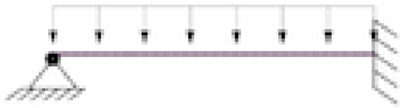
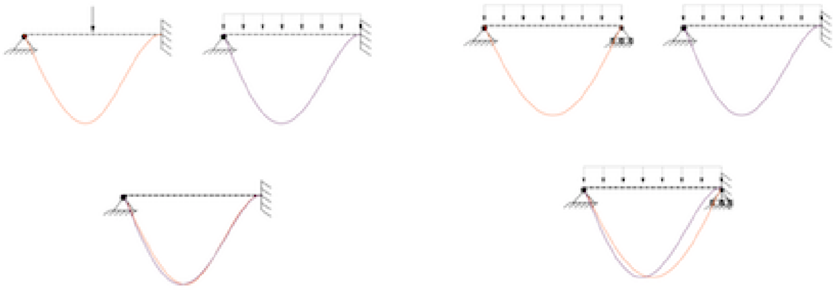
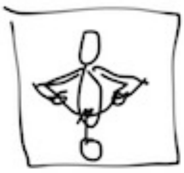
Scripts



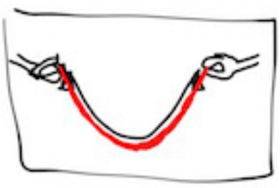
We look at a final basic configuration for a uniformly loaded beam. Namely, a hinge-fixed beam with a uniform load. This means there is a hinge on the left side and a fixed end on the right side and we apply a uniform load over the whole beam.



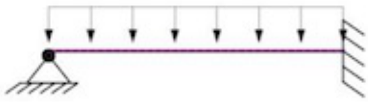
Now the deformed shape of a beam loaded with a central point force and a beam loaded with a uniform load are again practically visually indistinguishable if our aim is to sketch the shapes as we can see in the overlay comparison figure ... which we can see more clearly if we zoom in ... . To see what a visually significant difference might look like, we look at the uniformly loaded beam between the case where the supports are simply supported as compared to the case where the beam has hinge-fixed supports. Now, there is a significant change in the shapes that are clearly distinguishable.



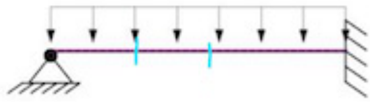
Therefore, I will demonstrate the behavior of a uniformly loaded straw with the visually near equivalent, namely the centrally loaded beam. Of course, I do this because it is easier to apply a single load than a distributed one. This straw shows the deformed shape. Notice that the shape has a positive curvature in the middle while it has a negative curvature near the right end. The negative curvature at the right end occurs because I am applying a resisting moment at that end in order to change the slope from going down to being horizontal.

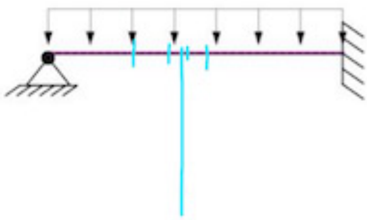


Again, if we compare the theoretical deformation with the deformation of the straw, we get a good match.

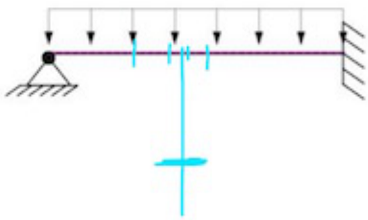


Now we consider how to sketch this case by hand.

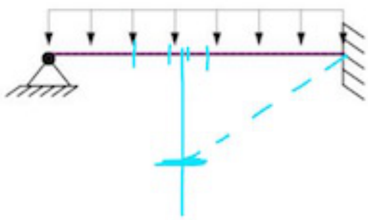




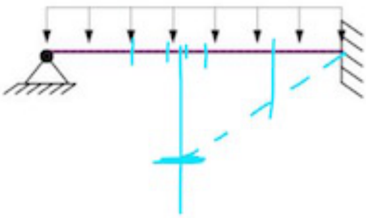
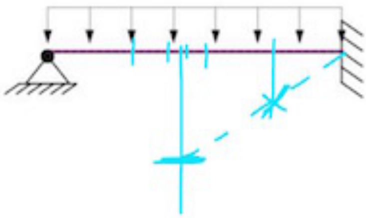
We start by locating the midpoint and then we have to move to the side of the hinge about 7.8% of the length of the beam. This could be approximated by moving half a quarter of the length which we locate by visually identifying half the half the half the half of the length and extend a line down. The maximum displacement will occur along this line.



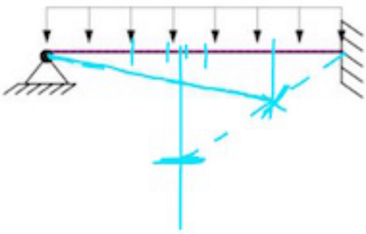
So along this line, we choose a level for the maximum displacement and draw a short horizontal guide line.



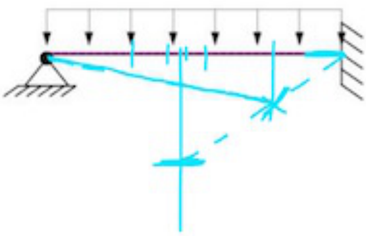
We next draw a diagonal guide-line that joins the point of maximum displacement with the fixed end.

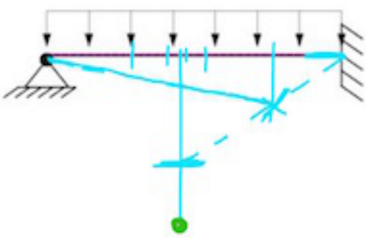
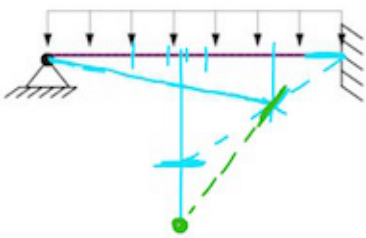
At exactly a quarter the length from the fixed end, we should extend a vertical line down to meet our diagonal guide line. In a sketch, this will be approximate. The point of intersection is approximately the location of the inflection point. Actually, the location of the inflection point is about 4% farther down along this vertical guide. So, let’s mark the approximate location of the inflection point.

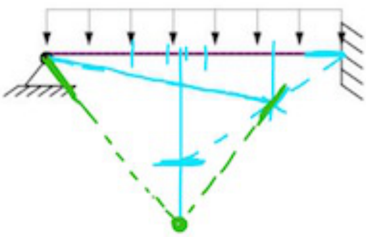
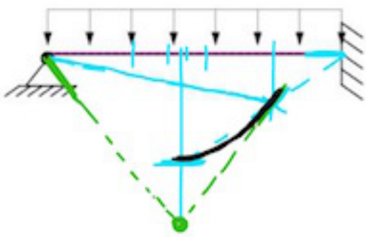


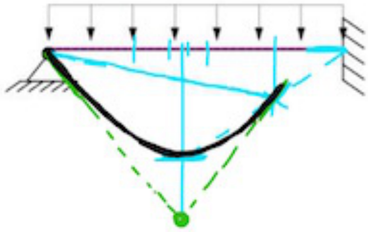
Now, draw a datum line joining the inflection point to the hinge support.



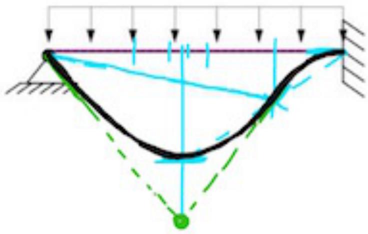
Next, draw a horizontal stub at the fixed end support. The deformed beam will be tangent to this stubs at the fixed end.

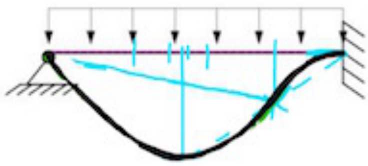
 



Relative to the datum line joining the inflection point with the hinge support, we have the shape of a simply supported beam but rotated according to that inclination. This means that it has the parabolic shape of the simply supported beam plus the inclined straight line shift of the datum line.
The steps for drawing this part of the deformed shape are the same as that of the simply supported beam but relative to an inclined datum line. So, we sketch the displaced beam between the inflection point and hinge according to the directions of the simply supported beam. Note that the maximum displacement is not at the center of the inclined datum but at the point identified by our first two steps and this is due to the rotational shift. However, the location of the maximum deflection and the middle of this datum line is only about 4.7% the length of the beam.

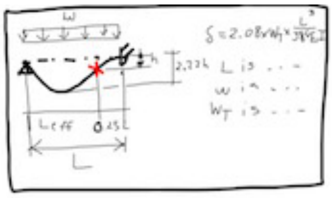


At the fixed support, draw a smooth curve tangent to the horizontal and tangent to the slope identified at the inflection points while sketching the right piece.

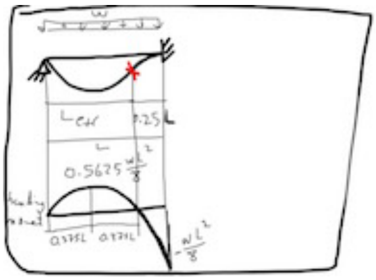


Finally, we remove or erase the extraneous lines, and further smooth out our curve to get our deformed shape.
Some or all of these steps may be omitted once we get a sufficient familiarity with sketching this shape. But this step-by-step approach makes us aware of all the salient details of the beam's deformation. One important insight that we will use is that the portion between the inflection point and the hinge support is exactly like a simply supported beam in regards to its relative deformation and thus also its bending moment diagram.

And now we compare with the theoretical shape, and, we again get a good match, this time between theory and sketch.



Now let's go back to the beam and look at some quantitative results. Looking at the deformed shape, the maximum deflection is at 7.8% (exactly $\frac{1}{16}\left(7-\sqrt{33}\right)$) times the length away from the center in the direction that brings it closer to the hinge end. That maximum deflection has a magnitude which is at about 2.22 times the deflection at the inflection point. In terms of the load, the maximum displacement is about 2.08 times the total load times $\frac{L^3}{384\,EI}$. This is about 2.4 times smaller than that of the simply supported case. By comparison with a centrally loaded point force case, the maximum displacement for this uniformly loaded case is about 60% (to be more precise 0.581 times) that of the centrally loaded point force with the same support conditions.
Finally, the inflection point is at a distance of exactly a quarter or 0.25 times the length from the fixed support. For any uniformly loaded beam connected to a passive structure with no side-sway, this is the farthest distance that an inflection point can be from a support.



The moment diagram for this basic case is again a parabola because the load is uniform. It is a parabola that has a maximum positive moment at the midpoint between the inflection point and the left hinge with a value of $\frac{9}{16}$ or $0.5625\ \frac{wL^2}{8}$. The maximum negative moment occurs at the fixed support with a value of exactly $\frac{wL^2}{8}$. We note that this negative moment is the largest negative moment that a uniformly loaded beam can have if it is only connected to passive structures at each end with no side-sway. So, the maximum possible positive and maximum possible negative bending moments for the uniformly loaded beam attached to passive structures and having no side-sway are both equal to $\frac{wL^2}{8}$.

It is interesting to note that the location of maximum positive bending moment and that of maximum displacement do not coincide. They generally don't. They will coincide when the support conditions are equivalent on the two sides. They will be equivalent when the corresponding rotary stiffness factors are equal.

Finally, as in the fixed-fixed case, we can obtain the bending moment diagram by knowing the location of the inflection point. For example, the maximum positive bending moment must equal

$\frac{w L_{\text{effective}}^2}{8}$ where $L_{\text{effective}}$ is the span between the inflection point and the hinge support. This length equals three quarters the length of the beam and squaring this gives us 9/16 the length square which gives the factor multiplying $\frac{w L^2}{8}$. Using statics, we can also obtain the maximum negative moments.