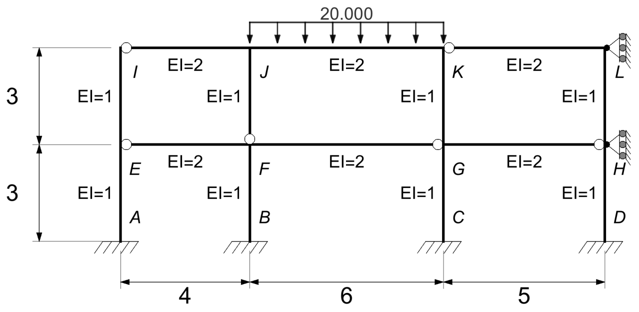
1. (70 points)



For the structure shown in the figure on the left:

a) Sketch the deformed shape and indicate numerically the approximate location of all inflection points.

b) Sketch the moment diagram of members JK and JI ONLY. Show the magnitudes of the maximum internal moments (if any) and the end moments in those members.

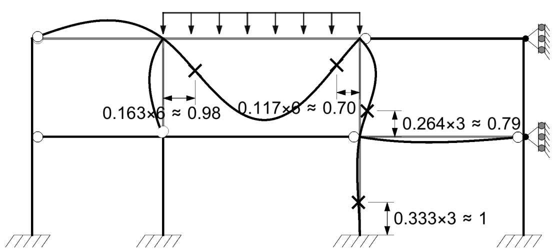
Note:

- There is an <u>internal</u> <u>hinge</u> in the beams slightly to the right of point I, K and E as shown.
- There is an <u>internal</u> <u>hinge</u> in the beams slightly to the left of point G and H as shown.
- There are <u>internal</u> <u>hinges</u> in the bottom of column FJ as shown.
- Use approximate analysis but the location of the inflection points and moments must have errors less than 20% for a full grade.
- Use SEPARATE figures for the deformed shape and moment diagram
- Show the main calculations, especially for member JK
- Values of EI are shown on the figure

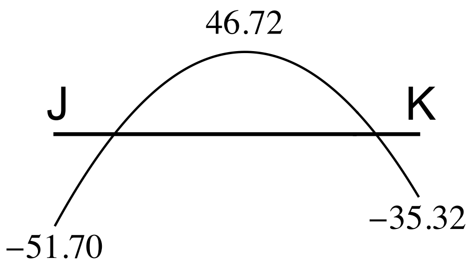
Solution:

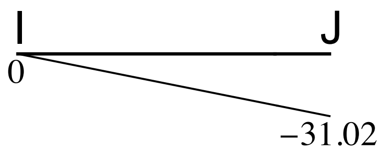
a) and b)

The deflected shapes of all members are shown in the figures below.



The moment diagrams of members JK and JI are shown in the figures below.





<u>Loaded member JK</u>: (inflection points and moment diagram)

First determine k_L and k_R:

$$k_L \approx \left(0.75 \times \left(\frac{EI}{L}\right)_{JI} + 0.75 \times \left(\frac{EI}{L}\right)_{JF}\right) \Big/ \left(\frac{EI}{L}\right)_{JK} \approx \left(0.75 \times \frac{2}{4} + 0.75 \times \frac{1}{3}\right) \Big/ \frac{2}{6} = 1.875$$

$$k_R \approx \left(\left(\frac{EI}{L}\right)_{KG}\right) \Big/ \left(\frac{EI}{L}\right)_{JK} \approx \left(\frac{1}{3}\right) \Big/ \frac{2}{6} = 1$$

Note:
JI and JF have reduced stiffness because there is an internal hinge at I and F

Now we have the following:

$$d_{IL} \approx \frac{0.92\,k_L}{3 + 4\,k_L} \times L_{JK} \approx \frac{0.92 \times 1.875}{3 + 4 \times 1.875} \times 6 \approx 0.164 \times 6 \approx 0.98 \quad \text{(exact is 0.98)}$$

$$d_{\mathrm{IR}} \approx \frac{0.92\,k_R}{3+4\,k_R}\times L_{\mathrm{JK}} \approx \frac{0.92\times 1}{3+4\times 1}\times 6 \approx 0.131\times 6 \approx 0.79 \quad \text{(exact is 0.70)}$$

$$L_{\mathrm{eff}} = \mathrm{L} - d_{\mathrm{IL}} - d_{\mathrm{IR}} \approx 6 - 0.98 - 0.79 \approx 4.23 \quad \text{(exact is 4.32)}$$

The maximum positive moment and end moments are then:

$$M_{\mathrm{positive}} = \frac{w\,L_{\mathrm{eff}}^2}{8} \approx \frac{20\times 4.23^2}{8} \approx 44.73 \quad \text{(exact result is} \sim 46.72 \text{ which implies} \approx 4.3\% \text{ error)}$$

$$M_{\mathrm{Left}} = \frac{w}{2}\times d_{\mathrm{IL}}\times(d_{\mathrm{IL}}+L_{\mathrm{eff}}) \approx \frac{20}{2}\times 0.98\times(0.98+4.23) \approx 51.06 \quad \text{(exact is} \sim 51.70 \text{ which implies} \sim 1.2\% \text{ error)}$$

$$M_{\mathrm{Right}} = \frac{w}{2}\times d_{\mathrm{IR}}\times(d_{\mathrm{IR}}+L_{\mathrm{eff}}) \approx \frac{20}{2}\times 0.79\times(0.79+4.23) \approx 39.66 \quad \text{(exact is} \sim 35.32 \text{ which implies} \sim -12.3\% \text{ error)}$$

<u>Member IJ</u>: (inflection point and moment diagram)

First the moment at the left of member JK, $M_{\mathrm{Left}} = 51.06$ is distributed to member JI and JF. We obtain the moment in member JI at J through a distribution factor:

$$M_{\mathrm{JI}} = \left(0.75\times\left(\frac{\mathrm{EI}}{L}\right)_{\mathrm{JI}} \Big/ \left(0.75\times\left(\frac{\mathrm{EI}}{L}\right)_{\mathrm{JI}} + 0.75\times\left(\frac{\mathrm{EI}}{L}\right)_{\mathrm{JF}}\right)\right)\times 51.06 =$$

$$\left(0.75\times\frac{2}{4}\Big/\left(0.75\times\frac{2}{4} + 0.75\times\frac{1}{3}\right)\right)\times 51.06 \approx 0.6\times 51.06 \approx 30.64 \quad \text{(exact is 31.02 which implies 1.2\% error)}$$

Note:

JI and JF have reduced stiffness because they have internal hinges

There are no inflection points on member IJ because of the internal hinge at I

The moment at end I of member IJ is zero because of the internal hinge.

<u>Member KG</u>: (inflection point only)

We determine the rotary stiffness factor at end G of member KG

$$k \approx \left(0.75\times\left(\frac{\mathrm{EI}}{L}\right)_{\mathrm{GH}} + \left(\frac{\mathrm{EI}}{L}\right)_{\mathrm{GC}}\right)\Big/\left(\frac{\mathrm{EI}}{L}\right)_{\mathrm{KG}} \approx \left(0.75\times\frac{2}{5}+\frac{1}{3}\right)\Big/\frac{1}{3} = 1.9$$

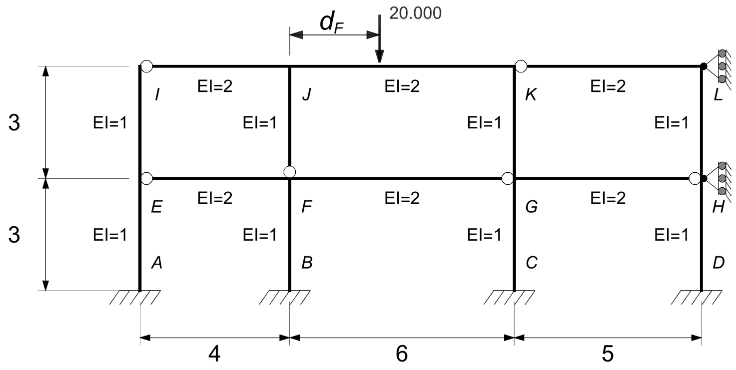
So the inflection point is at a distance from G equals to:

$$d_I = \frac{2\,k}{3+6\,k}\times L_{\mathrm{KG}} = \frac{2\times 1.9}{3+6\times 1.9}\times 3 \approx 0.264\times 3 = 0.79$$

<u>Member GC</u>: (inflection point only)

The end at C is fixed so that the inflection point is at one-third from the fixed end or $d_I = 1$ (from C)

2. (30 points)



For the structure shown in the figure on the left with the point load remaining on member JK:

a) (15 points)

(i) Identify the approximate location of d_F so that the positive moment in JK is the maximum possible.

(ii) Calculate the approximate value of this maximum moment.

b) (15 points)

(i) Identify the approximate location of d_F so that the negative moment in JK (in absolute value) is the maximum possible.

(ii) Calculate the approximate value of this largest possible negative moment.

Note:

• You must explain your choice of d_F in each case. The location you identify must be within ± 7% of the exact result.

• This is the SAME structure shown in problem 1 but the questions are different.

Solution:

Isolate member JK and determine k_L and k_R: (same as in problem 1)

$$k_L \approx \left(0.75 \times \left(\frac{EI}{L}\right)_{JI} + 0.75\times\left(\frac{EI}{L}\right)_{JF}\right) \Big/ \left(\frac{EI}{L}\right)_{JK} \approx \left(0.75\times\frac{2}{4} + 0.75\times\frac{1}{3}\right)\Big/\frac{2}{6} = 1.875$$

$$k_R \approx \left(\left(\frac{EI}{L}\right)_{KG}\right) \Big/ \left(\frac{EI}{L}\right)_{JK} \approx \left(\frac{1}{3}\right)\Big/\frac{2}{6} = 1$$

(a-i) Location of point force:

Point force should be at a distance of about 0.43 L ($\approx$ 2.58) from the less stiff side which is at point K.

This means that:

$$d_F = L - d_{F\,\text{Right}} = 6 - 2.58 = 3.42$$

$$d_{F\,\text{Right}} = 2.58$$

$\boxed{d_F = 3.42}$

(exact location is 3.15)

(a-ii) Maximum positive moment:

Now we have the following:

$$d_{IL} = \frac{3\,k_L}{2+4\,k_L}\times\frac{d_{F\,\text{Left}}}{L_{JK}+d_{F\,\text{Left}}}\times L_{JK} = \frac{3\times 1.875}{2+4\times 1.875}\times\frac{3.42}{6+3.42}\times 6 \approx 1.29$$

$$d_{IR} = \frac{3\,k_R}{2+4\,k_R}\times\frac{d_{F\,\text{Right}}}{L_{JK}+d_{F\,\text{Right}}}\times L_{JK} = \frac{3\times 1}{2+4\times 1}\times\frac{2.58}{6+2.58}\times 6 \approx 0.90$$

$$L_{\text{eff}} = \text{L} - d_{IL} - d_{IR} \approx 6 - 1.29 - 0.9 \approx 3.81$$

Therefore:

$$a_{\text{eff}} = d_{F\,\text{Left}} - d_{IL} = 3.42 - 1.29 = 2.13$$

$$b_{\text{eff}} = d_{F\,\text{Right}} - d_{IR} = 2.58 - 0.90 = 1.68$$

$$L_{\text{eff}} = a_{\text{eff}} + b_{\text{eff}} = 2.13 + 1.68 = 3.81 \text{ (checks)}$$

$$\Rightarrow M_{\text{positive max}} \approx \text{P}\times a_{\text{eff}}\times b_{\text{eff}} / L_{\text{eff}} = 20\times 2.13\times 1.68 / 3.81 \approx 18.78$$

$\boxed{M_{\text{positive max}} = 18.23}$

(exact max is 19.21)

(b-i) Location of point force:

Point force should be at a distance of about 0.38 L ($\approx 2.28 = d_{F\,\text{Left}}$) from the stiffer side which is at point J.

This means that $d_F = d_{F\,\text{Left}} \approx 2.28$

$\boxed{d_F = 2.28}$

(exact is 2.3)

(b-ii) Largest negative moment:

Now we have the following:

$$d_{\text{IL}} = \frac{3\,k_L}{2+4\,k_L} \times \frac{d_{F\,\text{Left}}}{L_{\text{JK}} + d_{F\,\text{Left}}} \times L_{\text{JK}} = \frac{3 \times 1.875}{2+4\times 1.875} \times \frac{2.28}{6+2.28} \times 6 \approx 0.978$$

$$d_{\text{IR}} = \frac{3\,k_R}{2+4\,k_R} \times \frac{d_{F\,\text{Right}}}{L_{\text{JK}} + d_{F\,\text{Right}}} \times L_{\text{JK}} = \frac{3 \times 1}{2+4\times 1} \times \frac{6-2.28}{6+(6-2.28)} \times 6 \approx 1.148$$

$$L_{\text{eff}} = \text{L} - d_{\text{IL}} - d_{\text{IR}} \approx 6 - 0.978 - 1.148 \approx 3.874$$

Therefore:

$$a_{\text{eff}} = d_{F\,\text{Left}} - d_{\text{IL}} = 2.28 - 0.978 = 1.302$$

$$b_{\text{eff}} = d_{F\,\text{Right}} - d_{\text{IR}} = (6 - 2.28) - 1.148 = 2.572$$

$$L_{\text{eff}} = a_{\text{eff}} + b_{\text{eff}} = 1.302 + 2.572 = 3.874 \quad \text{(consistent with above)}$$

$$\Rightarrow\ M_{\text{negative max}} \approx 20 \times d_{\text{IL}} \times b_{\text{eff}} / L_{\text{eff}} = 20 \times 0.978 \times 2.572 / 3.874 \approx 12.99$$

$$\boxed{\text{M}_{\text{negative max}} = 12.99}$$

(exact is 13.89)