A Model of Productivity Growth through Creative Destruction by Employee Spinouts

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Model overview

- Time t is continuous
- Agents:
  - Households
  - Intermediate goods firms
  - Final goods firm

Model overview

- Builds on (nests) standard quality ladders model of endogenous growth (Grossman & Helpman 1991)
- Endogenous productivity growth through improved quality of intermediate goods
- Quality improvements result from labor allocated to R&D
- Innovation by creative destruction
- **New ingredient:** R&D workers learn on the job how to form spinouts which compete in R&D race with incumbents

Model: Intermediate goods production

- Standard quality ladders model, step size $\lambda > 1$
- Continuum of intermediate goods, indexed by $j \in J = [0, 1]$
- Frontier quality of good j by q_j
- x_j is amount produced
- Each good produced with technology

  $$x_j = \overline{q} l_j$$

  where $\overline{q} = \int_0^1 q_j dj$ is the average quality level of the economy
- Each good j has monopolist, standard assumptions to guarantee no limit pricing
- Demand (final goods production) CES across goods j implies constant markup

Model: R&D race

At time t with average quality $\overline{q}_t$, incumbent in the R&D race for good j of quality q_j begins with monopoly on good j R&D

$\downarrow$

1. Hires R&D labor; at rate $\nu(q_j/\overline{q}_t)^{-1}$ per unit of R&D labor hired, employees learn, adding to mass of potential entrants (**scaling factor** $(q_j/\overline{q}_t)^{-1}$ **for BGP**)
2. Meanwhile, with arrival rate θ, "knowledge spillover" shock hits and makes knowledge public

$\downarrow$

At some point, either an incumbent or an entrant firm wins the race, and obtains a monopoly on production and R&D on good j of quality λq_j

Model: R&D technology

- Consider an intermediate j with relative quality $\tilde{q} = q/\overline{q}$
- **Scaling assumption for BGP:** flow cost of $\tilde{q}z$ units of labor yields z units of effective labor
- z,$\hat{z}$ units of R&D effort by incumbent and entrant respectively yields victory in the R&D race at Poisson rate

$$R(z) = \chi z \phi(z)$$
$$\hat{R}(\hat{z}; \overline{z}) = \hat{\chi} \hat{z} \eta(\overline{z})$$

  where $\overline{z} = \int_0^m \hat{z}(m')dm'$ is total good-j R&D effort by entrants.
- Entrant m' can perform $\hat{z} \leq \xi$ units of R&D effort (equilibrium does not pin down; look for equilibria where $\hat{z}(m') \in \{0, \xi\}$)
- Aggregate rate of innovations:

$$\tau = \chi z \phi(z) + \hat{\chi} \overline{z} \eta(\overline{z})$$

Model: R&D technology - congestion

- The reduced-form functions $\phi(z), \eta(z)$ capture diminishing returns and congestion in the R&D race, repsectively
- As such, assume $\phi(z), \eta(z)$ decreasing, $z\phi(z), z\eta(z)$ increasing
- For the incumbent, $\phi(z)$ captures *individual* decreasing returns in the R&D technology
- For the entrants, $\eta(z)$ captures *aggregate* decreasing returns due to the possibility that different entrants use the same approach
- Cross-congestion:
  - Incumbent and entrants do not congest each other (as in other models of innovation by entrants and incumbents, c.f. Acemoglu & Cao 2015, Akcigit & Kerr 2017)
  - Adds tractability and reflects empirical fact that spinouts often attempt different approaches
  - Can be relaxed

Intermediate goods firms optimization: incumbent static optimization

- Static optimization in product market: CES final goods production implies constant markup
- In equilibrium, flow profits $\pi(q) = \pi q$, with

$$\pi = L(1-\beta)\tilde{\beta}$$
$$\tilde{\beta} = \beta^{\beta}(1-\beta)^{1-2\beta}$$

Intermediate goods firms optimization: incumbent R&D decision

- HJB equation for incumbent:

$$
\begin{aligned}
\Big(\rho + \overbrace{\hat{\chi}\bar{z}(q,m,t)\eta(\bar{z}(q,m,t))}^{\text{Entrant innovation rate}}\Big)V(q,m,t) &= \overbrace{\pi q}^{\text{Flow profits}} + \overbrace{V_t(q,m,t)}^{\text{Changing aggregate state}} \\
&+ \overbrace{\nu\bar{z}(q,m,t)V_m(q,m,t)}^{\text{Knowledge spillovers from entrant R\&D}} + \overbrace{\theta\Big(V(q,M,t) - V(q,m,t)\Big)}^{\text{Knowledge becomes public}} \\
&+ \max_z \Bigg\{ \underbrace{\chi z\phi(z)}_{\text{Arrival rate of R\&D victory}} \overbrace{\Big(V(\lambda q,0,t) - V(q,m,t)\Big)}^{\text{NPV of successful innovation}} \\
&- \underbrace{z(q/\bar{q}_t)}_{\text{R\&D labor}} \Big(\underbrace{w(q,m,t)}_{\text{R\&D wage}} - \overbrace{\nu(q/\bar{q}_t)^{-1}V_m(q,m,t)}^{\text{Knowledge spillovers from own R\&D}}\Big)\Bigg\}
\end{aligned}
$$

Intermediate goods firms optimization: entrant R&D decision

- HJB equation for entrant:

$$
\begin{aligned}
(\rho + \overbrace{\hat{\chi}\bar{z}(q,m,t)\eta(\bar{z}(q,m,t))}^{\text{Entrant innovation rate}})W(q,m,t) = {} & \overbrace{W_t(q,m,t)}^{\text{Changing aggregate state}} \\
+ \quad \overbrace{\nu\bar{z}(q,m,t)W_m(q,m,t)}^{\text{Knowledge spillovers from entrant R\&D}} & + \overbrace{\theta\Big(W(q,M,t) - W(q,m,t)\Big)}^{\text{Knowledge becomes public}} \\
+ \max_{z}\Bigg\{ \quad \overbrace{\hat{\chi}z\eta(\bar{z}(q,m,t))}^{\text{Arrival rate of R\&D victory}} & \overbrace{\Big(V(\lambda q,0,t) - W(q,m,t)\Big)}^{\text{NPV of successful innovation}} \\
& - \underbrace{z(q/\bar{q}_t)}_{\text{R\&D labor}}\underbrace{w(q,m,t)}_{\text{R\&D wage}}\Bigg\}
\end{aligned}
$$

Model: Households

- Unit mass continuum of risk-neutral households indexed by $i \in I = [0,1]$, each with objective
$$U = \int_0^\infty \exp(-\rho t)c(t)dt$$
where $c(t)$ is final goods consumption at t.
- Instantaneous borrowing and lending at interest rate r; $r = \rho$ in equilibrium
- Individual i supplies labor to final goods production $\ell_i^F(t)$, intermediate good production $\ell_i^I(t)$ and R&D $\ell_i^{RD}(t)$ such that
$$\ell_i^F(t) + \ell_i^I(t) + \ell_i^{RD}(t) = 1$$
- Aggregate labor market satisfies (where $L^k(t) = \int_I \ell_i^k(t)idi$ for $k \in \{F, I, RD\}$)
$$L^F(t) + L^I(t) + L^{RD}(t) = 1$$

Household optimization timeline

Worker i allocates labor to R&D, intermediate and final goods production

$\downarrow$

While performing R&D for some good j of relative quality $\tilde{q}_j$, receives learning shock with Poisson intensity $\nu\tilde{q}_j^{-1}$ per flow unit of R&D labor supplied to j $\downarrow$

Provided it is still profitable, he opens entrant R&D lab performing R&D effort ξ and competing in developing the next step of good j

Household optimization

- Workers indifferent between occupations (Final goods production, intermediate goods production, R&D)
- In equilibrium, closed form for final goods wage $\overline{w}_t = \tilde{\beta}\overline{q}_t$ where $\overline{w}$ is a function of parameters
- Indifference condition intermediate goods wage $w_t^I = \overline{w}_t$
- R&D wage at product j depends on state of the product, which is (q, m)
- For now, no employment / entrepreneurship choice; hope to include eventually
- Household block of the model boils down to equilibrium condition

$$w(q, m, t) + \nu W^{NC}(q, m, t) = \overline{w}_t$$

where $W_t^{NC}(q, m)$ is the value of the knowledge to open an entrant in a good j in state (q, m) at time t

Model: Final good production

- Final good is produced using labor and a continuum of intermediate goods $j \in [0,1]$ with production technology

$$X(t) = L(t)^{\beta} \left(\left(\int_0^1 q_j(t)^{\beta} x_j(t)^{1-\beta} dj \right)^{1/(1-\beta)} \right)^{1-\beta}$$
$$= L(t)^{\beta} \int_0^1 q_j(t)^{\beta} x_j(t)^{1-\beta} dj$$

  where q_j is quality, x_j is quantity
- Restricts labor share to be related to markup $\mu = 1/(1-\beta)$
- Can relax this using Grossman et. al 2016
- CRS implies zero profits so no need to consider ownership

Aggregation: Kolmogorov Forward Equation

- Define $d\mu(q, m, t)$ as the distribution of intermediate goods j across states (q, m) at time t
- Kolmogorov Forward Equation (somewhat heuristic)

$$
\begin{aligned}
\mu_t(q,m,t) = & \overbrace{-\frac{d}{dq}(a^q(q,m,t)\mu(q,m,t))}^{\text{Drift in } q} - \overbrace{\frac{d}{dm}(a^m(q,m,t)\mu(q,m,t))}^{\text{Drift in } m} \\
- & \qquad \underbrace{\tau(q,m,t)\mu(q,m,t)}_{\text{Innovation arrival: jump } (q,m)\to(\lambda q,0)} \\
& \underbrace{+ \mathbb{1}_{\{m=0\}}\lambda^{-1}\int \tau(\lambda^{-1}q,m',t)d\mu(\lambda^{-1}q,m',t)}_{\text{Innovation arrival: jump } (\lambda^{-1}q,m')\to(q,0)}
\end{aligned}
$$

- $a^q(q, m, t), a^m(q, m, t)$ are drift in q, m direction, respectively, computed from $z(q, m, t)$ and $\overline{z}(q, m, t)$
- Last term for $m = 0$ arises because receiving inflows from $(\lambda^{-1}q, m')$ for all m'
- Factor λ^{-1} due to $d(\lambda q) = \lambda dq$

Recursive BGP Equilibrium

- For notation, below I sometimes omit dependence of functions on (q, m, t)
- Growth rate g of average quality $\overline{q}_t$, value functions V, W, individual R&D policies z and $\hat{z}$, aggregate R&D intensity τ, entrant R&D intensity $\overline{z}$, prices of intermediate goods, final and intermediate goods wage $\overline{w}$, and a distribution $d\mu$ such that:
  - Intermediate goods firms and final goods firms statically optimize production decisions
  - Value functions V, W solve HJB eqs, individual policy functions optimal given value functions
  - Distribution $\mu(q, m, t)$ satisfies KF equation (time dependent, haven't shown)
  - Final and intermediate goods wage satisfy $\overline{w} = \Gamma(\beta)$
  - R&D wages satisfy indifference condition $w(q, m, t) + \nu(q/\overline{q}_t)^{-1} W(q, m, t) = \overline{w}$
  - Labor resource constraint: $L^F + L^I + L^{RD} = 1$
  - Growth is constant at g, and consistent with R&D policy functions and distribution $\mu(q, m, t)$: $g = (\lambda - 1) \int \tau(q, m, t)(q/\overline{q}_t) d\mu(q, m, t)$

Finding a BGP

- Recall that in equilibrium, $w(q,m,t) = \tilde{\beta} \cdot \overline{q}_t - \nu W(q,m,t)$
- Taking this into account, in equilibrium the following holds:

$$
\begin{aligned}
\Big(\rho + \overbrace{\hat{\chi}\bar{z}(q,m,t)\eta(\bar{z}(q,m,t))}^{\text{Entrant innovation rate}}\Big)V(q,m,t) = {} & \overbrace{\pi q}^{\text{Flow profits}} + \overbrace{V_t(q,m,t)}^{\text{Changing aggregate state}} \\
& + \overbrace{\nu\bar{z}(q,m,t)V_m(q,m,t)}^{\text{Knowledge spillovers from entrant R\&D}} \\
& + \max_{z}\Bigg\{ \underbrace{\chi z\phi(z)}_{\text{Arrival rate of R\&D victory}} \overbrace{\Big(V(\lambda q,0,t) - V(q,m,t)\Big)}^{\text{NPV of successful innovation}} \\
& - \underbrace{z(q/\overline{q}_t)}_{\text{R\&D labor}}\underbrace{\big(\tilde{\beta}\cdot\overline{q}_t - \nu(q/\overline{q}_t)^{-1}W(q,m,t)}_{\text{Equilibrium R\&D wage}} - \overbrace{\nu(q/\overline{q}_t)^{-1}V_m(q,m,t)}^{\text{Knowledge spillovers from own R\&D}}\big)\Bigg\}
\end{aligned}
$$

Finding a BGP: Guess and verify

- Guess and verify: abusing notation, value of **incumbent** is $V(q, m, t) = qV(m)$, value of **entrant** is $W(q, m, t) = qW(m)$
- Given these guesses, makes sense to guess $\overline{z}(m, q, t) = \overline{z}(m) = \xi \min(m, M)$ for some $M > 0$
- Plugging guess into incumbent HJB yields: for $m < M$,

$$\begin{aligned}(\rho + \hat{\chi}\xi m\eta(\xi m))V(m) &= \pi + \nu\xi m V'(m) \\ &\quad + \max_z \Big\{\chi z\phi(z)(\lambda V(0) - V(m)) \\ &\quad - \chi z(\overline{w} - \nu(W(m) + V'(m))\Big\}\end{aligned}$$

  where $\overline{w} = \tilde{\beta}$.
- Boundary condition: $V'(m) = 0$ for $m \geq M$

Finding a BGP: Guess and verify (cont.)

- Similarly, HJB equation for entrant becomes: for $m < M$,

$$
\begin{aligned}
(\rho + \hat{\chi}\xi m\eta(\xi m))W(m) = \nu\xi m W'(m) \\
+ \max_{z} \Big\{ \chi z\eta(\xi m)(\lambda V(0) - W(m)) \\
- \chi z(\overline{w} - \nu W(m) \Big\}
\end{aligned}
$$

- Boundary condition: $W(M) = 0$
- Entrant optimality implies that M is determined by free-entry condition

$$
\eta(M)\lambda V(0) = \overline{w}
$$

Finding a BGP: Solving the HJBs numerically

1. Guess $W(\cdot)$
2. Solve for $V(\cdot), M$ given $W(\cdot)$:
    2.1 Guess $V(0)$
    2.2 Free entry condition and $V(0)$ determine M
    2.3 HJB and boundary condition $V'(M) = 0$ determine $V(M)$
    2.4 Integrate backward starting from $V(M)$, using $w(m) = \overline{w} - \nu W(m)$, to compute $\hat{V}(0)$
    2.5 If $\hat{V}(0) \neq V(0)$, update guess and return to Step 2.1
3. Using M computed from the previous step, solve for implied $W(\cdot)$
    3.1 Integrate entrant HJB backward starting from boundary condition $W(M) = 0$
    3.2 Denote resulting function by $\hat{W}(m)$
    3.3 Check $d(W, \hat{W})$ using some metric; if not converged, update guess and return to Step 1.

Finding a BGP: Stationary distribution $\mu(m)$

- No need to keep track of aggregate distribution across q
- Conjecture that a BGP exhibits a stationary distribution $d\mu(m,t) = d\mu(m)$
- For $m > 0$, the distribution $d\mu(m)$ will have no mass points, therefore its density $\mu(m)$ is well defined and satisfies the Kolmogorov Forward Equation:

$$0 = -\frac{d}{dm}(a(m)\mu(m)) - \tau(m)\mu(m)$$

- For $m > M$, have $a(m)$ and $\tau(m)$ constant, so the above becomes

$$\mu'(m) = -\frac{\tau(M)}{a(M)}\mu(m)$$

- Solution given by $\mu(m) = \mu(M)e^{-(\tau(M)/a(M))(m-M)}$ for $m \geq M$
- Probably solve the rest numerically

Finding a BGP: Algorithm

- The numerical algorithm I propose for computing a BGP is as follows:
  1. Guess L^F, the labor supply to final goods production
  2. This guess pins down equilibrium profit flow π and final goods / intermediate goods wage $\overline{w}$
  3. Given these, solve HJBs numerically using iterative procedure described above
  4. Next, solve KF equation to compute stationary distribution $\mu(m)$
  5. Using $\mu(m)$ and policy functions from previous step, integrate to compute aggregate labor demand
  6. Check that aggregate labor demand is equal to the unit labor supply. If not, adjust guess for L^F and return to Step 1.
  7. Finally, integrate innovation arrival rates to compute $g = (\lambda - 1) \int \tau(m) d\mu(m)$

Efficiency

- Worker indifference condition implies firm compensated (in expectation) for profits of future spinouts
- Franco-Filson 2006 logic suggests this may imply Pareto efficiency
- However, this ignores creative destruction
- Spinouts may reduce value of incumbent-worker pair by destroying monopoly power
- Equilibrium wage discount does not fully compensate R&D firm for knowledge produced
- So model exhibits standard Schumpeterian inefficiency from R&D firms not fully appropriating the returns to their investments
- Also standard: potential for inefficiently high creative destruction due to entrants not internalizing their "business stealing" effect

Next steps

- Existence and uniqueness of BGP; maybe closed forms for certain functional forms?
- Implement proposed algorithm for solving model numerically
- Calibrate, test the model
  - Microeconometric facts about employee spinouts from e.g., Starr et. al "Screening spinouts"
  - Klepper-Sleeper 2005
- Potential extensions
  - Learning and entrepreneurship decisions
  - Different step sizes for incumbents and entrants
  - Cross-congestion
  - Labor contracts: non-competes

Spinouts of Fairchild Semiconductor

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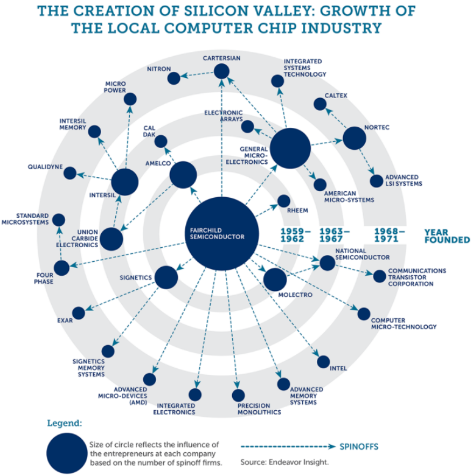

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