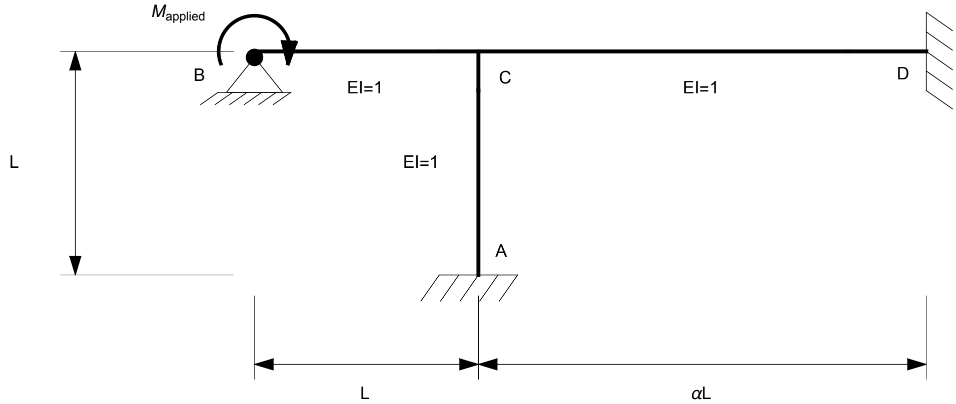
Problem 1



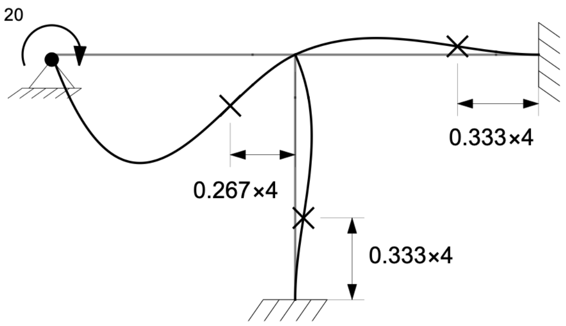
a) For $\alpha = 1$, sketch the approximate deflected shape and show the location of all inflection points.

b) Given $M_{\text{applied}} = 20$ and $\alpha = 1$, sketch the (exact) moment diagram.

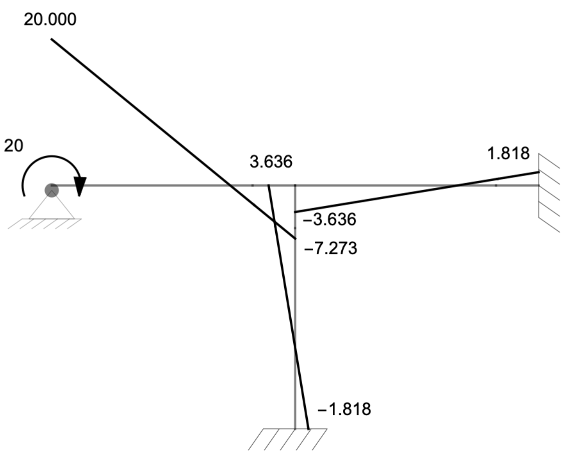
c) For any value of M_{applied} and any value of α, determine $M_{CA}/M_{\text{applied}}$ in terms of α and plot it.

Solution

a) For $\alpha = 1$, sketch the approximate deflected shape.



b) Given $M_{\text{applied}} = 20$ and $\alpha = 1$, sketch the (exact) moment diagram.



c) For any value of M_{applied} and any value of α, determine $M_{CA}/M_{\text{applied}}$ in terms of α and plot it.

$$M_{CB} = \frac{2k}{3+4k} \times M_B; \qquad k = \frac{4\left(\frac{EI}{L}\right)_{AC} + 4\left(\frac{EI}{L}\right)_{CD}}{4\left(\frac{EI}{L}\right)_{BC}} = 1 + \frac{1}{\alpha}$$

Therefore (after simplifying):

$$M_{BC} = \frac{2(1+\alpha)}{4+7\alpha} M_{applied}$$

The distribution factor for member CA is:

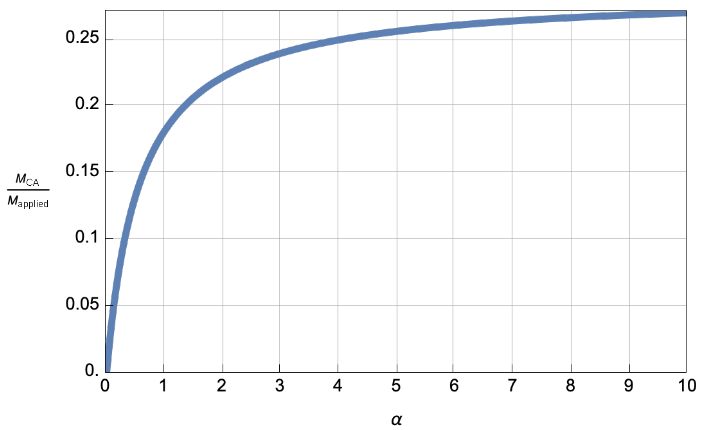
$$DF_{CA} = \frac{4\left(\frac{EI}{L}\right)_{CA}}{4\left(\frac{EI}{L}\right)_{CA} + 4\left(\frac{EI}{L}\right)_{CD}} = \frac{1}{1+\frac{1}{\alpha}} = \frac{\alpha}{1+\alpha}$$

Therefore, M_{CA} is given by:

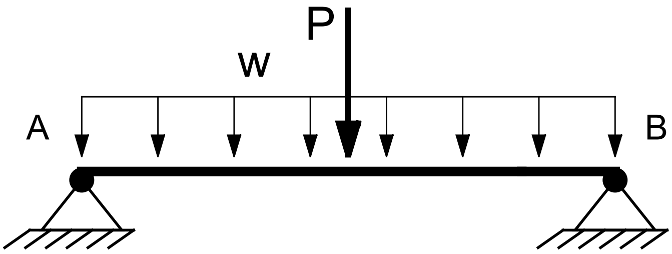
$$M_{CA} = DF_{CA} \times M_{BC} = \frac{\alpha}{1+\alpha} \times \frac{2(1+\alpha)}{4+7\alpha} M_{applied} = \frac{2\alpha}{4+7\alpha} M_{applied}$$

$$\Rightarrow \frac{M_{CA}}{M_{applied}} = \frac{2\alpha}{4+7\alpha}$$

Plot:



Problem 2



For a simple bridge with self-weight and a central force P applied, estimate the minimum total weight of the bridge in terms of:

σ_Y yield stress of the beam material
h height of the beam (or truss)
L length of the beam
γ specific gravity of the beam material
P applied central load

Estimate the minimum weight of the simple bridge for the following parameters:
σ_Y = 200 MPa, h = 1 m, L = 30 m, γ = 7.8, P = 2 × 10^3 × g N (g ≈ 9.8 m/s^2; so P is due to 2000 kg)

Solution

We consider the optimal section which has a moment of inertia:

$$I = \frac{A h^2}{4}$$

The failure is assumed to occur when yielding starts (true for an I-beam which has the optimal moment of inertia):

$$\sigma_Y = \frac{M_{\max} h}{2 I} = \frac{M_{\max} h}{2 A h^2/4} = 2 \frac{M_{\max}}{A h}$$

$M_{\max}$ for the combined loads are:

$$M_{\max} = \frac{\mathrm{PL}}{4} + \frac{\mathrm{wL}^2}{8}$$

The weight of the beam is given by:

W = w L

Therefore, $M_{\max}$ is given by:

$$M_{\max} = \frac{\mathrm{PL}}{4}\left(1 + \frac{\hat{W}}{2}\right) \quad \text{where } \hat{W} = W/P \text{ (What we're trying to calculate)}$$

Putting the failure with $M_{\max}$ together, we get:

$$\sigma_Y = \frac{1}{2} \frac{P L}{A h}\left(1 + \frac{\hat{W}}{2}\right)$$

If we multiply and divide by $\gamma \rho_{\text{water}} g L$ where $\rho_{\text{beam}} = \gamma \rho_{\text{water}}$, and noting that $W = \rho_{\text{beam}} A L g$, we get:

$$\sigma_Y = \frac{1}{2} \frac{\gamma \rho_{\text{water}} g L P L}{\gamma \rho_{\text{water}} g L A h}\left(1 + \frac{\hat{W}}{2}\right) = \frac{1}{2} \frac{\gamma \rho_{\text{water}} g P L^2}{W h}\left(1 + \frac{\hat{W}}{2}\right)$$

Define:

$$L_0 = 2\sqrt{\frac{\sigma_Y h}{\gamma \rho_{\text{water}} g}}$$

Note: We can show that L_0 is the maximum length that a beam can be built without failing under its own weight

Then:

$$\hat{W} = \frac{2}{\left(\frac{L_0}{L}\right)^2 - 1}$$

Given:

σ_Y = 200 MPa, h = 1 m, L = 30 m, γ = 7.8, P = 2 × 10^3 × g N (g ≈ 9.8 m/s^2; so P is due to 2000 kg)

We get:

$$W_{\text{minimum}} = 377 \text{ kg}$$