\mid\ ()_{\langle stage \rangle} \\
&\mid\ (\langle expr \rangle, \langle expr \rangle) \\
&\mid\ \pi_1\ \langle expr \rangle \\
&\mid\ \pi_2\ \langle expr \rangle \\
&\mid\ \texttt{let}\ \langle var \rangle = \langle expr \rangle\ \texttt{in}\ \langle expr \rangle \\
&\mid\ \langle var \rangle \\
&\mid\ \texttt{pause}\ \langle expr \rangle
\end{aligned}
$$

2 Stage Types

$$\frac{\cdot}{\Gamma \vdash ()_\sigma : \sigma} \tag{1}$$

$$\frac{\Gamma \vdash e : \sigma}{\Gamma \vdash f\ e : \sigma} \tag{2}$$

$$\frac{\Gamma \vdash e : \sigma}{\Gamma \vdash \pi_i\ e : \sigma} \tag{3}$$

$$\frac{\Gamma \vdash e_1 : \sigma \quad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash (e_1, e_2) : \sigma} \tag{4}$$

$$\frac{\Gamma \vdash e_1 : \sigma_1 \quad \Gamma, x : \sigma_1 \vdash e_2 : \sigma_2 \quad \sigma_1 \leq \sigma_2}{\Gamma \vdash \texttt{let}\ x = e_1\ \texttt{in}\ e_2 : \sigma_2} \tag{5}$$

$$\frac{\Gamma(x) = \sigma}{\Gamma \vdash x : \sigma} \tag{6}$$

$$\frac{\Gamma \vdash e : \mathbb{1}}{\Gamma \vdash \texttt{pause e} : \mathbb{2}} \tag{7}$$

3 Stage Splitting

3.1 Single Stage Conversion

3.2 Multiple Stage Splitting

$$\frac{\cdot}{\Gamma \vdash ()_{\mathbb{1}} \rightsquigarrow [(), x.x]} \tag{8}$$

$$\frac{\cdot}{\Gamma \vdash ()_{\mathbb{2}} \rightsquigarrow [(), \_.()]} \tag{9}$$

$$\frac{\Gamma \vdash e \rightsquigarrow [p, x.r]}{\Gamma \vdash f\ e \rightsquigarrow [p, x.f\ r]} \tag{10}$$

$$\frac{\Gamma \vdash e \rightsquigarrow [p, x.r]}{\Gamma \vdash \pi_i\ e \rightsquigarrow [p, x.\pi_i\ r]} \tag{11}$$

$$\frac{\Gamma \vdash e_1 \rightsquigarrow [p_1, x_1, r_1] \quad \Gamma \vdash e_2 \rightsquigarrow [p_2, x_2, r_2]}{\Gamma \vdash (e_1, e_2) \rightsquigarrow [(p_1, p_2), l.(\texttt{let}\ x_1 = \pi_1\ l\ \texttt{in}\ r_1, \texttt{let}\ x_2 = \pi_2\ l\ \texttt{in}\ r_2)]} \tag{12}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow e_1' \quad \Gamma, x : \mathbb{1} \vdash e_2 \rightsquigarrow [p_2, y_2.r_2]}{\Gamma \vdash \texttt{let}\ x = e_1\ \texttt{in}\ e_2 \rightsquigarrow [\texttt{let}\ x = e_1'\ \texttt{in}\ p_2, y_2.r_2]} \tag{13}$$

$$\frac{\Gamma \vdash e_1 \rightsquigarrow [p_1, y_1.r_1] \quad \Gamma, x : \sigma_1 \vdash e_2 \rightsquigarrow [p_2, y_2.r_2]}{\Gamma \vdash \texttt{let}\ x = e_1\ \texttt{in}\ e_2 \rightsquigarrow [(p_1, p_2), l.\texttt{let}\ x = (\texttt{let}\ y_1 = \pi_1\ l\ \texttt{in}\ r_1)\ \texttt{in let}\ y_2 = \pi_2\ l\ \texttt{in}\ r_2]} \tag{14}$$

$$\frac{\Gamma(x) = \mathbb{2}}{\Gamma \vdash x \rightsquigarrow [(), \_.x]} \tag{15}$$