Innovation by Entrants and Incumbents Supplementary Material

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Abstract

In this note, we present the algorithm to compute the stationary balanced growth path in Section 3 of Acemoglu and Cao (2014). The algorithm follows closely the constructive proof in the paper.

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1 Algorithm

This algorithm follows closely the steps in the constructive proof of Proposition 2 in Acemoglu and Cao (2014). First, given a conjectured growth rate, g, we solve for a value function of the incumbents as a solution to the functional equation (27). This value function implies the investment policy of the incumbents and the entrants. Second, we use these policy functions to solve for the stationary distribution of relative firm sizes in equations (30) and (31). Lastly, we combine the stationary distribution and the policies functions to calculate the implied growth rate (32) - $g'(g)$. We then use the bisection method to solve for the equilibrium growth rate g^* such that $g'(g) = g$.

We implement this algorithm using the following functional forms:

Preference: $u(c) = \frac{c^{1-\theta}}{1-\theta}$

Technology: $\phi(z) = Az^{1-\alpha}$ and $\eta(z) = Cz^{-\gamma}$

1.1 Value function

To compute the value function of the incumbents, we first calculate v_g defined in Lemma 1 in Acemoglu and Cao (2014): given g there exists an unique v_g solution of the equation (31) in the paper:

$$v = \frac{\beta L + I_e((\lambda - 1)v)}{(\rho + \theta g) + I_e(v)}$$

Given this constant, we look for the value function $\widehat{V}(\widetilde{q}) = \widetilde{q}U(\log(\widetilde{q}) - \log(\epsilon_g))$ in which U is the solution to the functional equation (34) in the paper by repeatedly applying the functional operator T_g in Definition 6. In particular, we start with $U_0(p) = v_g + v_g e^{-\theta p}$ and compute the sequence of $U_n(p)$ where

$$U_{n+1} = T_g U_n.$$

We solve for U_{n+1} by solving the ODE in equation (37).

We stop until the $\|U_{n+1} - U_n\|$ falls below some threshold.

1.2 Stationary distribution

Given the policy functions obtained from solving the value function of the incumbents, we look for the stationary density function under the form $f(\widetilde{q}) = \frac{h(\log(\widetilde{q}) - \log(\epsilon_g))}{\widetilde{q}}$ which satisfies functional equations (30) and (31) in the paper. We solve for $h(.)$ as a solution of the delayed ODE in Lemma 7.

1.3 Equilibrium growth rate:

Lastly, given the policy functions and the stationary distribution, we can compute the implied growth rate:

$$\begin{aligned} g' &= \frac{(\lambda-1)\int_{\epsilon_g}^{\infty}\widetilde{q}\phi\left(z\left(\widetilde{q}\right)\right)f_g\left(\widetilde{q}\right)d\widetilde{q}+(\kappa-1)\int_{\epsilon_g}^{\infty}\widetilde{q}\widehat{z}\left(\widetilde{q}\right)\eta\left(\widehat{z}\left(\widetilde{q}\right)\right)f_g\left(\widetilde{q}\right)d\widetilde{q}}{1-\epsilon_g f_g\left(\epsilon_g\right)\left(\omega-\epsilon_g\right)} \\ &= \epsilon_g\frac{(\lambda-1)\int_0^{\infty}e^p\phi\left(z_g\left(p\right)\right)h_g\left(p\right)dp+(\kappa-1)\int_0^{\infty}e^p\widehat{z}_g\left(p\right)\eta\left(\widehat{z}_g\left(p\right)\right)h_g\left(p\right)dp}{1-h_g\left(0\right)\left(\omega-\epsilon_g\right)} \end{aligned}$$

In equilibrium $g'=g$, so we can rewrite the growth decomposition as

$$\begin{aligned} g = &\underbrace{(\lambda-1)\int_{\epsilon_g}^{\infty}\widetilde{q}\phi\left(z\left(\widetilde{q}\right)\right)f_g\left(\widetilde{q}\right)d\widetilde{q}}_{\text{Innovation from incumbents}}+\underbrace{(\kappa-1)\int_{\epsilon_g}^{\infty}\widetilde{q}\widehat{z}\left(\widetilde{q}\right)\eta\left(\widehat{z}\left(\widetilde{q}\right)\right)f_g\left(\widetilde{q}\right)d\widetilde{q}}_{\text{Innovation from entrants}} \\ &+\underbrace{g\epsilon_g f_g\left(\epsilon_g\right)\left(\omega-\epsilon_g\right)}_{\text{Innovation from imitators}}. \end{aligned}$$

We then use the bisection method to solve for the equilibrium growth rate g^* such that $g'(g)=g$.