

Outline of Model

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1 Effect of non-compete enforcement: some stylized facts

1. Non-enforcing regions exhibit a larger response of entrepreneurship, patenting, employment, and income to exogenous increases in the supply of VC funding (Samila-Sorenson 2011).
 - (a) NB: I am skeptical of this, since they do not allow for population, or composition of population, to mediate the effect of VC funding increases. Since higher-impact, more collaborative entrepreneurs tend to relocate to non-enforcing regions (Marx 2015), local population of potential innovators / entrepreneurs etc. would be negatively correlated with non-compete enforcement, biasing the results in the direction of their conclusion.
2. Exogenous increase in enforcement leads to less frequent job switching; the effect is stronger for workers with firm-specific skills or who specialize in narrow technical occupation (Marx 2009).
3. Exogenous increase in enforcement leads to brain drain of inventors / knowledge workers towards non-enforcing regions; the effect is stronger for higher-impact, more collaborative workers (Marx 2015).
4. Non-enforcing states have higher mobility, higher wages and higher wage growth (Balasubramaniam 2017).
 - (a) NB: I am a bit skeptical of this result because are we really comparing the same worker, same firm? They take steps to justify this, need to re-read closely.

5. Wages only don't get a boost in non-Consideration enforcing states; wages and training higher in (consideration) enforcing states (Starr 2015 "Consider this...")

1.1 Caveat on stylized facts

- Difficult to explain Fact 5 in a rational model.
- Moreover, Fact 5 suggests that Facts 1-4 also can only be explained with a non-rational model.
- Would be nice to have a version of Facts 1-4, but focusing on the difference between non-enforcing states and on states that enforce but require due "consideration"

2 Model

The model builds on neo-Schumpeterian endogenous growth theory, specifically the work of Aghion-Howitt and Grossman-Helpman, via Hopenhayn 1992, Klette-Kortum 2004 and more recently Acemoglu et al 2013 and Akcigit-Kerr 2017. Time $t \geq 0$ is continuous. Essentially the model endogenizes the process of entry into the patent race for discovering the next level on the quality ladder: entrants are formed by former R&D employees of the incumbent firm.

2.1 Households

In this section I describe the general assumptions on households: their preferences and their labor endowment.

2.1.1 Preferences

There is a unit mass of households indexed by $i \in [0, 1]$. There is no representative household, as competition between businesses formed by households is crucial to my analysis.¹ Households are risk-neutral, maximizing objective function

$$U = \int_0^\infty \exp(-\rho t) C_t dt$$

where $C_t \geq 0$ is consumption of the final good, whose production I will describe in the next section. They borrow and lend short-term on bond markets at the endogenous risk-free interest rate $r_t = \rho$.

¹Equivalently, Akcigit & Kerr 2017 assume a representative household but maintain that the firms in which it holds stock do not cooperate.

2.1.2 Endowment

Household i is endowed with a unit flow of labor $l_t = 1$, which can be sold to final good firms (l_t^F), or to intermediate good firms for production (l_t^I) or for R&D (l_t^{RD}). Labor markets are competitive. The individual resource constraint is

$$l_{it}^F + l_{it}^I + l_{it}^{RD} \leq 1$$

Defining aggregate labor supply $L_t^m = \int_0^1 l_{it}^m di$ for $m = F, I, RD$, the aggregate resource constraint is

$$L_t^F + L_t^I + L_t^{RD} \leq 1$$

2.2 Final goods

As in Akcigit & Kerr 2017, final goods are produced by a competitive firm with a CRS production function,².

$$Y_t = (L_t^F)^\beta \int_0^1 q_{jt}^\beta y_{jt}^{1-\beta} dj$$

where q_{jt} is the leading edge quality of good j at time t .³ There is no storage technology, so $c_{jt} = y_{jt}$ and $C_t = Y_t$ in equilibrium. Ownership of the final goods firm is irrelevant as it has zero profits in equilibrium due to CRS and perfect competition.

2.3 Intermediate goods

Below I describe the assumptions on intermediate goods firms. I start by describing their production technology, then their R&D technology and the contracting problem / frictions that lead to non-competes being used when they are enforceable. I conclude by describing the creative destruction cycle that drives productivity growth in the economy.

²Labor here is difficult to interpret, but adds tractability by giving a closed form for the household's reservation wage

³Under certain conditions on technology and competition between providers of the same good, the leading edge quality is the only quality used in equilibrium. This is meant as an abstraction to improve tractability. Formal proof in the Appendix.

2.3.1 Production

At any given time t , each good j has a firm monopolizing its production, to which I refer as incumbent j .⁴ Define the average (leading) quality level in the economy by

$$\bar{q}_t = \int_0^1 q_{jt} dj$$

This firm has production function $y_{jt} = \bar{q} l_{jt}^I$.⁵

2.3.2 Research and development

Productivity growth in the model is driven by improvements to technology for producing the intermediate good. In turn, these are driven by R&D expenditures by incumbent firms and entrants (their spinouts).

Incumbents Incumbent j can hire a flow of R&D labor l_j , generating a Poisson flow probability of learning a production technology for quality $(1 + \lambda)q_j$. The technology for this process is

$$R_I(l_j, L_j^{RD}) \equiv \chi_I l_j \phi(L_j^{RD})$$

where $L_j^{RD,m}$ for $m = I(E)$ is the total flow R&D labor hired to work on improving good j by incumbents (entrants) and

$$L_j^{RD} = L_j^{RD,I} + L_j^{RD,E}$$

is total R&D labor allocated to improving good j . The function $\phi(\cdot)$ encodes congestion and is described below; for now note that it will be assumed to be decreasing.

Entrants At the same time, a mass m_j of entrants, indexed by $k \in [0, m_j]$, engages in R&D. Entrant k has technology

$$R_E(l_j, L_j^{RD}) \equiv \chi_{E,k} l_j \phi(L_j^{RD})$$

⁴As implied by the final goods production function, the intermediate goods market is monopolistically competitive

⁵An alternative setup is to assume that $y_{jt} = q_{jt} l_{jt}^I$ and that q_{jt} rather than q_{jt}^β enters the final good production function. The important point, for tractability, is that equilibrium prices be such that the equilibrium allocation is $y_{jt} \equiv y_t$.

where $\chi_{E,k} \geq 0$ is drawn from the distribution $G(\cdot)$.⁶ Total entrant R&D is calculated as

$$L_j^{RD,E} = \int_0^{m_j} l_{jk} dk$$

In this setup, entrants are infinitesimal and hence do not take into account their effect on congestion when choosing their optimal level of R&D activity. In order to pin down the distribution of activity across entrants, I assume an extreme form of decreasing-returns-to-scale production: entrants can hire a maximum flow of $\xi > 0$ units of R&D labor.⁷

Congestion The function $\phi(\cdot)$ is a reduced form representation of congestion in the R&D race.⁸ It will be assumed decreasing, so that more aggregate R&D decreases the individual marginal return to R&D. Two microfoundations of ϕ are relevant here. First, from the perspective of a large R&D lab, such as the incumbent, there are only so many approaches that can be tried to improve on a technology. As more R&D labor is hired, resources are allocated to less and less promising avenues, reducing marginal productivity of these resources. Second, from the perspective of any R&D lab, incumbent or entrant, more R&D by competitors attempting the same approach reduces the probability of being the first to succeed in innovating. This lowers the return of conducting R&D.⁹

2.3.3 Creative destruction

Endogenous entry by spinouts In the baseline Neo-Schumpeterian models of endogenous growth, there is free entry into R&D. The mass m_j is thus determined endogenously by a free-entry condition. An improvement to quality immediately spills over to the rest of the economy, becoming the base upon which all entrants innovate. Instead, in my model, entry into R&D can only occur by a worker who was previously employed at the incumbent *and* successfully learned the technology on the job. In other words, all entrants are spinouts.¹⁰ Entrants work at maximum capacity as long as R&D is profitable in expectation, that is as

⁶More on this in the sub-section below labeled "Non-competes"

⁷One interpretation of this assumption is that entrants have less access to capital than incumbents, making the constraint on managerial ability of the founder a more tightly binding constraint - he cannot hire more managers. Would be nice to have some empirical justification for this assumption.

⁸It is useful to assume Inada conditions on ϕ and that $L\phi(L)$ is non-decreasing.

⁹This ϕ construction is standard in the Neo-Schumpeterian endogenous growth literature; see Acemoglu 2008 (textbook). A choice $\phi(L) = L^{-1}$ roughly represents a zero-sum game, where the aggregate arrival rate of new technologies does not depend on R&D effort, only the allocation thereof; while a choice $\phi(L) \equiv \text{constant}$ represents no congestion, where technologies arrive in proportion to the aggregate spending on them. In the calibration, an intermediate case is taken; a typical choice is $\phi(L) \approx L^{-1/2}$. Typically, this is calibrated to match the elasticity of R&D output (e.g., citation-weighted patents) to R&D inputs (e.g. real expenditures on R&D or R&D employment).

¹⁰This assumption can be relaxed if we want to match data on entry by non-spinout firms

long as m_j is less than the mass that would prevail with free entry.

To be more specific

Suppose that there is a discovery in some good j with quality q_{jt} at time t . I make one key

1. At the beginning of t , the incumbent and mass $\tilde{m} = \lim_{t' \uparrow t} m_{jt'}$ of spinouts perform R&D
2. Either the incumbent or one of the spinouts wins race, discovers technology $(1 + \lambda)q$, becomes new incumbent
3. Knowledge held by losers of race expires: mass m of entrants jumps to $m_{jt} = 0$

2.4 Contracting: introducing non-competes, etc.

- Do I need to microfound the optimal contracting problem, or can I simply assume that "enforcement" means "everyone uses"?
 - Empirically, in tech industries most non-competes look pretty similar, and are simple contracts (e.g. no buyout menu)
 - However, people seem to want to know what the friction is that leads firms-employees to sign non-competes
 - My opinion is that it would be better to model it
 - To that end I have a discussion (Section 3) regarding under what circumstances the availability of non-compete clauses can increase bilateral value
- If we choose to model it, how do we model it? For tractability purposes, unless I switch to a framework where each firm only deals with one worker throughout its lifetime (as in Shi 2017), there are some constraints to prevent too many state variables:
 - Non-competes must be memoryless, i.e. expiry upon arrival of a Poisson process
 - Short-term contracts (contracts only stipulate what happens in different contingencies that happen "later this instant" so to speak), to avoid tracking promised utilities.
 - All workers hired must have the same contract, to avoid keeping track of mass of agents
 - Even in easiest setting, as long as non-competes are not permanent, the firm will have three state variables. Difficult problem.

The first version of the paper (in the slides) simply assumed a certain non-compete length in enforcing states without solving an optimal contracting problem.

Below I take a stab at solving a simple version of the optimal contracting problem. Throughout, I assume that ex-post renegotiation is impossible, so that non-compete simply prevent any and all spinouts. Empirically, this assumption seems to hold at least for R&D workers.¹¹

Optimal contract when $\mathcal{C} = (w, p)$ At each instant the firm offers a wage w which is coupled with a probability p that the worker is bound by a permanent non-compete. Both the firm and the worker learn whether the worker is bound by a non-compete at the moment the worker learns how to spin out. Unsurprisingly, the result will be that $p \in 0, 1$ except on a knife edge of the state space. But, at least the optimal contract can be solved pretty easily.

Consider a firm in state (q, m) at time t . I will use the fact that on a BGP, the firm's state is (\tilde{q}, m) and does not depend on t . Because of this, the endogenous growth rate g will appear in some of the equations below.¹²

The firm has endogenous value $V^I(\tilde{q}, m)$, which satisfies the HJB equation (omitting the arguments when clear)

$$rV^I = \pi(q) - g\tilde{q}V_{\tilde{q}}^I + \max_{\ell, \mathcal{C}} \left\{ \chi_I \ell \phi(\ell + L^E) \left[V^I((1 + \lambda)\tilde{q}, 0) - V^I \right] - \chi_E L^E \phi(\ell + L^E) V^I - (w + (1 - p)\nu V_m^I) \ell \right\}$$

where the contract \mathcal{C} satisfies the participation constraint

$$w + (1 - p)\nu V^E \geq \bar{w} \tag{1}$$

where \bar{w} is the wage the worker can earn in the production sector. At the firm's optimum, the participation constraint will be binding, since he can attract as many workers as he needs

¹¹Rauch 2015 shows how financial frictions on the part of the worker can prevent efficient ex-post renegotiation. As discussed in Shi 2017, executive contracts do often feature buyout menus. These could potentially mitigate some of the problem, but empirically we do not find buyout clauses in the typical non-compete contract for a scientist / tech worker. The interpretation is then that there is some unmodeled friction preventing this kind of contract from being written (to be decided.)

¹²Specifically, the firm will drift at rate $-g\tilde{q}$ in the \tilde{q} direction of the state space and this will be reflected in the HJBs

as long as he satisfies it. Substituting the IR constraint into the firm's HJB yields

$$rV^I = \pi(q) - g\tilde{q}V_{\tilde{q}}^I + \max_{\ell, p} \left\{ \chi_I \ell \phi(\ell + L^E) \left[V^I((1 + \lambda)\tilde{q}, 0) - V^I \right] \right. \\ \left. - \chi_E L^E \phi(\ell + L^E) V^I - \left(\bar{w} - (1 - p)\nu V^E + (1 - p)\nu V_m^I \right) \ell \right\}$$

The choice of contract reduces then to a choice of p . The firm wants to maximize the value it obtains from the R&D labor it hires. Increasing p to $p + \Delta$ increases the wage paid to the employee in proportion to the non-pecuniary benefits the worker accrues by working for the firm, $\nu V^E(\tilde{q}, m)\Delta$. At the same time, a higher p also increases the flow value of each unit of R&D labor in proportion to the non-pecuniary losses to the firm from the increased rate of spinouts, $\nu V_m^I(\tilde{q}, m)\Delta$. On a knife-edge, $V^E(\tilde{q}, m) = -V_m^I(\tilde{q}, m)$ and the firm chooses an interior optimum. If instead $|V_m^I| > V^E$, the firm sets $p = 1$ and if $|V_m^I| = V^E$ the firm sets $p = 0$.

Now suppose there is randomness in the realization of χ_E . Ex-post, the firm sometimes wants to allow competition, sometimes not. The only different is that the participation constraint changes to

$$w + (1 - p)\nu \mathbf{E}_{\chi_E}[V^E(\tilde{q}, m; \chi_E)] \geq \bar{w} \quad (2)$$

where the abuse of notation $V^E(\tilde{q}, m; \chi_E)$ denotes the value the entrant will obtain conditional on the realization χ_E of his R&D productivity. Proceeding exactly as before, conclude that the firm uses a non-compete as long as $\nu \mathbf{E}_{\chi_E}[V^E(\tilde{q}, m, \chi_E)] < |V_m^I|$.¹³

Since high quality entrants eventually drive out low quality entrants (they would do it immediately if they weren't size constrained), the quality increases over time. Hence, the firm is more likely to allow competition. Therefore, if the firm allows competition when $m = 0$, the firm will always allow competition. Thus, we only need to check whether the firm wants to allow competition initially, and then they will always allow competition. The one caveat, of course, is that \tilde{q} is drifting this entire time. If \tilde{q} affects whether the firm wants to allow competition, this could change things. Not sure how to think about this.

Optimal contract when $\mathcal{C} = (w, \tau)$ Now suppose that the contract consists of $C = (w, \tau)$ where τ is the Poisson rate at which non-competes expire. Now the firm's problem has three state variables, (\tilde{q}, m, n) , where n is the mass of agents who have the knowledge but are currently waiting for their non-competes to expire. One immediate problem that arises is

¹³In equilibrium, as m grows, the high χ_E entrants will drive out the low χ_E entrants. A prediction of the model, hence, is that the average quality of spinouts increases over time simply through selection

the fact that if the firm makes employees sign different contracts at different times, in order to forecast the evolution of entry the firm needs to keep track of the measure of each type of worker that is currently out there waiting for non-compete expiry. This immediately leads to the problem becoming completely intractable.

The HJB solved by the incumbent is then

$$rV^I = \pi(q) - g\tilde{q}V_{\tilde{q}}^I + \max_{\ell, \mathcal{C}} \left\{ \chi_I \ell \phi(\ell + L^E) [V^I((1 + \lambda)\tilde{q}, 0) - V^I] \right. \\ \left. - \chi_E L^E \phi(\ell + L^E) V^I - (w + \nu V_m^I) \ell \right\}$$

As before, the worker's participation constraint will be binding, so (omitting state space arguments, but leaving the τ argument for clarity)

$$w = \bar{w} - \nu V^{E,NC}(\tau)$$

where $V^{E,NC}$ is the value to the worker of the knowledge he learns from his employer, while still bound by the non-compete. Note the dependence on τ , which emerges from the HJB for $V^{E,NC}$. Let σ denote the (endogenous) aggregate rate of discoveries,

$$\sigma = (\chi_I L^I + \chi_E L^E) \phi(L^I + L^E)$$

and let $V^{E,F}$ denote the value of an entrant once his non-compete has expired. The HJB on $V^{E,NC}$ is

$$(r + \tau + \sigma)V^{E,NC} = \dot{\tilde{q}}V_{\tilde{q}}^{E,NC} + \tau V^{E,F}$$

As before, the participation constraint binds at the optimal contract. Substituting into the HJB, get

$$rV^I = \pi(q) + \dot{\tilde{q}}V_{\tilde{q}}^I + \dot{m}V_m^I \\ + \max_{\ell, \mathcal{C}} \left\{ \dot{n}(\tau)V_n^I + \chi_I \ell \phi(\ell + L^E) [V^I((1 + \lambda)\tilde{q}, 0) - V^I] \right. \\ \left. - \chi_E L^E \phi(\ell + L^E) V^I - (\bar{w} - \nu V^{E,NC}(\tau) + \nu V_n^I) \ell \right\}$$

Note that \dot{n} depends on τ . We have to be careful, though, because the firm cannot ex-post change the non-compete length of previously separated workers, so we need to take into account that it is only the marginal worker whose non-compete is being chosen. Analytically this is confusing, but intuitively, the firm considers whether the value it gains from paying

the worker less is worth the value it loses from the worker potentially competing one day in the future. As before, the firm-worker pair maximizes bilateral value. Hence the firm for sure uses a permanent non-compete as long as $\chi_E \leq \chi_I$, and pays the worker a flow wage of \bar{w} .

In fact, the firm-worker decide whether, in expectation, it pays bilaterally for the worker to compete. If not, the firm imposes a permanent non-compete. But I'm far from proving this, and it's not really a useful result either, because I want firms to make employees sign non-permanent non-competes.

3 Discussion: the role of non-competes

If the worker can commit not to compete "this instant"¹⁴ and ex-post¹⁵ renegotiation is frictionless, the ex-post bilaterally efficient outcome will obtain.¹⁶ Since the firm-worker pair's problem does not exhibit any time inconsistency, and preferences are linear, this implies that the ex-ante bilaterally efficient outcome will obtain as well. In addition, the distribution of surplus between the firm and the worker is unchanged. To be more concrete, ex-post the firm will offer the worker a wage high enough to buy his commitment not to compete that instant, forever after. In the previous period, the worker will accept a lower wage, knowing there is a possibility of a windfall in the next period. In expectation, agents have the same utility as though a non-compete were used.

Of course, in a non-enforcing state workers can only commit not to compete "this instant" if they continue to work for the original firm.¹⁷ If the firm is no longer the worker's best match, the ex-post bilateral value is lower relative than in the case where the worker could freely commit not to compete "this instant". Thus the ex-ante bilateral value is lower and the firm-worker pair would prefer to use an actual non-compete.

In addition, ex-post renegotiation is not frictionless. Asymmetric information can lead to adverse selection making the ex-post sale of the right to compete impossible. Financial frictions can prevent workers from buying out their non-competes (Rauch 2015). This makes non-competes matter much more for ex-post outcomes.¹⁸ If ex-post renegotiation is

¹⁴Imagine a costless version of committing not to compete by continuing to show up to work at the original firm.

¹⁵Once the worker learns the technology and is considering spinning out.

¹⁶In fact, if we allow e.g. households to negotiate with the incumbent-worker pair, Coase's theorem implies that the outcome will be ex-post *socially* efficient. This is an unrealistic setting, but it is a useful conceptual benchmark to keep in mind.

¹⁷Note that this is not possible in my model currently, as I have assumed entrepreneurship requires no resources

¹⁸Also, as is typical, differing preferences - between firm and worker - regarding consumption smoothing - across states or across time - can also make one or the other contract more bilaterally efficient. I abstract

completely shut down, non-competes prevent all spinouts and non-enforcement allows all spinouts. The firm and worker then must decide if eliminating spinouts maximizes their bilateral value.

Suppose $G(\cdot)$ is concentrated on $\chi_E = \chi_I$. In this case, it is ex-post bilaterally optimal for spinouts not to occur, since all they do is reduce monopoly rents. Still, a spinout can be socially optimal because the incumbent dedicates too few resources to R&D (he does not internalize the effect of R&D on consumer surplus). Non-competes give the incumbent the ability to stop these ex-post efficient spinouts, so non-competes are socially suboptimal ex-post. However, enforcement may still be optimal for the same reason that patents are granted: incentivizing the creation of intellectual property.

If instead $\chi_E > \chi_I$ by a sufficient amount, it is ex-post bilaterally (and socially) optimal for such a spinout to occur. If the entire support of $G(\cdot)$ has this property, non-competes will not be used in equilibrium and there will be no difference between enforcement and enforcement regimes.

Similarly, if $\chi_E < \chi_I$ by a sufficient amount, it is ex-post bilaterally (and socially) optimal for a spinout not to occur. That it is bilaterally optimal for it not to occur is clear from the above. The social suboptimality stems from the net effect of (1) more resources allocated to R&D by entrants, which leads to more R&D (2) misallocation of resources from more productive incumbents to entrants. In this case, non-competes will be used if enforceable, and this will be socially optimal.

What happens if the productivity of spinouts can fall in any of the above-described ranges? The incumbent then wants to use a non-compete to prevent entry by relatively unproductive spinouts (note: the maximum "unproductive" productivity will be some $\chi_E > \chi_I$), but will be willing to accept a payment ex-post to void the non-compete in order to screen the high-productivity spinouts.

As discussed above, without costs of renegotiation the outcome is ex-post bilaterally optimal. We are left with the extent to which this bilaterally optimal contract is socially efficient. Intuitively, since the incumbent-worker pair act as a monopolist, one might expect that unfettered application of non-competes would lead to inefficiently low entry, thus optimal policy might be expected to restrain to some degree the usage of non-competes.¹⁹

In reality, however, there are significant renegotiation costs. This means that whether or not a non-compete is signed affects greatly the ex-post outcome.

from these details for now, but they may be empirically relevant

¹⁹An outright *ban* on non-competes could still be welfare-reducing.