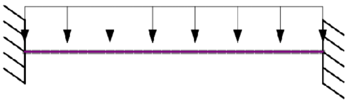
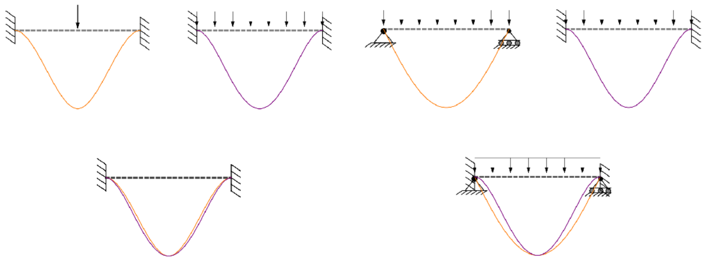
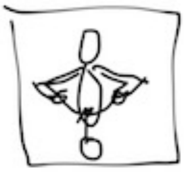
Scripts



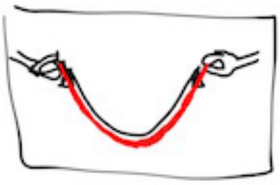
We look at another simple but statically indeterminate configuration for a uniformly loaded beam. Namely, a fixed-fixed beam with a uniform load. This means there are fixed ends that do not allow any displacements or rotations at both ends and we apply a uniform load over the whole beam.



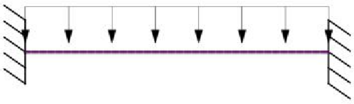
Now the deformed shape of a beam loaded with a central point force and a beam loaded with a uniform load are again practically visually indistinguishable if our aim is to sketch the shapes as we can see in the overlay comparison figure ... which we can see more clearly if we zoom in ... However, we do notice that there is more of a difference between these two cases than what we encountered with the simply-supported case. To see what a visually significant difference might look like, we look at the uniformly loaded beam between the case where the supports are simply supported as compared to the case where the beam has fixed supports at both ends. Now, there is a change in curvature near the ends that clearly makes the two shapes look different.



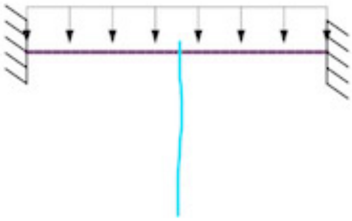
Therefore, I will demonstrate the behavior of a uniformly loaded straw with the visually near equivalent, namely the centrally loaded beam. Of course, I do this because it is easier to apply a single load than a distributed one. This straw shows the deformed shape. Notice that the shape has a positive curvature in the middle while it has a negative curvature near the ends. The negative curvatures occur because I am applying a resisting moment at each fixed support in order to change the slopes from going down like in the simply supported case to being horizontal near the ends.



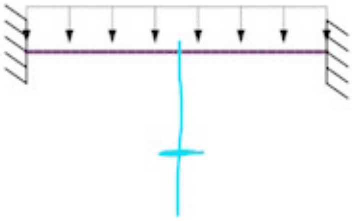
Again, if we compare the theoretical deformation with the deformation of the straw, we get a good match.



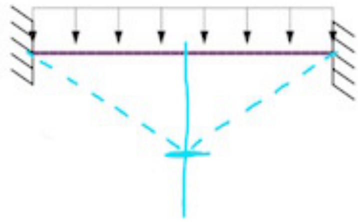
Now we consider how to sketch this case by hand.



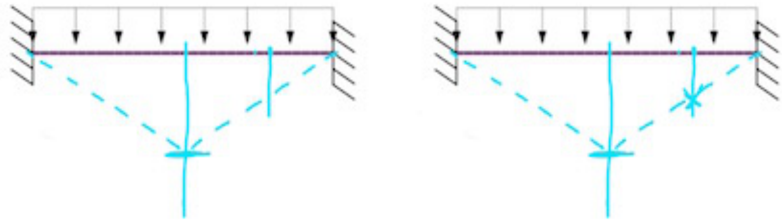
We start by locating the midpoint and extend a guide line down.



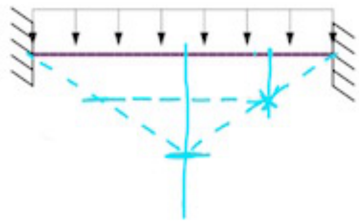
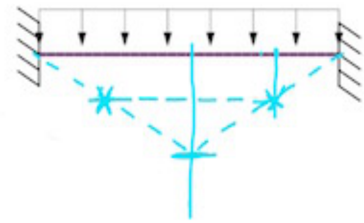
Along this line, we choose a level for the maximum displacement and draw a short horizontal guide line.



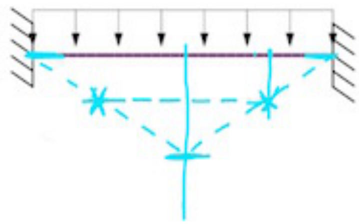
We next draw guide-lines that join the point of maximum displacement with each end.



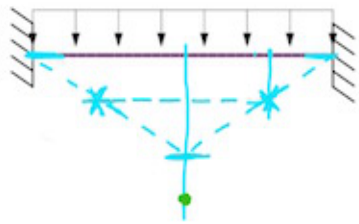
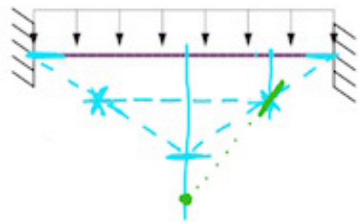
At about one fifth (the precise value is about 21.1% the length from one end), we extend a vertical line down to meet our diagonal guide line. The point of intersection is approximately the location of an inflection point. Actually, the location of inflection points are about 5% farther down along those vertical guides. So, let’s mark the approximate location of the inflection point.

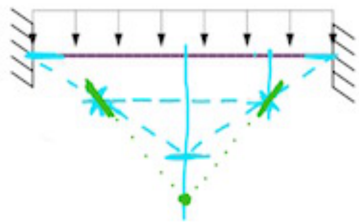
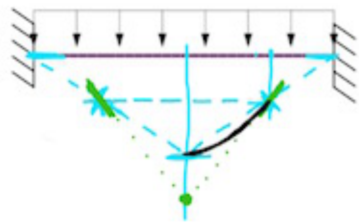



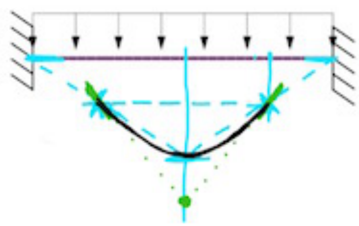
Now, to get a more symmetric figure, extend a horizontal guide line to intersect the diagonal guide line at the inflection point on the other side.



Next, draw two horizontal stubs at each end support. The deformed beam will be tangent to these stubs at both fixed ends.

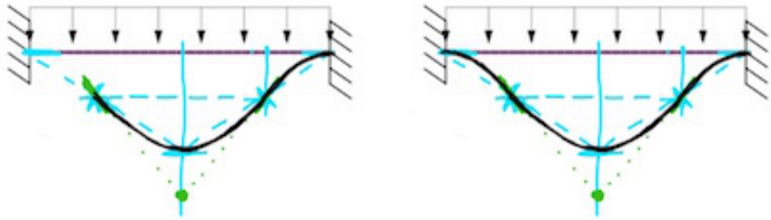



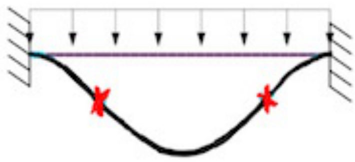


Relative to the guide line joining the two inflection points, we have exactly the shape of a simply supported beam because the bending moment is zero at the inflection points and it is also zero at the hinge or roller supports.

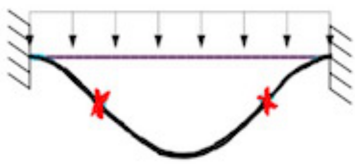
Using the directions described for the simply supported case, draw the deformed shape of the beam between the two inflection points. One small detail is to extend the slopes at the inflection points because these will guide us on how to sketch the end pieces. These slope extensions are shown in green in the figure.



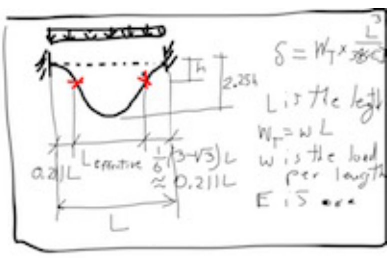
At each end, draw a smooth curve tangent to the horizontal at the support and tangent to the slopes identified at the inflection points while sketching the middle piece.



Finally, we remove or erase the extraneous lines, and further smooth out our curve to get our deformed shape.
Some or all of these steps may be omitted once we get a sufficient familiarity with sketching this shape. But this step-by-step approach makes us aware of all the salient details of the beam's deformation. One important insight that we will use is that the middle portion between inflection points is exactly like a simply supported beam in regards to its relative deformation and thus also its bending moment diagram.

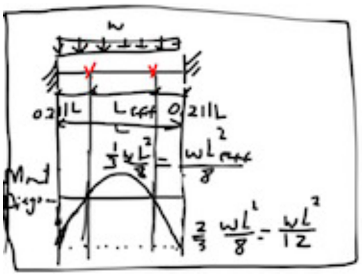


And now we compare with the theoretical shape, and, we again get a good match, this time between theory and sketch.



Now let's go back to the beam and look at some quantitative results. Looking at the deformed shape, the maximum deflection is at the middle, which of course it must be due to symmetry. That maximum deflection has a magnitude which is exactly 2.25 or '2 and a quarter' times the deflection at the inflection points. In terms of the load, the maximum displacement is 1 times the total load times $\frac{L^3}{384\,EI}$. This is 5 times smaller than that of the simply supported case. By comparison with a centrally loaded point force case, the displacement is half as much.

Finally, the inflection points are at a distance of about 0.211 times ($\frac{1}{6}\left(3-\sqrt{3}\right)$ L $\approx$ 0.211325 L) the length from each support. For symmetric support conditions and a uniform load, this is the farthest that the inflection points move away from the supports. Of course, the inflection points can be farther away with unsymmetric support conditions such as the fixed-hinge support case to be discussed later.



The moment diagram for this basic case is simple. It is a parabola having a maximum positive moment at the middle with a value of $\frac{1}{3}\frac{wL^2}{8}$ and maximum negative moment at each end with values of $\frac{2}{3}\frac{wL^2}{8}$. So, the largest negative bending moments are twice those of the largest positive bending moments. Furthermore, the maximum negative and positive bending moments are two-thirds and one-thirds those of the simply supported case. The value of the negative moments at the ends, namely $\frac{2}{3}\frac{wL^2}{8} = \frac{wL^2}{12}$ may be familiar to those who know the moment distribution method or the slope deflection method because these are known as the fixed end moments for a uniformly loaded beam.

It is interesting to note that we can obtain the bending moment diagram by knowing the location of the inflection points. For example, the maximum positive bending moment must equal $\frac{wL_{\text{effective}}^2}{8}$ where $L_{\text{effective}}$ is the span between the inflection points. This is the length minus about 21.1% the length from each side which is about 0.578 the length of the beam. If we use the exact location of the inflection point we get ‘one over the square root of 3’ times the length of the beam. Using that value for $L_{\text{effective}}$ we get the maximum positive moment $\frac{wL_{\text{effective}}^2}{8}$ equals to $\frac{1}{3}\frac{wL^2}{8}$. Using statics, we can also obtain the maximum negative moments.