FIND SOLUTIONS ON NEXT PAGE

CHAPTER ONE

Solutions for Section 1.1

Exercises

1. $f(6.9) = 2.9$.

2. $(2.2, 2.9)$; $(6.1, 4.9)$

3. Since $f(0) = f(4) = f(8) = 0$, the solutions are $x = 0, 4, 8$.

4. Since the graphs touch at $x = 2.2$ and $x = 6.1$, these are the solutions.

5. $m = f(v)$.

6. $w = f(c)$.

7. **(a)** The graphs in (I), (III), (IV), (V), (VII), and (VIII) are functions. The graphs in (II), (VI), and (IX) do not pass the vertical line test and so they cannot be the graphs of functions.

(b) (i) The graph of SAT Math score versus SAT Verbal score for a number of students will be a graph of a number of points. Graphs (V) and (VI) are of this type.

(ii) The graph of hours of daylight per day must be an oscillating function (since the number of hours of daylight fluctuates up and down throughout the year). Graph (VIII) represents this.

(c) If the train fare remains constant throughout the day, graph (III) describes the fare. If there are specific times of the day (rush hours, for example) when the train company raises its prices, then graph (IV) represents the train fare as a function of time of day.

8. These data are plotted in Figure 1.1. The independent variable is A and the dependent variable is n.

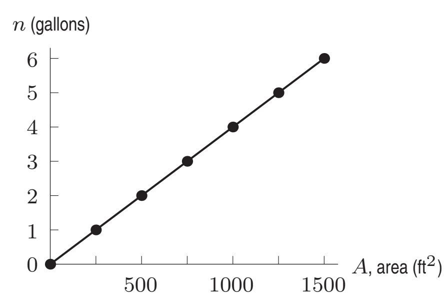

Figure 1.1

9. **(a)** Since the vertical intercept is $(0, 40)$, we have $f(0) = 40$.

(b) Since the horizontal intercept is $(2, 0)$, we have $f(2) = 0$.

10. **(a)** Since $f(x)$ is 4 when $x = 0$, we have $f(0) = 4$.

(b) Since $x = 3$ when $f(x) = 0$, we have $f(3) = 0$.

(c) $f(1) = 2$

(d) There are two x values leading to $f(x) = 1$, namely $x = 2$ and $x = 4$. So $f(2) = 1$ and $f(4) = 1$.

11. **(a)** w goes on the horizontal axis

(b) $(-4, 10)$

(c) $(6, 1)$

12. **(a)** We have

$$
\begin{aligned}
f(0) &= \frac{10}{1+0^2} = \frac{10}{1} = 10 \\
f(1) &= \frac{10}{1+1^2} = \frac{10}{2} = 5 \\
f(2) &= \frac{10}{1+2^2} = \frac{10}{5} = 2 \\
f(3) &= \frac{10}{1+3^2} = \frac{10}{10} = 1.
\end{aligned}
$$

See Table 1.1.

Table 1.1

x	0	1	2	3
$f(x)$	10	5	2	1

(b) For $x = 0$, we have $f(0) = 10$. This value is largest because the x-value is smallest.

13. Appropriate axes are shown in Figure 1.2.

14. Appropriate axes are shown in Figure 1.3.

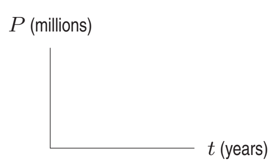

Figure 1.2

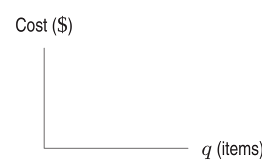

Figure 1.3

15. Appropriate axes are shown in Figure 1.4.

16. Appropriate axes are shown in Figure 1.5.

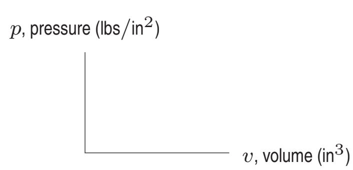

Figure 1.4

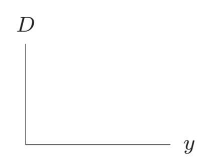

Figure 1.5

Problems

17. **(a)** When there is no snow, it is equivalent to no rain. Thus, the vertical intercept is 0. Since every ten inches of snow is equivalent to one inch of rain, we can specify the slope as

$$\frac{\Delta \text{rain}}{\Delta \text{snow}} = \frac{1}{10} = 0.1$$

We have a vertical intercept of 0 and a slope of 0.1. Thus, the equation is: $r = f(s) = 0.1s$.

(b) By substituting 5 in for s, we get $f(5) = 0.1(5) = 0.5$ This tells us that five inches of snow is equivalent to approximately $1/2$ inch of rain.

(c) Substitute 5 inches for $r = f(s)$ in the equation: $5 = 0.1s$. Solving gives $s = 50$. Five inches of rain is equivalent to approximately 50 inches of snow.

18. **(a)** One, because otherwise it would automatically fail the vertical-line test using the y-axis as the vertical line.
(b) Yes, it can cross an infinite number of times. For example, the graph in Figure 1.6 oscillates an infinite number of times across the x-axis.

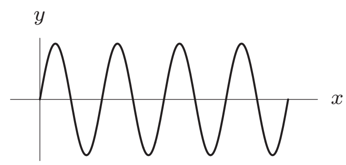

Figure 1.6

19. **(a)** The number of people who own cell phones in the year 2000 is $100{,}300{,}000$.
(b) There are $20{,}000{,}000$ people who own cell phones a years after 1990.
(c) There will be b million people who own cell phones in the year 2010.
(d) The number n is the number of people (in millions) who own cell phones t years after 1990.

20. From the table, $r(300) = 120$, which tells us that at a height of 300 m the wind speed is 120 mph.

21. Judging from the table, $r(s) \geq 116$ for $200 \leq s \leq 1000$. This tells us that the wind speed is at least 116 mph between 200 m and 1000 m above the ground.

22. The wind reaches its greatest speed, $v = 122$ mph, at a height of $s = 500$ m.

23. **(a)** 69°F
(b) July 17^{th} and 20^{th}
(c) Yes. For each date, there is exactly one low temperature.
(d) No, it is not true that for each low temperature, there is exactly one date: for example, 73° corresponds to both the 17^{th} and 20^{th}.

24. **(a)** From the table, we see that $f(100) = 524.5$. This means that there is approximately $524.5 billion worth of $100 bills in circulation in the United States.
(b) To determine the number of $5 bills, we divide 9.7 by 5. Thus, we have about 1.94 billion $5 bills in circulation. The number of $1 bills is the same as the value, so there are 8.4 billion $1 bills in circulation. There are more $1 bills.

25. **(a)** Figure 1.7 shows the plot of R versus t. R is a function of t because no vertical line intersects the graph in more than one place.
(b) Figure 1.8 shows the plot of F versus t. F is a function of t because no vertical line intersects the graph in more than one place.

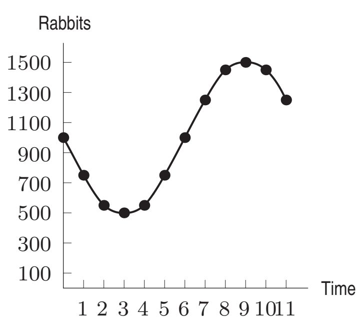

Figure 1.7: The graph of R versus t

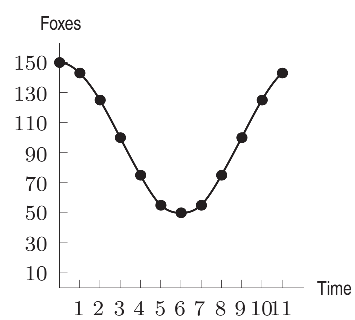

Figure 1.8: The graph of F versus t

(c) Figure 1.9 shows the plot of F versus R. We have also drawn the vertical line corresponding to $R = 567$. This tells us that F is not a function of R because there is a vertical line that intersects the graph twice. In fact the lines $R = 567$, $R = 750$, $R = 1000$, $R = 1250$, and $R = 1433$ all intersect the graph twice. However, the existence of any one of them is enough to guarantee that F is not a function of R.

(d) Figure 1.10 shows the plot of R versus F. We have drawn the vertical line corresponding to $F = 57$. This tells us that R is not a function of F because there is a vertical line that intersects the graph twice. In fact the lines $F = 57$, $F = 75$, $F = 100$, $F = 125$, and $F = 143$ all intersect the graph twice. However, the existence of any one of them is enough to guarantee that R is not a function of F.

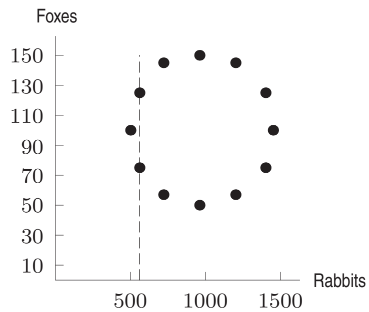

Figure 1.9: The graph of F versus R

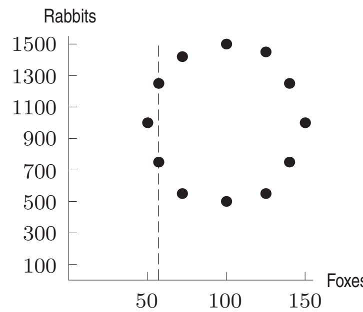

Figure 1.10: The graph of R versus F

26. **(a)** No, in the year 1954 there were two world records; in the year 1981 there were three world records.
(b) Yes, each world record occurred in only one year.
(c) The world record of 3 minutes and 47.33 seconds was set in 1981.
(d) The statement y(3:51.1) = 1967 tells us that the world record of 3 minutes, 51.1 seconds was set in 1967.

27. **(a)** Yes, if we know the Congress we can determine the number of female senators.
(b) No, there were 2 female senators in the 98^{th}, 100^{th} and 102^{nd} Congresses.
(c) The number of female senators who served the 104^{th} Congress is 8.
(d) The statement $S(108) = 14$ tells us that 14 female senators served the 108^{th} Congress.

28. A possible graph is shown in Figure 1.11.

29. A possible graph is shown in Figure 1.12.

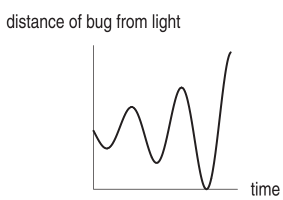

Figure 1.11

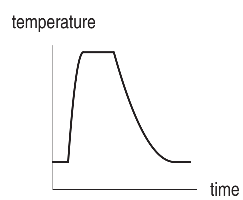

Figure 1.12

30. Since the tax is $0.06P$, the total cost would be the price of the item plus the tax,or

$$C = P + 0.06P = 1.06P.$$

31. The area of each end of the can is πr^2. To find the surface area of the cylindrical side, imagine making vertical cut from top to bottom and unfolding the cylinder into a rectangle. See Figure 1.13.

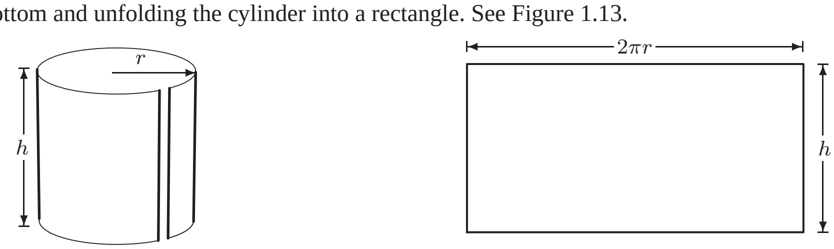

Figure 1.13

Thus, the surface area of the cylindrical side is $2\pi rh$.
The total surface area of the can is given by

$$S = 2(\text{Area of one end}) + \text{Area of cylindrical side}$$
$$S = 2(\pi r^2) + 2\pi rh.$$

Using the fact that height is twice radius, $h = 2r$, we get

$$S = 2\pi r^2 + 2\pi r(2r) = 6\pi r^2.$$

32. (a) It takes Charles Osgood 60 seconds to read 15 lines, so that means it takes him 4 seconds to read 1 line, 8 seconds for 2 lines, and so on. Table 1.2 shows this. From the table we see that it takes 36 seconds to read 9 lines.

Table 1.2 *The time it takes Charles Osgood to read*

Lines	0	1	2	3	4	5	6	7	8	9	10
Time	0	4	8	12	16	20	24	28	32	36	40

(b) Figure 1.14 shows the plot of the time in seconds versus the number of lines.

(c) In Figure 1.15 we have dashed in a line to see the trend. By drawing the vertical line at 9 lines, we see that this corresponds to approximately 36 seconds. By drawing a horizontal line at 30 seconds, we see that this corresponds to approximately 7.5 lines.

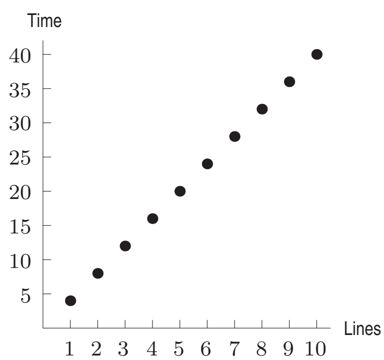

Figure 1.14: The graph of time versus lines

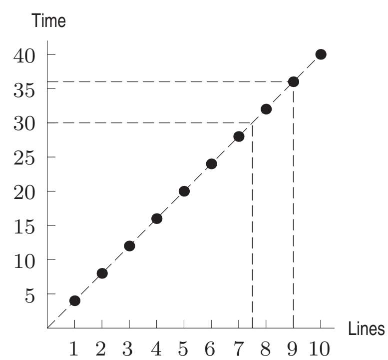

Figure 1.15: The graph of time versus lines

(d) If we let T be the time in seconds that it takes to read n lines, then $T = 4n$.

33. (a) Since the person starts out 5 miles from home, the vertical intercept on the graph must be 5. Thus, (i) and (ii) are possibilities. However, since the person rides 5 mph away from home, after 1 hour the person is 10 miles from home. Thus, (ii) is the correct graph.

(b) Since this person also starts out 5 miles from home, (i) and (ii) are again possibilities. This time, however, the person is moving at 10 mph and so is 15 miles from home after 1 hour. Thus, (i) is correct.

(c) The person starts out 10 miles from home so the vertical intercept must be 10. The fact that the person reaches home after 1 hour means that the horizontal intercept is 1. Thus, (v) is correct.

(d) Starting out 10 miles from home means that the vertical intercept is 10. Being half way home after 1 hour means that the distance from home is 5 miles after 1 hour. Thus, (iv) is correct.

(e) We are looking for a graph with vertical intercept of 5 and where the distance is 10 after 1 hour. This is graph (ii).

Notice that graph (iii), which depicts a bicyclist stopped 10 miles from home, does not match any of the stories.

34. **(a)**

Table 1.3 *Relationship between cost, C, and number of liters produced, l*

l (millions of liters)	0	1	2	3	4	5
C (millions of dollars)	2.0	2.5	3.0	3.5	4.0	4.5

(b) The cost, C, consists of a fixed cost of $\$2$ million plus a variable cost of $\$0.50$ million per million liters produced. If l millions of liters are produced, the total variable costs are $(0.5)l$. Thus, the total cost C in millions of dollars is given by

$$C = \text{Fixed cost} + \text{Variable cost},$$

so

$$C = 2 + (0.5)l.$$

35. The diagram is shown in Figure 1.16.

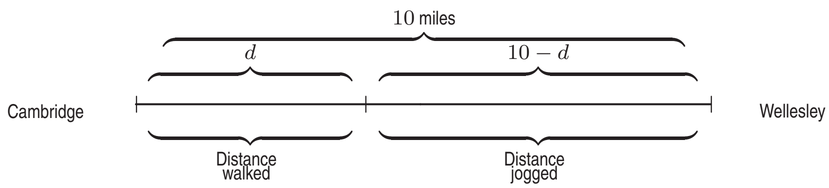

Figure 1.16

The total time the trip takes is given by the equation

$$\text{Total time} = \text{Time walked} + \text{Time jogged}.$$

The distance walked is d, and, since the total distance is 10, the remaining distance jogged is $(10 - d)$. See Figure 1.16. We know that time equals distance over speed, which means that

$$\text{Time walked} = \frac{d}{5} \quad \text{and} \quad \text{Time jogged} = \frac{10-d}{8}.$$

Thus, the total time is given by the equation

$$T(d) = \frac{d}{5} + \frac{10-d}{8}.$$

36. **(a)** Yes. If the person walks due west and then due north, the distance from home is represented by the hypotenuse of the right triangle that is formed (see Figure 1.17).

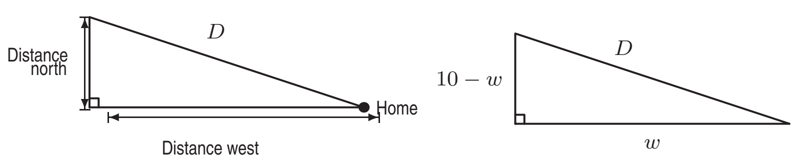

Figure 1.17

If the distance west is w miles and the total distance walked is 10 miles, then the distance north is $10 - w$ miles. We can use the Pythagorean Theorem to find that

$$D = \sqrt{w^2 + (10-w)^2}.$$

So, for each value of w, there is a unique value of D given by this formula. Thus, the definition of a function is satisfied.

(b) No. Suppose she walks 10 miles, that is, $x = 10$. She might walk 1 mile west and 9 miles north, or 2 miles west and 8 miles north, or 3 miles west and 7 miles north, and so on. The right triangles in Fig 1.18 show three different routes she could take and still walk 10 miles.

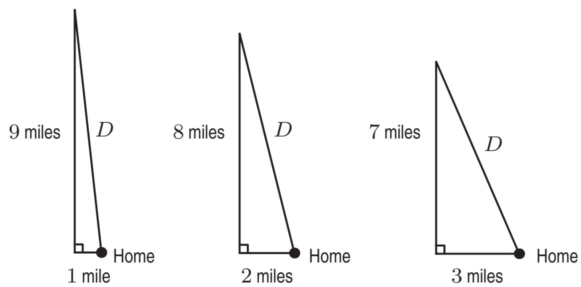

Figure 1.18

Each situation gives a different distance from home. The Pythagorean Theorem shows that the distances from home for these three examples are

$$D = \sqrt{1^2 + 9^2} = 9.06,$$
$$D = \sqrt{2^2 + 8^2} = 8.25,$$
$$D = \sqrt{3^2 + 7^2} = 7.62.$$

Thus, the distance from home cannot be determined from the distance walked.

Solutions for Section 1.2

Exercises

1. We have $G(3) - G(-1) > 0$.

2. We have $F(-2) > F(2)$.

3. We have

$$\begin{aligned}\frac{\Delta f}{\Delta x} &= \frac{f(6.1) - f(2.2)}{6.1 - 2.2} \\ &= \frac{4.9 - 2.9}{3.9} \\ &= 0.513.\end{aligned}$$

4. There are many such intervals. One way to find them is to look for points at which a horizontal line (a line of slope $m = 0$) intersects the graph. Here are some possible intervals: $0 \le x \le 4$, $0 \le x \le 8$, $4 \le x \le 8$, $2.2 \le x \le 5.2$, $2.2 \le x \le 6.9$, $5.2 \le x \le 6.9$.

5. Using the points on g

$$\text{Average rate of change } = \frac{g(6.1) - g(2.2)}{6.1 - 2.2} = \frac{4.9 - 2.9}{6.1 - 2.2} = 0.513.$$

6. They are equal; both are given by

$$\frac{4.9 - 2.9}{6.1 - 2.2}.$$

7. (a) Negative
(b) Positive

8. (a) Let $s = C(t)$ be the sales (in millions) of CDs in year t. Then

$$\begin{aligned}\begin{array}{l}\text{Average rate of change of } s\\ \text{from } t = 1982 \text{ to } t = 1984\end{array} &= \frac{\Delta s}{\Delta t} = \frac{C(1984) - C(1982)}{1984 - 1982}\\ &= \frac{5.8 - 0}{2}\\ &= 2.9 \text{ million discs/year.}\end{aligned}$$

Let $q = L(t)$ be the sales (in millions) of LPs in year t. Then

$$\begin{aligned}\begin{array}{l}\text{Average rate of change of } q\\ \text{from } t = 1982 \text{ to } t = 1984\end{array} &= \frac{\Delta q}{\Delta t} = \frac{L(1984) - L(1982)}{1984 - 1982}\\ &= \frac{205 - 244}{2}\\ &= -19.5 \text{ million records/year.}\end{aligned}$$

(b) By the same argument

$$\begin{aligned}\begin{array}{l}\text{Average rate of change of } s\\ \text{from } t = 1986 \text{ to } t = 1988\end{array} &= \frac{\Delta s}{\Delta t} = \frac{C(1988) - C(1986)}{1988 - 1986}\\ &= \frac{150 - 53}{2}\\ &= 48.5 \text{ million discs/year.}\end{aligned}$$

$$\begin{aligned}\begin{array}{l}\text{Average rate of change of } q\\ \text{from } t = 1986 \text{ to } t = 1988\end{array} &= \frac{\Delta q}{\Delta t} = \frac{L(1988) - L(1986)}{1988 - 1986}\\ &= \frac{72 - 125}{2}\\ &= -26.5 \text{ million records/year.}\end{aligned}$$

(c) The fact that $\Delta s/\Delta t = 2.9$ tells us that CD sales increased at an average rate of 2.9 million discs/year between 1982 and 1984. The fact that $\Delta s/\Delta t = 48.5$ tells us that CD sales increased at an average rate of 48.5 million discs/year between 1986 and 1988.

The fact that $\Delta q/\Delta t = -19.5$ means that LP sales decreased at an average rate of 19.5 million records/year between 1982 and 1984. The fact that the average rate of change is negative tells us that annual sales are decreasing.

The fact that $\Delta q/\Delta t = -26.5$ means that LP sales decreased at an average rate of 26.5 million records/year between 1986 and 1988.

9. To decide if CD sales are an increasing or decreasing function of LP sales, we must read the table in the direction in which LP sales increase. This means we read the table from right to left. As the number of LP sales increases, the number of CD sales decrease. Thus, CD sales are a decreasing function of LP sales.

10. (a) (i) After 2 hours 60 miles had been traveled. After 5 hours, 150 miles had been traveled. Thus on the interval from $t = 2$ to $t = 5$ the value of Δt is

$$\Delta t = 5 - 2 = 3$$

and the value of ΔD is

$$\Delta D = 150 - 60 = 90.$$

(ii) After 0.5 hours 15 miles had been traveled. After 2.5 hours, 75 miles had been traveled. Thus on the interval from $t = 0.5$ to $t = 2.5$ the value of Δt is

$$\Delta t = 2.5 - .5 = 2$$

and the value of ΔD is

$$\Delta D = 75 - 15 = 60.$$

(iii) After 1.5 hours 45 miles had been traveled. After 3 hours, 90 miles had been traveled. Thus on the interval from $t = 1.5$ to $t = 3$ the value of Δt is

$$\Delta t = 3 - 1.5 = 1.5$$

and the value of ΔD is

$$\Delta D = 90 - 45 = 45.$$

(b) For the interval from $t = 2$ to $t = 5$, we see

$$\text{Rate of change } = \frac{\Delta D}{\Delta t} = \frac{90}{3} = 30.$$

For the interval from $t = 0.5$ to $t = 2.5$, we see

$$\text{Rate of change } = \frac{\Delta D}{\Delta t} = \frac{60}{2} = 30.$$

For the interval from $t = 1.5$ to $t = 3$, we see

$$\text{Rate of change } = \frac{\Delta D}{\Delta t} = \frac{45}{1.5} = 30.$$

This suggests that the average speed is 30 miles per hour throughout the trip.

11. (a) For 1990 to 2000, the rate of change of P_1 is

$$\frac{\Delta P_1}{\Delta t} = \frac{83 - 53}{2000 - 1990} = \frac{30}{10} = 3 \text{ hundred people per year,}$$

while for P_2 we have

$$\frac{\Delta P_2}{\Delta t} = \frac{70 - 85}{2000 - 1990} = \frac{-15}{10} = -1.5 \text{ hundred people per year.}$$

(b) For 1995 to 2007,

$$\frac{\Delta P_1}{\Delta t} = \frac{93 - 73}{2007 - 1995} = \frac{20}{12} = 1.67 \text{ hundred people per year,}$$

and

$$\frac{\Delta P_2}{\Delta t} = \frac{65 - 75}{2007 - 1995} = \frac{-10}{12} = -0.83 \text{ hundred people per year.}$$

(c) For 1990 to 2007,

$$\frac{\Delta P_1}{\Delta t} = \frac{93 - 53}{2007 - 1990} = 2.35 \text{ hundred people per year,}$$

and

$$\frac{\Delta P_2}{\Delta t} = \frac{65 - 85}{2007 - 1990} = -1.18 \text{ hundred people per year.}$$

12. (a) (i) We find the average rate of change in the population as follows. For P_1 from 1990 to 2000,

$$\text{Rate of change} = \frac{\Delta P_1}{\Delta t} = \frac{62 - 42}{2000 - 1990} = 2 \text{ thousand people per year.}$$

Thus, P_1 is growing, on average, by two thousand people per year. For P_2 over the same period,

$$\text{Rate of change} = \frac{\Delta P_2}{\Delta t} = \frac{72 - 82}{2000 - 1990} = -1 \text{ thousand people per year.}$$

The negative sign tells us that P_2 is decreasing, on average, by one thousand people per year.

(ii) For 1990–2007, the average rate of change of P_1 is:

$$\text{Rate of change} = \frac{\Delta P_1}{\Delta t} = \frac{76-42}{2007-1990} = 2 \text{ thousand people per year.}$$

That is, the city is gaining 2 thousand people per year. The average rate of change of P_2 is:

$$\text{Rate of change} = \frac{\Delta P_2}{\Delta t} = \frac{65-82}{2007-1990} = -1 \text{ thousand people per year.}$$

That is, the city is losing a thousand people per year.

(iii) For 1995–2007, we have:

$$\frac{\Delta P_1}{\Delta t} = \frac{76-52}{2007-1995} = 2 \text{ thousand people per year.}$$

That is, the city is gaining 2 thousand people per year. The average rate of growth for the second population is:

$$\frac{\Delta P_2}{\Delta t} = \frac{65-77}{2007-1995} = -1 \text{ thousand people per year.}$$

That is, the city is losing a thousand people per year.

(b) The average rate of change of each population is the same on all three time intervals. Each population appears to be changing at a constant rate. The first population is growing, on average, by 2 thousand people per year in each time interval. The second population is dropping, on average, by 1 thousand people per year in each time interval.

Problems

13. According to the table in the text, the tree has 139μg of carbon-14 after 3000 years from death and 123μg of carbon-14 after 4000 years from death. Because the function $L = g(t)$ is decreasing, the tree must have died between $3{,}000$ and $4{,}000$ years ago.

14. **(a)** According to the table, a 200-lb person uses 5.4 calories per minute while walking. Since a half hour is 30 minutes, a half-hour walk uses $(5.4)(30) = 162$ calories.

(b) A 120-lb swimmer uses 6.9 calories per minute. Thus, in one hour the swimmer uses $(6.9)(60) = 414$ calories. A 220-lb bicyclist uses 11.9 calories per minute. In a half-hour, the bicyclist uses $(11.9)(30) = 357$ calories. Thus, the swimmer uses more calories.

(c) Increases, since the numbers 2.7, 3.2, 4.0, 4.6, 5.4, 5.9 are increasing.

15. **(a)** Between $(-2,-7)$ and $(3,3)$,

$$\text{Average rate of change } = \frac{\Delta y}{\Delta x} = \frac{3-(-7)}{3-(-2)} = \frac{10}{5} = 2.$$

(b) The function is increasing over this interval, since the average rate of change is positive.

(c) As the x-values increase, so do the y-values. See Figure 1.19. Thus, this function is increasing everywhere.

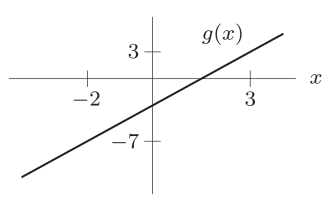

Figure 1.19

16. **(a)** (i) We have

$$\frac{f(2)-f(0)}{2-0} = \frac{16-2^2-(16-0)}{2} = -\frac{4}{2} = -2.$$

This means $f(x)$ decreases by an average of 2 units per unit change in x on the interval $0 \le x \le 2$.

(ii) We have

$$\frac{f(4)-f(2)}{4-2} = \frac{16-(4)^2-(16-2^2)}{2} = \frac{-16+4}{2} = -6.$$

This means $f(x)$ decreases by an average of 6 units per unit change in x on the interval $2 \le x \le 4$.

(iii) We have

$$\frac{f(4)-f(0)}{4-0} = \frac{16-(4)^2-(16-0)}{4} = -\frac{16}{4} = -4.$$

This means $f(x)$ decreases by an average of 4 units per unit change in x on the interval $0 \le x \le 4$.

(b) The graph of $f(x)$ is the solid curve in Figure 1.20. The secants corresponding to each rate of change are shown as dashed lines. The average rate of decrease is greatest on the interval $2 \le x \le 4$.

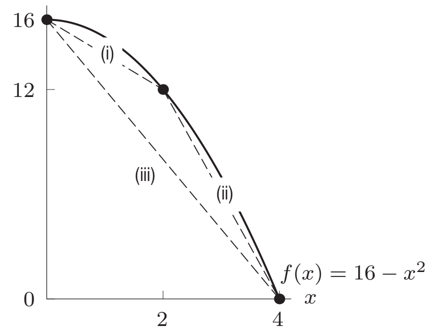

Figure 1.20

17. **(a)** From the graph, we see that $g(4) \approx 2$ and $g(0) \approx 0$. Thus,

$$\frac{g(4)-g(0)}{4-0} \approx \frac{2-0}{4-0} = \frac{1}{2}.$$

(b) The line segment joining the points in part (a), as well as the line segment in part (d), is shown on the graph in Figure 1.21.

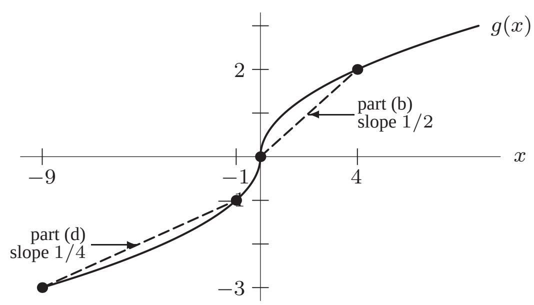

Figure 1.21

(c) From the graph, $g(-9) \approx -3$ and $g(-1) \approx -1$. Thus,

$$\frac{g(b)-g(a)}{b-a} \approx \frac{-1-(-3)}{-1-(-9)} = \frac{2}{8} = \frac{1}{4}.$$

(d) The line segment in part (c) with slope $(1/4)$ is shown in Figure 1.21.

18. **(a)** (i) Between $(-1, f(-1))$ and $(3, f(3))$

$$\text{Average rate of change } = \frac{f(3)-f(-1)}{3-(-1)} = \frac{(15-4)-(-5-4)}{4} = \frac{11-(-9)}{4} = \frac{20}{4} = 5.$$

(ii) Between $(a, f(a))$ and $(b, f(b))$

$$\text{Average rate of change } = \frac{f(b)-f(a)}{b-a} = \frac{(5b-4)-(5a-4)}{b-a} = \frac{5b-4-5a+4}{b-a} = \frac{5b-5a}{b-a} = \frac{5(b-a)}{b-a} = 5.$$

(iii) Between $(x, f(x))$ and $(x+h, f(x+h))$

$$\begin{aligned}\text{Average rate of change } &= \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{(5(x+h)-4)-(5x-4)}{(x+h)-x}\\ &= \frac{5x+5h-4-5x+4}{h} = \frac{5h}{h} = 5.\end{aligned}$$

(b) The average rate of change is always 5.

19. **(a)** (i) Between $(-1, f(-1))$ and $(3, f(3))$

$$\text{Average rate of change } = \frac{f(3)-f(-1)}{3-(-1)} = \frac{\left(\frac{3}{2}+\frac{5}{2}\right)-\left(\frac{-1}{2}+\frac{5}{2}\right)}{4} = \frac{4-2}{4} = \frac{2}{4} = \frac{1}{2}.$$

(ii) Between $(a, f(a))$ and $(b, f(b))$

$$\text{Average rate of change } = \frac{f(b)-f(a)}{b-a} = \frac{\left(\frac{b}{2}+\frac{5}{2}\right)-\left(\frac{a}{2}+\frac{5}{2}\right)}{b-a} = \frac{\frac{b}{2}+\frac{5}{2}-\frac{a}{2}-\frac{5}{2}}{b-a} = \frac{\frac{b}{2}-\frac{a}{2}}{b-a} = \frac{\frac{1}{2}(b-a)}{b-a} = \frac{1}{2}.$$

(iii) Between $(x, f(x))$ and $(x+h, f(x+h))$

$$\begin{aligned}\text{Average rate of change } &= \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{\left(\frac{x+h}{2}+\frac{5}{2}\right)-\left(\frac{x}{2}+\frac{5}{2}\right)}{(x+h)-x}\\ &= \frac{\frac{x+h}{2}+\frac{5}{2}-\frac{x}{2}-\frac{5}{2}}{x+h-x} = \frac{\frac{x+h-x}{2}}{h} = \frac{\frac{h}{2}}{h} = \frac{1}{2}.\end{aligned}$$

(b) The average rate of change is always $\frac{1}{2}$.

20. **(a)** (i) Between $(-1, f(-1))$ and $(3, f(3))$

$$\text{Average rate of change } = \frac{f(3)-f(-1)}{3-(-1)} = \frac{(3^2+1)-((-1)^2+1)}{4} = \frac{10-2}{4} = \frac{8}{4} = 2.$$

(ii) Between $(a, f(a))$ and $(b, f(b))$

$$\begin{aligned}\text{Average rate of change } &= \frac{f(b)-f(a)}{b-a} = \frac{(b^2+1)-(a^2+1)}{b-a}\\ &= \frac{b^2+1-a^2-1}{b-a} = \frac{b^2-a^2}{b-a} = \frac{(b+a)(b-a)}{b-a} = b+a.\end{aligned}$$

(iii) Between $(x, f(x))$ and $(x+h, f(x+h))$

$$\begin{aligned}\text{Average rate of change } &= \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{((x+h)^2+1)-(x^2+1)}{(x+h)-x}\\ &= \frac{x^2+2xh+h^2+1-x^2-1}{x+h-x} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = 2x+h.\end{aligned}$$

(b) The average rate of change is different each time. However, it seems to be the sum of the two x-coordinates.

21. **(a)** Average rate of change $= \dfrac{\Delta y}{\Delta x} = \dfrac{13-4}{2-1} = 9$

(b) Average rate of change $= \dfrac{\Delta y}{\Delta x} = \dfrac{n-k}{m-j}$

(c) The average rate of change is

$$
\begin{aligned}
\frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{(x+h)-x} &= \frac{(3(x+h)^2+1)-(3x^2+1)}{(x+h)-x} \\
&= \frac{(3(x^2+2xh+h^2)+1)-(3x^2+1)}{h} \\
&= \frac{3x^2+6xh+3h^2+1-3x^2-1}{h} \\
&= \frac{6xh+3h^2}{h} \\
&= 6x+3h.
\end{aligned}
$$

22. **(a)** The number of sunspots, s, is a function of the year, t, because knowing the year is enough to uniquely determine the number of sunspots. The graph passes the vertical line test.

(b) When read from left to right, the graph increases from $t = 1945$ to approximately $t = 1947$, from approximately $t = 1954$ to $t = 1957$, and from approximately $t = 1964$ to $t = 1969$. Thus, s is an increasing function of t on the approximate intervals $1945 < t < 1947$, $1954 < t < 1957$, $1964 < t < 1969$. For each of these intervals, the average rate of change on any subinterval must be positive.

23. **(a)** Since Δt refers to the change in the numbers of years, we calculate

$$\Delta t = 1970 - 1960 = 10, \qquad \Delta t = 1980 - 1970 = 10, \qquad \text{and so on.}$$

Since the entries in the table are all 10 years apart, we see that $\Delta t = 10$ for all consecutive entries.

(b) Since ΔG is the change in the amount of garbage produced per year, for the period 1960-1970 we have

$$\Delta G = 120 - 90 = 30.$$

Continuing in this way gives the Table 1.4:

Table 1.4

Time period	1960–70	1970-80	1980–90	1990–2000
ΔG	30	30	55	29

(c) Not all of the ΔG values are the same. We know that all the values of Δt are the same. If we knew that all the values of ΔG were the same, we could say that $\Delta G/\Delta t$, the average rate of change in the amount of garbage produced each year, is constant. Since, on the contrary, ΔG is not constant, we conclude that $\Delta G/\Delta t$ is not constant. This tells us that the amount of garbage being produced each year is changing, but not at a constant rate.

24. **(a)** Table 1.5 shows the average rate of change of distance, commonly called the average speed or average velocity.

Table 1.5 *Carl Lewis' times at 10 meter intervals*

Time (sec)	Distance (meters)	$\Delta d/\Delta t$ (meters/sec)
0.00 to 1.94	0 to 10	5.15
1.94 to 2.96	10 to 20	9.80
2.96 to 3.91	20 to 30	10.53
3.91 to 4.78	30 to 40	11.49
4.78 to 5.64	40 to 50	11.63
5.64 to 6.50	50 to 60	11.63
6.50 to 7.36	60 to 70	11.63
7.36 to 8.22	70 to 80	11.63
8.22 to 9.07	80 to 90	11.76
9.07 to 9.93	90 to 100	11.63

(b) He attained his maximum speed (11.76 meters/sec) between 80 and 90 meters. He does not appear to be running his fastest when he crossed the finish line.

25. **(a)** We have
$$\frac{f(150)-f(25)}{150-25}=\frac{5.50-5.50}{125}=0^\circ\text{C/meter}.$$
This tells us that on average the temperature changes by 0°C per meter of depth between 25 meters and 150 meters.

(b) We have
$$\frac{f(75)-f(25)}{75-25}=\frac{5.10-5.50}{50}=-0.008^\circ\text{C/meter}.$$
This tells us that on average the temperature drops by 0.008°C per meter of depth, or by 0.8°C per 100 meters of depth, on this interval.

(c) We have
$$\frac{f(200)-f(100)}{200-100}=\frac{6.00-5.10}{100}=0.009^\circ\text{C/meter}.$$
This tells us that on average the temperature rises by 0.009°C per meter of depth, or by 0.9°C per 100 meters of depth, on this interval.

Solutions for Section 1.3

Exercises

1. The function g is not linear even though $g(x)$ increases by $\Delta g(x)=50$ each time. This is because the value of x does not increase by the same amount each time. The value of x increases from 0 to 100 to 300 to 600 taking steps that get larger each time.

2. The function h is not linear even though the value of x increases by $\Delta x=10$ each time. This is because $h(x)$ does not increase by the same amount each time. The value of $h(x)$ increases from 20 to 40 to 50 to 55 taking smaller steps each time.

3. This table could not represent a linear function because the rate of change of $g(t)$ is not constant. We consider the first three points. Between $t=1$ and $t=2$, the value of $g(t)$ changes by $4-5=-1$. Between $t=2$ and $t=3$, the value of $g(t)$ changes by $5-4=1$. Thus, the rate of change is not constant ($-1\neq 1$), so the function is not linear.

4. The function f could be linear because the value of x increases by $\Delta x = 5$ each time and $f(x)$ increases by $\Delta f(x) = 10$ each time. Assuming that any values of f not shown by the table follow this same pattern, the function f is linear.

5. This table could represent a linear function because the rate of change of $p(\gamma)$ is constant. Between consecutive data points, $\Delta\gamma = -1$ and $\Delta p(\gamma) = 10$. Thus, the rate of change is $\Delta p(\gamma)/\Delta\gamma = -10$. Since this is constant, the function could be linear.

6. The function j could be linear if the pattern continues for values of x that are not shown, because we see that a one unit increase in x corresponds to a constant decrease of two units in $j(x)$.

7. The vertical intercept is 54.25, which tells us that in 1970 ($t = 0$) the population was $54,250$ (54.25 thousand) people. The slope is $-\frac{2}{7}$. Since

$$\text{Slope} = \frac{\Delta\text{population}}{\Delta\text{years}} = -\frac{2}{7},$$

we know that every seven years the population decreases by 2000 people. That is, the population decreases by 2/7 thousand per year.

8. The vertical intercept is 17.75, which tells us that the stalactite was 17.75 inches long when it was first measured. The slope is $\frac{1}{250}$. Since

$$\text{Slope} = \frac{\Delta\text{inches}}{\Delta\text{years}} = \frac{1}{250},$$

it means that, for every 250 years, the stalactite grows 1 inch. That is, the stalactite grows 1/250 inch per year.

9. The vertical intercept is -3000, which tells us that if no items are sold, the company loses $\$3000$. The slope is 0.98. Since

$$\text{Slope} = \frac{\Delta\text{profit}}{\Delta\text{number}} = \frac{0.98}{1},$$

this tells us that, for each item the company sells, their profit increases by $\$0.98$.

10. The vertical intercept is 29.99, which tells us that the company charges $\$29.99$ per month for the phone service, even if the person does not talk on the phone. The slope is 0.05. Since

$$\text{Slope} = \frac{\Delta\text{cost}}{\Delta\text{minutes}} = \frac{0.05}{1},$$

we see that, for each minute the phone is used, it costs an additional $\$0.05$.

Problems

11. Table 1.6 shows the population of the town as a function of the number of years since 2006. So, a formula is $P = 18{,}310 + 58t$.

Table 1.6

t	P
0	$18{,}310$
1	$18{,}310 + 58$
2	$18{,}310 + 2 \cdot 58$
3	$18{,}310 + 3 \cdot 58$
4	$18{,}310 + 4 \cdot 58$
$\cdots$	
t	$18{,}310 + t \cdot 58$

12. **(a)** See Table 1.7.

Table 1.7

t	0	10	20	30	40	50
$P(t)$	22	25	28	31	34	37

(b) See Figure 1.22.

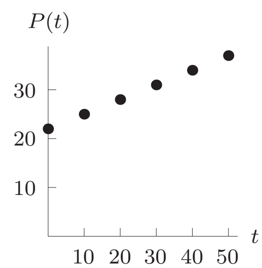

Figure 1.22

(c) From Table 1.7, we see that $P(0) = 22$, so the initial population is 22 million.

(d) The rate of change of the population is the difference in the number of people from one year to the next. The change from year 0 to year 10 is $P(10) - P(0) = 3$ million people, so the change from year 0 to year 1 is

$$\frac{P(10) - P(0)}{10 - 0} = 0.3 \text{ million people.}$$

The change from one year to the next will be the same no matter which year we choose since P is linear, so the rate of change is 0.3 million people/year.

13. The 78.9 tells us that there were approximately 79 cases on March 17. The 30.1 tells us that the number of cases increased by about 30 a day.

14. **(a)** If the relationship is linear we must show that the rate of change between any two points is the same. That is, for any two points (x_0, C_0) and (x_1, C_1), the quotient

$$\frac{C_1 - C_0}{x_1 - x_0}$$

is constant. From Table 1.27 we have taken the data $(0, 50)$, $(10, 52.50)$; $(5, 51.25)$, $(100, 75.00)$; and $(50, 62.50)$, $(200, 100.00)$.

$$\frac{52.50 - 50.00}{10 - 0} = \frac{2.50}{10} = 0.25$$
$$\frac{75.00 - 51.25}{100 - 5} = \frac{23.75}{95} = 0.25$$
$$\frac{100.00 - 62.50}{200 - 50} = \frac{37.50}{150} = 0.25$$

You can verify that choosing any one other pair of data points will give a slope of 0.25. The data are linear.

(b) The data from Table 1.27 are plotted below.

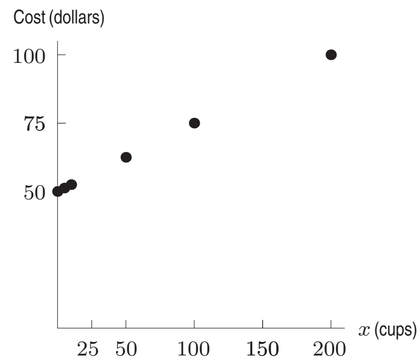

Figure 1.23

(c) Place a ruler on these points. You will see that they appear to lie on a straight line. The slope of the line equals the rate of change of the function, which is 0.25. Using units, we note that

$$\frac{\$52.50 - \$50.00}{10 \text{ cups} - 0 \text{ cups}} = \frac{\$2.50}{10 \text{ cups}} = \frac{\$0.25}{\text{cup}}.$$

In other words, the price for each additional cup of coffee is $0.25.

(d) The vendor has fixed start-up costs for this venture, i.e. cart rental, insurance, salary, etc.

15. (a) One horse costs the woodworker $5000 in start-up costs plus $350 for labor and materials, a total of $5350. Thus, if $n = 1$, we have

$$C = \underbrace{5000}_{\text{Start-up costs}} + \underbrace{350}_{\text{Extra cost for 1 horse}} = 5350.$$

Similarly, for 2 horses

$$C = \underbrace{5000}_{\text{Start-up costs}} + \underbrace{350 \cdot 2}_{\text{Extra cost for 2 horses}} = 5700,$$

and for 5 horses

$$C = \underbrace{5000}_{\text{Start-up costs}} + \underbrace{350 \cdot 5}_{\text{Extra cost for 5 horses}} = 6750.$$

Similarly, for 10 horses, $C = 5000 + 350 \cdot 10 = 8500$, and for 20 horses, $C = 12{,}000$. See Table 1.8 and Figure 1.24.

Table 1.8 *Total cost of carving n horses*

n, number of horses	C, total cost ($)
0	5000
1	5350
2	5700
5	6750
10	8500
20	12,000

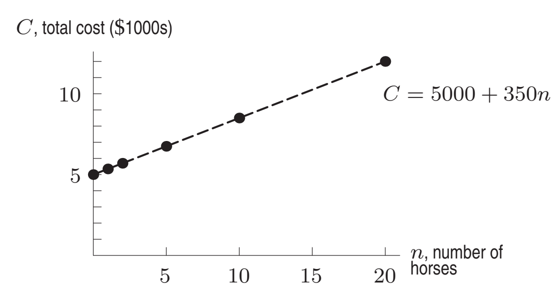

Figure 1.24: Total cost of carving n horses

Notice that it costs the woodworker $5000 to carve 0 horses since he buys the tools, plans, and advertising even if he never carves a single horse.

(b) From part (a), a formula for C, as a function of n, is

$$C = \underbrace{5000}_{\text{Start-up costs}} + \underbrace{350 \cdot n}_{\text{Extra cost for } n \text{ horses}} = 5000 + 350n.$$

(c) The average rate of change of this function is

$$\text{Rate of change} = \frac{\Delta C}{\Delta n} = \frac{\text{Change in cost}}{\text{Change in number of horses carved}}.$$

Each additional horse costs an extra \$350, so

$$\Delta C = 350 \qquad \text{if} \qquad \Delta n = 1.$$

Thus, the rate of change is given by

$$\frac{\Delta C}{\Delta n} = \frac{\$350}{1 \text{ horse}} = \$350 \text{ per horse.}$$

The rate of change of C gives the additional cost to carve one additional horse. Since the total cost increases at a constant rate (\$350 per horse), the graph of C against n is a straight line sloping upward.

16. (a) Looking at the data from Table 1.28 and calculating the rate of change of area versus side length between various points, we see that the function is not linear. For example, the rate of change between the points $(0,0)$ and $(1,1)$ is

$$\frac{\Delta\text{area}}{\Delta\text{length}} = \frac{1-0}{1-0} = \frac{1}{1} = 1$$

while the rate of change between the points $(1,1)$ and $(2,4)$ is

$$\frac{\Delta\text{area}}{\Delta\text{length}} = \frac{4-1}{2-1} = \frac{3}{1} = 3.$$

The rates of change are different. The relationship is not linear. On the other hand, when we view the data from Table 1.28, we see that the rate of change of perimeter versus side length between any two points is always constant. Thus, that function could be linear. For example, let's look at the pairs of points $(0,0)$, $(3,12)$; $(1,4)$, $(4,16)$ and $(2,8)$, $(5,20)$. For $(0,0)$, $(3,12)$ the rate of change is

$$\frac{\Delta\text{perimeter}}{\Delta\text{length}} = \frac{12-0}{3-0} = \frac{12}{3} = 4.$$

For $(1,4)$, $(4,16)$ the rate of change is

$$\frac{\Delta\text{perimeter}}{\Delta\text{length}} = \frac{16-4}{4-1} = \frac{12}{3} = 4.$$

For $(2,8)$, $(5,20)$ the rate of change is

$$\frac{\Delta\text{perimeter}}{\Delta\text{length}} = \frac{20-8}{5-2} = \frac{12}{3} = 4.$$

Check that using any two of the data points in Table 1.28 to calculate the rate of change gives a rate of change of 4.

(b) See Figures 1.25 and 1.26.

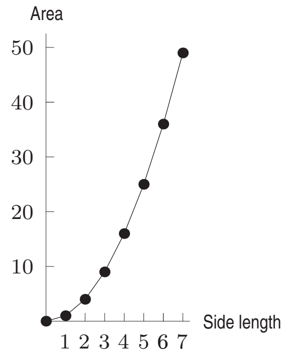

Figure 1.25: Area and side length

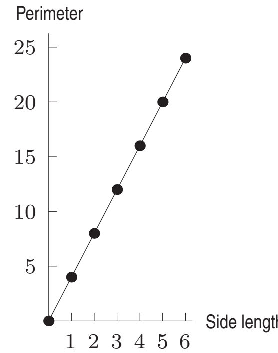

Figure 1.26: Perimeter and side length

(c) From part (a) we see that the rate of change of the function giving perimeter versus side length is 4. This tells us that for a given square, when we increase the length of each side by one unit, the length of the perimeter increases by four units.

17. We know that the area of a circle of radius r is

$$\text{Area } = \pi r^2$$

while its circumference is given by

$$\text{Circumference } = 2\pi r.$$

Thus, a table of values for area and circumference is

Table 1.9

Radius	0	1	2	3	4	5	6
Area	0	π	4π	9π	16π	25π	36π
Circumference	0	2π	4π	6π	8π	10π	12π

(a) In the area function we see that the rate of change between pairs of points does not remain constant and thus the function is not linear. For example, the rate of change between the points $(0,0)$ and $(2,4\pi)$ is not equal to the rate of change between the points $(3,9\pi)$ and $(6,36\pi)$. The rate of change between $(0,0)$ and $(2,4\pi)$ is

$$\frac{\Delta \text{area}}{\Delta \text{radius}} = \frac{4\pi - 0}{2-0} = \frac{4\pi}{2} = 2\pi$$

while the rate of change between $(3,9\pi)$ and $(6,36\pi)$ is

$$\frac{\Delta \text{area}}{\Delta \text{radius}} = \frac{36\pi - 9\pi}{6-3} = \frac{27\pi}{3} = 9\pi.$$

On the other hand, if we take only pairs of points from the circumference function, we see that the rate of change remains constant. For instance, for the pair $(0,0)$, $(1,2\pi)$ the rate of change is

$$\frac{\Delta \text{circumference}}{\Delta \text{radius}} = \frac{2\pi - 0}{1-0} = \frac{2\pi}{1} = 2\pi.$$

For the pair $(2,4\pi)$, $(4,8\pi)$ the rate of change is

$$\frac{\Delta \text{circumference}}{\Delta \text{radius}} = \frac{8\pi - 4\pi}{4-2} = \frac{4\pi}{2} = 2\pi.$$

For the pair $(1,2\pi)$, $(6,12\pi)$ the rate of change is

$$\frac{\Delta \text{circumference}}{\Delta \text{radius}} = \frac{12\pi - 2\pi}{6-1} = \frac{10\pi}{5} = 2\pi.$$

Picking any pair of data points would give a rate of change of 2π.

(b) The graphs for area and circumference as indicated in Table 1.9 are shown in Figure 1.27 and Figure 1.28.

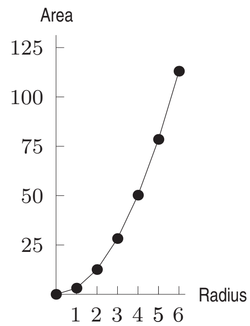

Figure 1.27

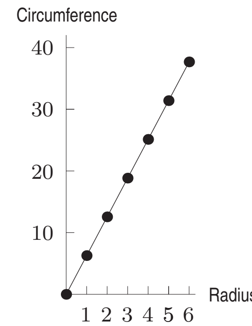

Figure 1.28

(c) From part (a) we see that the rate of change of the circumference function is 2π. This tells us that for a given circle, when we increase the length of the radius by one unit, the length of the circumference would increase by 2π units. Equivalently, if we decreased the length of the radius by one unit, the length of the circumference would decrease by 2π.

18. (a) We see that the population of Country B grows at the constant rate of roughly 2.4 million every ten years. Thus Country B must be Sri Lanka. The population of country A did not change at a constant rate: In the ten years of 1970–1980 the population of Country A grew by 2.7 million while in the ten years of 1980–1990 its population dropped. Thus, Country A is Afghanistan.

(b) The rate of change of Country B is found by taking the population increase and dividing it by the corresponding time in which this increase occurred.Thus

$$\text{Rate of change of population } = \frac{9.9 - 7.5}{1960 - 1950} = \frac{2.4 \text{ million people}}{10 \text{ years}} = 0.24 \text{ million people/year.}$$

This rate of change tells us that on the average, the population of Sri Lanka increases by 0.24 million people every year. The rate of change for the other intervals is the same or nearly the same.

(c) In 1980 the population of Sri Lanka was 14.9 million. If the population grows by 0.24 million every year, then in the eight years from 1980 to 1988

$$\text{Population increase } = 8 \cdot 0.24 \text{ million } = 1.92 \text{ million.}$$

Thus in 1988

$$\text{Population of Sri Lanka} = 14.9 + 1.92 \text{ million } \approx 16.8 \text{ million.}$$

19. (a) Any line with a slope of 2.1, using appropriate scales on the axes. The horizontal axis should be labeled ”days” and the vertical axis should be labeled ”inches.” See Figure 1.29.

(b) Any line with a slope of -1.3, using appropriate scales on the axes. The horizontal axis should be labeled ”miles” and the vertical axis should be labeled ”gallons.” See Figure 1.30.

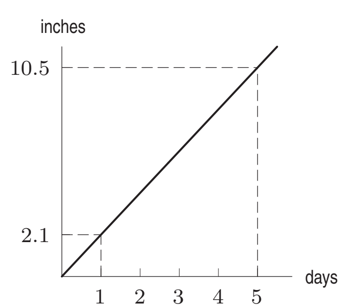

Figure 1.29

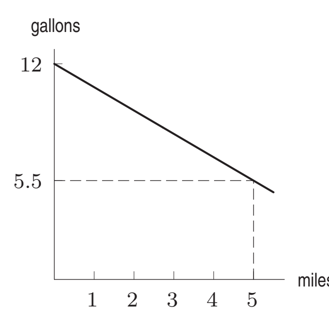

Figure 1.30

20. Since the depreciation can be modeled linearly, we can write the formula for the value of the car, V, in terms of its age, t, in years, by the following formula:

$$V = b + mt.$$

Since the initial value of the car is \$21,000, we know that $b = 21{,}000$.

Hence,

$$V = 21{,}000 + mt.$$

To find m, we know that $V = 10,500$ when $t = 3$, so

$$\begin{aligned} 10{,}500 &= 21{,}000 + m(3) \\ -10{,}500 &= 3m \\ \frac{-10{,}500}{3} &= m \\ -3500 &= m. \end{aligned}$$

So, $V = 21{,}000 - 3500t$.

21. **(a)** No. The values of $f(d)$ first drop, then rise, so f is not linear.

(b) For $d \geq 150$, the graph looks linear. See Figure 1.31.

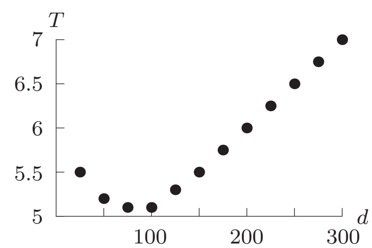

Figure 1.31

(c) For $d \geq 150$ the average rate of change appears to be constant. Each time the depth goes up by $\Delta d = 25$ meters, the temperature rises by $\Delta T = 0.25°$C, so the average rate of change is $\Delta T/\Delta d = 0.25/25 = 0.01°$C/meter. In other words, the temperature rises by $0.01°$C for each extra meter in depth.

22. **(a)** $F = 2C + 30$

(b) Since we are finding the difference for a number of values, it would perhaps be easier to find a formula for the difference:

$$\begin{aligned} \text{Difference} &= \text{Approximate value} - \text{Actual value} \\ &= (2C + 30) - \left(\frac{9}{5}C + 32\right) = \frac{1}{5}C - 2. \end{aligned}$$

If the Celsius temperature is $-5°$, $(1/5)C - 2 = (1/5)(-5) - 2 = -1 - 2 = -3$. This agrees with our results above.

Similarly, we see that when $C = 0$, the difference is $(1/5)(0) - 2 = -2$ or 2 degrees too low. When $C = 15$, the difference is $(1/5)(15) - 2 = 3 - 2 = 1$ or 1 degree too high. When $C = 30$, the difference is $(1/5)(30) - 2 = 6 - 2 = 4$ or 4 degrees too high.

(c) We are looking for a temperature C, for which the difference between the approximation and the actual formula is zero.

$$\begin{aligned} \frac{1}{5}C - 2 &= 0 \\ \frac{1}{5}C &= 2 \\ C &= 10 \end{aligned}$$

Another way we can solve for a temperature C is to equate our approximation and the actual value.

$$\begin{aligned}\text{Approximation} &= \text{Actual value}\\ 2C + 30 &= 1.8C + 32,\\ 0.2C &= 2\\ C &= 10\end{aligned}$$

So the approximation agrees with the actual formula at 10° Celsius.

23. (a) Since C is 8, we have $T = 300 + 200C = 300 + 200(8) = 1900$. Thus, taking 8 credits costs $1900.

(b) Here, the value of T is 1700 and we solve for C.

$$\begin{aligned}T &= 300 + 200C\\ 1700 &= 300 + 200C\\ 7 &= C\end{aligned}$$

Thus, $\$1,700$ is the cost of taking 7 credits.

(c) Table 1.10 is the table of costs.

Table 1.10

C	1	2	3	4	5	6	7	8	9	10	11	12
T	500	700	900	1100	1300	1500	1700	1900	2100	2300	2500	2700
$\frac{T}{C}$	500	350	300	275	260	250	243	238	233	230	227	225

(d) The largest value for C, that is, 12 credits, gives the smallest value of T/C. In general, the ratio of tuition cost to number of credits is getting smaller as C increases.

(e) This cost is independent of the number of credits taken; it might cover fixed fees such as registration, student activities, and so forth.

(f) The 200 represents the rate of change of cost with the number of credit hours. In other words, one additional credit hour costs an additional $\$200$.

24. (a) Since for each additional $\$5000$ spent the company will sell 20 more units, we have

$$m = \frac{\Delta y}{\Delta x} = \frac{20}{5000}.$$

Also, since 300 units will be sold even if no money is spent on advertising, the y–intercept, b, is 300. Our formula is

$$y = 300 + \frac{20}{5000}x = 300 + \frac{1}{250}x.$$

(b) If $x = \$25{,}000$, the number of units it sells will be

$$y = 300 + \frac{1}{250}(25000) = 300 + 100 = 400.$$

If $x = \$50{,}000$, the number of units it sells will be

$$y = 300 + \frac{1}{250}(50000) = 300 + 200 = 500.$$

(c) If $y = 700$, we need to solve for x:

$$\begin{aligned}300 + \frac{1}{250}x &= 700\\ \frac{1}{250}x &= 700 - 300 = 400\\ x &= 250 \cdot 400 = 100{,}000.\end{aligned}$$

Thus, the firm would need to spend $100,000 to sell 700 units.

(d) The slope is the change in the value of y, the number of units sold, for a given change in x, the amount of money spent on ads. Thus, an interpretation of the slope is that for each additional $\$250$ spent on ads, one additional unit is sold.

25. **(a)** The slope is $m = -0.062$, which tells us that the glacier area in the park goes down by 0.062 km^2/year. The A-intercept is $b = 16.2$, which tells us that in the year 2000, the area was 16.2 km^2.

(b) We have $f(15) = 16.2 - 0.062(15) = 15.27$, which tells us that in the year 2015, the area is predicted to be 15.27 km^2.

(c) Letting $\Delta t = 15$, we have

$$\begin{aligned} m = \frac{\Delta A}{\Delta t} &= -0.062 \\ \frac{\Delta A}{15} &= -0.062 \\ \Delta A &= -0.062(15) = -0.93\text{km}^2, \end{aligned}$$

so about 0.93 acres disappears in 15 years.

(d) We have

$$\begin{aligned} 16.2 - 0.062t &= 12 \\ -0.062t &= -4.2 \\ t &= 67.7, \end{aligned}$$

and so the glacier area will drop to 12 km^2 about midway through the year 2067.

26. In the window $-10 \le x \le 10, -10 \le y \le 10$, the graph of this equation looks like a line. By choosing a larger viewing window, however, you can see that it is not a line. For example, using $-500 \le x \le 500, -500 \le y \le 500$ produces Figure 1.32.

27. As Figure 1.33 shows, the graph of $y = 2x + 400$ does not appear in the window $-10 \le x \le 10, -10 \le y \le 10$. This is because all the corresponding y-values are between 380 and 420, which are outside this window. The graph can be seen by using a different viewing window: for example, $380 \le y \le 420$.

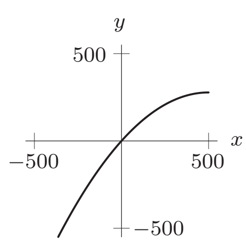

Figure 1.32

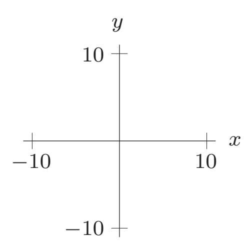

Figure 1.33

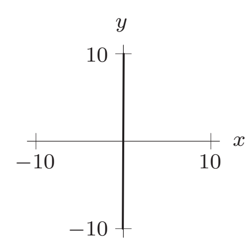

Figure 1.34

28. As Figure 1.34 shows, the graph is not visible in the window $10 \le x \le 10, -10 \le y \le 10$. The reason is that the graph of $y = 200x + 4$ is nearly vertical and almost coincides with the y-axis in this window. To see more clearly that the graph is not vertical, use a much larger y-range. For example, a window of $-10 \le x \le 10$, $-2000 \le y \le 2000$ gives a more informative graph. Alternatively, use a much smaller x-range; for example, try a window of $-0.1 \le x \le 0.1$, $-10 \le y \le 10$.

29. Most functions look linear if viewed in a small enough window. This function is not linear. We see this by graphing the function in the larger window $-100 \le x \le 100, -20 \le y \le 20$.

30. Since the radius is 10 miles, the longest ride will not be more than 20 miles. The maximum cost will therefore occur when $d = 20$, so the maximum cost is $C = 1.50 + 2d = 1.50 + 2(20) = 1.50 + 40 = 41.50$. Therefore, the window should be at least $0 \le d \le 20$ and $0 \le C \le 41.50$. See Figure 1.35.

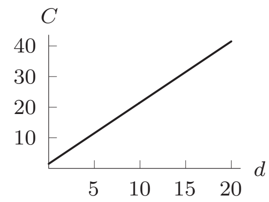

Figure 1.35

31. (a) The inequalities place b to the right of a and $f(b)$ higher than $f(a)$. Since the function is linear, the graph is a line. See Figure 1.36.

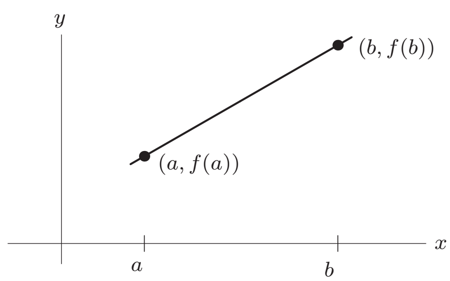

Figure 1.36

(b) The slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}.$$

32. (a) (i)

$$\begin{aligned}\frac{f(2) - f(1)}{2 - 1} &= \frac{(0.003 - (1.246(2) + 0.37)) - (0.003 - (0.1246(1) + 0.37))}{1} \\ &= -2.859 - (-1.613) = -1.246\end{aligned}$$

(ii)

$$\frac{f(1) - f(2)}{1 - 2} = \frac{-1.613 - (-2.859)}{-1} = -1.246$$

(iii)

$$\frac{f(3) - f(3)}{3 - 4} = \frac{-4.105 - (-5.351)}{-1} = \frac{1.246}{-1} = -1.246$$

(b)

$$\begin{aligned}f(x) &= 0.003 - (1.246x + 0.37) \\ &= 0.003 - 0.37 - 1.246x \\ f(x) &= -0.367 - 1.246x\end{aligned}$$

Solutions for Section 1.4

Exercises

1. Rewriting in slope-intercept form:

$$\begin{aligned} 5(x+y) &= 4 \\ 5x+5y &= 4 \\ 5y &= 4-5x \\ \frac{5y}{5} &= \frac{4}{5}-\frac{5x}{5} \\ y &= \frac{4}{5}-x \end{aligned}$$

2. Rewriting in slope-intercept form:

$$\begin{aligned} 3x+5y &= 20 \\ 5y &= 20-3x \\ y &= \frac{20}{5}-\frac{3x}{5} \\ y &= 4-\frac{3}{5}x \end{aligned}$$

3. Rewriting in slope-intercept form:

$$\begin{aligned} 0.1y+x &= 18 \\ 0.1y &= 18-x \\ y &= \frac{18}{0.1}-\frac{x}{0.1} \\ y &= 180-10x \end{aligned}$$

4. Rewriting in slope-intercept form:

$$\begin{aligned} 5x-3y+2 &= 0 \\ -3y &= -2-5x \\ y &= \frac{-2}{-3}-\frac{5}{-3}x \\ y &= \frac{2}{3}+\frac{5}{3}x \end{aligned}$$

5. Rewriting in slope-intercept form:

$$\begin{aligned} y-0.7 &= 5(x-0.2) \\ y-0.7 &= 5x-1 \\ y &= 5x-1+0.7 \\ y &= 5x-0.3 \\ y &= -0.3+5x \end{aligned}$$

6. Writing $y = 5$ as $y = 5 + 0x$ shows that $y = 5$ is the form $y = b + mx$ with $b = 5$ and $m = 0$.

7. Rewriting in slope-intercept form:

$$\begin{aligned} 3x + 2y + 40 &= x - y \\ 2y + y &= x - 3x - 40 \\ 3y &= -40 - 2x \\ y &= -\frac{40}{3} - \frac{2}{3}x \end{aligned}$$

8. Not possible, the slope is not defined (vertical line).

9. Rewriting in slope-intercept form:

$$\begin{aligned} \frac{x+y}{7} &= 3 \\ x + y &= 21 \\ y &= 21 - x \end{aligned}$$

10. Yes. Write the function as

$$g(w) = -\frac{1-12w}{3} = -\left(\frac{1}{3} - \frac{12}{3}w\right) = -\frac{1}{3} + 4w,$$

so $g(w)$ is linear with $b = -1/3$ and $m = 4$.

11. Yes. Write the function as

$$F(P) = 13 - \frac{2^{-1}}{4}P = 13 - \frac{1}{8}P = 13 + \left(\frac{-1}{8}\right)P,$$

so $F(P)$ is linear with $b = 13$ and $m = -1/8$.

12. The function is not linear because the power of s is not 1.

13. Yes. Write the function as

$$C(r) = 0 + 2\pi r,$$

so $C(r)$ is linear with $b = 0$ and $m = 2\pi$.

14. The function $h(x)$ is not linear because the 3^x term has the variable in the exponent and is not the same as $3x$ which would be a linear term.

15. Yes. Write the function as $f(x) = n^2 + m^2x$. The constant term is $b = n^2$ and the coefficient of x is m^2.

16. We have the slope $m = -4$ so

$$y = b - 4x.$$

The line passes through $(7, 0)$ so

$$\begin{aligned} 0 &= b + (-4)(7) \\ 28 &= b \end{aligned}$$

and

$$y = 28 - 4x.$$

17. We can put the slope $m = 3$ and y-intercept $b = 8$ directly into the general equation $y = b + mx$ to get $y = 8 + 3x$.

18. We first use the points to find the slope m:

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{5 - (-1)}{-1 - 2} = \frac{6}{-3} = -2.$$

Next we use the equation:

$$y = b + mx.$$

Substituting -2 for m, we have

$$y = b + (-2)x.$$

Using the point $(-1, 5)$, we have:

$$\begin{aligned} 5 &= b + (-2)(-1) \\ 5 &= b + 2 \\ 3 &= b \end{aligned}$$

so,

$$y = 3 - 2x.$$

19. We can put $m = \frac{2}{3}$ and $(x_0, y_0) = (5, 7)$ into the equation

$$\begin{aligned} y &= b + mx \\ 7 &= b + \frac{2}{3}(5) \\ \frac{21}{3} &= b + \frac{10}{3} \\ \frac{11}{3} &= b \end{aligned}$$

so

$$y = \frac{11}{3} + \frac{2}{3}x.$$

20. Since we know the x-intercept and y-intercepts are $(3, 0)$ and $(0, -5)$ respectively, we can find the slope:

$$\text{slope} = m = \frac{-5 - 0}{0 - 3} = \frac{-5}{-3} = \frac{5}{3}.$$

We can then put the slope and y-intercept into the general equation for a line.

$$y = -5 + \frac{5}{3}x.$$

21. Since the slope is $m = 0.1$, the equation is

$$y = b + 0.1x.$$

Substituting $x = -0.1$, $y = 0.02$ to find b gives

$$\begin{aligned} 0.02 &= b + 0.1(-0.1) \\ 0.02 &= b - 0.01 \\ b &= 0.03. \end{aligned}$$

The equation is $y = 0.03 + 0.1x$.

22. We have $f(0.3) = 0.8$ and $f(0.8) = -0.4$. This gives $y = b + mx$ where

$$m = \frac{f(0.8) - f(0.3)}{0.8 - 0.3} = \frac{-0.4 - 0.8}{0.5} = -2.4.$$

Solving for b, we have

$$\begin{aligned} f(0.3) &= b - 2.4(0.3) \\ b &= f(0.3) + 2.4(0.3) = 0.8 + 2.4(0.3) = 1.5, \end{aligned}$$

so $y = 1.5 - 2.4x$.

23. Two points on the graph of f are $(x_0, y_0) = (-2, 7)$ and $(x_1, y_1) = (3, -3)$. We can then find

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{-3-7}{3-(-2)} = \frac{-10}{5} = -2.$$

Therefore, $y = b - 2x$. To solve for b, we could use the point $(x_0, y_0) = (-2, 7)$:

$$\begin{aligned} 7 &= b - 2(-2) = b + 4 \\ b &= 3. \end{aligned}$$

This gives $y = f(x) = 3 - 2x$. We can check this formula by plugging in $x = 3$:

$$f(3) = 3 - 2(3) = -3.$$

This is the y-value we expected.

24. We have a V intercept of 2000. Since the value is decreasing by \$500 per year, our slope is -500 dollars per year. So one possible equation is

$$V = 2000 - 500t.$$

25. Since the function is linear, we can choose any two points to find its formula. We use the form

$$q = b + mp$$

to get the number of bottles sold as a function of the price per bottle. We use the two points $(0.50, 1500)$ and $(1.00, 500)$. We begin by finding the slope, $\Delta q/\Delta p = (500 - 1500)/(1.00 - 0.50) = -2000$. Next, we substitute a point into our equation using our slope of -2000 bottles sold per dollar increase in price and solve to find b, the q-intercept. We use the point $(1.00, 500)$:

$$\begin{aligned} 500 &= b - 2000 \cdot 1.00 \\ 2500 &= b. \end{aligned}$$

Therefore,

$$q = 2500 - 2000p.$$

26. Since the function is linear, we can use any two points to find its formula. We use the form

$$y = b + mx$$

to get temperature in $^\circ$C, y, as a function of temperature in $^\circ$F, x. We use the two points, $(32, 0)$ and $(41, 5)$. We begin by finding the slope, $\Delta y/\Delta x = (5-0)/(41-32) = 5/9$. Next, we substitute a point into our equation using our slope of $5/9^\circ$C per $^\circ$F and solve to find b, the y-intercept. We use the point $(32, 0)$:

$$\begin{aligned} 0 &= b + \frac{5}{9} \cdot 32 \\ -\frac{160}{9} &= b. \end{aligned}$$

Therefore,

$$y = -\frac{160}{9} + \frac{5}{9}x.$$

Traditionally, we give this formula as $y = (5/9)(x - 32)$, which is often easier to manipulate. You might want to check to see if the two are the same.

27. Since we are told that the function is linear, any two points will define the line for us. We will use the form

$$y = b + mx$$

to get temperature in $^\circ$ Rankine, y, as a function of temperature in $^\circ$ Fahrenheit, x. (Rankine is a rarely used absolute temperature scale.) We choose the two points, $(0, 459.7)$ and $(10, 469.7)$. We begin by finding the slope, $\Delta R/\Delta F = (469.7 - 459.7)/(10 - 0) = 10/10 = 1$. Next, we substitute a point into our equation using our slope of 1 $^\circ$R per $^\circ$F and solve to find b, the y-intercept. We use the point $(0, 459.7)$:

$$\begin{aligned} 459.7 &= b + 1 \cdot 0 \\ 459.7 &= b. \end{aligned}$$

Therefore,

$$y = 459.7 + 1x.$$

Note that each $^\circ$R is the same as each $^\circ$F, but the two systems choose different starting points (zero $^\circ$R is absolute zero, while zero $^\circ$F is an arbitrary point).

28. Since the function is linear, we can choose any two points (from the graph) to find its formula. We use the form

$$p = b + mh$$

to get the price of an apartment as a function of its height. We use the two points $(10, 175{,}000)$ and $(20, 225{,}000)$. We begin by finding the slope, $\Delta p/\Delta h = (225{,}000 - 175{,}000)/(20 - 10) = 5000$. Next, we substitute a point into our equation using our slope of 5000 dollars per meter of height and solve to find b, the p-intercept. We use the point $(10, 175{,}000)$:

$$\begin{aligned} 175{,}000 &= b + 5000 \cdot 10 \\ 125{,}000 &= b. \end{aligned}$$

Therefore,

$$p = 125{,}000 + 5000h.$$

29. Since the function is linear, we can use any two points (from the graph) to find its formula. We use the form

$$u = b + mn$$

to get the meters of shelf space used as a function of the number of different medicines stocked. We use the two points $(60, 5)$ and $(120, 10)$. We begin by finding the slope, $\Delta u/\Delta n = (10 - 5)/(120 - 60) = 1/12$. Next, we substitute a point into our equation using our slope of $1/12$ meters of shelf space per medicine and solve to find b, the u-intercept. We use the point $(60, 5)$:

$$\begin{aligned} 5 &= b + (1/12) \cdot 60 \\ 0 &= b. \end{aligned}$$

Therefore,

$$u = (1/12)n.$$

The fact that $b = 0$ is not surprising, since we would expect that, if no medicines are stocked, they should take up no shelf space.

30. Since the function is linear, we can use any two points (from the graph) to find its formula. We use the form

$$s = b + mq$$

to get the number of hours of sleep obtained as a function of the quantity of tea drunk. We use the two points $(4, 7)$ and $(12, 3)$. We begin by finding the slope, $\Delta s/\Delta q = (3 - 7)/(12 - 4) = -0.5$. Next, we substitute a point into our equation using our slope of -0.5 hours of sleep per cup of tea and solve to find b, the s-intercept. We use the point $(4, 7)$:

$$\begin{aligned} 7 &= b - 0.5 \cdot 4 \\ 9 &= b. \end{aligned}$$

Therefore,

$$s = 9 - 0.5q.$$

Problems

31. Point P is on the curve $y = x^2$ and so its coordinates are $(2, 2^2) = (2, 4)$. Since line l contains point P and has slope 4, its equation is

$$y = b + mx.$$

Using $P = (2, 4)$ and $m = 4$, we get

$$\begin{aligned} 4 &= b + 4(2) \\ 4 &= b + 8 \\ -4 &= b \end{aligned}$$

so,

$$y = -4 + 4x.$$

32. Using the point-slope form, we have $y = y_0 + m(x - x_0)$ where the slope m is given by the average rate of change of f on the interval $1 \le x \le 3$:

$$\begin{aligned} m &= \frac{\Delta f}{\Delta x} \\ &= \frac{f(3) - f(1)}{3 - 1} \\ &= \frac{1 - 5}{2} = -2. \end{aligned}$$

Letting $(x_0, y_0) = (1, f(1)) = (1, 5)$, we have

$$\begin{aligned} y &= 5 + (-2)(x - 1) \\ &= 7 - 2x. \end{aligned}$$

To verify that this line contains the other point, $(3, f(3))$, we let $x = 3$:

$$y = 7 - 2(3) = 1.$$

Since $f(3) = 1$, we see that the line $y = 7 - 2x$ is correct.

33. Using our formula for j, we have

$$\begin{aligned} h(-2) &= j(-2) = 30(0.2)^{-2} = 750 \\ h(1) &= j(1) = 30(0.2)^{1} = 6. \end{aligned}$$

This means that $h(t) = b + mt$ where

$$m = \frac{h(1) - h(-2)}{1 - (-2)} = \frac{6 - 750}{3} = -248.$$

Solving for b, we have

$$\begin{aligned} h(-2) &= b - 248(-2) \\ b &= h(-2) + 248(-2) = 750 + 248(-2) = 254, \end{aligned}$$

so $h(t) = 254 - 248t$.

34. First we find the coordinates of the squares' corners on l. Because the area of the left square is 13, the length of a side is $\sqrt{13}$. Similarly for the right square with area equal to 8, the side length is $\sqrt{8}$.

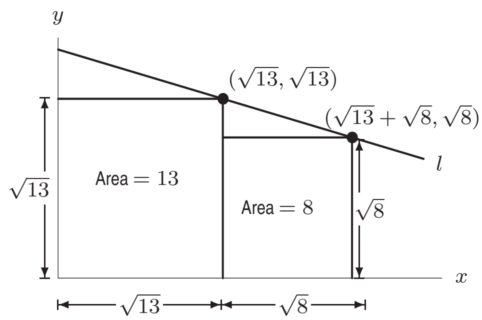

Figure 1.37

From Figure 1.37 we see that the coordinates of the corners on the line are $(\sqrt{13}+\sqrt{8}, \sqrt{8})$ and $(\sqrt{13}, \sqrt{13})$. We use this to find the slope of the line l:

$$m = \frac{\Delta y}{\Delta x} = \frac{\sqrt{8}-\sqrt{13}}{(\sqrt{13}+\sqrt{8})-\sqrt{13}} = \frac{\sqrt{8}-\sqrt{13}}{\sqrt{8}}.$$

So

$$y = b + \left(\frac{\sqrt{8}-\sqrt{13}}{\sqrt{8}}\right)x.$$

To find the y-intercept b, we put one of the known points into the equation and solve for b. Since $(\sqrt{13}, \sqrt{13})$ is on line l we have

$$\begin{aligned}
\sqrt{13} &= b + \left(\frac{\sqrt{8}-\sqrt{13}}{\sqrt{8}}\right)\sqrt{13}\\
\sqrt{13} &= b + \frac{\sqrt{8}\sqrt{13}-13}{\sqrt{8}}\\
b &= \sqrt{13} - \frac{\sqrt{8}\sqrt{13}-13}{\sqrt{8}}\\
&= \frac{\sqrt{13}\sqrt{8}}{\sqrt{8}} - \frac{\sqrt{8}\sqrt{13}-13}{\sqrt{8}}\\
&= \frac{\sqrt{13}\sqrt{8}-\sqrt{13}\sqrt{8}+13}{\sqrt{8}} = \frac{13}{\sqrt{8}}.
\end{aligned}$$

So the equation of the line is

$$y = \frac{13}{\sqrt{8}} + \left(\frac{\sqrt{8}-\sqrt{13}}{\sqrt{8}}\right)x.$$

Optionally, this can be simplified (including rationalizing denominators) to

$$y = \frac{13\sqrt{2}}{4} + \frac{4-\sqrt{26}}{4}x.$$

35. Both P and Q lie on $y = x^2+1$, so their coordinates must satisfy that equation. Point Q has x-coordinate 2, so $y = 2^2+1 = 5$. Point P has y-coordinate 8, so

$$\begin{aligned}
8 &= x^2+1\\
x^2 &= 7,
\end{aligned}$$

and $x = -\sqrt{7}$ because we know from the graph that $x < 0$.

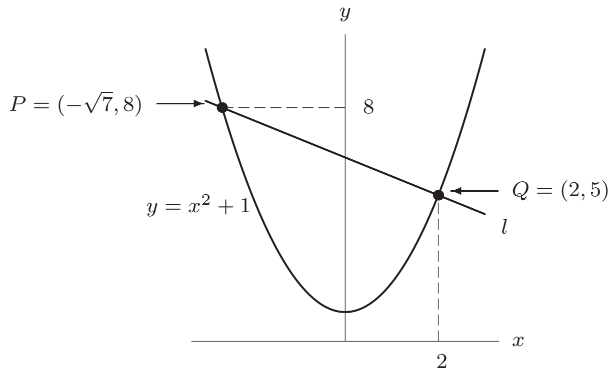

Figure 1.38

Using the coordinates of P and Q, we know that

$$m = \frac{\Delta y}{\Delta x} = \frac{5-8}{2-(-\sqrt{7})} = \frac{-3}{2+\sqrt{7}}.$$

Since $y = 5$ when $x = 2$, we have

$$\begin{aligned}
5 &= b - \frac{3}{2+\sqrt{7}} \cdot 2 \\
5 &= b - \frac{6}{2+\sqrt{7}} \\
b &= 5 + \frac{6}{2+\sqrt{7}} = \frac{5(2+\sqrt{7})}{(2+\sqrt{7})} + \frac{6}{2+\sqrt{7}} \\
&= \frac{10+5\sqrt{7}+6}{2+\sqrt{7}} = \frac{16+5\sqrt{7}}{2+\sqrt{7}}.
\end{aligned}$$

So the equation of the line is

$$y = \frac{16+5\sqrt{7}}{2+\sqrt{7}} - \frac{3}{2+\sqrt{7}}x.$$

Optionally, this can be simplified (by rationalizing denominators) to

$$y = 1 + 2\sqrt{7} + (2-\sqrt{7})x.$$

36.

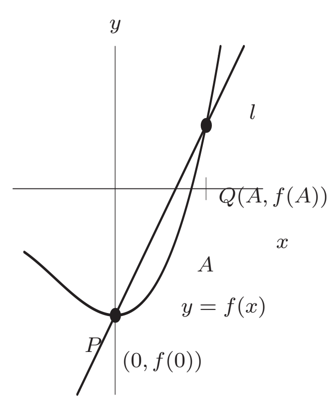

Figure 1.39

Using points $P = (0, f(0))$ and $Q = (A, f(A))$, we can find the slope of the line to be

$$m = \frac{\Delta y}{\Delta x} = \frac{f(A) - f(0)}{A - 0} = \frac{f(A) - f(0)}{A}.$$

Since the y-intercept of l is $b = f(0)$, we have

$$y = f(0) + \frac{f(A) - f(0)}{A} x.$$

37. **(a)** The results are in Table 1.11.

Table 1.11

t	0	1	2	3	4
$v = f(t)$	1000	990.2	980.4	970.6	960.8

(b) The speed of the bullet is decreasing at a constant rate of 9.8 meters/sec every second. To confirm this, calculate the rate of change in velocity over every second. We get

$$\frac{\Delta v}{\Delta t} = \frac{990.2 - 1000}{1 - 0} = \frac{980.4 - 990.2}{2 - 1} = \frac{970.6 - 980.4}{3 - 2} = \frac{960.8 - 970.6}{4 - 3} = -9.8.$$

Since the value of $\Delta v/\Delta t$ comes out the same, -9.8, for every interval, we can say that the bullet is slowing down at a constant rate. This makes sense as the constant force of gravity acts to slow the upward moving bullet down at a constant rate.

(c) The slope, -9.8, is the rate at which the velocity is changing. The v-intercept of 1000 is the initial velocity of the bullet. The t-intercept of $1000/9.8 = 102.04$ is the time at which the bullet stops moving and starts to head back to Earth.

(d) Since Jupiter's gravitational field would exert a greater pull on the bullet, we would expect the bullet to slow down at a faster rate than a bullet shot from earth. On earth, the rate of change of the bullet is -9.8, meaning that the bullet is slowing down at the rate of 9.8 meters per second. On Jupiter, we expect that the coefficient of t, which represents the rate of change, to be a more negative number (less than -9.8). Similarly, since the gravitational pull near the surface of the moon is less, we expect that the bullet would slow down at a lesser rate than on earth. So, the coefficient of t should be a less negative number (greater than -9.8 but less than 0).

38. **(a)** This can be solved by finding the formula for the line through the two points $(30, 152.50)$ and $(60, 250)$. Here is an alternate approach. The membership fee will be the same for the 30-meal and 60-meal plans, while the fee for the meals themselves will depend on the number of meals. Thus,

$$\text{Total fee} = \text{Membership fee} + \text{Number of meals} \cdot \text{Price per meal}.$$

This gives us the two equations:

$$\begin{aligned} 152.50 &= M + 30 \cdot F \\ 250.00 &= M + 60 \cdot F, \end{aligned}$$

where M is the membership fee, and F is the fixed price per meal. Subtracting the first equation from the second and solving for F gives us:

$$\begin{aligned} 97.50 &= 30 \cdot F \\ F &= \frac{97.50}{30} = 3.25. \end{aligned}$$

Now that we know the fixed price per meal, we can use either of our original equations to solve for the membership fee, M:

$$\begin{aligned} 152.50 &= M + 30 \cdot 3.25 \\ M &= 152.50 - 97.50 = 55. \end{aligned}$$

Thus, the membership fee is \$55 and the price per meal is \$3.25.

(b) The cost of a meal plan is the membership fee plus n times the cost of a meal. Using our results from part (a):

$$C = 55 + 3.25 \cdot n.$$

(c) Using our formula for the cost of a meal plan:

$$C = 55 + 3.25 \cdot n = 55 + 3.25 \cdot 50 = \$217.50.$$

(d) Rewriting our expression for the cost of a meal plan:

$$\begin{aligned} 55 + 3.25 \cdot n &= C \\ 3.25 \cdot n &= C - 55 \\ n &= \frac{C-55}{3.25}. \end{aligned}$$

(e) Given $C = \$300$ you can buy:

$$n = \frac{C-55}{3.25} = \frac{300-55}{3.25} \approx 75.38.$$

Since the college is unlikely to sell you a fraction of a meal, we round this number down. Thus, 75 is the maximum number of meals you can buy for $300.

39. (a) A table of the allowable combinations of sesame and poppy seed rolls is shown below.

Table 1.12

s, sesame seed rolls	0	1	2	3	4	5	6	7	8	9	10	11	12
p, poppy seed rolls	12	11	10	9	8	7	6	5	4	3	2	1	0

(b) The sum of s and p is 12. So we can write $s + p = 12$, or $p = 12 - s$.

(c)

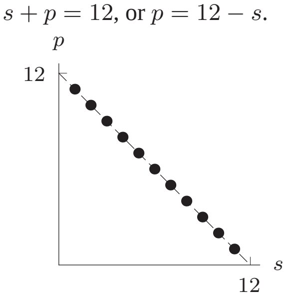

Figure 1.40

40. (a) The general equation for a line is

$$y = b + mx,$$

so we must find m, the slope, and b, the y-intercept of the line. Since we have two points on the line, we can find the slope. The coordinates of the two points are $(1324, 11328)$ and $(1529, 13275.50)$. The slope is then:

$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{13275.50 - 11328}{1529 - 1324} = \frac{1947.50}{205} = 9.50.$$

We can put our value for the slope into the general equation to get:

$$y = b + (9.50)x.$$

To find b, we use either of the points and solve for b. Using the point $(1324, 11328)$, we get

$$\begin{aligned} (11328) &= b + (9.50)(1324) \\ b &= 11328 - (9.50)(1324) = -1250. \end{aligned}$$

We now have the slope and the intercept and can use them to get the equation of the line in part (a):

$$y = -1250 + (9.50)x.$$

(b) The y-intercept of the line is the profit the movie theater makes if zero patrons attend the theater during a week. Since this number is negative, we see that the theater loses \$1250 if nobody attends. The slope of the line represents the increase in profit the theater receives for each patron. The theater makes an additional $\$9.50$ in profit per patron.

(c) To find the break-even point, we find the number of patrons, x, that makes the profit, y, equal to 0, i.e. we set y equal to zero and solve for x.

$$\begin{aligned} 0 &= -1250 + (9.50)x \\ 1250 &= (9.50)x \\ x &= \frac{1250}{9.50} \approx 131.58 \end{aligned}$$

Therefore the theater needs 132 patrons per week to break even.

(d) The equation we found in part (a) gives the profit as a function of the number of patrons. To find the number of patrons as a function of profit, we solve this equation for x in terms of y.

$$\begin{aligned} y &= -1250 + (9.50)x \\ y + 1250 &= (9.50)x \\ 9.50x &= y + 1250 \\ x &= \frac{y}{9.50} + \frac{1250}{9.50} \end{aligned}$$

(e) Putting $y = 17759.50$ into the equation found in (d), we get

$$x = \frac{17759.50}{9.50} + \frac{1250}{9.50} = 2001.$$

So 2001 patrons attended the theater.

41. (a) Since q is linear, $q = b + mp$, where

$$\begin{aligned} m &= \frac{\Delta q}{\Delta p} = \frac{65 - 45}{2.10 - 2.50} \\ &= \frac{20}{-0.40} = -50 \text{ gallons/dollar}. \end{aligned}$$

Thus, $q = b - 50p$ and since $q = 65$ if $p = 2.10$,

$$\begin{aligned} 65 &= b - 50(2.10) \\ 65 &= b - 105 \\ b &= 65 + 105 = 170. \end{aligned}$$

So,

$$q = 170 - 50p.$$

(b) The slope is $m = -50$ gallons per dollar, which tells us that the quantity of gasoline demanded in one time period decreases by 50 gallons for each \$1 increase in price.

(c) If $p = 0$ then $q = 170$, which means that if the price of gas were \$0 per gallon, then the quantity demanded in one time period would be 170 gallons per month. This means if gas were free, a person would want 170 gallons. If $q = 0$ then $170 - 50p = 0$, so $170 = 50p$ and $p = 170/50 = 3.40$. This tells us that (according to the model), at a price of $\$3.40$ per gallon there will be no demand for gasoline. In the real world, this is not likely.

42. (a) The bottle travels upward at first, and then begins to fall toward the ground. As it falls, it falls faster and faster. Negative values of v represent falling toward the ground.

(b) Notice that v decreases by 32 ft/sec each second. Since $m = \Delta v/\Delta t$,

$$m = \frac{-32}{1} = -32.$$

We have

$$v = b + mt = b - 32t,$$

and since at $t = 0$, $v = 40$, this gives

$$\begin{aligned} 40 &= b - 32 \cdot 0 \\ b &= 40 \end{aligned}$$

and so $v = 40 - 32t$.

(c) The slope is

$$\frac{\Delta v}{\Delta t} = \frac{-32 \text{ ft/sec}}{\text{sec}},$$

which tells us that the velocity of the bottle decreases by 32 ft/sec for each second elapsed.

(d) The v-axis intercept occurs when $t = 0$. If $t = 0$, then $v = 40$, which means that the bottle's initial velocity is $v = 40$ ft/sec. The t-axis intercept occurs when $v = 0$. If $v = 0$, then

$$\begin{aligned} 0 &= 40 - 32t \\ 32t &= 40 \\ t &= \frac{40}{32} = 1.25 \text{ seconds.} \end{aligned}$$

This means that at $t = 1.25$ seconds, the bottle's velocity is zero, meaning that it stopped rising and began to fall.

43. (a) If she holds no client meetings, she can hold 30 co-worker meetings. On the other hand, if she holds no co-worker meetings, she can hold 20 client meetings. A graph that describes the relationship is shown in Figure 1.41.

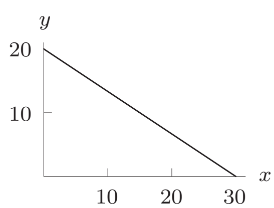

Figure 1.41

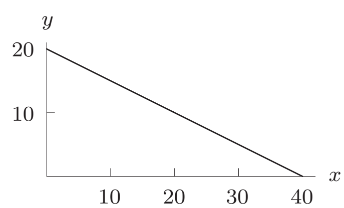

Figure 1.42

(b) Since $(0, 20)$ and $(30, 0)$ are on the line, $m = (20 - 0)/(0 - 30) = -(2/3)$. Using the slope intercept form of the line, we have $y = 20 - (2/3)x$.

(c) Since the slope is $-(2/3)$, we know that for every two additional client meetings she must sacrifice three co-worker meetings. Equivalently, for every two fewer client meetings, she gains time for three additional co-worker meetings. The x-intercept is 30. This means that she does not have time for any client meetings at all when she's scheduled 30 co-worker meetings. The y-intercept is 20. This means that she does not have time for any co-worker meetings at all when she's scheduled 20 client meetings.

(d) Instead of 2 hours, co-worker meetings now take $3/2$ hours. If all of her 60 hours are spent in co-worker meetings, she can have $60/(3/2) = 40$ co-worker meetings. The new graph is shown in Figure 1.42. The y-intercept remains at 20. However, the x-intercept is changed to 40. The slope changes, too, from $-(2/3)$ to $-(1/2)$. The new slope is still negative but is less steep because there is less of a decrease in the amount of time available for client meetings due to each extra co-worker meeting.

44. (a) The organism never matures at a development rate of $r = 0$. Solving for t, we have $b + kH_{\text{min}} = 0$, so $H_{\text{min}} = -b/k$. This is the t-intercept of the graph of r.

(b) We know that $r = 1/t$ and so $t = 1/r$. We have $S = (H - H_{\text{min}})t$, so

$$S = (H - H_{\text{min}})t = (H - H_{\text{min}})/r.$$

We know that $r = b + kH$ and, from part (a), that $H_{\text{min}} = -b/k$. We have

$$S = \frac{H - (-b/k)}{b + kH} = \frac{H + b/k}{b + kH} = \frac{k}{k} \cdot \frac{H + b/k}{b + kH} = \frac{1}{k}.$$

(c) We are told that $H_{\text{min}} = 15°$C and that at a temperature of $H = 20°$C, development takes $t = 25$ days, so the required number of degree-days is given by

$$S = (H - H_{\text{min}})t = (20 - 15)25 = 125.$$

Since S is constant, we see that at a temperature of $H = 25°$C,

$$125 = (H - H_{\text{min}})t = (25 - 15)t = 10t,$$

so the organism requires $125/10 = 12.5$ days to develop.

(d) From Table 1.13, we see that after 1 day the total number of degree-days is 5. The total rises to 12 after 2 days, to 24 after 3 days, and so on. The total reaches 126 degree-days on the eleventh day, and since the organism requires only 125 degree-days, it has reached maturity by this point.

Table 1.13 *Accumulated number of degree-days over a twelve-day period*

Day	1	2	3	4	5	6	7	8	9	10	11	12
Degree-day	5	12	24	37	49	65	79	94	107	117	126	137

45. (a) We know that $r = 1/t$. Table 1.14 gives values of r. From the table, we see that $\Delta r/\Delta H \approx 0.01/2 = 0.005$, so $r = b + 0.005H$. Solving for b, we have

$$0.070 = b + 0.005 \cdot 20$$
$$b = 0.070 - 0.1 = -0.03.$$

Thus, a formula for r is given by $r = 0.005H - 0.03$.

Table 1.14 *Development time t (in days) for an organism as a function of ambient temperature H (in °C)*

H, °C	20	22	24	26	28	30
r, rate	0.070	0.080	0.090	0.100	0.110	0.120

(b) From Problem 44, we know that if $r = b + kH$ then the number of degree-days is given by $S = 1/k$. From part (a) of this problem, we have $k = 0.005$, so $S = 1/0.005 = 200$.

46. There are many possible answers. For example, when you buy something, the amount of sales tax depends on the sticker price of the item bought. Let's say Tax $= 0.05\times$Price. This means that the sales tax rate is 5%.

Solutions for Section 1.5

Exercises

1. (a) $f(x)$ has a y-intercept of 1 and a positive slope. Thus, (ii) must be the graph of $f(x)$.
(b) $g(x)$ has a y-intercept of 1 and a negative slope. Thus, (iii) must be the graph of $g(x)$.
(c) $h(x)$ is a constant function with a y intercept of 1. Thus, (i) must be the graph of $h(x)$.

2. (a) is (V), because slope is positive, vertical intercept is negative
(b) is (IV), because slope is negative, vertical intercept is positive
(c) is (I), because slope is 0, vertical intercept is positive
(d) is (VI), because slope and vertical intercept are both negative
(e) is (II), because slope and vertical intercept are both positive
(f) is (III), because slope is positive, vertical intercept is 0
(g) is (VII), because it is a vertical line with positive x-intercept.

3. (a) Since the slopes are 2 and 3, we see that $y = -2 + 3x$ has the greater slope.
(b) Since the y-intercepts are -1 and -2, we see that $y = -1 + 2x$ has the greater y-intercept.

4. **(a)** Since the slopes are 4 and -2, we see that $y = 3 + 4x$ has the greater slope.
(b) Since the y-intercepts are 3 and 5, we see that $y = 5 - 2x$ has the greater y-intercept.

5. **(a)** Since the slopes are $\frac{1}{4}$ and -6, we see that $y = \frac{1}{4}x$ has the greater slope.
(b) Since the y-intercepts are 0 and 1, we see that $y = 1 - 6x$ has the greater y-intercept.

6. The functions f and g have the same y-intercept, $b = 20$. u and v both have y-intercept $b = 60$. f and g are increasing functions, with slopes $m = 2$ and $m = 4$, respectively. u and v are decreasing functions, with slopes $m = -1$ and $m = -2$, respectively.

The figure shows that graphs A and B describe increasing functions with the same y-intercept. The functions f and g are good candidates since they are both linear functions with positive slope and their y-intercepts coincide. Since graph A is steeper than graph B, the slope of A is greater than the slope of B. The slope of g is larger than the slope of f, so graph A corresponds to g and graph B corresponds to f.

Graphs D and E describe decreasing functions with the same y-intercept. u and v are good candidates since they both have negative slope and their y-intercepts coincide. Graph E is steeper than graph D. Thus, graph D corresponds to u, and graph E to v. Note that graphs D and E start at a higher point on the y-axis than A and B do. This corresponds to the fact that the y-intercept $b = 60$ of u and v is above the y-intercept $b = 20$ of f and g.

This leaves graph C and the function h. The y-intercept of h is -30, corresponding to the fact that graph C starts below the x-axis. The slope of h is 2, the same slope as f. Since graph C appears to climb at the same rate as graph B, it seems reasonable that f and h should have the same slope.

7. In Figure 7, we see that lines A and B both represent increasing functions with the same y-intercept. Thus, since f and h have positive slope and the same y-intercept, $b = 5$, lines A and B correspond to the functions f and h. Since line A is steeper than line B, its slope is greater. The slope of h is 3, while the slope of f is 2. Therefore, line A is $h(x) = 5 + 3x$ and line B is $f(x) = 5 + 2x$.

Line C also represents an increasing function. Furthermore, since it crosses the y-axis below the x-axis, it has a negative y-intercept. Since $g(x) = -5 + 2x$ is an increasing function with a negative y-intercept, it corresponds to line C.

Finally, lines D and E both represent decreasing functions, and so both have negative slopes. Since line E is steeper than line D, its slope is steeper—that is, more negative—than the slope of line D. Thus, line E represents $k(x) = 5 - 3x$ and line D represents $j(x) = 5 - 2x$.

8. **(a)** See Figures 1.43 and 1.44.

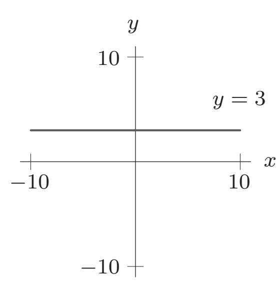

Figure 1.43

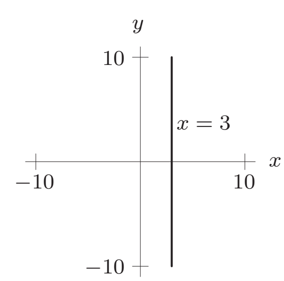

Figure 1.44

(b) Yes for $y = 3$: $y = 3 + 0x$. No for $x = 3$, since the slope is undefined, and there is no y-intercept.

9. These lines are parallel because they have the same slope, 5.

10. These lines are perpendicular because one slope, $-\frac{1}{4}$, is the negative reciprocal of the other, 4.

11. These line are parallel because they have the same slope, 2.

12. These lines are neither parallel nor perpendicular. They do not have the same slope, nor are their slopes negative reciprocals (if they were, one of the slopes would be negative).

13. These lines are neither parallel nor perpendicular. They do not have the same slopes, nor are their slopes negative reciprocals (if they were, one of the slopes would be negative).

14. Rewriting as $y = 8 - \frac{1}{2}x$ and $y = -2 - \frac{1}{2}x$ shows that these lines are parallel because their slopes are the same, $-\frac{1}{2}$.

Problems

15. Since the graph is parallel to the line $y = 20 - 4x$, the slope is the same, so $m = -4$. Using the point-slope formula with $(x_0, y_0) = (3, 12)$ we have

$$\begin{aligned} y &= 12 - 4(x - 3) \\ &= 12 - 4x + 12 \\ &= 24 - 4x. \end{aligned}$$

16. First we place $5x - 3y = 6$ into slope-intercept form:

$$\begin{aligned} 5x - 3y &= 6 \\ 3y &= -6 + 5x \\ y &= -2 + \frac{5}{3}x. \end{aligned}$$

The slope of this line is $5/3$. Since the graph of g is perpendicular to it, the slope of g is

$$m = -\frac{1}{5/3} = -\frac{3}{5}.$$

We see from the first equation that at $x = 15$,

$$y = \frac{5}{3} \cdot 15 - 2 = 25 - 2 = 23,$$

so the graph of the first equation contains the point $(15, 23)$. Since the graph of g also contains this point, we have

$$\begin{aligned} g(15) = b + m \cdot 15 &= 23 \\ b - \frac{3}{5}(15) &= 23 \\ b - 9 &= 23 \\ b &= 32, \end{aligned}$$

so $g(x) = 32 - (3/5)x$.

17. **(a)** This line, being parallel to l, has the same slope. Since the slope of l is $-\frac{2}{3}$, the equation of this line is

$$y = b - \frac{2}{3}x.$$

To find b, we use the fact that $P = (6, 5)$ is on this line. This gives

$$\begin{aligned} 5 &= b - \frac{2}{3}(6) \\ 5 &= b - 4 \\ b &= 9. \end{aligned}$$

So the equation of the line is

$$y = 9 - \frac{2}{3}x.$$

(b) This line is perpendicular to line l, and so its slope is given by

$$m = \frac{-1}{-2/3} = \frac{3}{2}.$$

Therefore its equation is

$$y = b + \frac{3}{2}x.$$

We again use point P to find b:

$$5 = b + \frac{3}{2}(6)$$
$$5 = b + 9$$
$$b = -4.$$

This gives

$$y = -4 + \frac{3}{2}x.$$

(c) Figure 1.45 gives a graph of line l together with point P and the two lines we have found.

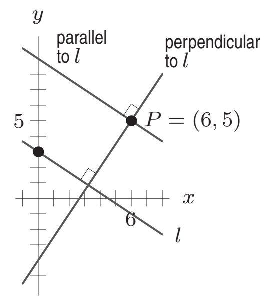

Figure 1.45: Line l and two lines through P, one parallel and one perpendicular to l

18. **(a)** $P = (a, 0)$
(b) $A = (0, b)$, $B = (-c, 0)$
$C = (a + c, b)$, $D = (a, 0)$

19. The graphs are shown in Figure 1.46.

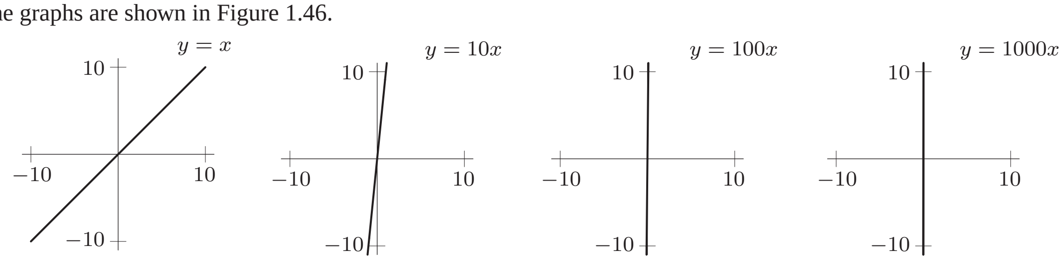

Figure 1.46

(a) As the slopes become larger, the lines become steeper, getting very close to the y-axis.
(b) We start with the equation of a line $y = b + mx$. Because the line passes through the origin, we want the graph to have a y-intercept of zero, so $b = 0$. Because the line is horizontal, we want a slope of zero, so $m = 0$. Thus, our equation is

$$y = 0.$$

20. The graphs are shown in Figure 1.47.

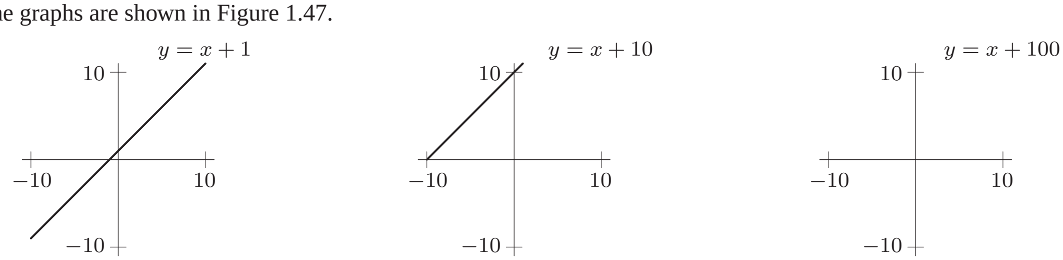

Figure 1.47

(a) As b becomes larger, the graph moves higher and higher up, until it disappears from the viewing rectangle.
(b) There are many correct answers, one of which is $y = x - 100$.

21.

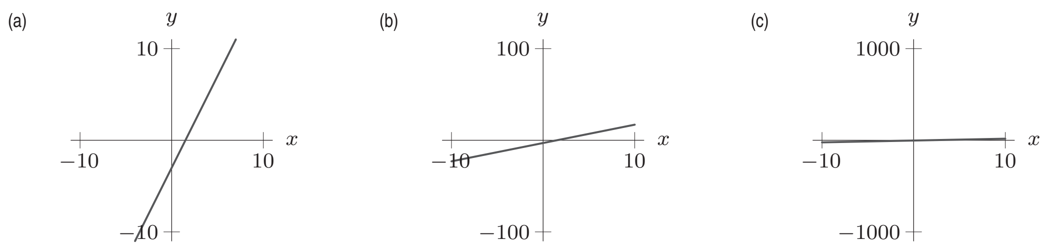

Figure 1.48

(d) If the width of the window remains constant and the height of the window increases, then the graph will appear less steep.

22. The sloping line has $m = 1$, so its equation is $y = x - 1$. The horizontal line is $y = 3$.

Solving simultaneously gives

$$3 = x - 1 \quad \text{so} \quad x = 4.$$

Thus, the point of intersection is $(4, 3)$.

23. Note that the x and y scales are different and the intercepts appear to be (0,3) and (7.5,0), giving

$$\text{Slope } = \frac{-3}{7.5} = -\frac{6}{15} = -\frac{2}{5}.$$

The y-intercept is at (0,3), so

$$y = -\frac{2}{5}x + 3$$

is a possible equation for the line (answers may vary).

24. Since P is the x-intercept, we know that point P has y-coordinate $= 0$, and if the x-coordinate is x_0, we can calculate the slope of line l using $P(x_0, 0)$ and the other given point $(0, -2)$.

$$m = \frac{-2 - 0}{0 - x_0} = \frac{-2}{-x_0} = \frac{2}{x_0}.$$

We know this equals 2, since l is parallel to $y = 2x + 1$ and therefore must have the same slope. Thus we have

$$\frac{2}{x_0} = 2.$$

So $x_0 = 1$ and the coordinates of P are $(1, 0)$.

25. We see in the figure from the problem that line l_2 is perpendicular to line l_1. We can find the slope of line l_1 because we are given the x-intercept $(3, 0)$ and the y-intercept $(0, 2)$.

$$m_1 = \frac{2 - 0}{0 - 3} = \frac{2}{-3} = \frac{-2}{3}$$

Therefore, we know that the slope of line l_2 is

$$m_2 = \frac{-1}{\frac{-2}{3}} = \frac{3}{2}$$

We also know that l_2 passes through the origin $(0, 0)$ and therefore has a y-intercept of zero.

Hence, the equation of l_2 is

$$y = \frac{3}{2}x.$$

26. Writing this equation as

$$y = \underbrace{\frac{1}{\beta - 3}}_{m} \cdot x + \underbrace{\frac{1}{6 - \beta}}_{b},$$

we see that $m = 1/(\beta - 3)$ and $b = 1/(6 - \beta)$. For the slope to be positive, we require the denominator of m to be positive, so $\beta > 3$. For the y-intercept to be positive, we require the denominator of b to be positive, so $\beta < 6$. Putting together these two requirements gives $3 < \beta < 6$.

27. Writing this equation as

$$y = \underbrace{\frac{1}{\beta - 3}}_{m} \cdot x + \underbrace{\frac{1}{6 - \beta}}_{b},$$

we see that $m = 1/(\beta - 3)$ and $b = 1/(6 - \beta)$. The line $y = (\beta - 7)x - 3$ has slope $m_1 = \beta - 7$. For these two lines to be perpendicular, we require

$$\begin{aligned} m \cdot m_1 &= -1 \\ \frac{1}{\beta - 3} \cdot (\beta - 7) &= -1 \\ \beta - 7 &= -(\beta - 3) \\ \beta - 7 &= -\beta + 3 \\ 2\beta &= 10 \\ \beta &= 5. \end{aligned}$$

We can verify this by confirming that the resulting equations describe perpendicular lines:

$$\begin{aligned} y &= \frac{1}{\beta - 3} \cdot x + \frac{1}{6 - \beta} && \text{First equation} \\ &= \frac{1}{5 - 3} \cdot x + \frac{1}{6 - 5} \\ &= \frac{1}{2} \cdot x + 1 \\ y &= (\beta - 7)x - 3 && \text{Second equation} \\ &= (5 - 7)x - 3 \\ &= -2x - 3. \end{aligned}$$

Since the slopes are negative reciprocals, the lines are perpendicular, as required.

28. **(a)** Use the point-slope formula:

$$y - 0 = \frac{\sqrt{3}}{-1}(x - 0),$$

so $y = -\sqrt{3}x$.

(b) The slope of the tangent line is the negative reciprocal of $-\sqrt{3}$, so $m = \dfrac{1}{\sqrt{3}}$, and

$$y - \sqrt{3} = \frac{1}{\sqrt{3}}(x - (-1)),$$

or

$$y = \frac{1}{\sqrt{3}}x + \frac{4}{\sqrt{3}}.$$

29. The altitude is perpendicular to the side $\overline{BC}$. The slope of $\overline{BC}$ is

$$\frac{8-2}{9-(-3)} = \frac{6}{12} = \frac{1}{2}.$$

Therefore the slope of the altitude is the negative reciprocal of $1/2$, which is -2, and it passes through the point $(-4, 5)$. Using the point-slope formula, we find the equation $y - 5 = -2(x + 4)$, or $y = -2x - 3$.

30. **(a)** After one year, the value of the Frigbox refrigerator is $\$950-\$50 = \$900$; after two years, its value is $\$950-2\cdot\$50 = \$850$; after t years, the value, V, of the Frigbox is given by

$$V = 950 - t\cdot 50 \quad \text{or} \quad V = 950 - 50t.$$

Similarly, after t years, the value of the ArcticAir refrigerator is

$$V = 1200 - 100t.$$

The two refrigerators have equal value when

$$\begin{aligned} 950 - 50t &= 1200 - 100t \\ -250 &= -50t \\ 5 &= t. \end{aligned}$$

In five years the two refrigerators have equal value.

(b) According to the formula, in 20 years time, the value of the Frigbox refrigerator will be

$$\begin{aligned} V &= 950 - 50(20) \\ &= 950 - 1000 = -50 \end{aligned}$$

This negative value is not realistic, so after some time, the linear model is no longer appropriate. Similarly, the value of the ArcticAir refrigerator is predicted to be $V = 1200 - 100(20) = 1200 - 2000 = -800$, which is also not realistic.

31. **(a)** The three formulas are linear with b being the fixed rate and m being the cost per mile. The formulas are,

$$\begin{aligned} \text{Company A} &= 20 + 0.2x \\ \text{Company B} &= 35 + 0.1x \\ \text{Company C} &= 70. \end{aligned}$$

(b)

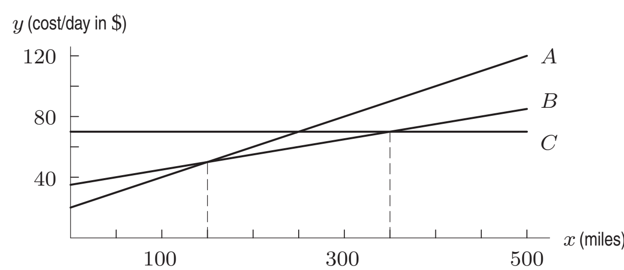

Figure 1.49

(c) The slope is the rate charged for each mile, and its units are dollars per mile. The vertical intercept is the fixed cost—what you pay for renting the car for a day, not considering mileage charges.

(d) By reading Figure 1.49 we see *A* is cheapest if you drive less than 150 miles; *B* is cheapest if you drive between 150 and 350 miles; *C* is cheapest if you drive more than 350 miles. We would expect A to be the cheapest for a small number of miles since it has the lowest fixed rate and C to be the cheapest for a large number of miles since it does not charge per mile.

32. (a) Company A charges $\$0.37$ per minute. So, the cost with company A is simply 0.37 times the number of minutes, or

$$Y_A = 0.37x.$$

Company B charges $\$13.95$ per month plus $\$0.22$ per minute. So, the cost for company B is

$$Y_B = 13.95 + 0.22x.$$

Company C charges a fixed rate of $\$50$ per month. So,

$$Y_C = 50.$$

(b) Using the fixed costs for each company – $\$0$ for company A, $\$13.95$ for company B, and $\$50$ for company C – we know that Y_A goes through the origin, that Y_B goes through the point $(0, 13.95)$ and that Y_C goes through $(0, 50)$. We also know that Y_A has a rate of change of 0.35, Y_B has a rate of change of 0.22 and Y_C is a constant function. So, Y_A has the steepest slope and Y_C has a slope of zero. So, we can label the graphs as follows:

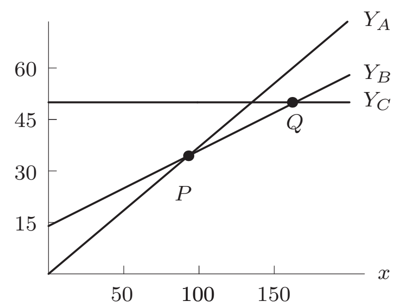

Figure 1.50

(c) From Figure 1.50 we see that between the points P and Q, the graph of Y_B is below the graphs of Y_A and Y_C. This means that company B is the cheapest in the interval between P and Q. We need to find the x-coordinates of P and Q.

To find the x-coordinate of point P, we note that the graphs of Y_A and Y_B intersect at P. So, we can set the formulas for Y_A and Y_B equal to each other and solve for x.

$$\begin{aligned} Y_A &= Y_B \\ 0.37x &= 13.95 + 0.22x \\ 0.15x &= 13.95 \\ x &= \frac{13.95}{0.15} = 93 \end{aligned}$$

Therefore, for $x > 93$ minutes, the graph of Y_B is below the graph of Y_A, meaning that company B is cheaper than company A.

To find the x-coordinate of point Q, we note that Q is the intersection of the graphs of Y_B and Y_C. By setting the formulas for Y_B and Y_C equal to each other, we can solve for x.

$$\begin{aligned} Y_B &= Y_C \\ 13.95 + 0.22x &= 50 \\ 0.22x &= 50 - 13.95 \\ 0.22x &= 36.05 \\ x &= \frac{36.05}{0.22} \approx 163.86 \end{aligned}$$

Thus, for $x \leq 163$ minutes, company B is cheaper than company C.

Putting these two results together, we conclude that company B is the cheapest for values of x between 93 and 163 minutes.

33. (a) We are looking at the amount of municipal solid waste, W, as a function of year, t, and the two points are $(1960, 88.1)$ and $(2000, 234)$. For the model, we assume that the quantity of solid waste is a linear function of year. The slope of the line is

$$m = \frac{234 - 88.1}{2000 - 1960} = \frac{145.9}{40} = 3.65 \frac{\text{millions of tons}}{\text{year}}.$$

This slope tells us that the amount of solid waste generated in the cities of the US has been going up at a rate of 3.65 million tons per year. To find the equation of the line, we must find the vertical intercept. We substitute the point $(1960, 88.1)$ and the slope $m = 3.65$ into the equation $W = b + mt$:

$$\begin{aligned} W &= b + mt \\ 88.1 &= b + (3.65)(1960) \\ 88.1 &= b + 7154 \\ -7065.9 &= b. \end{aligned}$$

The equation of the line is $W = -7065.9 + 3.65t$, where W is the amount of municipal solid waste in the US in millions of tons, and t is the year.

(b) How much solid waste does this model predict in the year 2020? We can graph the line and find the vertical coordinate when $t = 2020$, or we can substitute $t = 2020$ into the equation of the line, and solve for W:

$$\begin{aligned} W &= -7065.9 + 3.65t \\ W &= -7065.9 + (3.65)(2020) = 307.1. \end{aligned}$$

The model predicts that in the year 2020, the solid waste generated by cities in the US will be 307.1 million tons.

34. When $x = 1$, $y = \sqrt{1} = 1$, and when $x = 4$, $y = \sqrt{4} = 2$, so the points of intersection are $(1, 1)$ and $(4, 2)$. See Figure 1.51.

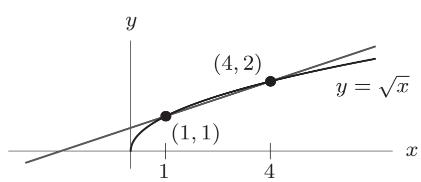

Figure 1.51

The line connecting $(1, 1)$ and $(4, 2)$ has slope $m = \frac{2-1}{4-1} = \frac{1}{3}$. To find the y-intercept, we can substitute one of the points, for example, $x = 1,\ y = 1$:

$$\begin{aligned} y &= \frac{1}{3}x + b \\ 1 &= \frac{1}{3}(1) + b \\ b &= \frac{2}{3} \end{aligned}$$

The equation of the line is $y = \frac{1}{3}x + \frac{2}{3}$. Now we'll solve the system

$$\begin{aligned} y &= \sqrt{x} \\ y &= \frac{1}{3}x + \frac{2}{3} \end{aligned}$$

by setting the equations equal to each other:

$$\sqrt{x} = \frac{1}{3}x + \frac{2}{3}.$$

Squaring both sides gives

$$\begin{aligned} x &= \left(\frac{1}{3}x+\frac{2}{3}\right)^2 \\ x &= \frac{1}{9}x^2+\frac{4}{9}x+\frac{4}{9} \\ \frac{1}{9}x^2-\frac{5}{9}x+\frac{4}{9} &= 0 \\ x^2-5x+4 &= 0 \quad \text{after multiplying both sides by 9} \\ (x-4)(x-1) &= 0 \\ x=4 \ \text{or}\ x &= 1 \end{aligned}$$

When $x = 4$, $y = \sqrt{4} = 2$, giving the point $(4, 2)$. When $x = 1$, $y = \sqrt{1} = 1$, giving the point $(1, 1)$. The results are consistent with the original problem

35. **(a)** To have no points in common the lines will have to be parallel and distinct. To be parallel their slopes must be the same, so $m_1 = m_2$. To be distinct we need $b_1 \neq b_2$.
(b) To have all points in common the lines will have to be parallel and the same. To be parallel their slopes must be the same, so $m_1 = m_2$. To be the same we need $b_1 = b_2$.
(c) To have exactly one point in common the lines will have to be nonparallel. To be nonparallel their slopes must be distinct, so $m_1 \neq m_2$.
(d) It is not possible for two lines to meet in just two points.

36. **(a)** Since the first option remains constant, an equation describing it is $y_1 = 100$. The second option has a base of 50 and increases at a rate of 10% for every dollar in sales. An equation is: $y_2 = 50 + 0.10x$. The break-even point is found by setting the two expressions equal to each other and solving:

$$100 = 50 + 0.10x,$$

so $x = 500$. The first option is better if the person has sales below $\$500$, and the second option is better if the person has sales above $\$500$.
(b) The first option has a base of 175 and increases at a rate of 7% for every dollar in sales. The second option also has a base of 175, but increases at a rate of 8% for every dollar in sales. Since both options have the same starting point, and the second option increases at a faster rate, it is better. Thus, choose the second option.
(c) The first option has a base of 145 and increases at a rate of 7% for every dollar in sales. The second option has a base of 165 and also increases at a rate of 7% for every dollar in sales. Since the two options have different starting points, but increase at the same rate, the one with the higher starting point is a higher salary. Thus, choose the second option.
(d) The first option has a base of $\$225$ and increases at a rate of 3% for every dollar in sales. An equation describing it is $y_1 = 225 + 0.03x$ The second option has a base of 180 and increases at a rate of 6% for every dollar in sales. An equation is: $y_2 = 180 + 0.06x$. The break-even point is found by setting the two equations equal to each other and solving:

$$225 + 0.03x = 180 + 0.06x,$$

so $x = 1500$. The first option is better if the person has sales below $\$1500$, and the second option is better if the person has sales above $\$1500$.

Solutions for Section 1.6

1. **(a)** Since the points lie on a line of positive slope, $r = 1$.
(b) Although the points do not lie on a line, they are tending upward as x increases. So, there is a positive correlation and a reasonable guess is $r = 0.7$.
(c) The points are scattered all over. There is neither an upward nor a downward trend, so there is probably no correlation between x and y, so $r = 0$.
(d) These points are very close to lying on a line with negative slope, so the best correlation coefficient is $r = -0.98$.
(e) Although these points are quite scattered, there is a downward slope, so $r = -0.25$ is probably a good answer.
(f) These points are less scattered than those in part (e). The best answer here is $r = -0.5$.

2. **(a)** The points are graphed in Figure 1.52.
(b) See Figure 1.52.
(c) Since the points all seem to lie on a line, the correlation coefficient is close to one.

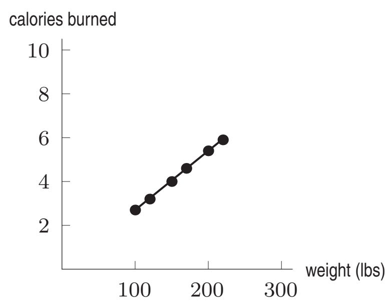

Figure 1.52

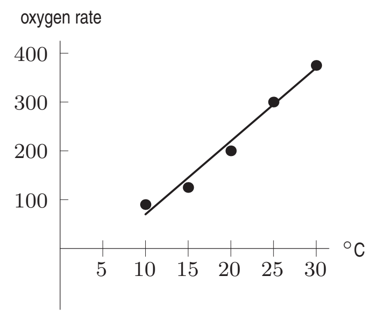

Figure 1.53

3. **(a)** See Figure 1.53.
(b) Estimates vary, but should be roughly $y = 15x - 80$.
(c) A calculator gives $y = 15x - 80$. Without it, results will vary.
(d) The slope of 15 tells us that for every rise of one degree, the consumption rate increases by 15. The horizontal intercept value (5.33) tells us the temperature when the rate of consumption is 0 (the beetle stops breathing). The vertical intercept value would give us the oxygen rate at $0°$C (freezing) but in this case a negative value $(-80°)$ tells us that the model breaks down and is not valid for cold temperatures.
(e) There is a strong positive correlation $(r \approx 0.99)$.

4. **(a)** See Figure 1.54.
(b) The scatterplot suggests that as IQ increases, the number of hours of TV viewing decreases. The points, though, are not close to being on a line, so a reasonable guess is $r \approx -0.5$.
(c) A calculator gives the regression equation $y = 27.5139 - 0.1674x$ with $r = -0.5389$.

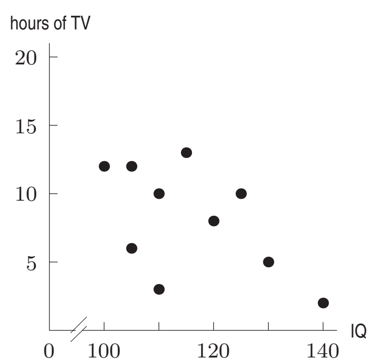

Figure 1.54

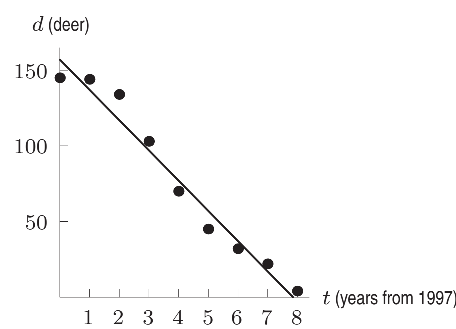

Figure 1.55

5. **(a)** See Figure 1.55.
(b) Estimates vary, but should be roughly $d = -20t + 157$.
(c) A calculator gives $d = -20t + 157$.
(d) The slope of -20 tells us that, on average, 20 deer die per year. The vertical intercept value is the initial population and should be close to 145. The horizontal intercept is the number of years until all the deer have died.
(e) There is a strong negative correlation, $(r \approx -0.98)$.

6. (a) See Figure 1.56.

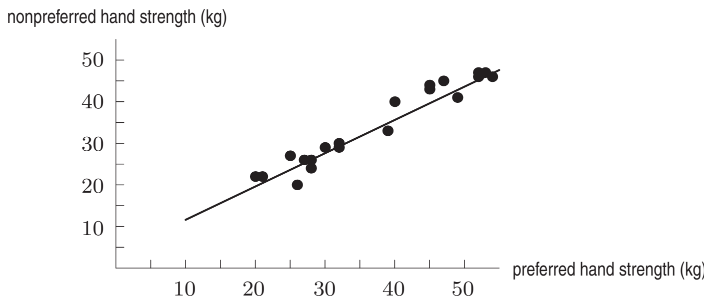

Figure 1.56

(b) Answers vary, but should be close to $y = 3.6 + 0.8x$.

(c) Answers may vary slightly. A possible equation is: $y = 3.623 + 0.825x$.

(d) The preferred hand strength is the independent quantity, so it is represented by x. Substituting $x = 37$ gives

$$y = 3.623 + 0.825(37) \approx 34.$$

So, the nonpreferred hand strength is about 34 kg.

(e) If we predict strength of the nonpreferred hand based on the strength of the preferred hand for values within the observed values of the preferred hand (such as 37), then we are interpolating. However, if we chose a value such as 10, which is below all the actual measurements, and use this to predict the nonpreferred hand strength, then we are extrapolating. Predicting from a value of 100 would be another example of extrapolation. In this case of hand strength, it seems safe to extrapolate; in other situations, extrapolation can be inaccurate.

(f) The correlation coefficient is positive because both hand strengths increase together, so the line has a positive slope. The value of r is close to 1 because the hand strengths lie close to a line of positive slope.

(g) The two clusters suggest that there are two distinct groups of students. These might be men and women, or perhaps students who are involved in college athletics (and therefore in excellent physical shape) and those who are not involved.

7. (a) We would expect a race of length 0 meters to take 0 seconds for both men and women, so we should insert a column of zeros at the start of the table.

(b) Figure 1.57 shows the men's and women's times against distance together with the regression lines $t = 0.587d - 9.069$ (men) and $t = 0.637d - 8.260$ (women). The slopes represent the average change in record time per meter increase in the length of the race. The women's line is steeper than the men's. This means that for a given increase in race length, the women's record time increases by more than the men's. The vertical intercepts are -9.069 and -8.260 seconds. We expect the vertical intercept to represent an estimate of the world record for swimming 0 meters. However, the practical interpretation of the fact that the intercepts are negative is that the model is not valid for $d = 0$.

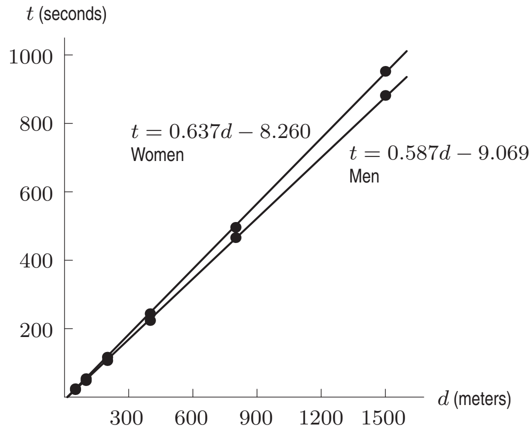

Figure 1.57: The men's and women's times against distance and the lines $t = 0.587d - 9.069$ and $t = 0.637d - 8.260$

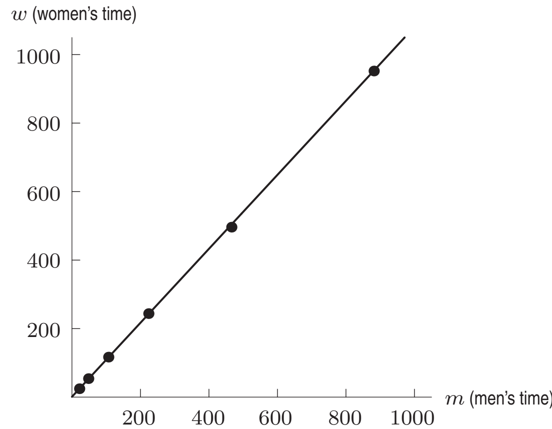

Figure 1.58: The men's and women's times and the line $w = 1.085m + 1.603$

(c) Suppose m = men's time and w = women's time. From part (b) we found $t = 0.587d - 9.069$ (men) and $t = 0.637d - 8.260$ (women), so that $m = 0.587d - 9.069$ and $w = 0.637d - 8.260$. We solve the first equation for d, obtaining $d = (m + 9.069)/0.587$, and substitute in the second to find $w = 0.637(m + 9.069)/0.587 - 8.260 = 1.085m + 1.603$. The slope is 1.085 and represents the average change in women's times per unit change in men's times. Figure 1.58 shows the men's and women's times and the line $w = 1.085m + 1.603$. Another way to say this is that the changes in women's times are 108.5% of changes in the men's. However this does not mean that women's times are 108.5% of men's times (only that *changes* are). For large times (that is, large enough to make the 1.603 negligible), women's times are 108.5% of men's times. Thus

$$952.10 \text{ is } 108.866\% \text{ of } 874.56$$

but

$$24.13 \text{ is } 111.506\% \text{ of } 21.64.$$

Thus the reporter's statement that the women's records are about 8% higher than the men's—that is, 108% of the men's—is approximately correct for large distances, but not for small.

The vertical intercept is 1.603 seconds. This would mean that 1.603 seconds would be the women's world record for swimming a race that took the men 0 seconds to swim. The practical interpretation is that the linear model is not valid for $m = 0$.

8. (a) Figure 1.59 shows the data.

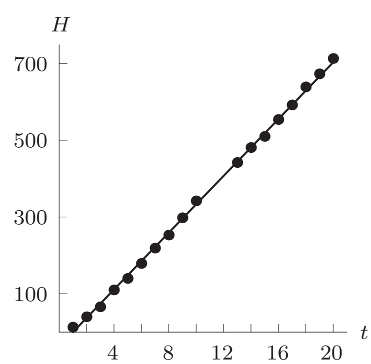

Figure 1.59: Aaron's home-run record from 1954 to 1973

(b) Estimates will vary but the equation $H = 37t - 37$ is typical.

(c) A calculator gives $H = 37.26t - 39.85$, with correlation coefficient $r = 0.9995$, which rounds to $r = 1$. The data set lies very close indeed to the regression line, which has a positive slope. In other words, Aaron's home-runs grew at a constant rate over his career.

(d) The slope gives the average number of home-runs per year, about 37.

(e) From the answer to part (d) we expect Henry Aaron to hit about 37 home-runs in each of the years 1974, 1975, 1976, and 1977. However, the knowledge that Aaron retired in 1976 means that he scored 0 home-runs in 1977. Also, people seldom retire at the peak of their abilities, so it is likely that Aaron's performance dropped off in the last few years. In fact he scored 20, 12, and 10 home-runs in the years 1974, 1975, and 1976, well below the average of 37.

Solutions for Chapter 1 Review

Exercises

1. Any w value can give two z values. For example, if $w = 0$,

$$\begin{aligned} 7 \cdot 0^2 + 5 &= z^2 \\ \pm\sqrt{5} &= z, \end{aligned}$$

so there are two z values (one positive and one negative) for $w = 0$. Thus, z is not a function of w.

A similar argument shows that w is not a function of z.

2. Here, y is a function of x, because any particular x value gives one and only one y value. For example, if we input the constant a as the value of x, we have $y = a^4 - 1$, which is one particular y value.

However, some values of y lead to more than one value of x. For example, if $y = 15$, then $15 = x^4 - 1$, so $x^4 = 16$, giving $x = \pm 2$. Thus, x is not a function of y.

3. Here, m is a function of t. For any t, there is only one possible value of m. In addition, for any m, there is only one possible value of t, given by $t = m^2$. Thus, t is a function of m.

4. At the two points where the graph breaks (marked A and B in Figure 1.60), there are two y values for a single x value. The graph does not pass the vertical line test. Thus, y is not a function of x.

Similarly, x is not a function of y because there are many y values that give two x values (For example, $y = 0$.)

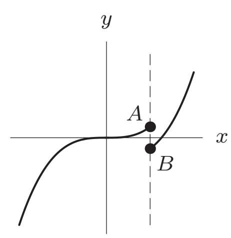

Figure 1.60

5. Both of the relationships are functions because any quantity of gas determines the quantity of coffee that can be bought, and vice versa. For example, if you buy 30 gallons of gas, spending \$60, you buy 4 pounds of coffee.

6. We take the change in number, n, of CDs divided by the change in time, t years, to determine the average rate of change.

(a) $\dfrac{\Delta n}{\Delta t} = \dfrac{120 - 40}{2008 - 2005} = \dfrac{80}{3}$, so the average rate of change is $80/3$ CDs per year.

(b) $\dfrac{\Delta n}{\Delta t} = \dfrac{40 - 120}{2012 - 2008} = \dfrac{-80}{4} = -20$, so the average rate of change is -20 CDs per year. That is, you have on average 20 fewer CDs per year.

(c) $\dfrac{\Delta n}{\Delta t} = \dfrac{40 - 40}{2012 - 2005} = \dfrac{0}{7} = 0$, so the average rate of change is zero CDs per year.

7. (a) Between $(1,4)$ and $(2,13)$,

$$\text{Average rate of change } = \frac{\Delta y}{\Delta x} = \frac{13-4}{2-1} = 9.$$

(b) Between (j,k) and (m,n),

$$\text{Average rate of change } = \frac{\Delta y}{\Delta x} = \frac{n-k}{m-j}.$$

(c) Between $(x, f(x))$ and $(x+h, f(x+h))$,

$$\begin{aligned}\text{Average rate of change } = \frac{\Delta y}{\Delta x} &= \frac{(3(x+h)^2+1)-(3x^2+1)}{(x+h)-x} = \frac{(3(x^2+2xh+h^2)+1)-(3x^2+1)}{h}\\ &= \frac{3x^2+6xh+h^2+1-3x^2-1}{h} = \frac{6xh+h^2}{h} = 6x+h.\end{aligned}$$

8. This table could not represent a linear function, because the rate of change of $q(\lambda)$ is not constant. Consider the first three points in the table. Between $\lambda = 1$ and $\lambda = 2$, we have $\Delta\lambda = 1$ and $\Delta q(\lambda) = 2$, so the rate of change is $\Delta q(\lambda)/\Delta\lambda = 2$. Between $\lambda = 2$ and $\lambda = 3$, we have $\Delta\lambda = 1$ and $\Delta q(\lambda) = 4$, so the rate of change is $\Delta q(\lambda)/\Delta\lambda = 4$. Thus, the function could not be linear.

9. This table could represent a linear function, because, for the values shown, the rate of change of $a(t)$ is constant. For the given data points, between consecutive points, $\Delta t = 3$, and $\Delta a(t) = 2$. Thus, in each case, the rate of change is $\Delta a(t)/\Delta t = 2/3$. Since the rate of change is constant, the function could be linear.

10. The function g is linear, and its formula can be written as $y = b + mx$. We can find m using any pair of data points from the table. For example, letting $(x_0, y_0) = (200, 70)$ and $(x_1, y_1) = (230, 68.5)$, we have

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}\\ &= \frac{68.5-70}{230-200} = \frac{-1.5}{30} = -0.05.\end{aligned}$$

Alternatively, we could have picked any pair of data points such as $(x_0, y_0) = (400, 60)$ and $(x_1, y_1) = (300, 65)$. This gives

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}\\ &= \frac{65-60}{300-400} = \frac{5}{-100} = -0.05,\end{aligned}$$

the same answer as before.

Having found that the slope is $m = -0.05$, we know that the equation is of the form $y = b - 0.05x$. We still need to find b. The value of $g(0)$ is not given in the table but we don't need it. We can use any data point in the table to determine b. For example, we know that $y = 70$ if $x = 200$. Using our equation, we have

$$\begin{aligned} 70 &= b - 0.05 \cdot 200\\ 70 &= b - 10\\ b &= 80.\end{aligned}$$

You can check for yourself to see that any other data point in the table gives the same value of b. Thus, the formula for g is $g(x) = 80 - 0.05x$.

11. We know that the function $f(t)$ is linear, so knowing the coordinates of two data points from the table gives us sufficient information to determine the formula. Let's use the points $(1.2, 0.736)$ and $(1.4, 0.492)$. We know that the slope is given by

$$m = \frac{0.492 - 0.736}{1.4 - 1.2} = -1.22.$$

We must now solve for the y-intercept, b. We know that the function is of the form

$$f(t) = b - 1.22t.$$

Using the point $(1.4, 0.492)$, we get

$$\begin{aligned} 0.492 &= b - 1.22(1.4) \\ 0.492 &= b - 1.708 \end{aligned}$$

giving us

$$b = 2.2$$

and

$$f(t) = 2.2 - 1.22t.$$

12. We know that the function is linear so it is of the form $f(t) = b + mt$. We can choose any two points to find the slope. We use $(5.4, 49.2)$ and $(5.5, 37)$, so

$$m = \frac{37 - 49.2}{5.5 - 5.4} = -122.$$

Thus $f(t)$ is of the form $f(t) = b - 122t$. Substituting the coordinates of the point $(5.5, 37)$ we get

$$37 = b - 122 \cdot 5.5.$$

In other words,

$$b = 37 + 122 \cdot 5.5 = 708.$$

Thus

$$f(t) = 708 - 122t.$$

13. **(a)** Since the slopes are 2 and -15, we see that $y = 7 + 2x$ has the greater slope.
(b) Since the y-intercepts are 7 and 8, we see that $y = 8 - 15x$ has the greater y-intercept.

14. **(a)** Since the slopes are -2 and -3, we see that $y = 5 - 2x$ has the greater slope.
(b) Since the y-intercepts are 5 and 7, we see that $y = 7 - 3x$ has the greater y-intercept.

15. These lines are neither parallel nor perpendicular. They do not have the same slope, nor are their slopes negative reciprocals (if they were, one of the slopes would be negative).

16. These lines are perpendicular because one slope, $-\frac{1}{14}$, is the negative reciprocal of the other, 14.

17. These lines are perpendicular because one slope, $-\frac{1}{3}$, is the negative reciprocal of the other, 3.

18. Rewriting as $y = \frac{8}{7} + 3x$ and $y = \frac{77}{9} - \frac{1}{3}x$ shows that these lines are perpendicular. The slope of the first is 3, and the slope of the second is $-\frac{1}{3}$. These slopes are negative reciprocals of each other.

Problems

19. We have $f(-3) = -8$ and $f(5) = -20$. This gives $f(x) = b + mx$ where

$$m = \frac{f(5) - f(-3)}{5 - (-3)} = \frac{-20 - (-8)}{8} = -\frac{12}{8} = -1.5.$$

Solving for b, we have

$$\begin{aligned} f(-3) &= b - 1.5(-3) \\ b &= f(-3) + 1.5(-3) \\ &= -8 + 1.5(-3) = -12.5, \end{aligned}$$

so $f(x) = -12.5 - 1.5x$.

20. We have $g(100) = 2000$ and $g(400) = 3800$. This gives $g(x) = b + mx$ where

$$m = \frac{g(400) - g(100)}{400 - 100} = \frac{3800 - 2000}{300} = 6.$$

Solving for b, we have

$$\begin{aligned} g(100) &= b + 6(100) \\ b &= g(100) - 6(100) = 2000 - 6(100) = 1400, \end{aligned}$$

so $g(x) = 1400 + 6x$.

21. The starting value is $b = 12{,}000$, and the growth rate is $m = 225$, so $h(t) = 12{,}000 + 225t$.

22. The graph contains the points $(-2, (-2)^2) = (-2, 4)$ and $(3, 3^2) = (3, 9)$, so $h(-2) = 4$ and $h(3) = 9$. This gives $y = b + mx$ where

$$m = \frac{h(3) - h(-2)}{3 - (-2)} = \frac{9 - 4}{5} = 1.$$

Solving for b, we have

$$\begin{aligned} h(-2) &= b + 1(-2) \\ b &= h(-2) - 1(-2) = 4 - 1(-2) = 6, \end{aligned}$$

so $h(x) = 6 + x$.

23. We can write the equation in slope-intercept form

$$\begin{aligned} 3x + 5y &= 6 \\ 5y &= 6 - 3x \\ y &= \frac{6}{5} - \frac{3}{5}x. \end{aligned}$$

The slope is $\frac{-3}{5}$. Lines parallel to this line all have slope $\frac{-3}{5}$. Since the line passes through $(0, 6)$, its y-intercept is equal to 6. So $y = 6 - \frac{3}{5}x$.

24. $y = 5x - 3$. Since the slope of this line is 5, we want a line with slope $-\frac{1}{5}$ passing through the point $(2, 1)$. The equation is $(y - 1) = -\frac{1}{5}(x - 2)$, or $y = -\frac{1}{5}x + \frac{7}{5}$.

25. The line $y + 4x = 7$ has slope -4. Therefore the parallel line has slope -4 and equation $y - 5 = -4(x - 1)$ or $y = -4x + 9$. The perpendicular line has slope $\frac{-1}{(-4)} = \frac{1}{4}$ and equation $y - 5 = \frac{1}{4}(x - 1)$ or $y = 0.25x + 4.75$.

26. You start with nothing. Each year, on average, your net worth increases by \$5000, so in forty years, you have $40 \text{ yrs} \cdot \$5000/\text{yr} = \$200{,}000$.

27. If zero passengers take the flight, there is no revenue, and the cost of operation is $\$10{,}000$, then the profit is $\pi = -10{,}000$, which is our π-intercept. Since each passenger pays \$127, the slope is 127 dollars per passenger, so

$$\pi(n) = -10{,}000 + 127n.$$

28. **(a)** In 1995, or year $t = 0$, the ranking for Hannah was 7, making it most popular, and the ranking for Madison was 29, making it least popular.

(b) In 2004, or year $t = 9$, the ranking for Madison was 3, making it the most popular, and the ranking for Alexis was 11, making it least popular.

29. **(a)** We have $r_m(0) - r_h(0) = 29 - 7 = 22$. This tells us that in 1995 (year $t = 0$), the name Hannah was ranked 22 places higher than Madison on the list of most popular names. (Recall that the lower the ranking, the higher a name's position on the list.)

(b) We have $r_m(9) - r_h(9) = 3 - 5 = -2$. This tells us that in 2004 (year $t = 9$), the name Hannah was ranked 2 places lower than Madison on the list of most popular names.

(c) We have $r_m(t) < r_a(t)$ for $t = 5$ to $t = 9$. This tells us that the name Madison was ranked higher than the name Alexis on the list of most popular names in the years 2000 to 2004.

30. **(a)** On the graph, the high tides occur when the graph is at its highest points. On this particular day, there were two high tides.

(b) The low tides occur when the graph is at its lowest points. There were two low tides on this day.

(c) To find the amount of time elapsed between high tides, find the distance between the two highest points on the graph. It is about 12 hours.

31. **(a)** Yes. For each value of s, there is exactly one area.

(b) No. Suppose $s = 4$ represents the length of the rectangle. The width could have any other value, say 7 or 1.5 or π or .... In this case, for one value of s, there are infinitely many possible values for A, so the area of a rectangle is not a function of the length of one of its sides.

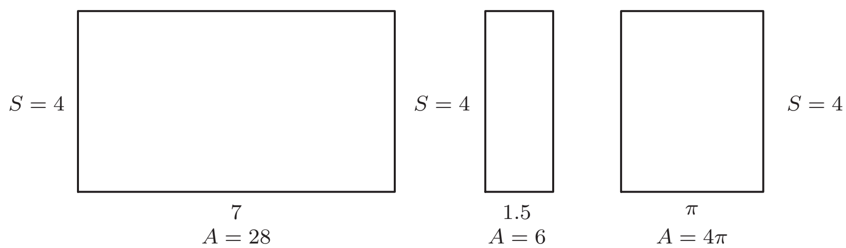

Figure 1.61

32. Figure 1.62 shows a possible graph of blood sugar level as a function of time over one day. Note that the actual curve is smooth, and does not have any sharp corners.

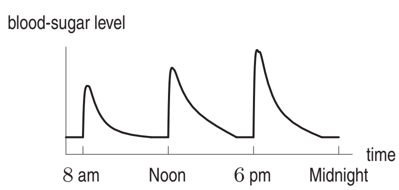

Figure 1.62

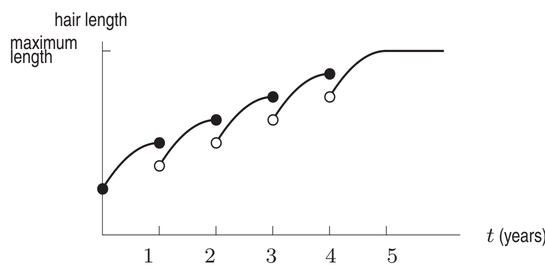

Figure 1.63

33. In Figure 1.63 the graph of the hair length is steepest just after each haircut, assumed to be at the beginning of each year. As the year progresses, the growth is slowed by split ends. By the end of the year, the hair is breaking off as fast as it is growing, so the graph has leveled off. At this time the hair is cut again. Once again it grows until slowed by the split ends. Then it is cut. This continues for five years when the longest hairs fall out because they have come to the end of their natural lifespan.

34. The value of N is not necessarily a function of G, since each value of G does not need to have a unique value of N associated to it. For example, suppose we choose the value of G to be a B. There may be more than one student who received a B, so there may be more than one ID number corresponding to B.

The value of G must be a function of N, because each ID number (each student) receives exactly one grade. Therefore each value of N has a unique value of G associated with it. Writing $G = f(N)$ indicates that the ID number is the input which uniquely determines the grade, the output.

35. The original price is P. Inflation causes a 5% increase, giving

$$\text{Inflated price} = P + 0.05P = 1.05P.$$

Then there is a 10% decrease, giving

$$\begin{aligned}\text{Final price} &= 90\%(\text{Inflated price}) \\ &= 0.9(1.05P) \\ &= 0.945P.\end{aligned}$$

36. Figure 1.64 shows the tank.

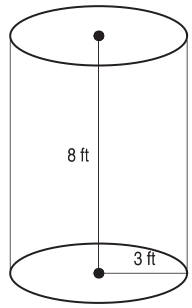

Figure 1.64: Cylindrical water tank

(a) The volume of a cylinder is equal to the area of the base times the height, where the area of the base is πr^2. Here, the radius of the base is $(1/2)(6) = 3$ ft, so the area is $\pi \cdot 3^2 = 9\pi$ ft^2. Therefore, the capacity of this tank is $(9\pi)8 = 72\pi$ ft^3.

(b) If the height of the water is 5 ft, the volume becomes $(9\pi)5 = 45\pi$ ft^3.

(c) In general, if the height of water is h ft, the volume of the water is $(9\pi)h$. If we let $V(h)$ be the volume of water in the tank as a function of its height, then

$$V(h) = 9\pi h.$$

Note that this function only makes sense for a non-negative value of h, which does not exceed 8 feet, the height of the tank.

37. (a) At 40 mph, fuel consumption is about 28 mpg, so the fuel used is $300/28 = 10.71$ gallons.

(b) At 60 mph, fuel consumption is about 29 mpg. At 70 mph, fuel consumption is about 28 mpg. Therefore, on a 200 mile trip

$$\text{Fuel saved } = \frac{200}{28} - \frac{200}{29} = 0.25 \text{ gallons.}$$

(c) The most fuel-efficient speed is where mpg is a maximum, which is about 55 mph.

38. (a) We know that 75% of David Letterman's 7 million person audience belongs to the nation's work force. Thus

$$\begin{pmatrix}\text{Number of people from the}\\ \text{work force in Dave's audience}\end{pmatrix} = 75\% \text{ of 7 million} = 0.75 \cdot (\text{7 million }) = 5.25 \text{ million.}$$

Thus the percentage of the work force in Dave's audience is

$$\begin{aligned}\begin{pmatrix}\text{\% of work force}\\ \text{in audience}\end{pmatrix} &= \left(\frac{\text{People from work force in audience}}{\text{Total work force}}\right) \cdot 100\% \\ &= \left(\frac{5.25}{118}\right) \cdot 100\% = 4.45\%.\end{aligned}$$

(b) Since 4.45% of the work force belongs to Dave's audience, David Letterman's audience must contribute 4.45% of the GDP. Since the GDP is estimated at $\$6.325$ trillion,

$$\begin{pmatrix}\text{Dave's audience's contribution}\\ \text{to the G.D.P.}\end{pmatrix} = (0.0445) \cdot (6.325 \text{ trillion}) \approx 281 \text{ billion dollars.}$$

(c) Of the contributions by Dave's audience, 10% is estimated to be lost. Since the audience's total contribution is $\$281$ billion, the "Letterman Loss" is given by

$$\text{Letterman loss } = 0.1 \cdot (281 \text{ billion dollars}) = \$28.1 \text{ billion.}$$

39. (a) Adding the male total to the female total gives $x + y$, the total number of applicants.

(b) Of the men who apply, 15% are accepted. So $0.15x$ male applicants are accepted. Likewise, 18% of the women are accepted so we have $0.18y$ women accepted. Summing the two tells us that $0.15x + 0.18y$ applicants are accepted.

(c) The number accepted divided by the number who applied times 100 gives the percentage accepted. This expression is

$$\frac{(0.15)x + (0.18)y}{x+y}(100), \quad \text{or} \quad \frac{15x + 18y}{x+y}.$$

40. Since you are moving in a straight line away from Pittsburgh, your total distance is the initial distance, 60 miles, plus the additional miles covered. In each hour, you will travel fifty miles as shown in Figure 1.65.

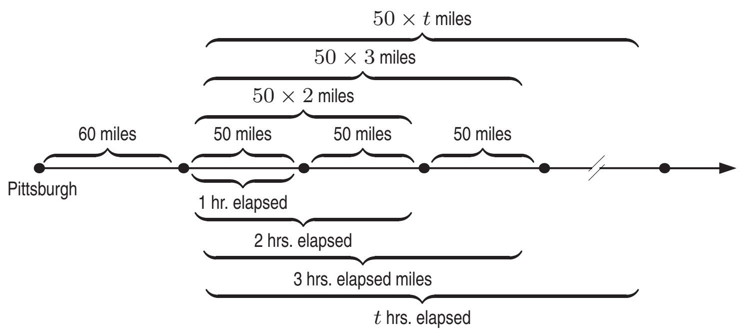

Figure 1.65

So, the total distance from Pittsburgh can be expressed as $d = 60 + 50t$.

41. **(a)** $C(175) = 11{,}375$, which means that it costs \$11,375 to produce 175 units of the good.

(b) $C(175) - C(150) = 125$, which means that the cost of producing 175 units is \$125 greater than the cost of producing 150 units. That is, the cost of producing the additional 25 units is an additional \$125.

(c) $\dfrac{C(175) - C(150)}{175 - 150} = \dfrac{125}{25} = 5$, which means that the average per-unit cost of increasing production to 175 units from 150 units is \$5.

42. We would like to find a table value that corresponds to $n = 0$. The pattern from the table, is that for each decrease of 25 in n, $C(n)$ goes down by 125. It takes four decreases of 25 to get from $n = 100$ to $n = 0$, and $C(100) = 11{,}000$, so we might estimate $C(0) = 11{,}000 - 4 \cdot 125 = 10{,}500$. This means that the fixed cost, before any goods are produced, is \$10,500.

43. We found in Problem 42 that the fixed cost of this good is \$10,500. We found the unit cost in Problem 41(c) to be \$5. (In that problem, we used $n = 150$ and $n = 175$, but since this is a linear total-cost function, any pair of values of n will give the same rate of change or cost per unit.) Thus,

$$C(n) = 10{,}500 + 5n.$$

44. This family of lines all have y–intercept equal to -2. Furthermore, the slopes of these lines are positive. A possible family is shown in Figure 1.66

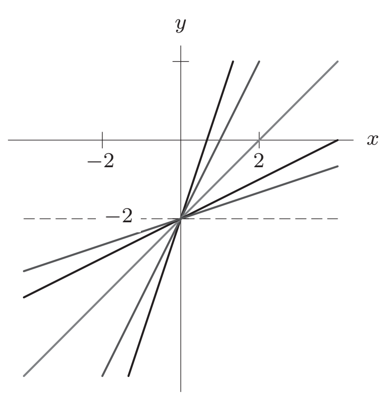

Figure 1.66

45. **(a)** Since $y = f(x)$, to show that $f(x)$ is linear, we can solve for y in terms of A, B, C, and x.

$$\begin{aligned} Ax + By &= C \\ By &= C - Ax, \text{ and, since } B \neq 0, \\ y &= \frac{C}{B} - \frac{A}{B}x \end{aligned}$$

Because C/B and $-A/B$ are constants, the formula for $f(x)$ is of the linear form:

$$f(x) = y = b + mx.$$

Thus, f is linear, with slope $m = -(A/B)$ and y-intercept $b = C/B$.

To find the x-intercept, we set $y = 0$ and solve for x:

$$\begin{aligned} Ax + B(0) &= C \\ Ax &= C, \text{ and, since } A \neq 0, \\ x &= \frac{C}{A}. \end{aligned}$$

Thus, the line crosses the x–axis at $x = C/A$.

(b) (i) Since $A > 0, B > 0, C > 0$, we know that C/A (the x-intercept) and C/B (the y-intercept) are both positive and we have Figure 1.67.

(ii) Since only $C < 0$, we know that C/A and C/B are both negative, and we obtain Figure 1.68.

(iii) Since $A > 0, B < 0, C > 0$, we know that C/A is positive and C/B is negative. Thus, we obtain Figure 1.69.

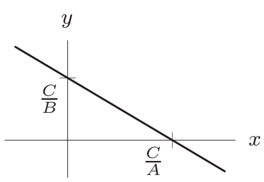

Figure 1.67

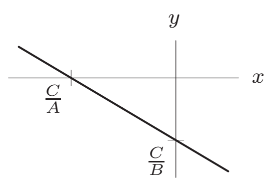

Figure 1.68

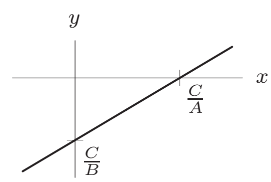

Figure 1.69

CHECK YOUR UNDERSTANDING

1. False. $f(t)$ is functional notation, meaning that f is a function of the variable t.

2. False. A rule need not be a formula.

3. True. The $P = f(x)$ notation means that P is the dependent variable and x is the independent variable.

4. False. The independent variable is commonly denoted by the letter x or t, but any letter can be used.

5. True. The number of people who enter a store in a day and the total sales for the day are related, but neither quantity is uniquely determined by the other.

6. True. This is the definition of a function.

7. True. If one column of values has no repeated values then that column can be defined as the input values of a function and the other column as the output.

8. False. For example, if Q is a constant function, then P is not a function of Q.

9. True. A circle does not pass the vertical line test.

10. False. Just the reverse; the equation tells us that 100 angels can dance on a pin head of 10 square millimeters.

11. True. This is the definition of average speed.

12. False. The average rate of change of Q with respect to t is written $\Delta Q/\Delta t$.

13. True. This is the definition of an increasing function.

14. True. A decreasing function decreases on all intervals for which it is defined.

15. True. For a function $f(x)$ over the interval $a \leq x \leq b$ the slope is $\dfrac{f(b)-f(a)}{b-a}$. This is the slope of the line through the points $(a, f(a))$ and $(b, f(b))$ on the graph of f.

16. False. $\Delta y = (3 \cdot 6 - 4) - (3 \cdot 2 - 4) = 12$ and $\Delta x = 6 - 2 = 4$, so the average rate of change is $\Delta y/\Delta x = 3$.

17. False. Parentheses must be inserted. The correct ratio is $\dfrac{(10-2^2)-(10-1^2)}{2-1} = -3$.

18. False. The first slope is 5, the second slope is -5.

19. False. If $y = mx + b$ then m, the slope, is the rate of change over every interval.

20. True. A linear function has a constant rate of change.

21. False. Writing the equation as $y = (-3/2)x + 7/2$ shows that the slope is $-3/2$.

22. True. This ratio is the slope, which is constant for a linear function.

23. True. If a linear function is decreasing, then for all positive Δx, the change, Δy, is negative, so the slope, $\Delta y/\Delta x$, is negative.

24. True. The slope is $\Delta y/\Delta x$ and this quotient can only be negative when either $\Delta x > 0$ and $\Delta y < 0$ or $\Delta x < 0$ and $\Delta y > 0$.

25. True. A constant function has slope zero. Its graph is a horizontal line.

26. False. It can be written $y = 2x - 3$.

27. False. The slope is $-3/5$.

28. False. The line $y = 4x + 5$ has slope 4 but the given point is not on the line since $3 \neq 4 \cdot (-2) + 5$.

29. True. At $y = 0$, we have $4x = 52$, so $x = 13$. The x-intercept is $(13, 0)$.

30. True. Evaluate $f(2) = -2 \cdot 2 + 7 = 3$

31. False. Since the slope is $\Delta y/\Delta x = (-10-2)/(4-1) = -12/3 = -4$.

32. False. First simplify the right side to $y - 5 = 4x + 4$ then add 5 to both sides to get $y = 4x + 9$.

33. False. Substitute the point's coordinates in the equation: $-3 - 4 \neq -2(4 + 3)$.

34. True. The given equation is in the slope-intercept form so that the slope is the coefficient of x.

35. True. The given equation is in the slope-intercept form so that the y-intercept is the constant term.

36. True. The given equation is in the slope-intercept form for a line.

37. False. The first line does but the second, in slope-intercept form, is $y = (1/8)x + (1/2)$, so it crosses the y-axis at $y = 1/2$.

38. False. The graph is a horizontal line. The slope is zero.

39. False. The slopes are $-4/5$ and $4/5$. Thus the lines are not parallel because they have different slopes.

40. True. The slopes, 9 and $-1/9$, are negative reciprocals of one another.

41. True. The point $(1, 3)$ is on both lines because $3 = -2 \cdot 1 + 5$ and $3 = 6 \cdot 1 - 3$.

42. True. Parallel lines have equal slopes.

43. False. The line $y = -3/4$ is parallel to the x-axis.

44. True. A line parallel to the x-axis represents a constant function. Since $\Delta y = 0$ for any two points on the line, its slope is $\Delta y/\Delta x = 0$.

45. True. The slope, $\Delta y/\Delta x$ is undefined because Δx is zero for any two points on a vertical line.

46. True.

47. True. Interpolation estimates values within the range of data values. Extrapolation estimates values outside the range for which data is available.

48. False. An interpolation value is calculated between known data values so it is, in general, more reliable than extrapolation.

49. False. For example, in children there is a high correlation between height and reading ability, but it is clear that neither causes the other.

50. False. There can be a relationship, but if it is not linear then the correlation coefficient can be close to zero.

51. True. All values from -1.0 to 1.0 are possible values of a correlation coefficient.

52. True.

53. True. There is a perfect fit of the line to the data.

54. True. The least squares line refers to the criterion by which the regression line was chosen.

Solutions to Tools for Chapter 1

1.

$$\begin{aligned} 3x &= 15 \\ \frac{3x}{3} &= \frac{15}{3} \\ x &= 5 \end{aligned}$$

2.

$$\begin{aligned} -2y &= 12 \\ \frac{-2y}{-2} &= \frac{12}{-2} \\ y &= -6 \end{aligned}$$

3.

$$\begin{aligned} 4z &= 22 \\ \frac{4z}{4} &= \frac{22}{4} \\ z &= \frac{11}{2} \end{aligned}$$

4.

$$\begin{aligned} x + 3 &= 10 \\ x &= 7 \end{aligned}$$

5.

$$\begin{aligned} y - 5 &= 21 \\ y &= 26 \end{aligned}$$

6.

$$\begin{aligned} w - 23 &= -34 \\ w &= -11 \end{aligned}$$

7.

$$\begin{aligned} 2x - 5 &= 13 \\ 2x &= 18 \\ \frac{2x}{2} &= \frac{18}{2} \\ x &= 9 \end{aligned}$$

8.

$$\begin{aligned} 7 - 3y &= -14 \\ -3y &= -21 \\ \frac{-3y}{-3} &= \frac{-21}{-3} \\ y &= 7 \end{aligned}$$

9.
$$\begin{aligned} 13t + 2 &= 47 \\ 13t &= 45 \\ \frac{13t}{13} &= \frac{45}{13} \\ t &= \frac{45}{13} \end{aligned}$$

10.
$$\begin{aligned} 2x - 5 &= 4x - 9 \\ 2x &= 4x - 4 \\ -2x &= -4 \\ x &= 2 \end{aligned}$$

11.
$$\begin{aligned} 0.5x - 3 &= 7 \\ 0.5x &= 10 \\ x &= 20. \end{aligned}$$

12.
$$\begin{aligned} 17 - 28y &= 13y + 24 \\ -28y &= 13y + 7 \\ -41y &= 7 \\ y &= -\frac{7}{41} \end{aligned}$$

13. We first distribute $\frac{5}{3}(y+2)$ to obtain:
$$\begin{aligned} \frac{5}{3}(y+2) &= \frac{1}{2} - y \\ \frac{5}{3}y + \frac{10}{3} &= \frac{1}{2} - y \\ \frac{5}{3}y + y &= \frac{1}{2} - \frac{10}{3} \\ \frac{5}{3}y + \frac{3y}{3} &= \frac{3}{6} - \frac{20}{6} \\ \frac{8y}{3} &= -\frac{17}{16} \\ \left(\frac{3}{8}\right)\frac{8y}{3} &= \left(\frac{3}{8}\right)\left(-\frac{17}{6}\right) \\ y &= -\frac{17}{16}. \end{aligned}$$

14. The common denominator for this fractional equation is 3. If we multiply both sides of the equation by 3, we obtain:
$$\begin{aligned} 3\left(3t - \frac{2(t-1)}{3}\right) &= 3(4) \\ 9t - 2(t-1) &= 12 \\ 9t - 2t + 2 &= 12 \\ 7t + 2 &= 12 \\ 7t &= 10 \\ t &= \frac{10}{7}. \end{aligned}$$

15.

$$\begin{aligned} 2(r+5)-3 &= 3(r-8)+21 \\ 2r+10-3 &= 3r-24+21 \\ 2r+7 &= 3r-3 \\ 2r &= 3r-10 \\ -r &= -10 \\ r &= 10 \end{aligned}$$

16. Solving for B,

$$\begin{aligned} B-4[B-3(1-B)] &= 42 \\ B-4[B-3+3B] &= 42 \\ B-4[4B-3] &= 42 \\ B-16B+12 &= 42 \\ -15B+12 &= 42 \\ -15B &= 30 \\ B &= -2. \end{aligned}$$

17. Expanding yields

$$\begin{aligned} 1.06s-0.01(248.4-s) &= 22.67s \\ 1.06s-2.484+0.01s &= 22.67s \\ -21.6s &= 2.484 \\ s &= -0.115. \end{aligned}$$

18. Dividing by w gives $l = A/w$.

19. Dividing by $w\pi$ gives

$$r = \frac{C}{2\pi}.$$

20. Dividing by rt gives

$$P = \frac{I}{rt}.$$

21. We have

$$\begin{aligned} C &= \frac{5}{9}(F-32) \\ \frac{9C}{5} &= F-32 \\ F &= \frac{9}{5}C+32 \end{aligned}$$

22. Solving for w,

$$\begin{aligned} l &= l_0 + \frac{k}{2}w \\ l-l_0 &= \frac{k}{2}w \end{aligned}$$

$$\begin{aligned}2(l-l_0) &= kw\\ \frac{2}{k}(l-l_0) &= w.\end{aligned}$$

23. Putting v_0t on the other side of the equation:

$$\begin{aligned}h - v_0t &= \frac{1}{2}at^2\\ \frac{2}{t^2}(h-v_0t) &= a\end{aligned}$$

24. We collect all terms involving y and then factor out the y.

$$\begin{aligned}by-d &= ay+c\\ by-ay &= c+d\\ y(b-a) &= c+d\\ y &= \frac{c+d}{b-a}.\end{aligned}$$

25. We collect all terms involving x and then divide by $2a$:

$$\begin{aligned}ab+ax &= c-ax\\ 2ax &= c-ab\\ x &= \frac{c-ab}{2a}.\end{aligned}$$

26. We have

$$\begin{aligned}3xy+1 &= 2y-5x\\ 3xy-2y &= -5x-1\\ y(3x-2) &= -5x-1\\ y &= \frac{-5x-1}{3x-2}.\end{aligned}$$

27. We collect all terms involving v and then factor out the v.

$$\begin{aligned}u(v+2)+w(v-3) &= z(v-1)\\ uv+2u+wv-3w &= zv-z\\ uv+wv-zv &= 3w-2u-z\\ v(u+w-z) &= 3w-2u-z\\ v &= \frac{3w-2u-z}{u+w-z}.\end{aligned}$$

28. Multiplying by $(r-1)$:

$$\begin{aligned}S(r-1) &= rL-a\\ Sr-S &= rL-a\\ Sr-rL &= S-a\\ r(S-L) &= S-a\\ r &= \frac{S-a}{S-L}.\end{aligned}$$

29. Solving for x:

$$\begin{aligned}
\frac{a-cx}{b+dx}+a&=0\\
\frac{a-cx}{b+dx}&=-a\\
a-cx&=-a(b+dx)=-ab-adx\\
adx-cx&=-ab-a\\
(ad-c)x&=-a(b+1)\\
x&=-\frac{a(b+1)}{ad-c}.
\end{aligned}$$

30. Multiplying on both sides by $C-B(1-2t)$ gives

$$\begin{aligned}
At-B&=3(C-B+2Bt)\\
At-B&=3C-3B+6Bt\\
At-6Bt&=3C-3B+B=3C-2B\\
t(A-6B)&=3C-2B\\
t&=\frac{3C-2B}{A-6B}.
\end{aligned}$$

31. Solving for y',

$$\begin{aligned}
y'y^2+2xyy'&=4y\\
y'(y^2+2xy)&=4y\\
y'&=\frac{4y}{y^2+2xy}\\
y'&=\frac{4}{y+2x}\text{ if } y\neq 0.
\end{aligned}$$

Note that if $y=0$, then y' could be any real number.

32. We collect all terms involving the variable y' and factor out the y'.

$$\begin{aligned}
2x-(xy'+yy')+2yy'&=0\\
2x-xy'-yy'+2yy'&=0\\
2x-xy'+yy'&=0\\
2x-y'(x-y)&=0\\
-y'(x-y)&=-2x\\
y'(x-y)&=2x\\
y'&=\frac{2x}{x-y}.
\end{aligned}$$

33. Adding the two equations to eliminate y, we have

$$\begin{aligned}
2x&=8\\
x&=4.
\end{aligned}$$

Using $x=4$ in the first equation gives

$$4+y=3,$$

so

$$y=-1.$$

34. Substituting the value of y from the first equation into the second equation, we obtain

$$\begin{aligned} x + 2(2x - 10) &= 15 \\ x + 4x - 20 &= 15 \\ 5x &= 35 \\ x &= 7. \end{aligned}$$

Now we substitute $x = 7$ into the first equation, obtaining $2(7) - y = 10$, hence $y = 4$.

35. Substituting the value of y from the second equation into the first, we obtain

$$\begin{aligned} 3x - 2(2x - 5) &= 6 \\ 3x - 4x + 10 &= 6 \\ -x &= -4 \\ x &= 4. \end{aligned}$$

From the second equation, we have

$$y = 2(4) - 5 = 3$$

so

$$y = 3.$$

36. Substituting the value of x from the first equation into the second equation, we obtain

$$\begin{aligned} 4(7y - 9) - 15y &= 26 \\ 28y - 36 - 15y &= 26 \\ 13y &= 62 \\ y &= \frac{62}{13}. \end{aligned}$$

From the first equation, we have

$$\begin{aligned} x &= 7\left(\frac{62}{13}\right) - 9 \\ &= \frac{434}{13} - \frac{117}{13} = \frac{317}{13}. \end{aligned}$$

37. We substitute the expression $-\frac{3}{5}x + 6$ for y in the first equation.

$$\begin{aligned} 2x + 3y &= 7 \\ 2x + 3\left(-\frac{3}{5}x + 6\right) &= 7 \\ 2x - \frac{9}{5}x + 18 &= 7 \quad \text{or} \\ \frac{10}{5}x - \frac{9}{5}x + 18 &= 7 \\ \frac{1}{5}x + 18 &= 7 \\ \frac{1}{5}x &= -11 \\ x &= -55 \\ y &= -\frac{3}{5}(-55) + 6 \\ y &= 39 \end{aligned}$$

38. One way to solve this system is by substitution. Solve the first equation for y:

$$\begin{aligned} 3x - y &= 17 \\ -y &= 17 - 3x \\ y &= 3x - 17. \end{aligned}$$

In the second equation, substitute the expression $3x - 17$ for y:

$$\begin{aligned} -2x - 3y &= -4 \\ -2x - 3(3x - 17) &= -4 \\ -2x - 9x + 51 &= -4 \\ -11x &= -4 - 51 = -55 \\ x &= \frac{-55}{-11} = 5. \end{aligned}$$

Since $x = 5$ and $y = 3x - 17$, we have

$$y = 3(5) - 17 = 15 - 17 = -2.$$

Thus, the solution to the system is $x = 5$ and $y = -2$.

Check your results by substituting the values into the second equation:

$$\begin{aligned} -2x - 3y &= -4 \\ \text{Substituting, we get } -2(5) - 3(-2) &= -4 \\ -10 + 6 &= -4 \\ -4 &= -4. \end{aligned}$$

39. From the first equation, we get $2x + 2y = 3$. From the second equation we get $-2x - y = -15$. So,

$$\begin{aligned} 2x + 2y &= 3 \\ -2x - y &= -15 \end{aligned}$$

Adding the two equations gives $y = -12$, and solving for x in either equation gives $x = 13.5$.

40. We regard a as a constant. Multiplying the first equation by a and subtracting the second gives

$$\begin{aligned} a^2x + ay &= 2a^2 \\ x + ay &= 1 + a^2 \end{aligned}$$

so, subtracting

$$(a^2 - 1)x = a^2 - 1.$$

Thus $x = 1$ (provided $a \neq \pm 1$). Solving for y in the first equation gives $y = 2a - a(1)$, so $y = a$.

41. We set the equations $y = x$ and $y = 3 - x$ equal to one another.

$$\begin{aligned} x &= 3 - x \\ 2x &= 3 \\ x &= \frac{3}{2} \quad \text{and} \quad y = \frac{3}{2} \end{aligned}$$

So the point of intersection is $x = 3/2$, $y = 3/2$.

42. Substituting $y = x + 1$ into $2x + 3y = 12$ gives

$$\begin{aligned} 2x + 3(x+1) &= 12 \\ 5x + 3 &= 12 \\ 5x &= 9 \\ x &= \frac{9}{5} = 1.8. \end{aligned}$$

If $x = 1.8$, then $y = 1.8 + 1 = 2.8$. Thus, the point of intersection is $x = 1.8$, $y = 2.8$.

43. Substituting $y = 2x$ into $2x + y = 12$ gives

$$\begin{aligned} 2x + 2x &= 12 \\ 4x &= 12 \\ x &= 3. \end{aligned}$$

Thus, substituting $x = 3$ into $y = 2x$ gives $y = 6$, so the point of intersection is $x = 3$, $y = 6$.

44. The point of intersection lies on the two lines

$$y = 2x - 3.5 \qquad \text{and} \qquad y = -\frac{1}{2}x + 4.$$

To find the point, we solve this system of equations simultaneously. Setting these two equations equal to each other and solving for x, we have

$$\begin{aligned} 2x - 3.5 &= -\frac{1}{2}x + 4 \\ 2x + \frac{1}{2}x &= 3.5 + 4 = 7.5 = \frac{15}{2} \\ \frac{5}{2}x &= \frac{15}{2} \\ x &= \frac{15}{2} \cdot \frac{2}{5} = 3. \end{aligned}$$

Since $x = 3$, we have

$$y = 2x - 3.5 = 2(3) - 3.5 = 6 - 3.5 = 2.5.$$

Thus, the point of intersection is $(3, 2.5)$.

45. The figure is a square, so $A = (17, 23)$; $B = (0, 40)$; $C = (-17, 23)$.

46. The figure is a parallelogram, so $A = (-2, 8)$.

47. The figure is a parallelogram, so $A = (3, 21)$.

48. The figure is a parallelogram, so $A = (-4, 7)$.

49. The radius is 4, so $B = (7, 4)$, $A = (11, 0)$.

50. The radius is 8, so $A = (2, 9)$, $B = (10, 1)$.

51. Since A is a corner of the rectangle, $A = (2, 5)$. The radius of the circle is $(8 - 2)/2 = 3$, so $B = (5, 8)$.

52. The radius is 4 so $A = (-7, 8)$, $B = (-3, 4)$