

Mockingbird Semantics

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1 MbFilter

1.1 Grammar

$$\begin{aligned}\langle breakMethodG \rangle &= 1G \mid 2.GP \\ \langle breakMethodS \rangle &= 1S \mid 16^2.SP \\ \langle breakMethodH \rangle &= 1H\end{aligned}$$
$$\begin{aligned}\langle expr \rangle &= \langle expr \rangle > \langle expr \rangle \\ &\mid \textbf{fix } x. \langle expr \rangle \\ &\mid x \\ &\mid \textbf{test} \{ \langle expr \rangle \} \\ &\mid \textbf{hit} \\ &\mid \textbf{Shade} \\ &\mid \textbf{if } |g| > n \textbf{ then } \langle expr \rangle \textbf{ else } \langle expr \rangle \\ &\mid \textbf{BreakMapReduceG}(\langle breakMethodG \rangle) \{ \langle expr \rangle \} \\ &\mid \textbf{BreakMapReduceS}(\langle breakMethodS \rangle) \{ \langle expr \rangle \} \\ &\mid \textbf{BreakMapReduceH}(\langle breakMethodH \rangle) \{ \langle expr \rangle \}\end{aligned}$$
$$\begin{aligned}\langle sizeRange \rangle &= [a, b] \mid [a+] \\ \langle entity-type \rangle &= \textbf{GeoSamp } \langle sizeRange \rangle \langle sizeRange \rangle \\ &\mid \textbf{Hit } \langle sizeRange \rangle \\ &\mid \textbf{Frag } \langle sizeRange \rangle \\ \langle transition-type \rangle &= \langle entity-type \rangle \rightarrow \langle entity-type \rangle \\ \langle breakG-type \rangle &= (\langle sizeRange \rangle \rightarrow \langle sizeRange \rangle) \langle sizeRange \rangle \\ \langle breakS-type \rangle &= \langle sizeRange \rangle (\langle sizeRange \rangle \rightarrow \langle sizeRange \rangle) \\ \langle breakH-type \rangle &= \langle sizeRange \rangle \rightarrow \langle sizeRange \rangle\end{aligned}$$

1.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \tau \rightarrow \tau' \quad \Gamma \vdash e_2 : \tau' \rightarrow \tau''}{\Gamma \vdash e_1 >> e_2 : \tau \rightarrow \tau''} \quad (1)$$

$$\frac{\Gamma, x : \phi \vdash e : \phi}{\Gamma \vdash \text{fix } x.e : \phi} \quad (2)$$

$$\frac{\Gamma(x) = \phi}{\Gamma \vdash \text{call } x : \phi} \quad (3)$$

$$\frac{\Gamma \vdash e : \text{GeoSamp } \sigma_1 \sigma_2 \rightarrow \tau}{\Gamma \vdash \text{test}\{e\} : \text{GeoSamp } \sigma_1 \sigma_2 \rightarrow \tau} \quad (4)$$

$$\frac{\cdot}{\Gamma \vdash \text{hit} : \text{GeoSamp } [1, 1] [1, 1] \rightarrow \text{Hit}} \quad (5)$$

$$\frac{\cdot}{\Gamma \vdash \text{Shade} : \text{Hit } [1, 1] \rightarrow \text{Frag}} \quad (6)$$

$$\frac{\Gamma \vdash e_1 : \text{GeoSamp } [n+1, b] \sigma \rightarrow \tau \quad \Gamma \vdash e_2 : \text{GeoSamp } [a, n] \sigma \rightarrow \tau}{\Gamma \vdash \text{if } |g| > n \text{ then } e_1 \text{ else } e_2 : \text{GeoSamp } [a, b] \sigma \rightarrow \tau} \quad (7)$$

$$\frac{\Gamma \vdash e : \text{GeoSamp} \rightarrow \tau}{\Gamma \vdash \text{BreakMapReduceG}(d)\{e\} : \text{GeoSamp} \rightarrow \tau} \quad (8)$$

$$\frac{\Gamma \vdash e : \text{GeoSamp} \rightarrow \tau}{\Gamma \vdash \text{BreakMapReduceS}(d)\{e\} : \text{GeoSamp} \rightarrow \tau} \quad (9)$$

$$\frac{\Gamma \vdash e : \text{Hit} \rightarrow \tau}{\Gamma \vdash \text{BreakMapReduceH}(d)\{e\} : \text{Hit} \rightarrow \tau} \quad (10)$$

1.3 Denotational Semantics

$$|e_1 > e_2| = |e_2| \circ |e_1| \quad (11)$$

$$|\mathbf{fix} \ x.e| = |[(\mathbf{fix} \ x.e)/(\mathbf{call} \ x)]e| \quad (12)$$

$$|\mathbf{test}\{e\}| = |e| \circ id_{\emptyset} \cup (\lambda(G, S). \{(\perp, k) : (s, k) \in S\}) \circ id_{\emptyset} \quad (13)$$

$$|\mathbf{hit}| = \{(\{(g), \{(s, k)\}), \{(h, k)\}) : h = isect(g, s)\} \quad (14)$$

$$|\mathbf{Shade}| = \{(\{(s, k)\}, \{(f, k)\}) : f = shade(h)\} \quad (15)$$

$$|\mathbf{if} \ |g| > n \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2| = |e_1| \circ id_{|g| > n} \cup |e_2| \circ id_{|g| \leq n} \quad (16)$$

$$|\mathbf{BreakMapReduceG}(d)\{e\}| = \begin{aligned} & \{((G, S), \{F_1 \oplus \dots \oplus F_n\}) \\ & : ((G_i, S), f_i) \in |e|, (G, \{G_1, \dots, G_n\}) \in |d|\} \end{aligned} \quad (17)$$

$$|\mathbf{BreakMapReduceS}(d)\{e\}| = \begin{aligned} & \{((G, S), \{F_1 \cup \dots \cup F_n\}) \\ & : ((G, S_i), f_i) \in |e|, (S, \{S_1, \dots, S_n\}) \in |d|\} \end{aligned} \quad (18)$$

$$|\mathbf{BreakMapReduceH}(d)\{e\}| = \begin{aligned} & \{(H, \{F_1 \cup \dots \cup F_n\}) \\ & : (H_i, F_i) \in |e|, (H, \{H_1, \dots, H_n\}) \in |d|\} \end{aligned} \quad (19)$$

1.4 Theorems

1. For any expression e , if $((G, S), F) \in |e|$, then $keys(S) = keys(F)$.
2. For any expression $e : \mathbf{GeoSamp} \rightarrow \mathbf{Frag}$, if $((G, S), F) \in |e|$, then for all $(s, k) \in S$, $(\bigoplus_{g \in G} shade(isect(g, s)), k) \in F$.

2 MbOrder

2.1 Grammar

```
 $e = e \gg e$   
| fix  $x. e$   
|  $x$   
| hit  
| test{ $e$ }  
| filt{ $e$ }  
| ifsizeG(>  $n$ )  $e$  else  $e$   
| buildG{ $d_G$ }  
| buildS{ $d_S$ }  
| unboundG{ $e$ }  
| unboundS{ $e$ }  
| mmrG{ $e$ }  
| mmrS{ $e$ }
```

```
 $d_\alpha = \text{flat}$   
|  $p_\alpha \Rightarrow d_\alpha$   
| fix  $x. d_\alpha$   
|  $x$   
|  $\text{bound}(d_\alpha)$   
| ifsize $\alpha$ (>  $n$ )  $d_\alpha$  else  $d_\alpha$ 
```

```
 $p_G = 1G \mid 2\_GP$   
 $p_S = 1S \mid 16^2\_SP$ 
```

```
 $\tau = \sigma \rightarrow \sigma$   
 $\sigma = \text{GeoSamp } \delta \ \delta$   
| Hit  $\delta$   
 $\delta = *$   
| [ $\delta$ ]  
|  $\# \delta$   
|  $\delta + \delta$   
| fix  $x. \delta$   
|  $x$ 
```

2.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \sigma \rightarrow \sigma' \quad \Gamma \vdash e_2 : \sigma' \rightarrow \sigma''}{\Gamma \vdash e_1 > e_2 : \sigma \rightarrow \sigma''} \quad (20)$$

$$\frac{\Gamma, x : \tau \vdash e : \tau}{\Gamma \vdash \mathbf{fix} \ x.e : \tau} \quad (21)$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \quad (22)$$

$$\frac{\Gamma \vdash e : \mathbf{GeoSamp} \ \# \delta_1 \ \# \delta_2 \rightarrow \sigma}{\Gamma \vdash \mathbf{test}\{e\} : \mathbf{GeoSamp} \ \# \delta_1 \ \# \delta_2 \rightarrow \sigma} \quad (23)$$

$$\frac{\cdot}{\Gamma \vdash \mathbf{hit} : \mathbf{GeoSamp} \ * \ * \rightarrow \mathbf{Hit}*} \quad (24)$$

$$\frac{\Gamma \vdash e : \mathbf{GeoSamp} \ \delta_1 \ \delta_2 \rightarrow \mathbf{Hit}*}{\Gamma \vdash \mathbf{mmrG}(d)\{e\} : \mathbf{GeoSamp} \ [\delta_1] \ \delta_2 \rightarrow \mathbf{Hit}*} \quad (25)$$