Parameters taken as given are $\chi_I, \psi_I, r, \nu, \sigma, \tau(q,m), w(q,m)$. Terminal conditions are $B(0) = A(0,m) = 0$, and $\lim_{m\to\infty} A(q,m) = B(q)$. The "max" stuff on the RHS is annoying and confusing. If it helps, just assume you have some functions $z_A(q,m)$ and $z_B(q)$ that you can just plug in (having a "max" on the RHS is not a complication for the weird method at all since the RHS will be known at the time I calculate the maximum).

$$\begin{aligned}
R_I(z) &= \chi_I z^{\psi_I} \\
(r+\sigma)B(q) &= \pi(q) - gqB'(q) \\
&\quad + \max_z \left\{ z[A((1+\lambda)q, 0) - B(q)] - wR_I(z) \right\} \\
(r+\theta+\tau(q,m)-g)A(q,m) &= \pi(q) - gq\partial_q A(q,m) + \theta B(q) \\
&\quad + \max_z \left\{ z[A((1+\lambda)q,0) - A(q,m)] + \partial_m A(q,m)\nu R_I(z) - w(q,m)R_I(z) \right\}
\end{aligned}$$

The weird method consists roughly of augmenting the functions $A(q,m), B(q)$ with a time argument, adding a partial to the PDE, guessing an arbitrary initial condition $A(q,m,0), B(q,0)$, integrating forward by finite difference ("implicit" vs "explicit" pops up here, it's pretty opaque but it saves orders of magnitude of computational time somehow), solving for $\lim_{t\to\infty} A(q,m,t)$ and $\lim_{t\to\infty} B(q,t)$. The idea is that these limits solve the original PDEs above (you argue this using "von Neumann stability analysis" in the text I just sent you) Concretely, the new system is

$$\begin{aligned}
R_I(z) &= \chi_I z^{\psi_I} \\
-\partial_t B(q,t) + (r+\sigma)B(q,t)) &= \pi(q) - gq\partial_q B(q,t) \\
&\quad + \max_z \left\{ z[A((1+\lambda)q,0,t) - B(q,t)] - wR_I(z) \right\} \\
-\partial_t A(q,t) + (r+\theta+\tau(q,m)-g)A(q,m,t) &= \pi(q) - gq\partial_q A(q,m,t) + \theta B(q,t) \\
&\quad + \max_z \left\{ z[A((1+\lambda)q,0,t) - A(q,m,t)] \right. \\
&\quad \left. + \partial_m A(q,m,t)\nu R_I(z) - w(q,m)R_I(z) \right\}
\end{aligned}$$