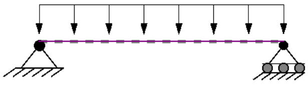
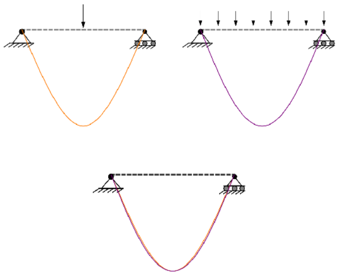
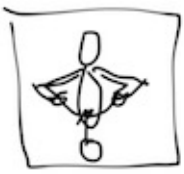
Scripts



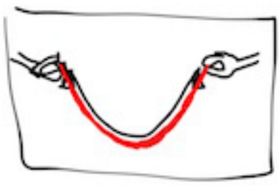
We start with the simplest configuration for a uniformly loaded beam. Namely, a simply supported beam with a uniform load. This means there are hinges at both ends and we apply a uniform load over the whole beam.



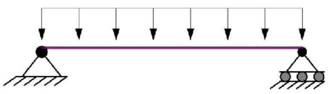
Now the deformed shape of a beam loaded with a central point force and a beam loaded with a uniform load are visually indistinguishable as we can see in the overlay comparison figure ... which we can see more clearly if we zoom in.



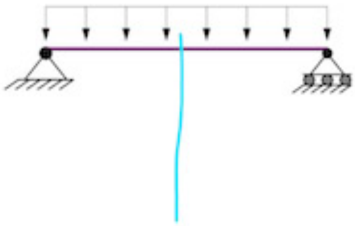
Therefore, I will demonstrate the behavior of a uniformly loaded straw with the visually near equivalent, namely the centrally loaded beam. Of course, I do this because it is easier to apply a single load than a distributed one. This straw shows the deformed shape. Notice that the shape has a positive curvature throughout, using the convention of positive curvature described in the lecture on chapter 1 part 1. We also notice that the beam is rather straight at each end.



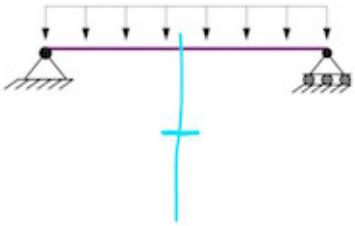
If we compare the theoretical deformation with the deformation of the straw, we get a good match.



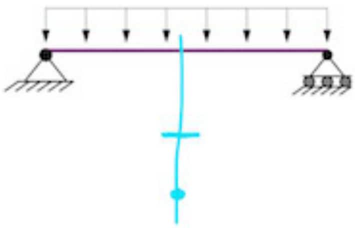
Now we consider how to sketch this case by hand.



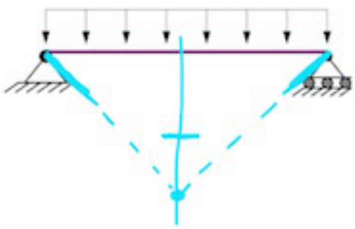
We start by locating the midpoint and extend a guide line down.



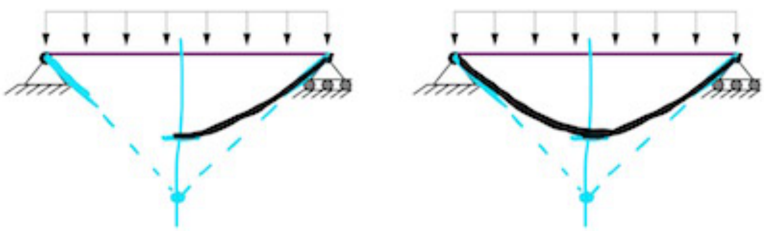
Along this line, we choose a level for the maximum displacement and draw a short horizontal guide line.



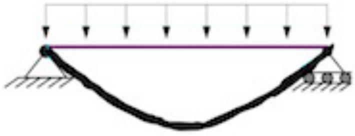
Now we choose a point below this maximum displacement at about 60% further down and mark a point.



The slopes of the deflected shape at the supports is along the line joining this point and the supports. We mark those with small stubs or lines.

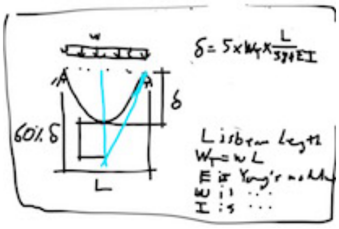


Now, we start at each end initially moving along the tangent and then smoothly reaching the maximum tangentially at the level indicated by the horizontal guide line.

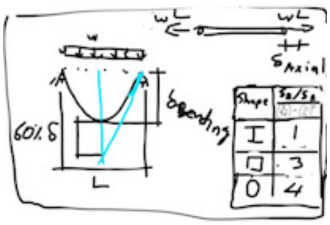


Finally, we remove or erase the extraneous lines, and further smooth out our curve to get our deformed shape.
Some or all of these steps may be omitted once we get a sufficient familiarity with sketching this shape. But this step-by-step approach makes us aware of all the salient details of the beam's deformation.

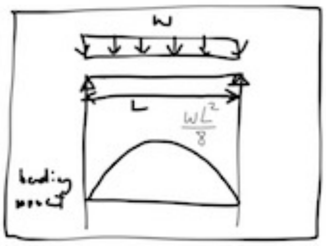
And now we compare with the theoretical shape, and, yes, we again get a good match, this time between theory and sketch.



Now let's go back to the beam and look at some quantitative results. Looking at the deformed shape. The maximum deflection is at the middle, which of course it must be due to symmetry. That maximum deflection has a height which is 60% of the height obtained by extending straight lines from the ends. In terms of the load, the maximum displacement is 5 times the total load times $\frac{L^3}{384\ EI}$. This last factor is common to many displacement formulas and we will factor it out for easier comparisons. For example, a point load applied at the center will be 8 times the load magnitude times that same factor. Therefore a centrally loaded point force causes 60% more deflection than a uniformly loaded simply supported beam.



Compared to an axially applied load, if we applied the same total load axially, then the ratio of bending to axial displacement for a rectangular section is $\frac{5}{32}$ multiplied by the ratio of the beam length over sectional height squared. For a beam to height ratio of 8, 12 and 16, the ratio is 10, 22.5 and 40. For an I-beam, these ratios would be three times smaller.



The moment diagram for this basic case is simple. It is a parabola having a maximum at the middle with a value of $w\,L^2/8$. This value is important because for a uniformly loaded beam connected to any passive structure with no side-sway, we find that this is the maximum possible positive moment.