

# Mockingbird Semantics

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## 1 MbFilter

### 1.1 Grammar

$$\begin{aligned}\langle breakMethodG \rangle &= 1G \mid 2.GP \\ \langle breakMethodS \rangle &= 1S \mid 16^2.SP \\ \langle breakMethodH \rangle &= 1H\end{aligned}$$
$$\begin{aligned}\langle expr \rangle &= \langle expr \rangle > \langle expr \rangle \\ &\mid \textbf{fix } x. \langle expr \rangle \\ &\mid x \\ &\mid \textbf{test} \{ \langle expr \rangle \} \\ &\mid \textbf{hit} \\ &\mid \textbf{Shade} \\ &\mid \textbf{if } |g| > n \textbf{ then } \langle expr \rangle \textbf{ else } \langle expr \rangle \\ &\mid \textbf{BreakMapReduceG}(\langle breakMethodG \rangle) \{ \langle expr \rangle \} \\ &\mid \textbf{BreakMapReduceS}(\langle breakMethodS \rangle) \{ \langle expr \rangle \} \\ &\mid \textbf{BreakMapReduceH}(\langle breakMethodH \rangle) \{ \langle expr \rangle \}\end{aligned}$$
$$\begin{aligned}\langle sizeRange \rangle &= [a, b] \mid [a+] \\ \langle entity-type \rangle &= \textbf{GeoSamp } \langle sizeRange \rangle \langle sizeRange \rangle \\ &\mid \textbf{Hit } \langle sizeRange \rangle \\ &\mid \textbf{Frag } \langle sizeRange \rangle \\ \langle transition-type \rangle &= \langle entity-type \rangle \rightarrow \langle entity-type \rangle \\ \langle breakG-type \rangle &= (\langle sizeRange \rangle \rightarrow \langle sizeRange \rangle) \langle sizeRange \rangle \\ \langle breakS-type \rangle &= \langle sizeRange \rangle (\langle sizeRange \rangle \rightarrow \langle sizeRange \rangle) \\ \langle breakH-type \rangle &= \langle sizeRange \rangle \rightarrow \langle sizeRange \rangle\end{aligned}$$

## 1.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \tau \rightarrow \tau' \quad \Gamma \vdash e_2 : \tau' \rightarrow \tau''}{\Gamma \vdash e_1 >> e_2 : \tau \rightarrow \tau''} \quad (1)$$

$$\frac{\Gamma, x : \phi \vdash e : \phi}{\Gamma \vdash \mathbf{fix} \ x.e : \phi} \quad (2)$$

$$\frac{\Gamma(x) = \phi}{\Gamma \vdash \mathbf{call} \ x : \phi} \quad (3)$$

$$\frac{\Gamma \vdash e : \mathbf{GeoSamp} \ \sigma_1 \ \sigma_2 \rightarrow \tau}{\Gamma \vdash \mathbf{test}\{e\} : \mathbf{GeoSamp} \ \sigma_1 \ \sigma_2 \rightarrow \tau} \quad (4)$$

$$\frac{\cdot}{\Gamma \vdash \mathbf{hit} : \mathbf{GeoSamp} \ [1, 1] \ [1, 1] \rightarrow \mathbf{Hit}} \quad (5)$$

$$\frac{\cdot}{\Gamma \vdash \mathbf{Shade} : \mathbf{Hit} \ [1, 1] \rightarrow \mathbf{Frag}} \quad (6)$$

$$\frac{\Gamma \vdash e_1 : \mathbf{GeoSamp} \ [n+1, b] \ \sigma \rightarrow \tau \quad \Gamma \vdash e_2 : \mathbf{GeoSamp} \ [a, n] \ \sigma \rightarrow \tau}{\Gamma \vdash \mathbf{if} \ |g| > n \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2 : \mathbf{GeoSamp} \ [a, b] \ \sigma \rightarrow \tau} \quad (7)$$

$$\frac{\Gamma \vdash e : \mathbf{GeoSamp} \rightarrow \tau}{\Gamma \vdash \mathbf{BreakMapReduceG}(d)\{e\} : \mathbf{GeoSamp} \rightarrow \tau} \quad (8)$$

$$\frac{\Gamma \vdash e : \mathbf{GeoSamp} \rightarrow \tau}{\Gamma \vdash \mathbf{BreakMapReduceS}(d)\{e\} : \mathbf{GeoSamp} \rightarrow \tau} \quad (9)$$

$$\frac{\Gamma \vdash e : \mathbf{Hit} \rightarrow \tau}{\Gamma \vdash \mathbf{BreakMapReduceH}(d)\{e\} : \mathbf{Hit} \rightarrow \tau} \quad (10)$$

### 1.3 Denotational Semantics

$$|e_1 > e_2| = |e_2| \circ |e_1| \quad (11)$$

$$|\mathbf{fix} \ x.e| = |[(\mathbf{fix} \ x.e)/(\mathbf{call} \ x)]e| \quad (12)$$

$$|\mathbf{test}\{e\}| = |e| \circ id_{\emptyset} \cup (\lambda(G, S). \{( \perp, k) : (s, k) \in S\}) \circ id_{\emptyset} \quad (13)$$

$$|\mathbf{hit}| = \{(\{(g), \{(s, k)\}), \{(h, k)\}) : h = isect(g, s)\} \quad (14)$$

$$|\mathbf{Shade}| = \{(\{(s, k)\}, \{(f, k)\}) : f = shade(h)\} \quad (15)$$

$$|\mathbf{if} \ |g| > n \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2| = |e_1| \circ id_{|g| > n} \cup |e_2| \circ id_{|g| \leq n} \quad (16)$$

$$|\mathbf{BreakMapReduceG}(d)\{e\}| = \begin{aligned} & \{((G, S), \{F_1 \oplus \dots \oplus F_n\}) \\ & : ((G_i, S), f_i) \in |e|, (G, \{G_1, \dots, G_n\}) \in |d|\} \end{aligned} \quad (17)$$

$$|\mathbf{BreakMapReduceS}(d)\{e\}| = \begin{aligned} & \{((G, S), \{F_1 \cup \dots \cup F_n\}) \\ & : ((G, S_i), f_i) \in |e|, (S, \{S_1, \dots, S_n\}) \in |d|\} \end{aligned} \quad (18)$$

$$|\mathbf{BreakMapReduceH}(d)\{e\}| = \begin{aligned} & \{(H, \{F_1 \cup \dots \cup F_n\}) \\ & : (H_i, F_i) \in |e|, (H, \{H_1, \dots, H_n\}) \in |d|\} \end{aligned} \quad (19)$$

### 1.4 Theorems

1. For any expression  $e$ , if  $((G, S), F) \in |e|$ , then  $keys(S) = keys(F)$ .
2. For any expression  $e : \mathbf{GeoSamp} \rightarrow \mathbf{Frag}$ , if  $((G, S), F) \in |e|$ , then for all  $(s, k) \in S$ ,  $(\bigoplus_{g \in G} shade(isect(g, s)), k) \in F$ .

## 2 MbOrder

### 2.1 Grammar

```
e = e >> e
  | fix x. e
  | x
  | hit
  | test{e}
  | filt{e}
  | ifsizeG(> n) e else e
  | buildG(dG)
  | buildS(dS)
  | unboundG
  | unboundS
  | mmrG{e}
  | mmrS{e}
```

```
dα = id
  | pα >=> dα
  | fix x. dα
  | x
  | bound(dα)
  | ifsizeα(> n) dα else dα
```

```
pG = 1G | 2_GP
pS = 1S | 162_SP
```

```
τ = σ → σ
σ = GeoSamp δ δ
  | Hit δ
δ = *
  | [δ]
  | #δ
  | δ + δ
  | μ α. δ
  | α
```

## 2.2 Typing Rules

$$\frac{\Gamma \vdash e_1 : \sigma \rightarrow \sigma' \quad \Gamma \vdash e_2 : \sigma' \rightarrow \sigma''}{\Gamma \vdash e_1 > e_2 : \sigma \rightarrow \sigma''} \quad (20)$$

$$\frac{\Gamma, x : \tau \vdash e : \tau}{\Gamma \vdash \text{fix } x.e : \tau} \quad (21)$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \quad (22)$$

$$\frac{\Gamma \vdash e : \text{GeoSamp } \# \delta_1 \# \delta_2 \rightarrow \sigma}{\Gamma \vdash \text{test}\{e\} : \text{GeoSamp } \# \delta_1 \# \delta_2 \rightarrow \sigma} \quad (23)$$

$$\frac{\cdot}{\Gamma \vdash \text{hit} : \text{GeoSamp } * * \rightarrow \text{Hit } *} \quad (24)$$

$$\frac{\Gamma \vdash e : \text{GeoSamp } \delta_1 \delta_2 \rightarrow \text{Hit } *}{\Gamma \vdash \text{mmrG}\{e\} : \text{GeoSamp } [\delta_1] \delta_2 \rightarrow \text{Hit } *} \quad (25)$$

$$\frac{\Gamma \vdash e_1 : \text{GeoSamp } \delta_1 \delta_2 \rightarrow \sigma \quad \Gamma \vdash e_2 : \text{GeoSamp } \delta_1 \delta_2 \rightarrow \sigma}{\text{ifsizeG}( > n ) e_1 \text{ else } e_2 : \text{GeoSamp } \delta_1 \delta_2 \rightarrow \sigma} \quad (26)$$

$$\frac{\cdot}{\text{unboundG} : \text{GeoSamp } \# \delta_1 \delta_2 \rightarrow \text{GeoSamp } \delta_1 \delta_2} \quad (27)$$

$$\frac{\cdot}{\text{unboundS} : \text{GeoSamp } \delta_1 \# \delta_2 \rightarrow \text{GeoSamp } \delta_1 \delta_2} \quad (28)$$

$$\frac{d : \delta_1}{\text{buildG}(d) : \text{GeoSamp } * \delta_2 \rightarrow \text{GeoSamp } \delta_1 \delta_2} \quad (29)$$

$$\frac{d : \delta_2}{\text{buildS}(d) : \text{GeoSamp } \delta_1 * \rightarrow \text{GeoSamp } \delta_1 \delta_2} \quad (30)$$

## 2.3 Big Step Semantics

$$\frac{e_1 \Downarrow v' \quad e_2 \Downarrow v''}{(e_1 > e_2) \Downarrow v''} \quad (31)$$

$$\frac{h = \text{intersect}(g, s)}{\text{Hit}(d) ([g], [s]) \Downarrow [h]} \quad (32)$$

$$\frac{d \Downarrow_G g'}{\text{buildG}(d) (g, s) \Downarrow (g', s)} \quad (33)$$

$$\frac{d \Downarrow_G g'}{\text{bound}(d) \Downarrow_G (\text{box}(g'), v)} \quad (34)$$

$$\frac{p(g) = [g_1, g_2, \dots, g_n] \quad d \Downarrow_G g'_i}{\text{bound}(p >=> d) \Downarrow_G [g'_1, g'_2, \dots, g'_n]} \quad (35)$$

$$\frac{\text{intersects}(b_1, b_2) \quad e ((b_1, g), (b_2, s)) \Downarrow v}{\text{test}\{e\} ((b_1, g), (b_2, s)) \Downarrow v} \quad (36)$$

$$\frac{e (g_i, s) \Downarrow v_i \quad v = v_1 \oplus \dots \oplus v_n}{\text{mmrG}\{e\} ([g_1, \dots, g_n], s) \Downarrow v} \quad (37)$$

$$\frac{e (g, s_i) \Downarrow v_i \quad v = v_1 \cup \dots \cup v_n}{\text{mmrS}\{e\} (g, [s_1, \dots, s_n]) \Downarrow v} \quad (38)$$