Non-competes paper - Bullet points

October 21, 2017

Preferences and labor endowment

- Continuum of individuals indexed by $i \in [0,1]$. Individuals are risk-neutral with discount rate ρ: maximize
$$U = \int_0^\infty \exp(-\rho t) C(t) dt$$
- Individuals endowed with unit of labor, which can be supplied to final good production (l^F), intermediate good production (l^I), and R&D (l^{RD}).
- Aggregate labor market satisfies
$$L_t^F + L_t^I + L_t^{RD} = 1$$

Final good technology

- Consumption consists of a final good. Produced using labor and a continuum of intermediate goods $j \in [0,1]$ with production technology
$$Y(t) = \frac{L^\beta(t)}{1-\beta} \int_0^1 q_j^\beta(t) k_j^{1-\beta}(t) dj$$
  - q_j is quality, k_j is quantity
  - CES production structure ensures constant markups –> tractability

Intermediate good technology

- Each good j is produced with a linear technology
$$k_j = \overline{q} l_j$$
where $\overline{q} = \int_0^1 q_j dj$.
  - Assumptions guarantee that only the technology leader produces in equilbrium –> no limit pricing

Research and development

- Successful R&D project on quality q renders its owner an incument with quality $(1+\lambda)q$.
- Incumbent and mass m_j of entrants work on R&D on machine j

- Innovations effort z requires z units R&D labor generates innovations at Poisson rate

$$
\begin{aligned}
R_I(z_I;\overline{z}) &= \chi_I z_I \phi(\overline{z}) \\
R_E(z_E :;\overline{z}) &= \chi_E z_E \phi(\overline{z})
\end{aligned}
$$

for $\phi(z)$ decreasing such that $z\phi(z)$ is increasing, and where

$$
\overline{z} = \int_0^m z(l)dl + z_I
$$

is the total innovation effort in that machine line

  - Incumbents are large and take into account their effect on $\overline{z}$, but entrants are small and do not

- An entrant can only hire $\xi > 0$ units of R&D labor; in equilibrium, total entrant effort is $\overline{z}_j^E = \min(M, m)\xi$, where M is the mass that is present when there is free entry

Knowledge spillovers and non-competes

- Individual supplying R&D labor to either an incumbent or an entrant on a project on quality q acquires the ability to spin-off and form his own R&D project at Poisson rate ν per unit of R&D labor

- Therefore, if l_j^{RD} is the total amount of R&D labor used by the incumbent and entrants in machine line j,

$$
\begin{aligned}
\dot{n}_j &= \nu l_j^{RD} - \upsilon n_j \\
\dot{m}_j &= \upsilon n_j
\end{aligned}
$$

- This means that we actually have three state variables, (q, n, m), which is a problem. No way around this - will simply need to use few grid points on (q, n, m) and interpolate. Should still be feasible...

- R&D race j becomes free entry at exogenous rate θ

  - Easier model to solve: assume non-competes are permanent. Then θ becomes really important for fitting data (otherwise there is no entry in the model, just incumbents incrementally improving their quality).

Equilibrium

- Focus on recursive BGP where state variables are either constant (e.g. distributions, productivity-normazlied wages) or grow at constant rate g (e.g. output, productivity, wages), and where the only aggregate state variable is the joint distribution of $(\tilde{q}, m, n)$ in the economy (and where individual $(\tilde{q}, m, n)$ is the individual state of each firm). Here $\tilde{q} = q/\overline{q}$, that is, it is the quality relative to the economy average. From now on I will simply use the notation q for $\tilde{q}$ where there is no confusion.

- After doing some math (e.g. solving all static optimization problems, normalizing non-stationary BGPs by $\overline{q}$ and rewriting in stationary form, etc.), we can say that an equilibrium is defined by a set of functions and numbers satisfying certain conditions. These are summarized below:

  1. Prices:

- $\overline{w} = \tilde{\beta} = \beta^{\beta}[1-\beta]^{1-2\beta}$ is the constant normalized wage in the final goods production and intermediate goods production occupations.
- $w(q,m,n)$ is the normalized wage in the R&D occupation, for an intermediate good at frontier quality q with mass $n+m$ of workers who have learned the R&D technology, a mass m of which has been released from their non-competes.

2. Aggregate states (non-price):
   - $\overline{z}_E(q,m,n)$,$\overline{z}_E^B$
   - No need to include $\overline{z}_I(q,m,n)$ here, consistency below is enough.
3. Incumbent firm flow profit function:
   - $\pi(q,m,n) = \pi q$ where $\pi = L^F(1-\beta)\tilde{\beta}$.
4. Firm functions $A(q,m,n), B(q,m,n), W^{NC}(q,m,n), W^F(q,m,n)$ and policy functions $z_I(q,m,n), z_I^B(q), z_E(q,$ such that, given aggregate states and prices, are the maximizers of the HJBs below:

$$
\begin{aligned}
(r-g)B(q) &= \pi q - gqB'(q) \\
&\quad + \max_{z\geq 0} z\phi(\overline{z}_E^B + z)\big[A((1+\lambda)q,0,0) - B(q)\big] - \overline{w}z \\
&\quad -\overline{z}_E^B\phi(\overline{z}_E^B + z)B(q) \\
(r+\theta-g)A(q,m,n) &= \pi q + \theta B(q) - gqA_q(q,m,n) \\
&\quad + \max_{z\geq 0}\Big\{z\phi(\overline{z}_E(q,m,n)+z)\big[A((1+\lambda)q,0,0) - A(q,m,n)\big] \\
&\quad -w(q,m,n)z + \big(\nu(z+\overline{z}_E(q,m,n)) - vn\big)A_n(q,m,n) \\
&\quad +vnA_m(q,m,n) - \overline{z}_E(q,m,n)\phi(\overline{z}_E(q,m,n)+z)A(q,m,n)\Big\} \\
(r+\theta-g)W^F(q,m,n) &= \max_{0\leq z\leq \xi}\Big\{z\phi(z_I(q,m,n)+\overline{z}_E(q,m,n))\big[A((1+\lambda)q,0,0) - W^F(q,m,n)\big] \\
&\quad -w(q,m,n)z\Big\} \\
&\quad -(z_I(q,m,n)+\overline{z}_E(q,m,n))\phi(z_I(q,m,n)+\overline{z}_E(q,m,n))W^F(q,m,n) \\
&\quad -gqW^F(q,m,n) \\
&\quad +\big(\nu(z_I(q,m,n)+\overline{z}_E(q,m,n)-vn)\big)A_n(q,m,n) + vnA_m(q,m,n) \\
(r+\theta-g)W^{NC}(q,m,n) &= vW^F(q,m,n) - gqW_q^{NC}(q,m,n) \\
&\quad +\big(\nu(z_I(q,m,n)+\overline{z}_E(q,m,n)) - vn\big)W_n^F(q,m,n) \\
&\quad +vnW_m^F(q,m,n)
\end{aligned}
$$

5. Helps to write down entrant optimality conditions. Due to the fact that entrant R&D functions are individually CRS, if an entrant is producing R&D they are producing at a corner, $z_E(q,m,n) = \xi$ and $z_E^B = \xi$. When R&D knowledge q is common knowledge, free entry determines the mass $M(q)$ of firms attempting innovation. Since there is free entry, we have $W^F = 0$ Thus, the free entry condition boils down to

$$\phi(z_I^B(q) + M(q)\xi)\big[A((1+\lambda)q,0,0)\big] = w(q,m,n)$$

6. Worker optimality: workers must be indifferent between all types of labor in equilibrium. Hence,

$$w(q,m,n) = \overline{w} - \nu W^{NC}(q,m,n)$$

7. Labor allocation satisfying

$$L^F + L^I + L^{RD} = 1$$

where

$$L^{RD} = \int_0^1 l_j^{RD} dj$$

and so on.