Derivative of $\cos x$.

What is the specific formula for the derivative of the function $\cos x$?

This calculation is very similar to that of the derivative of $\sin(x)$. If you get stuck on a step here it may help to go back and review the corresponding step there.

As in the calculation of $\frac{d}{dx}\sin x$, we begin with the definition of the derivative:

$$\frac{d}{dx}\cos x = \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos(x)}{\Delta x}$$

Use the angle sum formula $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ and then simplify:

$$\begin{aligned}
\frac{d}{dx}\cos x &= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left[\frac{\cos x \cos \Delta x - \cos x}{\Delta x} + \frac{-\sin x \sin \Delta x}{\Delta x} \right] \\
&= \lim_{\Delta x \to 0} \left[\frac{\cos x (\cos \Delta x - 1)}{\Delta x} + \frac{-\sin x \sin \Delta x}{\Delta x} \right] \\
&= \lim_{\Delta x \to 0} \left[\cos x \left(\frac{\cos \Delta x - 1}{\Delta x} \right) + (-\sin x) \left(\frac{\sin \Delta x}{\Delta x} \right) \right] \\
\frac{d}{dx}\cos x &= \lim_{\Delta x \to 0} \cos x \left(\frac{\cos \Delta x - 1}{\Delta x} \right) + \lim_{\Delta x \to 0} (-\sin x) \left(\frac{\sin \Delta x}{\Delta x} \right)
\end{aligned}$$

Once again we use the following (unproven) facts:

$$\begin{aligned}
\lim_{\Delta x \to 0} \frac{\cos \Delta x - 1}{\Delta x} &= 0 \text{ (A)} \\
\lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} &= 1. \text{ (B)}
\end{aligned}$$

And we conclude:

$$\begin{aligned}
\frac{d}{dx}\cos x &= \lim_{\Delta x \to 0} \cos x \left(\frac{\cos \Delta x - 1}{\Delta x} \right) + \lim_{\Delta x \to 0} (-\sin x) \left(\frac{\sin \Delta x}{\Delta x} \right) \\
&= \cos x \cdot 0 + (-\sin x) \cdot 1 \\
\frac{d}{dx}\cos x &= -sin(x).
\end{aligned}$$