Equilibrium production wage determination

Nicolas Fernandez-Arias

April 25, 2018

Have

$$Y = L_F^{\beta}\left(\left(\int_0^1 q_j^{\beta} x_j^{1-\beta} dj\right)^{1/(1-\beta)}\right)^{1-\beta}$$

Maximization:

$$\max_{\{x_j\}_{j\in[0,1]}} \int_0^1 q_j^{\beta} x_j^{1-\beta}$$

subject to

$$\int_0^1 p_j x_j dj \leq E$$

Lagrangean has FOCs: for each $j \in [0,1]$,

$$\begin{aligned}(1-\beta)q_j^{\beta}x_j^{-\beta} &= \sigma p_j \\ q_j^{\beta} &= \sigma p_j (1-\beta)^{-1} x_j^{\beta}\end{aligned}$$

where σ is a Lagrange multiplier. In equilibrium, every j will charge the same price

$$p_j = \frac{w}{\overline{q}(1-\beta)}$$

Therefore, for all i, j, we get

$$x_i = \frac{q_i}{q_j} x_j$$

Substituting into budget constraint and solving for x_i yields

$$x_i = \frac{q_i}{\overline{q}} \times \frac{E}{p}$$

Lab equipment model If R&D were done using final goods, we can write E as a function of L_F using the equation:

$$\begin{aligned} L_F &= 1 - \int_0^1 l_j dj \\ &= 1 - \frac{E}{p} \end{aligned}$$

Further, we can substitute to obtain an expression for production in terms of L_F, E, assuming expenditures on capital goods are optimal given the quality distribution. First, do some algebra to get an expression for the optimal CES aggregator given price p, qualities $\{q_j\}_{j\in[0,1]}$ and spending E:

$$\begin{aligned} \Big(\Big(\int_0^1 q_j^\beta x_j^{1-\beta} dj\Big)^{1/(1-\beta)}\Big)^{1-\beta} &= \Big(\Big(\int_0^1 q_j^\beta \Big(\frac{q_j}{\overline{q}}\frac{E}{p}\Big)^{1-\beta} dj\Big)^{1/(1-\beta)}\Big)^{1-\beta} \\ &= \Big(\frac{1}{\overline{q}}\frac{E}{p}\Big)^{1-\beta}\Big(\Big(\int_0^1 q_j dj\Big)^{1/(1-\beta)}\Big)^{1-\beta} \\ &= \Big(\frac{1}{\overline{q}p}\Big)^{1-\beta}\overline{q}E^{1-\beta} \\ &= \overline{q}^\beta p^{\beta-1}E^{1-\beta} \end{aligned}$$

Substitute this into the final goods production function:

$$Y(L_F, E; \overline{q}) = \overline{q}^\beta p^{\beta-1} L_F^\beta E^{1-\beta}$$

This yields FOCs for L_F and E:

$$\begin{aligned} \beta\overline{q}^\beta p^{\beta-1} L_F^{\beta-1} E^{1-\beta} &= w \\ (1-\beta)\overline{q}^\beta p^{\beta-1} L_F^\beta E^{-\beta} &= 1 \end{aligned}$$

because the price of one unit of E is, by definition, equal to 1.

Finally recall our equation for p:

$$p = \frac{w}{\overline{q}(1-\beta)}$$

Hence we have four equations in four unknowns $\{L_F, E, w, p\}$ and parameters:

$$L_F = 1 - \frac{E}{p} \tag{1}$$

$$\beta \overline{q}^{\beta} p^{\beta-1} L_F^{\beta-1} E^{1-\beta} = w \tag{2}$$

$$(1-\beta)\overline{q}^{\beta} p^{\beta-1} L_F^{\beta} E^{-\beta} = 1 \tag{3}$$

$$p = \frac{w}{\overline{q}(1-\beta)} \tag{4}$$

This part of the model is therefore determined separately from the R&D side of the model. Intuitively, I haven't proven that there exists a closed-form solution – this is shown by Akcigit & Kerr 2017, which is exactly the same framework. To check these conditions we could substitute that solution and check there is no contradiction.

My model In my model, R&D is done using labor drawn from the same pool as intermediate and final goods production. Now we cannot derive (1) because

$$L_F = 1 - \int_0^1 l_j^I dj - \int_0^1 l_j^{RD} dj$$

Hence, we cannot derive a formula relating E and L without appealing to $z(m), \hat{z}(m)$ in order to compute the last term in the equation above. But those require solving the HJBs, etc. The static and dynamic aspects of the model now interact.

Possible solutions The only way to eliminate this feature is to entirely decouple the production and R&D labor markets. In addition, we must assume elastic labor supply in the R&D market in order to make the model an endogenous growth model. Also note that we can't endogenize the elasticity of R&D labor supply by using some kind of decision to specialize in final goods production or R&D with some initial heterogeneity in relative productivities in each form of employment, because this couples the labor markets, eliminating the tractability. Hence, the only way to have a tractable, non-trivial model is to assume a separate population of potential R&D workers with some aggregate labor supply elasticity.

New algorithm In light of this, we need a new algorithm.

1. Guess L^{RD}, the BGP labor supply to R&D

2. Now we know the labor supply available to production, hence can solve for all static production variables L^F, L^I, w, p, π in closed form

3. Given these, solve HJBs numerically using iterative procedure described above
4. Next, solve KF equation to compute stationary distribution $\mu(m)$
5. Using $\mu(m)$ and policy functions from previous step, integrate to compute aggregate labor demand
6. Check market clearing in R&D market $L^{RD} = \int l(m) + \hat{l}(m) d\mu(m)$. If market does not clear, update guess L^{RD} and go back to Step 1

My original algorithm was needlessly complex.