A Model of Productivity Growth through Creative Destruction by Employee Spinout: Updates

Nicolas Fernandez-Arias

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Updates - Overview

- Preliminary proposal for nesting model in general model
  - For now: exogenous shock that makes knowledge public
  - Potential improvement: immediately allow free entry into R&D race, but endow spinouts with some extra productivity relative to these entrants. Over time, spinouts drive entrants out of the market.
  - Very different implications, but hard to observe
- Ideas for calibration / identification of parameters

Model overview

- Time t is continuous
- Agents:
  - Households
  - Intermediate goods firms
  - Final goods firm

Model: Intermediate goods production

- Standard quality ladders model, step size $\lambda > 1$
- Continuum of intermediate goods, indexed by $j \in J = [0, 1]$
- Frontier quality of good j by q_j
- x_j is amount produced
- Each good produced with technology

$$x_j = \bar{q} l_j$$

  where $\bar{q} = \int_0^1 q_j dj$ is the average quality level of the economy
- Each good j has monopolist, standard assumptions to guarantee no limit pricing
- Demand (final goods production) CES across goods j implies constant markup

Model: R&D race

At time t with average quality $\bar{q}_t$, incumbent in the R&D race for good j of quality q_j begins with monopoly on good j R&D

$\downarrow$

- Hires R&D labor; at rate $\nu(q_j/\bar{q}_t)^{-1}$ per unit of R&D labor hired, employees learn, adding to mass of potential entrants (**scaling factor** $(q_j/\bar{q}_t)^{-1}$ **for BGP**))
- Simultaneously, with arrival rate θ, "knowledge spillover" shock hits $\rightarrow$ **free entry** into R&D race

$\downarrow$

At some point, either an incumbent or an entrant firm wins the race, and obtains a monopoly on production and R&D on good j of quality λq_j

Model: R&D technology

- Consider an intermediate j with relative quality $\tilde{q} = q/\bar{q}$
- **Scaling assumption for BGP:** flow cost of $\tilde{q}z$ units of labor yields z units of effective labor
- z,$\hat{z}$ units of R&D effort by incumbent and entrant respectively yields victory in the R&D race at Poisson rate

$$R(z) = \chi z \phi(z)$$
$$\hat{R}(\hat{z}; \bar{z}) = \hat{\chi} \hat{z} \eta(\bar{z})$$

  where $\bar{z} = \int_0^m \hat{z}(m')dm'$ is total good-j R&D effort by entrants.
- Entrant m' can perform $\hat{z} \leq \xi$ units of R&D effort (equilibrium does not pin down; look for equilibria where $\hat{z}(m') \in \{0, \xi\}$)
- Aggregate rate of innovations:

$$\tau = \chi z \phi(z) + \hat{\chi} \bar{z} \eta(\bar{z})$$

Model: R&D technology - congestion

- The reduced-form functions $\phi(z), \eta(z)$ capture diminishing returns and congestion in the R&D race, repsectively
- As such, assume $\phi(z), \eta(z)$ decreasing, $z\phi(z), z\eta(z)$ increasing
- For the incumbent, $\phi(z)$ captures *individual* decreasing returns in the R&D technology
- For the entrants, $\eta(z)$ captures *aggregate* decreasing returns due to the possibility that different entrants use the same approach
- Cross-congestion:
  - Incumbent and entrants do not congest each other (as in other models of innovation by entrants and incumbents, c.f. Acemoglu & Cao 2015, Akcigit & Kerr 2017)
  - Adds tractability and reflects empirical fact that spinouts often attempt different approaches
  - Can be relaxed

Intermediate goods firms optimization: incumbent static optimization

- Static optimization in product market: CES final goods production implies constant markup
- Regardless of the aggregate supply of labor, have

$$\bar{w} = \tilde{\beta}\bar{q}$$
$$\tilde{\beta} = \beta^{\beta}(1-\beta)^{2-2\beta}$$

  where L^F is labor allocated to final goods production.
- Taking as given research labor allocation L^{RD}, have closed forms for L^F, L^I and static profits $\pi(q)$ for an incumbent with technology q,

$$L^F = \frac{\beta(1-L^{RD})}{\beta+(1-\beta)^2}$$
$$L^I = 1 - L^{RD} - L^F$$
$$\pi = \beta(1-\beta)^{\frac{2-\beta}{\beta}}\tilde{\beta}^{-1}L^F q$$

Intermediate goods firms optimization: incumbent R&D decision

- HJB equation for incumbent:

$$
\begin{aligned}
\Big(\rho + \overbrace{\hat{\chi}\bar{z}(q,m,t)\eta(\bar{z}(q,m,t))}^{\text{Entrant innovation rate}}\Big)V(q,m,t) &= \overbrace{\pi q}^{\text{Flow profits}} + \overbrace{V_t(q,m,t)}^{\text{Changing aggregate state}} \\
&+ \overbrace{\nu\bar{z}(q,m,t)V_m(q,m,t)}^{\text{Knowledge spillovers from entrant R\&D}} + \theta\overbrace{\Big(V(q,M,t) - V(q,m,t)\Big)}^{\text{Knowledge becomes public}} \\
&+ \max_{z}\Bigg\{ \underbrace{\chi z\phi(z)}_{\text{Arrival rate of R\&D victory}} \overbrace{\Big(V(\lambda q,0,t) - V(q,m,t)\Big)}^{\text{NPV of successful innovation}} \\
&- \underbrace{z(q/\bar{q}_t)}_{\text{R\&D labor}}\Big(\underbrace{w(q,m,t)}_{\text{R\&D wage}} - \overbrace{\nu(q/\bar{q}_t)^{-1}V_m(q,m,t)}^{\text{Knowledge spillovers from own R\&D}}\Big)\Bigg\}
\end{aligned}
$$

Intermediate goods firms optimization: entrant R&D decision

- HJB equation for entrant:

$$
\begin{aligned}
&(\rho + \overbrace{\hat{\chi}\bar{z}(q,m,t)\eta(\bar{z}(q,m,t)+)}^{\text{Entrant innovation rate}})W(q,m,t) = \overbrace{W_t(q,m,t)}^{\text{Changing aggregate state}} \\
&+ \overbrace{\nu\bar{z}(q,m,t)W_m(q,m,t)}^{\text{Knowledge spillovers from entrant R\&D}} + \overbrace{\theta\Big(W(q,M,t) - W(q,m,t)\Big)}^{\text{Knowledge becomes public}} \\
&+ \max_{z}\Big\{ \overbrace{\hat{\chi}z\eta(\bar{z}(q,m,t))}^{\text{Arrival rate of R\&D victory}} \overbrace{\Big(V(\lambda q,0,t) - W(q,m,t)\Big)}^{\text{NPV of successful innovation}} \\
&\qquad - \underbrace{z(q/\bar{q}_t)}_{\text{R\&D labor}}\underbrace{w(q,m,t)}_{\text{R\&D wage}}\Big\}
\end{aligned}
$$

Model: Households

- Unit mass continuum of risk-neutral households indexed by $i \in I = [0, 1]$, each with objective

$$U = \int_0^\infty \exp(-\rho t) c(t) dt$$

  where $c(t)$ is final goods consumption at t.
- Instantaneous borrowing and lending at interest rate r; $r = \rho$ in equilibrium
- Individual i supplies labor to final goods production $\ell_i^F(t)$, intermediate good production $\ell_i^I(t)$ and R&D $\ell_i^{RD}(t)$ such that

$$\ell_i^F(t) + \ell_i^I(t) + \ell_i^{RD}(t) = 1$$

- Aggregate labor market satisfies (where $L^k(t) = \int_I \ell_i^k(t) i di$ for $k \in \{F, I, RD\}$)

$$L^F(t) + L^I(t) + L^{RD}(t) = 1$$

Household optimization timeline

Worker i allocates labor to R&D, intermediate and final goods production

$\downarrow$

While performing R&D for some good j of relative quality $\tilde{q}_j$, receives learning shock with Poisson intensity $\nu \tilde{q}_j^{-1}$ per flow unit of R&D labor supplied to j $\downarrow$

Provided it is still profitable, he opens entrant R&D lab performing R&D effort ξ and competing in developing the next step of good j

Household optimization

- Workers indifferent between occupations (Final goods production, intermediate goods production, R&D).
- **For production:** Indifference requires requires same wage in final goods and intermediate goods production: $w_t^I = w_t^F = \bar{w}$.
- **For R&D:** Indifference now requires **total compensation** from R&D – including **expected value of knowledge earned** – to be equal to $\bar{w}$, i.e.

$$w(m,t) + \nu W(m,t) = \bar{w}_t$$

where $W(m,t)$ is the value at time t of the knowledge to open an entrant in a good which is currently in state m

Model: Final good production

- Final good is produced using labor and a continuum of intermediate goods $j \in [0,1]$ with production technology

$$X(t) = L(t)^{\beta} \left(\left(\int_0^1 q_j(t)^{\beta} x_j(t)^{1-\beta} dj \right)^{1/(1-\beta)} \right)^{1-\beta}$$
$$= L(t)^{\beta} \int_0^1 q_j(t)^{\beta} x_j(t)^{1-\beta} dj$$

  where q_j is quality, x_j is quantity
- Restricts labor share to be related to markup $\mu = 1/(1-\beta)$
- Can relax this using Grossman et. al 2016
- CRS implies zero profits so no need to consider ownership

Aggregation: Kolmogorov Forward Equation

- Define $d\mu(q, m, t)$ as the distribution of intermediate goods j across states (q, m) at time t
- Kolmogorov Forward Equation (somewhat heuristic)

$$
\begin{aligned}
\mu_t(q,m,t) = & \overbrace{-\frac{d}{dq}\left(a^q(q,m,t)\mu(q,m,t)\right)}^{\text{Drift in } q} - \overbrace{\frac{d}{dm}\left(a^m(q,m,t)\mu(q,m,t)\right)}^{\text{Drift in } m} \\
& - \underbrace{\tau(q,m,t)\mu(q,m,t)}_{\text{Innovation arrival: jump } (q,m)\to(\lambda q,0)} \\
& + \underbrace{\mathbb{1}_{\{m=0\}}\lambda^{-1}\int \tau(\lambda^{-1}q, m', t)\,d\mu(\lambda^{-1}q, m', t)}_{\text{Innovation arrival: jump } (\lambda^{-1}q,m')\to(q,0)}
\end{aligned}
$$

- $a^q(q, m, t), a^m(q, m, t)$ are drift in q, m direction, respectively, computed from $z(q, m, t)$ and $\bar{z}(q, m, t)$
- Last term for $m = 0$ arises because receiving inflows from $(\lambda^{-1}q, m')$ for all m'
- Factor λ^{-1} due to $d(\lambda q) = \lambda dq$

Recursive BGP Equilibrium

- For notation, below I sometimes omit dependence of functions on (q, m, t)
- Growth rate g of average quality $\bar{q}_t$, value functions V, W, individual R&D policies z and $\hat{z}$, aggregate R&D intensity τ, entrant R&D intensity $\bar{z}$, prices of intermediate goods, final and intermediate goods wage $\bar{w}$, and a distribution $d\mu$ such that:
  - Intermediate goods firms and final goods firms statically optimize production decisions
  - Value functions V, W solve HJB eqs, individual policy functions optimal given value functions
  - Distribution $\mu(q, m, t)$ satisfies KF equation (time dependent, haven't shown)
  - Final and intermediate goods wage satisfy $\bar{w} = \Gamma(\beta)$
  - R&D wages satisfy indifference condition $w(q, m, t) + \nu(q/\bar{q}_t)^{-1} W(q, m, t) = \bar{w}$
  - Labor resource constraint: $L^F + L^I + L^{RD} = 1$
  - Growth is constant at g, and consistent with R&D policy functions and distribution $\mu(q, m, t)$: $g = (\lambda - 1) \int \tau(q, m, t)(q/\bar{q}_t) d\mu(q, m, t)$

Finding a BGP

- Recall that in equilibrium, $w(q,m,t) = \tilde{\beta} \cdot \bar{q}_t - \nu W(q,m,t)$
- Taking this into account, in equilibrium the following holds:

$$\Big(\rho + \overbrace{\hat{\chi}\bar{z}(q,m,t)\eta(\bar{z}(q,m,t))}^{\text{Entrant innovation rate}}\Big)V(q,m,t) = \overbrace{\pi q}^{\text{Flow profits}} + \overbrace{V_t(q,m,t)}^{\text{Changing aggregate state}}$$

$$+ \overbrace{\nu \bar{z}(q,m,t) V_m(q,m,t)}^{\text{Knowledge spillovers from entrant R\&D}}$$

$$+ \max_{z} \Bigg\{ \underbrace{\chi z \phi(z)}_{\text{Arrival rate of R\&D victory}} \overbrace{\Big(V(\lambda q, 0, t) - V(q,m,t)\Big)}^{\text{NPV of successful innovation}}$$

$$- \underbrace{z(q/\bar{q}_t)}_{\text{R\&D labor}} \underbrace{\left(\tilde{\beta} \cdot \bar{q}_t - \nu (q/\bar{q}_t)^{-1} W(q,m,t)\right.}_{\text{Equilibrium R\&D wage}} - \overbrace{\nu (q/\bar{q}_t)^{-1} V_m(q,m,t)}^{\text{Knowledge spillovers from own R\&D}} \Big) \Bigg\}$$

Finding a BGP: Guess and verify

- Guess and verify: abusing notation, value of **incumbent** is $V(q, m, t) = qV(m)$, value of **entrant** is $W(q, m, t) = qW(m)$
- Given these guesses, makes sense to guess $\bar{z}(m, q, t) = \bar{z}(m) = \xi \min(m, M)$ for some $M > 0$
- Plugging guess into incumbent HJB yields: for $m < M$,

$$
\begin{aligned}
(\rho + \hat{\chi}\xi m \eta(\xi m))V(m) = \pi + {} & \overbrace{\theta\Big(V(M) - V(m)\Big)}^{\text{Knowledge becomes public}} + \nu \xi m V'(m) \\
& + \max_z \Big\{ \chi z \phi(z)(\lambda V(0) - V(m)) \\
& - z(\bar{w} - \nu(W(m) + V'(m))\Big\}
\end{aligned}
$$

  where $\bar{w} = \tilde{\beta}$.
- Boundary condition: $V'(m) = 0$ for $m \geq M$

Finding a BGP: Guess and verify (cont.)

- Similarly, HJB equation for entrant becomes: for $m < M$,

$$(\rho + \hat{\chi}\xi m\eta(\xi m))W(m) = \overbrace{\theta\Big(W(M) - W(m)\Big)}^{\text{Knowledge becomes public}} + \nu\xi m W'(m)$$
$$+ \max_z \Big\{\chi z\eta(\xi m)(\lambda V(0) - W(m))$$
$$- z(\bar{w} - \nu W(m)\Big\}$$

- Boundary condition: $W(M) = 0$
- Equilibrium M pinned down by free-entry condition

$$\eta(M)\lambda V(0) = \bar{w}$$

Alternative nesting of model

- The above feels ad-hoc. I am arbitrarily restricting entrants until a reduced form shock occurs.
- Another possibility: endow employees who learn on the job with a more efficient R&D
- Specifically, if $\bar{z}$ is total innovation effort by entrants (spinouts or not), the R&D production functions are (for spinouts, non-spinouts, respectively):

$$\hat{R}_S(\hat{z}_S; \bar{z}) = \hat{\chi}_S \hat{z}_S \eta(\bar{z})$$
$$\hat{R}_E(\hat{z}_E; \bar{z}) = \hat{\chi}_E \hat{z}_E \eta(\bar{z})$$

- Nests standard model when $\hat{\chi}_E \geq \hat{\chi}_S$

Identification

- Specialize $\phi(z) = \eta(z) = z^{-\psi}$
- Parameters in baseline model: $\{\beta, \rho, \lambda, \chi, \psi, \nu, \xi, \theta\}$
- No closed forms so even when a certain moment "identifies" a parameter, I will have to do identification with indirect inference.
- General parameters: $\{\rho, \beta\}$
- R&D parameters: $\{\psi, \lambda, \chi\}$
- Spinout parameters: $\{\nu, \xi, \theta\}$
    - Empirical component of my paper
    - Attempt to identify using data on spinouts and creative destruction (details on next slide)

Identification: ρ, β

- Calibrating ρ:
  - Agents in model are risk-neutral $\Rightarrow$ Interest rate $= \rho$
  - To get realistic interest rate, have to assume unrealistic discount factor
- Calibrating β:
  - In model, β determines elasticity of substitution across intermediate goods varieties labor share of final goods firm value-added
  - In my model, makes more sense to have realistic markups than realistic labor share of final goods production
  - Idea: follow AK 2017 in identifying based on profit / sales ratio (of, say, incumbent firms)
  - How to interpret final goods labor in this model if I think of intermediate goods as actually goods? Retail workers? Does this pose a problem given that I have required R&D workers to be indifferent?

Identification: R&D parameters ψ, λ, χ

- Difficult to disentangle using data generated from a single BGP
- Setting ψ:
  - Literature has identified using (1) elasticity of R&D spending to tax changes, (2) elasticity of patents to R&D spending
  - Both suggest $\psi \approx 2$
- Calibrating λ:
  - Literature has typically tried various values (approx 1.2) and checked robustness
  - AK 2017: identify based on patent citation distribution
- Calibrating χ:
  - Given other R&D parameters, can identify from measures of R&D intensity

Identification of spinout parameters

- Identifying θ:
  - Idea: fraction of existing firms which are spinouts?
  - Idea: fraction of startups which are spinouts?
- What fraction of winners of R&D race are spinouts?
  - How to identify winners of R&D race in the data? (1) entrants that are still around after n years, for some n; (2) entrants that grow to a certain percentile in the firm-size distribution
- Employer-employee matched data (LEHD, Brazilian data, German data): identify spinouts as in Muendler et al. 2012
- Establishment level data (same datasets): identify entrant firms, rate of entry, etc.
- **Problem:** entrants in data are not just creative destruction. Will overestimate number of entrants, hence overestimate role of within-industry spinouts in entry.
- **Problem:** firms have more than one product.
- Average R&D expenditure given the state m of an industry?
- What fraction of winners of R&D race are spinouts?
- Average R&D expenditure given the state m of an industry?

Identification of Alternative Nesting

- To the extent that $\hat{\chi}_S > \hat{\chi}_E$, the model has predictions on:
  1. The fraction of firms which started out as spinouts vs. ordinary entrants
  2. How the likelihood of successful overtaking by a spinout increases with the time since the last innovation
  3. Stark model will likely have trouble matching data - eventually, create some attenuation by adding heterogeneity in R&D productivity of both spinouts and entrants...but I am getting ahead of myself.
- If I can come up with a proxy for m or "time since last innovation" in the data, I can identify this mechanism.
- Need a proxy for "how many products are currently being developed that compete with this product".
- Idea: Extend Muendler 2012 logic. Entrants that hire many employees of an existing firm are classified as spinouts. But entrants that hire some employees are still more likely to be competing firms than entrants that hire no employees.
- **I can distinguish much more reliably between these two using VentureSource data.**
- Ideas in the literature for identifying these things?

Identification: next steps

- Previous slides were for baseline model which assumed same R&D technology for incumbents and entrants
- May be helpful to allow ψ, λ, χ to differ between incumbents and entrants
  - Empirical evidence suggests incumbents focus on incremental innovations while entrants focus on more substantial innovations
  - Probably reasonable to assume ψ the same for incumbents and entrants, but can do robustness checks.
  - χ, λ should be allowed to vary: can identify using patent data as in AK 2017.

Spinouts of Fairchild Semiconductor

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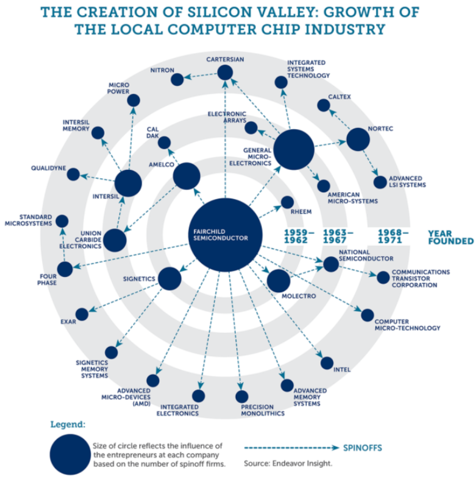

Source: Endeavor Insights