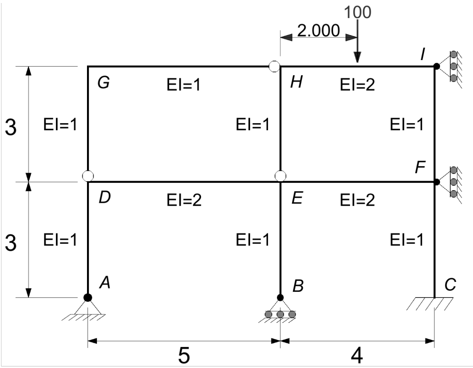
Exam solution

Pr 1

1. (70 points)



For the structure shown in the figure on the left:

a) Sketch the deformed shape and indicate with a big × the approximate location of ALL inflection points. You must also calculate and LABEL on the figure the approximate location of inflection points in members HI, IF and FC ONLY. 30 points

b) Sketch the moment diagram of members HI and IF ONLY. Show the magnitudes of the maximum internal moments (if any) and the end moments in those members. 40 points

Note:

- There is an internal hinge in the beam slightly to the left of point H as shown.
- There is an internal hinge in the bottom of column HE as shown.
- There is an internal hinge in the bottom of column GD as shown.
- Supports A is a hinge, B is a roller while support C is fixed as shown.
- Use approximate analysis but the location of the inflection points and moments must have errors less than 20% for a full grade.
- Use SEPARATE figures for the deformed shape and moment diagram
- Show the main calculations, especially for member HI
- Values of EI are shown on the figure

Basic correction approach:

Solution:

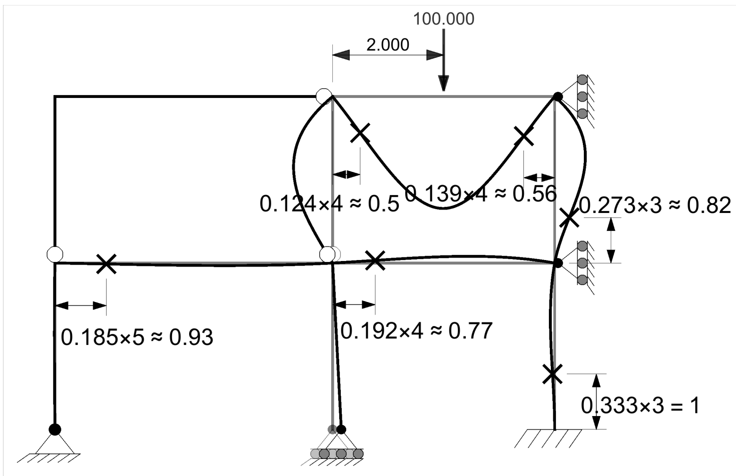
1- Each item has set points

a) and b)

2 - Errors remove grades up to the maximum points for item

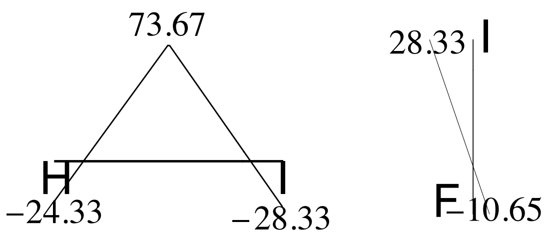
3 - Propagated errors penalty limited to immediate results except when specific rule applies

The deflected shapes of all members are shown in the figure below (only 2 inflection points).

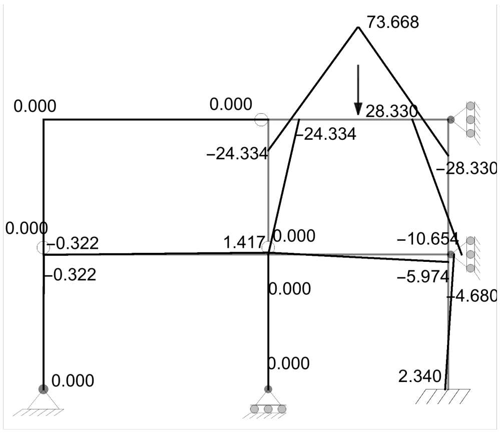


DEFLECTED SHAPE 30 pts
- loaded beam (max -10 pts): - 10 pts if wrong (-3 pts per inflection point error)
- connection not 90 but should be (-3 pts) up to maximum - 6 pts (first two penalty)
- moving columns up, down, side (max - 12 pts): - 3 pts if wrong (-4 pts also inflection point error)
- Members GH or GD moving; - 10 points if either or both.
- lower beams floor (max -6): - 4 pts if wrong (-2 pts if inflection point error)
- side columns (max - 6): - 3 pts / member up to - 12 pts total
- support columns (max - 3): -2 first error; - 3 pts if column on roller not straight; max -3
- If each member drawn separately then -10 points on deflected shape

The moment diagrams of members EF and DE are shown in the figures below (exact).



For reference (NOT REQUIRED), the moment diagram for whole structure is shown below:



MOMENT DIAGRAM: 40 pts
Member HI (25 points):
Shape 10 pts
3 values for 15 pts:
- 5 pts at H
- 5 pts at max positive
- 5 pts at I

Member IF (15 points):
Shape: 5 pts
Value at I: 4 pts
Value at F: 6 pts

<u>Loaded member HI</u>: (inflection points and moment diagram)

First determine k_L and k_R:

$$k_L \approx \left(0 \text{ for HG } + 3\left(\frac{EI}{L}\right)_{HE}\right) \Big/ \left(4\left(\frac{EI}{L}\right)_{HI}\right) \approx \left(3\times\frac{1}{3}\right)\Big/\left(4\times\frac{2}{4}\right) = 0.5$$

$$k_R \approx \left(4\left(\frac{EI}{L}\right)_{HE}\right) \Big/ \left(4\left(\frac{EI}{L}\right)_{HI}\right) \approx \left(4\times\frac{1}{3}\right)\Big/\left(4\times\frac{2}{4}\right) = 0.67$$

Note:
HG is zero because of the internal hinge at H and
HE has reduced stiffness because of the internal hinge at E

ADDITIONAL RULES
* If sign of moment diagram flipped (shape and values correct) then -1 pts
* If inflection point value wrong, penalty taken here -2 pts and not in sketching
* Error propagated from main member end moments is NOT penalized if done consistently

Now we have the following:

$$d_{IL} \approx \left(\frac{d_{FL}}{L+d_{FL}}\right)\left(\frac{3\,k_L}{2+4\,k_L}\right)\times L_{HI} \approx \left(\frac{2}{4+2}\right)\left(\frac{3\times0.5}{2+4\times0.5}\right)\times 4 \approx 0.125\times 4 \approx 0.5 \quad \text{(exact is 0.496)}$$

$$d_{IR} \approx \left(\frac{d_{FR}}{L+d_{FR}}\right)\left(\frac{3\,k_R}{2+4\,k_R}\right)\times L_{HI} \approx \left(\frac{2}{4+2}\right)\left(\frac{3\times0.67}{2+4\times0.67}\right)\times 4 \approx 0.143\times 4 \approx 0.57 \quad \text{(exact is 0.556)}$$

$$a_{eff} = d_{FL} - d_{IL} = 2 - 0.5 = 1.5$$
$$b_{eff} = d_{FR} - d_{IR} = 2 - 0.57 = 1.43$$
$$L_{eff} = a_{eff} + b_{eff} = 1.5 + 1.43 = 2.93$$

The maximum positive moment and end moments are then:

$$M_{positive} = \frac{P\,L_{eff}}{4} \approx \frac{100\times2.93}{4} \approx 73.25 \quad \text{(exact result is} \sim 73.67 \text{ which implies} \approx \text{- 0.6\% error)}$$
$$M_{Right} = P\,d_{IR}\,a_{eff} \,/\, L_{eff} = 100\times0.57\times1.5 \,/\, 2.93 \approx 29.2 \quad \text{(exact is 28.33)}$$
$$M_{Left} = P\,d_{IL}\,b_{eff} \,/\, L_{eff} = 100\times0.5\times1.43 \,/\, 2.93 \approx 24.4 \quad \text{(exact is} \sim 24.3)$$

<u>Member IF</u>: (inflection point and moment diagram)

The moment at I in member IF is equal to the moment at I in member HI because it is the only connected member.

The rotary stiffness factor at F is:

$$k = \left(4\times\left(\frac{EI}{L}\right)_{FE} + 4\times\left(\frac{EI}{L}\right)_{FC}\right)\Big/\left(4\times\left(\frac{EI}{L}\right)_{FI}\right) = \left(4\times\frac{2}{4} + 4\times\frac{1}{3}\right)\Big/\left(4\times\frac{1}{3}\right) \approx 2.5$$

This implies the inflection point from point F is at a distance of:

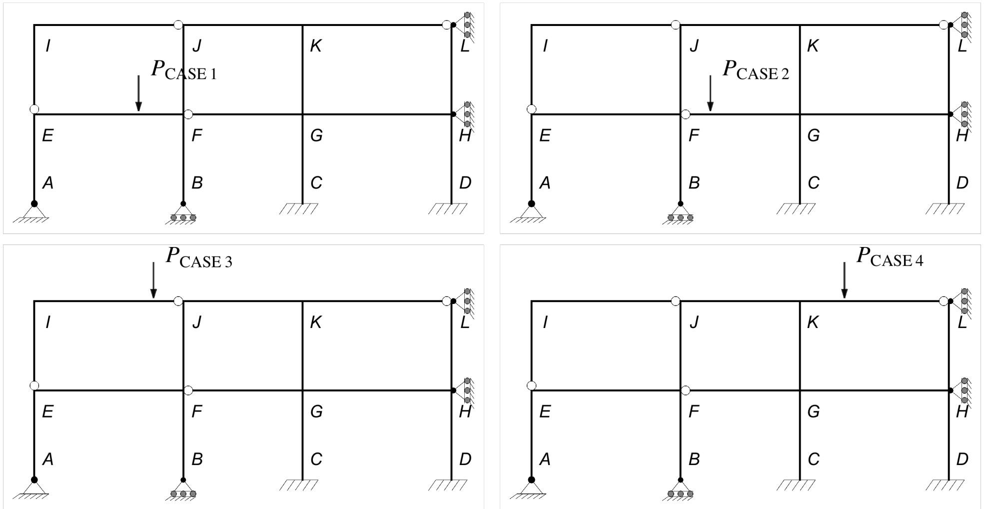
$$d_I = \frac{2\,k}{3+6\,k}\times L_{IF} = \frac{2\times2.5}{3+6\times2.5}\times 3 \approx 0.28\times 3 \approx 0.84 \quad \text{(exact is about 0.82)}$$

The bending moment at F has a value of:

$$M_{FI} = \frac{2\,k}{3+4\,k}\times M_{ED} = \frac{2\times2.5}{3+4\times2.5}\times 29.2 \approx 0.385\times 29.2 \approx 11.2 \quad \text{(exact is 10.65)}$$

Pr 2

2. (30 points)



Consider the frame shown and the four loading cases 1, 2, 3 and 4 shown.
For each of the three loading cases:

20 points i) Specify the SIGN (positive, negative or zero) of the moment at the MIDPOINT of member JK and briefly explain your choice (you may sketch rough figures)

10 points ii) For each case, specify whether the bending moment at the MIDPOINT of member JK is closer to 0, 1/4, 1/10 or 1/100 the maximum moment on the loaded member. Hint: Each case corresponds to a different one of those factors.

Note :
- The convention for positive bending moment is for tension to be at the bottom of the beam.
- For part (ii), you only need to match the loading case with the factor (no explanations needed).

Solution:

i) 1 + 1/10, 2 - 1/100, 3 0 0, 4 - 1/4

Case 1: Positive moment. Beam EF goes down, node F rotates counterclockwise, node J rotates clockwise because of member FJ and the rotation of node J. This implies that beam JK will move down producing a positive moment in the middle of it.
Case 2: Negative moment. Beam FG goes down, node G rotates counterclockwise, node K rotates clockwise because of member GK and the rotation of node G. This implies that beam JK will move up producing a negative moment in the middle of it.
Case 3: Zero moment. Beam IJ cannot cause rotation of node J because of the internal hinge at J and it cannot indirectly cause rotation to members after node E because of the internal hinge at E.
Case 4: Negative moment. Beam KL goes down, node K rotates clockwise and this makes member JK go up producing a negative moment in the middle of it.

ii)

Case 1: 1/10 (the load is close to F where it is more effective at producing a negative moment. Because of the internal hinge at F, the negative moment is distributed all of it to member FJ. That moment is transmitted to J which distributes it all to member JK. (moment at J is about 0.3 that at F and in the middle of JK is about half that at J so that gives a total reduction factor of about 1/10)
Case 2: 1/100 (the load is far from edge G and also the moment at G has to be distributed to 3 members, then transmitted to K, then distributed again to 2 members. Finally, the moment at the middle is about half the edge. This means we multiply about $1/2 \times 1/3 \times 0.3 \times 1/2 \times 1/2 \approx 1.2\%$)
Case 3: 0 (beam IJ and column IE are isolated from the rest of the frame because of the internal hinges. This means exactly 0 moment)
Case 4: 1/4 (the load is nearest in this case and has the most influence. We only need to distribute once to two members and take about half the edge value)

Pr ii) If case 2 and 4 interchanged then -2 points (close)
If case 1 and 2 interchanged then -4 points