FIND SOLUTIONS ON NEXT PAGE

CHAPTER FIVE

Solutions for Section 5.1

Exercises

1. **(a)**

x	-1	0	1	2	3
$g(x)$	-3	0	2	1	-1

The graph of $g(x)$ is shifted one unit to the right of $f(x)$.

(b)

x	-3	-2	-1	0	1
$h(x)$	-3	0	2	1	-1

The graph of $h(x)$ is shifted one unit to the left of $f(x)$.

(c)

x	-2	-1	0	1	2
$k(x)$	0	3	5	4	2

The graph $k(x)$ is shifted up three units from $f(x)$.

(d)

x	-1	0	1	2	3
$m(x)$	0	3	5	4	2

The graph $m(x)$ is shifted one unit to the right and three units up from $f(x)$.

2. See Figure 5.1.

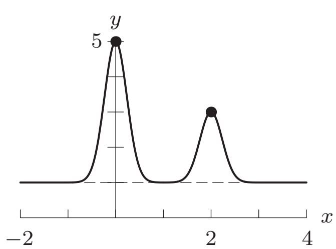

Figure 5.1

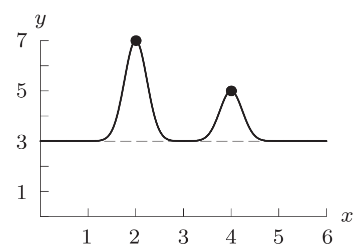

Figure 5.2

3. See Figure 5.2.

4. See Figure 5.3.

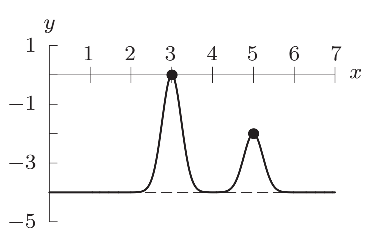

Figure 5.3

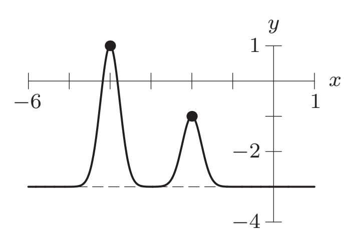

Figure 5.4

5. See Figure 5.4.

6. For all x, the graph of $g(x)$ is two units higher than the graph of $f(x)$, while the graph of $h(x)$ is 3 units lower than the graph of $f(x)$. See Figure 5.5.

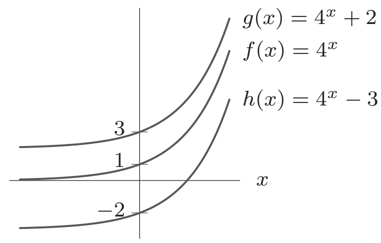

Figure 5.5

7. The graph of $g(x)$ is shifted four units to the left of $f(x)$, and the graph of $h(x)$ is shifted two units to the right of $f(x)$.

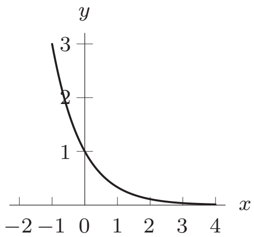

Figure 5.6: $f(x) = \left(\frac{1}{3}\right)^x$

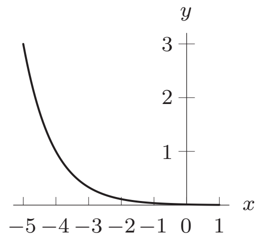

Figure 5.7: $g(x) = \left(\frac{1}{3}\right)^{x+4}$

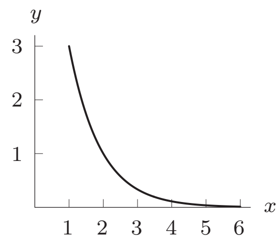

Figure 5.8: $h(x) = \left(\frac{1}{3}\right)^{x-2}$

8. **(a)** This is the graph of the function $y = |x|$ shifted both up and to the right. Thus the formula is (vi).
(b) This is the graph of the function $y = |x|$ shifted to the right. Thus the formula is (iii).
(c) This is the graph of the function $y = |x|$ shifted down. Thus formula is (ii).
(d) This is the graph of the function $y = |x|$ shifted to the left. Thus the formula is (v).
(e) This is the graph of the function $y = |x|$. Thus the formula is (i).
(f) This is the graph of the function $y = |x|$ shifted up. Thus formula is (iv).

9. **(a)** The translation $f(x) + 5$ moves the graph up 5 units. The x-coordinate is not changed, but the y-coordinate is $-4 + 5 = 1$. The new point is $(3, 1)$.
(b) The translation $f(x + 5)$ shifts the graph to the left 5 units. The y-coordinate is not changed, but the x-coordinate is $3 - 5 = -2$. The new point is $(-2, -4)$.
(c) This translation shifts both the x and y coordinates; 3 units right and 2 units down resulting in $(6, -6)$.

10. The translation shifts the graph to the right 2 units, so the new domain is $0 < x < 9$.

11. The range shifts the graph down 150 units, so the new range is $-50 \le R(s) - 150 \le 50$.

12. $m(n) + 1 = \dfrac{1}{2}n^2 + 1$

To graph this function, shift the graph of $m(n) = \frac{1}{2}n^2$ one unit up. See Figure 5.9.

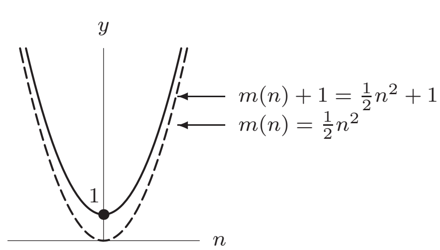

Figure 5.9

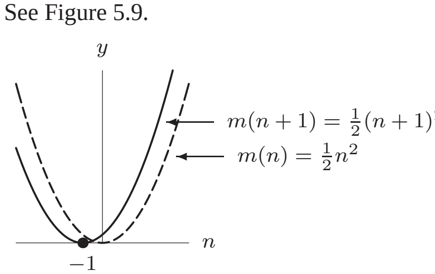

Figure 5.10

13. $m(n + 1) = \dfrac{1}{2}(n + 1)^2 = \dfrac{1}{2}n^2 + n + \dfrac{1}{2}$

To sketch, shift the graph of $m(n) = \frac{1}{2}n^2$ one unit to the left, as in Figure 5.10.

14. $m(n) - 3.7 = \dfrac{1}{2}n^2 - 3.7$

Sketch by shifting the graph of $m(n) = \frac{1}{2}n^2$ down by 3.7 units, as in Figure 5.11.

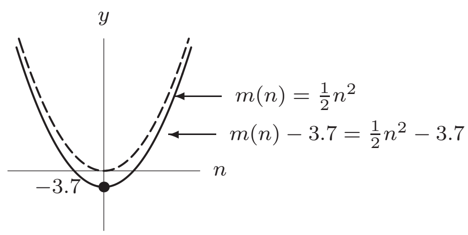

Figure 5.11

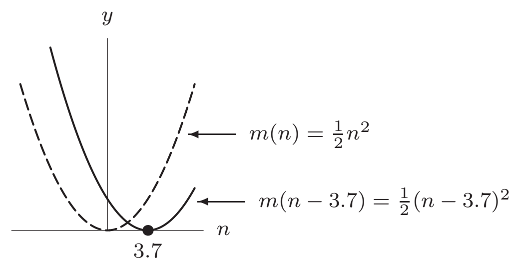

Figure 5.12

15. $m(n - 3.7) = \dfrac{1}{2}(n - 3.7)^2 = \dfrac{1}{2}n^2 - 3.7n + 6.845$

To sketch, shift the graph of $m(n) = \frac{1}{2}n^2$ to the right by 3.7 units, as in Figure 5.12.

16. $m(n) + \sqrt{13} = \dfrac{1}{2}n^2 + \sqrt{13}$

To sketch, shift the graph of $m(n) = \frac{1}{2}n^2$ up by $\sqrt{13}$ units, as in Figure 5.13.

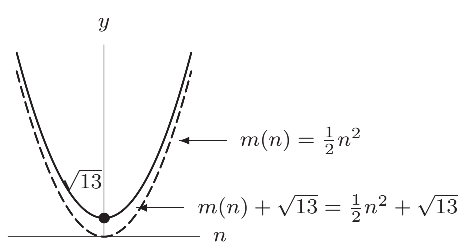

Figure 5.13

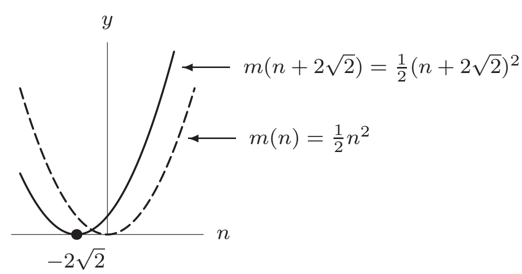

Figure 5.14

17. $m(n + 2\sqrt{2}) = \dfrac{1}{2}(n + 2\sqrt{2})^2 = \dfrac{1}{2}n^2 + 2\sqrt{2}n + 4$

To sketch, shift the graph of $m(n) = \frac{1}{2}n^2$ by $2\sqrt{2}$ units to the left, as in Figure 5.14.

18. $m(n + 3) + 7 = \dfrac{1}{2}(n + 3)^2 + 7 = \left(\dfrac{1}{2}n^2 + 3n + \dfrac{9}{2}\right) + 7 = \dfrac{1}{2}n^2 + 3n + \dfrac{23}{2}.$

To sketch, shift the graph of $m(n) = \frac{1}{2}n^2$ by 3 units to the left and 7 units up, as in Figure 5.15.

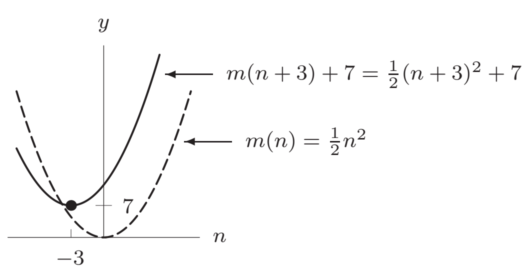

Figure 5.15

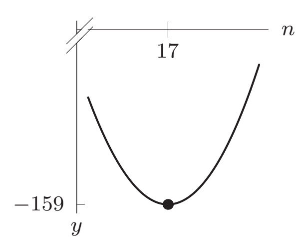

Figure 5.16

19. $m(n - 17) - 159 = \frac{1}{2}(n - 17)^2 - 159 = \left(\frac{1}{2}n^2 - 17n + \frac{289}{2}\right) - 159 = \frac{1}{2}n^2 - 17n - \frac{29}{2}$

To sketch, shift the graph of $m(n) = \frac{1}{2}n^2$ by 17 units to the right and 159 units down, as in Figure 5.16.

20. $k(w) - 3 = 3^w - 3$
To sketch, shift the graph of $k(w) = 3^w$ down 3 units, as in Figure 5.17.

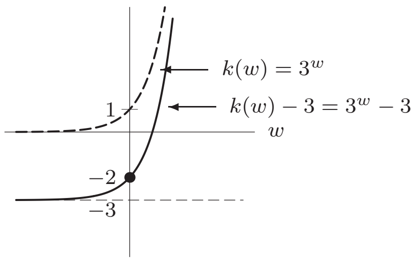

Figure 5.17

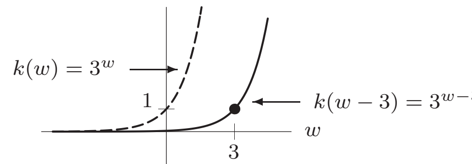

Figure 5.18

21. $k(w-3) = 3^{w-3}$
To sketch, shift the graph of $k(w) = 3^w$ to the right by 3 units, as in Figure 5.18.

22. $k(w) + 1.8 = 3^w + 1.8$
To sketch, shift the graph of $k(w) = 3^w$ up by 1.8 units, as in Figure 5.19.

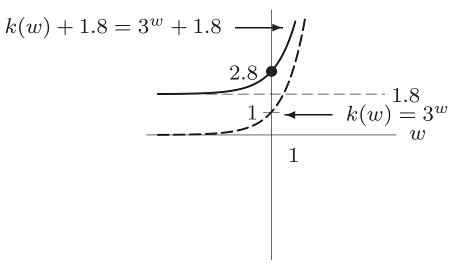

Figure 5.19

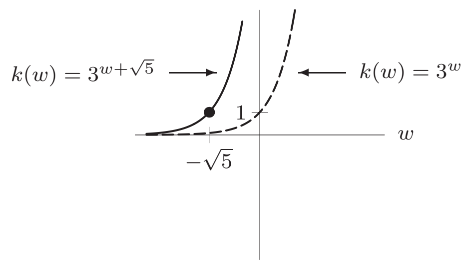

Figure 5.20

23. $k(w + \sqrt{5}) = 3^{w+\sqrt{5}}$
To sketch, shift the graph of $k(w) = 3^w$ to the left by $\sqrt{5}$ units, as in Figure 5.20.

24. $k(w + 2.1) - 1.3 = 3^{w+2.1} - 1.3$
To sketch, shift the graph of $k(w) = 3^w$ to the left by 2.1 units and down 1.3 units, as in Figure 5.21.

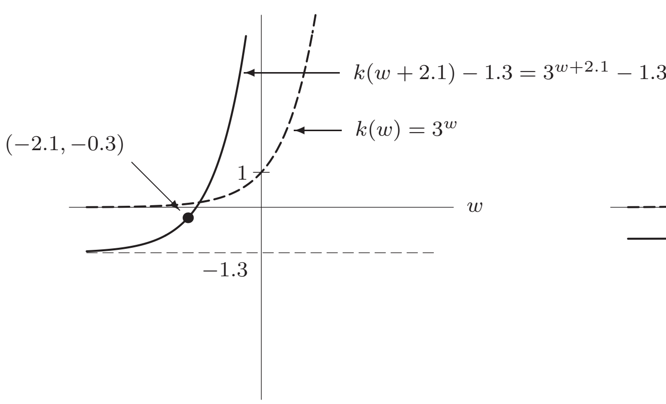

Figure 5.21

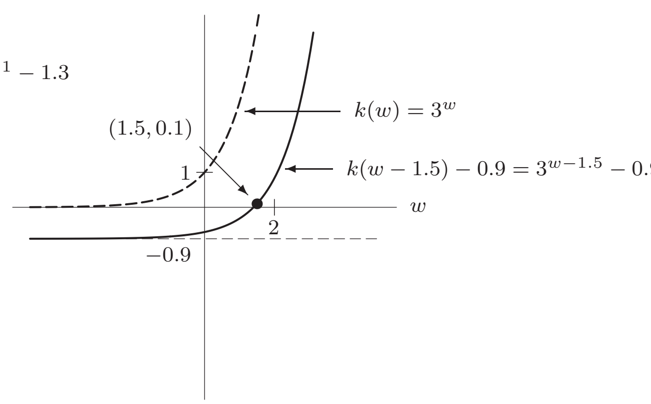

Figure 5.22

25. $k(w - 1.5) - 0.9 = 3^{w-1.5} - 0.9$
To sketch, shift the graph of $k(w) = 3^w$ to the right by 1.5 units and down by 0.9 units, as in Figure 5.22.

Problems

26. (a) (i) To evaluate $f(x)$ for $x = 6$, we find from the table the value of $f(x)$ corresponding to an x-value of 6. In this case, the corresponding value is 248. Thus, $f(x)$ at $x = 6$ is 248.

(ii) $f(5)$ equals the value of $f(x)$ corresponding to $x = 5$, or 145. $f(5) - 3 = 145 - 3 = 142$.

(iii) $f(5-3)$ is the same thing as $f(2)$, which is the value of $f(x)$ corresponding to $x = 2$. Since $f(5-3) = f(2)$, and $f(2) = 4$, $f(5-3) = 4$.

(iv) $g(x) + 6$ for $x = 2$ equals $g(2) + 6$. $g(2)$ is the value of $g(x)$ corresponding to an x-value of 2, thus $g(2) = 6$. $g(2) + 6 = 6 + 6 = 12$.

(v) $g(x+6)$ for $x = 2$ equals $g(2+6) = g(8)$. Looking at the table in the problem, we see that $g(8) = 378$. Thus, $g(x+6)$ for $x = 2$ equals 378.

(vi) $g(x)$ for $x = 0$ equals $g(0) = -6$. $3 \cdot (g(0)) = 3 \cdot (-6) = -18$.

(vii) $f(3x)$ for $x = 2$ equals $f(3 \cdot 2) = f(6)$. From part (a), we know that $f(6) = 248$; thus, $f(3x)$ for $x = 2$ equals 248.

(viii) $f(x) - f(2)$ for $x = 8$ equals $f(8) - f(2)$. $f(8) = 574$ and $f(2) = 4$, so $f(8) - f(2) = 574 - 4 = 570$.

(ix) $g(x+1) - g(x)$ for $x = 1$ equals $g(1+1) - g(1) = g(2) - g(1)$. $g(2) = 6$ and $g(1) = -7$, so $g(2) - g(1) = 6 - (-7) = 6 + 7 = 13$.

(b) (i) To find x such that $g(x) = 6$, we look for the entry in the table at which $g(x) = 6$ and then see what the corresponding x-value is. In this case, it is 2. Thus, $g(x) = 6$ for $x = 2$.

(ii) We use the same principle as that in part (i): $f(x) = 574$ when $x = 8$.

(iii) Again, this is just like part (i): $g(x) = 281$ when $x = 7$.

(c) Solving $x^3 + x^2 + x - 10 = 7x^2 - 8x - 6$ involves finding those values of x for which both sides of the equation are equal, or where $f(x) = g(x)$. Looking at the table, we see that $f(x) = g(x) = -7$ for $x = 1$, and $f(x) = g(x) = 74$ for $x = 4$.

27. (a) The translation should leave the x-coordinate unchanged, and shift the y-coordinate up 3; so $y = g(x) + 3$.

(b) The translation should leave the y-coordinate unchanged, and shift the x-coordinate right by 2; so $y = g(x - 2)$.

28. (a) $f(-6) = ((-6)/2)^3 + 2 = (-3)^3 + 2 = -27 + 2 = -25$

(b) We are trying to find x so that $f(x) = 6$. Setting $f(x) = -6$, we have

$$\begin{aligned} -6 &= \left(\frac{x}{2}\right)^3 + 2 \\ -8 &= \left(\frac{x}{2}\right)^3 \\ -2 &= \left(\frac{x}{2}\right) \\ -4 &= x. \end{aligned}$$

Thus, $f(x) = -6$ for $x = -4$.

(c) In part (a), we found that $f(-6) = -25$. This means that the point $(-6, f(-6))$, or $(-6, -25)$ is on the graph of $f(x)$. We call this point A in Figure 5.23. In part (b), we found that $f(x) = -6$ at $x = -4$. This means the point $(-4, -6)$ is also on the graph of $f(x)$. We call this point B in Figure 5.23.

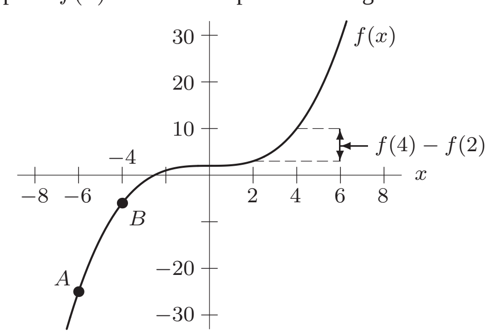

Figure 5.23

(d) We have $f(4) = (4/2)^3 + 2 = 8 + 2 = 10$ and $f(2) = (2/2)^3 + 2 = 1^3 + 2 = 3$. Thus $f(4) - f(2) = 10 - 3 = 7$. This is shown in Fig 5.23.

(e) If $a = -2$, we have $f(a + 4) = f(-2 + 4) = f(2) = 3$. Thus, $f(a + 4) = 3$ for $a = -2$. $f(-2) + 4 = (-2/2)^3 + 2 + 4 = -1 + 2 + 4 = 5$. Thus, $f(a) + 4 = 5$ for $a = -2$.

(f) $f(a+4) = f(-2+4) = f(2)$. Thus, an x-value of 2 corresponds to $f(a+4)$ for $a = -2$. $f(a)+4 = f(-2)+4 = 5$ for $a = -2$. To find an x-value which corresponds to $f(a) + 4$, we need to find the value of x for which $f(x) = 5$. Setting $f(x) = 5$,

$$\begin{aligned} (\frac{x}{2})^3 + 2 &= 5 \\ \frac{x^3}{8} + 2 &= 5 \\ \frac{x^3}{8} &= 3 \\ x^3 &= 24 \\ x &= \sqrt[3]{24} = 2\sqrt[3]{3} \\ &\approx 2.884. \end{aligned}$$

29. **(a)** $P(t) + 100$ describes a population that is always 100 people larger than the original population.

(b) $P(t + 100)$ describes a population that has the same number of people as the original population, but the number occurs 100 years earlier.

30. To compensate for the down shift, we shift up 1. To compensate for the left shift by 3, we shift right by 3.

31. The graph is in Figure 5.24. Notice that the domain of $f(x)$ is all real numbers except 3 and the domain of $g(x)$ is all real numbers except 0. The vertical asymptotes of $f(x)$ is $x = 3$ and the vertical asymptote of $g(x)$ is $x = 0$.

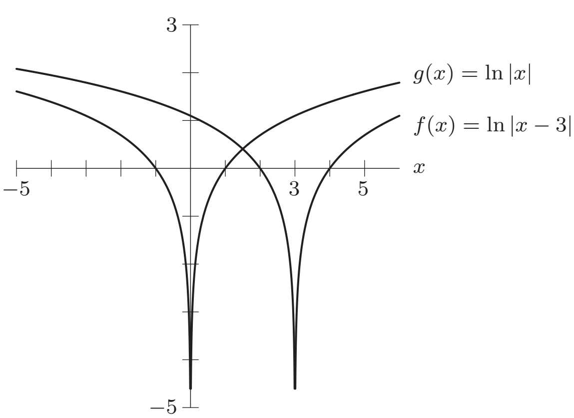

Figure 5.24

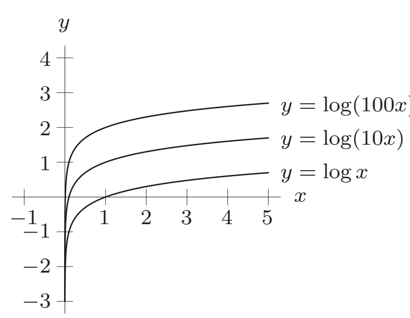

Figure 5.25

32. The graphs in Figure 5.25 appear to be vertical shifts of each other. The explanation for this relies on the property of logs which says that $\log(ab) = \log a + \log b$. Since

$$y = \log(10x) = \log 10 + \log x = 1 + \log x,$$

the graph of $y = \log(10x)$ is the graph of $y = \log x$ shifted up 1 unit. Similarly,

$$y = \log(100x) = \log 100 + \log x = 2 + \log x,$$

the graph of $y = \log(100x)$ is the graph of $y = \log x$ shifted up 2 units. Thus, we see that the graphs are indeed vertical shifts of one another.

33. Since the $+3$ is an outside change, this transformation shifts the entire graph of $q(z)$ up by 3 units. That is, for every z, the value of $q(z)+3$ is three units greater than $q(z)$.

34. Since the $-a$ is an outside change, this transformation shifts the entire graph of $q(z)$ down by a units. That is, for every z, the value of $q(z)-a$ is a units less than $q(z)$.

35. Since this is an inside change, the graph is four units to the left of $q(z)$. That is, for any given z value, the value of $q(z+4)$ is the same as the value of the function q evaluated four units to the right of z (at $z+4$).

36. Since this is an inside change, the graph is a units to the right of $q(z)$. That is, for any given z value, the value of $q(z-a)$ is the same as the value of the function q evaluated a units to the left of z (at $z-a$).

37. From the inside change, we know that the graph is shifted b units to the left. From the outside change, we know that it is shifted a units down. So, for any given z value, the graph of $q(z+b)-a$ is b units to the left and a units below the graph of $q(z)$.

38. From the inside change, we know that the graph is shifted $2b$ units to the right. From the outside change, we know that it is shifted ab units up. So, for any given z value, the graph of $q(z-2b)+ab$ is $2b$ units to the right and ab units above the graph of $q(z)$.

39. **(a)** On day d, high tide in Tacoma, $T(d)$, is 1 foot higher than high tide in Seattle, $S(d)$. Thus, $T(d)=S(d)+1$.
(b) On day d, height of the high tide in Portland equals high tide of the previous day, i.e. $d-1$, in Seattle. Thus, $P(d)=S(d-1)$.

40. **(a)** Notice that the value of $h(x)$ at every value of x is 2 less than the value of $f(x)$ at the same x value. Thus

$$h(x)=f(x)-2.$$

(b) Observe that $g(0)=f(1)$, $g(1)=f(2)$, and so on.In general,

$$g(x)=f(x+1).$$

(c) The values of $i(x)$ are two less than the values of $g(x)$at the same x value. Thus

$$i(x)=f(x+1)-2.$$

41. **(a)** If

$$\begin{aligned} H(t) &= 68+93(0.91)^t \\ \text{then} \quad H(t+15) &= 68+93(0.91)^{(t+15)} = 68+93(0.91)^{t+15} \\ \text{and} \quad H(t)+15 &= (68+93(.91)^t)+15 = 83+93(0.91)^t \end{aligned}$$

(b)

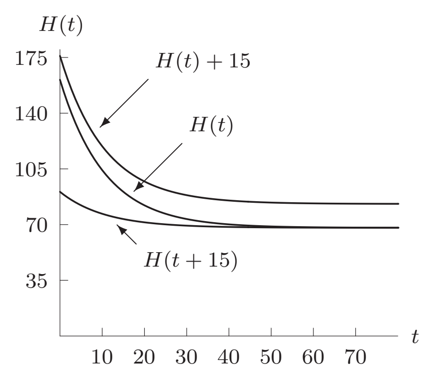

Figure 5.26

(c) $H(t+15)$ is the function $H(t)$ shifted 15 units to the left. This function could describe the temperature of the cup of coffee if it had been brought to class fifteen minutes earlier. $H(t)+15$ is the function $H(t)$ shifted upward 15 units, or $15^\circ F$. This function could describe the temperature of the coffee if it had been brought into a warmer classroom.
(d) As t gets very large, both $H(t+15)$ and $H(t)$ approach a final temperature of 68°F. In contrast, $H(t)+15$ approaches $68^\circ\text{F} + 15^\circ\text{F} = 83^\circ\text{F}$.

42. (a) If each drink costs \$7 then x drinks cost \$$7x$. Adding this to the \$20 cover charge gives $20 + 7x$. So

$$t(x) = 20 + 7x, \quad \text{for } x \geq 0.$$

(b) The cover charge is now \$25, so we have

$$\begin{aligned} n(x) &= 25 + 7x \\ &= 5 + \underbrace{20 + 7x} \\ &= 5 + t(x). \end{aligned}$$

Alternatively, notice that for any number of drinks the new cost, $n(x)$, is \$5 more than the old cost, $t(x)$. So

$$n(x) = t(x) + 5.$$

Thus $n(x)$ is the vertical shift of $t(x)$ up 5 units.

(c) Since 2 drinks are free, a customer who orders x drinks pays for only $(x - 2)$ drinks at $\$7/\text{drink}$ if $x \geq 2$. Thus

$$p(x) = 30 + 7(x - 2), \ \text{ if } x \geq 2.$$

The formula for $p(x)$ if $x \geq 2$ can be written in terms of $t(x)$ as follows:

$$\begin{aligned} p(x) &= 10 + \underbrace{20 + 7(x - 2)}_{t(x-2)} \\ &= 10 + t(x - 2) \text{if } x \geq 2. \end{aligned}$$

Another way to think of this is to subtract two from your total number of drinks, x. Use $t(x-2)$ to determine the cost of two fewer drinks with the initial cover charge. Then add this 10 dollar increase in the cover charge to the result, so $p(x) = t(x - 2) + 10$. This shows that the cover charge is $\$10$ more but you are charged for 2 fewer drinks.

43. Since the difference in temperatures decays exponentially, first we find a formula describing that difference over time. Let $D(t)$ represent the difference between the temperature of the brick and the temperature of the room.

When the brick comes out of the kiln, the difference between its temperature and room temperature is $350^\circ - 70^\circ = 280^\circ$. This difference will decay at the constant rate of 3% per minute. Therefore, a formula for $D(t)$ is

$$D(t) = 280(0.97)^t.$$

Since $D(t)$ is the difference between the brick's temperature, $H(t)$, and room temperature, 70°, we have

$$D(t) = H(t) - 70.$$

Add 70 to both sides of the equation so that

$$H(t) = D(t) + 70,$$

Since $D(t) = 280(0.97)^t$,

$$H(t) = 280(0.97)^t + 70.$$

This function, $H(t)$, is *not* exponential because it is not of the form $y = ab^x$. However, since $D(t) = 280(0.97)^t$ *is* exponential, and since

$$H(t) = D(t) + 70,$$

$H(t)$ is a transformation of an exponential function. The graph of $H(t)$ is the graph of $D(t)$ shifted upward by 70. Figures 5.27 and 5.28 give the graphs of $D(t)$ and $H(t)$ for the first 4 hours, or 240 minutes, after the brick is removed from the kiln—that is, for $0 \leq t \leq 240$. As you can see, the brick cools off rapidly at first, and then levels off toward 70°, or room temperature, where the graph of $H(t)$ has a horizontal asymptote. Notice that, by shifting the graph of $D(t)$upward by 70, the horizontal asymptote is also shifted, resulting in the asymptote at $T = 70$ for $H(t)$.

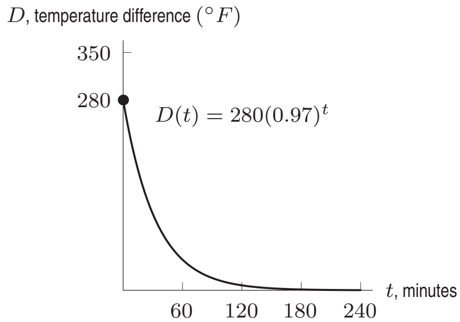

Figure 5.27

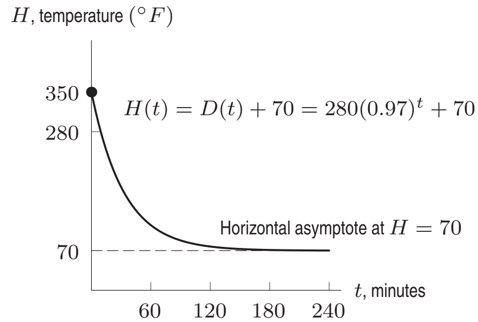

Figure 5.28

44. (a) There are many possible graphs, but all should show seasonally-related cycles of temperature increases and decreases, as in Figure 5.29.

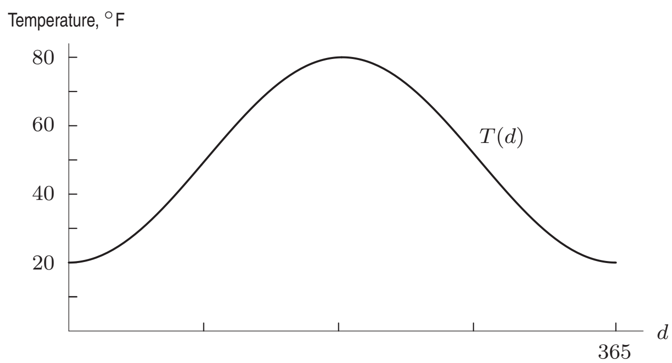

Figure 5.29

(b) While there are a wide variety of correct answers, the value of $T(6)$ is a temperature for a day in early January, $T(100)$ for a day in mid-April, and $T(215)$ for a day in early August. The value for $T(371) = T(365+6)$ should be close to that of $T(6)$.

(c) Since there are usually 365 days in a year, $T(d)$ and $T(d+365)$ represent average temperatures on days which are a year apart.

(d) $T(d+365)$ is the average temperature on the same day of the year a year earlier. They should be about the same value. Therefore, the graph of $T(d+365)$ should be about the same as that of $T(d)$.

(e) The graph of $T(d)+365$ is a shift upward of $T(d)$, by 365 units. It has no significance in practical terms, other than to represent a temperature that is 365° hotter than the average temperature on day d.

45. Since $g(x) = 5e^x$ and $g(x) = f(x-h) = e^{x-h}$, we have

$$5e^x = e^{x-h}.$$

Solve for h by taking the natural log of both sides

$$\begin{aligned}
\ln(5e^x) &= \ln(e^{x-h}) \\
\ln 5 + x &= x - h \\
h &= -\ln 5.
\end{aligned}$$

Solutions for Section 5.2

Exercises

1. (a) The y-coordinate is unchanged, but the x-coordinate is the same distance to the left of the y-axis, so the point is $(-2,-3)$.

(b) The x-coordinate is unchanged, but the y-coordinate is the same distance above the x-axis, so the point is $(2,3)$.

2. (a) Even symmetry means that the function is symmetric about the P-axis, so the P-coordinate is unchanged, but the t-coordinate is the same distance to the right of the P-axis, so the point is $(1,-5)$.

(b) The negative sign causes a reflection across the t-axis giving $(-1,5)$.

3. Since $H(x)$ is symmetric about the origin, $H(-x)=-H(x)$. So $H(3)=-H(-3)=-7$.

4. The negative sign reflects the graph about the x-axis, so the highest point on the original graph becomes the lowest on the new graph, similarly, the lowest becomes the highest, giving a new range of $-12 \le -Q(x) \le 2$.

5. To reflect about the x-axis, we make all the y-values negative, getting $y=-e^x$ as the formula.

6. To reflect about the y-axis, we substitute $-x$ for x in the formula getting $y=e^{-x}$.

7.

Table 5.1

p	-3	-2	-1	0	1	2	3
$f(p)$	0	-3	-4	-3	0	5	12

Table 5.2

p	-3	-2	-1	0	1	2	3
$g(p)$	12	5	0	-3	-4	-3	0

Table 5.3

p	-3	-2	-1	0	1	2	3
$h(p)$	0	3	4	3	0	-5	-12

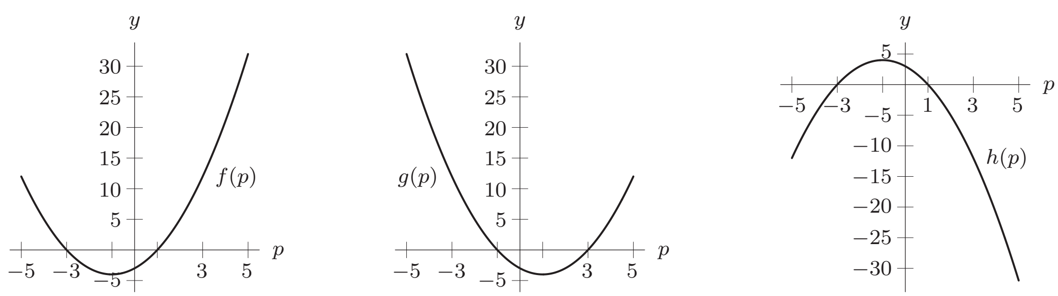

Figure 5.30: Graphs of $f(p)$, $g(p)$, and $h(p)$

Since $g(p)=f(-p)$, the graph of g is a horizontal reflection of the graph of f across the y-axis. Since $h(p)=-f(p)$, the graph of h is a reflection of the graph of f across the p-axis.

8. See Figure 5.31. The graph of $y=f(-x)$ is the graph of $y=f(x)$ reflected across the y-axis. The formula for $y=f(-x)$ is $y=4^{-x}$.

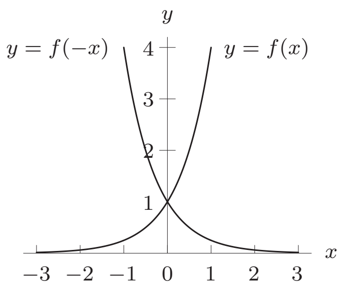

Figure 5.31

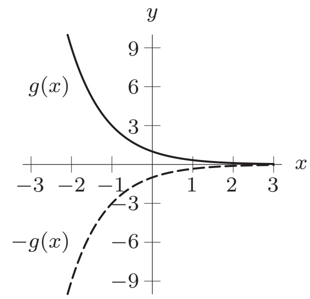

Figure 5.32

9. The graph of $y = -g(x) = -(1/3)^x$ is the graph of $y = g(x)$ reflected across the x-axis. See Figure 5.32.

10.

$$\begin{aligned} y = m(-n) &= (-n)^2 - 4(-n) + 5 \\ &= n^2 + 4n + 5 \end{aligned}$$

To graph this function, reflect the graph of m across the y-axis.

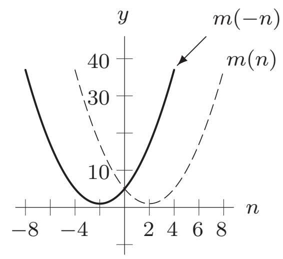

Figure 5.33: $y = m(-n)$

11.

$$y = -m(n) = -(n)^2 + 4n - 5$$

To graph this function, reflect the graph of m across the n-axis.

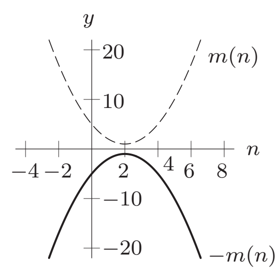

Figure 5.34: $y = -m(n)$

12.

$$y = -m(-n) = -(-n)^2 + 4(-n) - 5 = -n^2 - 4n - 5$$

To graph this function, first reflect the graph of m across the y-axis, then reflect it again across the n-axis.

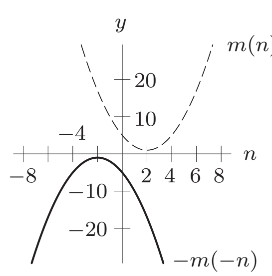

Figure 5.35: $y = -m(-n)$

13.

$$y = -m(-n) + 3 = -(-n)^2 + 4(-n) - 5 + 3$$
$$= -n^2 - 4n - 2.$$

To graph this function, first reflect the graph of m across the y-axis, then reflect it across the n-axis, and finally shift it up by 3 units.

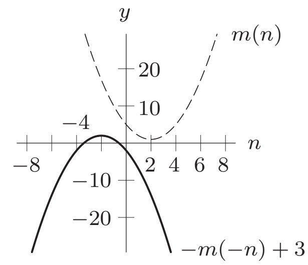

Figure 5.36: $y = -m(-n) + 3$

14.

$$y = k(-w) = 3^{-w}$$

To graph this function, reflect the graph of k across the y-axis.

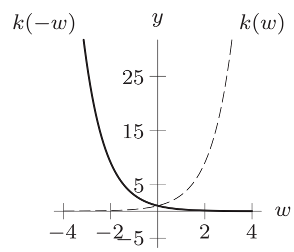

Figure 5.37: $y = k(-w)$

15.

$$y = -k(w) = -3^{w}$$

To graph this function, reflect the graph of k across the w-axis.

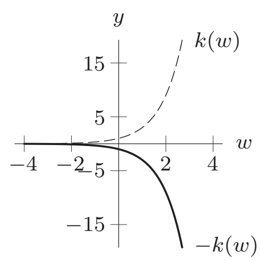

Figure 5.38: $y = -k(w)$

16.

$$y = -k(-w) = -3^{-w}$$

To graph this function, first reflect the graph of k across the y-axis, then reflect it again across the w-axis.

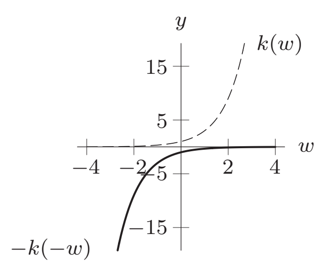

Figure 5.39: $y = -k(-w)$

17.

$$y = -k(w - 2) = -3^{w-2}$$

To graph this function, first reflect the graph of k across the w-axis, then shift it to the right by 2 units.

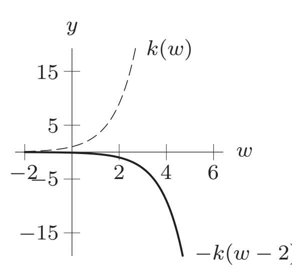

Figure 5.40: $y = -k(w - 2)$

18.

$$y = k(-w) + 4 = 3^{-w} + 4$$

To graph this function, first reflect the graph of k across the y-axis, then shift it up by 4 units.

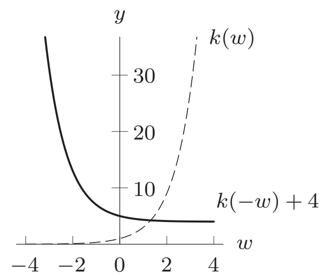

Figure 5.41: $y = k(-w) + 4$

19.

$$y = -k(-w) - 1 = -3^{-w} - 1$$

To graph this function, first reflect the graph of k across the y-axis, then reflect it across the w-axis, finally shift it down by 1 unit.

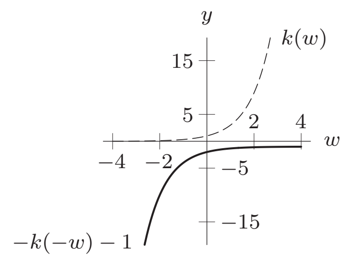

Figure 5.42: $y = -k(-w) - 1$

20. Since $f(-x) = 7(-x)^2 - 2(-x) + 1 = 7x^2 + 2x + 1$ is equal to neither $f(x)$ or $-f(x)$, the function is neither even nor odd.

21. The definition of an odd function is that $f(-x) = -f(x)$. Since $f(-x) = 4(-x)^7 - 3(-x)^5 = -4x^7 + 3x^5$, we see that $f(-x) = -f(x)$, so the function is odd.

22. The definition of an even function is that $f(-x) = f(x)$. Since $f(-x) = 8(-x)^6 + 12(-x)^2 = 8x^6 + 12x^2$, we see that $f(-x) = f(x)$, so the function is even.

23. Since $f(-x) = (-x)^5 + 3(-x)^3 - 2 = -x^5 - 3x^3 - 2$ is equal to neither $f(x)$ or $-f(x)$, the function is neither even nor odd.

Problems

24. **(a)** See Figure 5.43.
(b) See Figure 5.44.

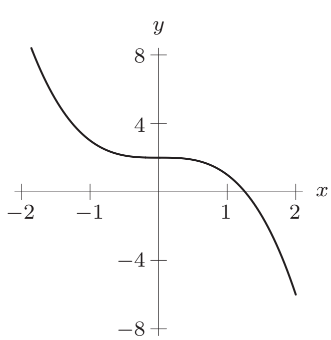

Figure 5.43: $y = -x^3 + 2$

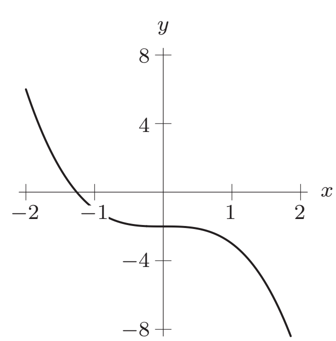

Figure 5.44: $y = -(x^3 + 2)$

(c) The two functions are not the same.

25. **(a)** See Figure 5.45.
(b) See Figure 5.45.

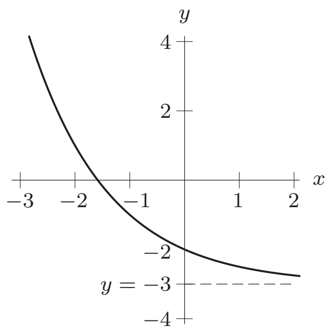

Figure 5.45: $y = 2^{-x} - 3$

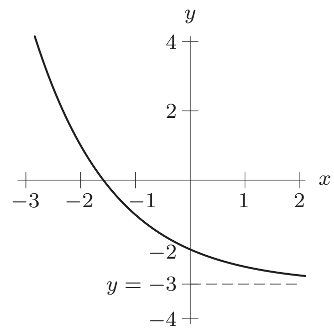

Figure 5.46: $y = 2^{-x} - 3$

(c) The two functions are the same in this case. Note that you will not always obtain the same result if you change the order of the transformations.

26. The equation of the reflected line is

$$y = b + m(-x) = b - mx.$$

The reflected line has the same y-intercept as the original; that is b. Its slope is $-m$, the negative of the original slope, and its x-intercept is b/m, the negative of the original x-intercept. A possible graph is in Figure 5.47.

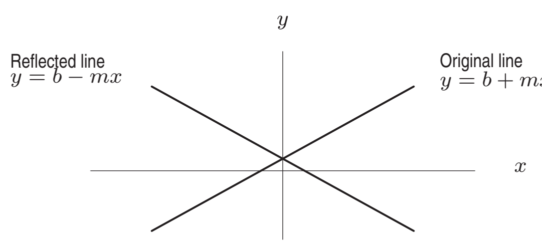

Figure 5.47

27. The graphs in Figure 5.48 are reflections of each other across the x-axis. To see this algebraically, note that

$$y = \log\left(\frac{1}{x}\right) = \log 1 - \log x = -\log x.$$

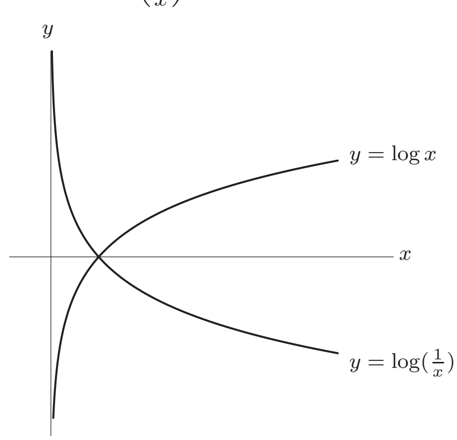

Figure 5.48

28. **(a)** The building is kept at 60° F until 5 am when the heat is turned up. The building heats up at a constant rate until 7 am when it is 68° F. It stays at that temperature until 3 pm when the heat is turned down. The building cools at a constant rate until 5 pm. At that time, the temperature is 60° F and it stays that level through the end of the day.

(b) The graph of $c(t)$ will look like the graph of $d(t)$ reflected across the t-axis and raised 142 units.

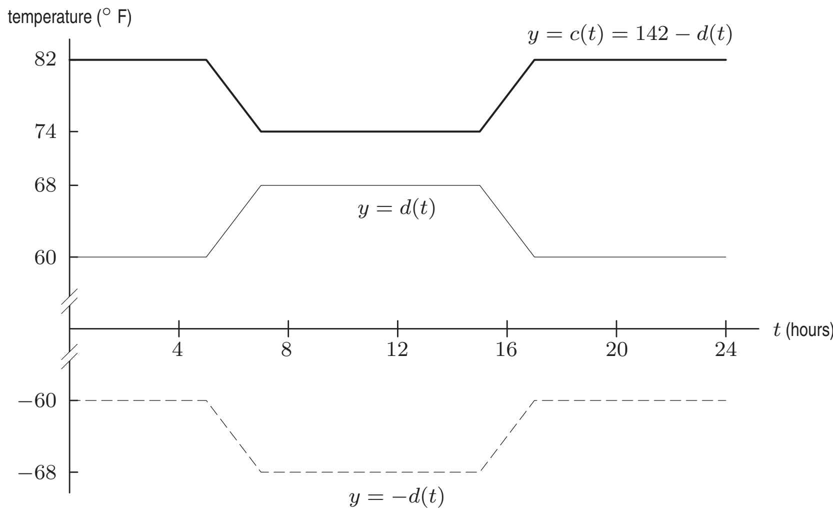

Figure 5.49

(c) This could describe the cooling schedule in the summer months when the temperature is kept at 82° F at night and cooled down to 74° during the day.

29. The answers are

(i) c (ii) d (iii) e
(iv) f (v) a (vi) b

30. **(a)**

Table 5.4

x	-3	-2	-1	0	1	2	3
y	5	-8	-4	?	-4	-8	5

(b)

Table 5.5

x	-3	-2	-1	0	1	2	3
y	5	8	-4	0	4	-8	-5

31. **(a)** Figure 5.50 shows the graph of a function f that is symmetric across the y-axis.

(b) Figure 5.51 shows the graph of function f that is symmetric across the origin.

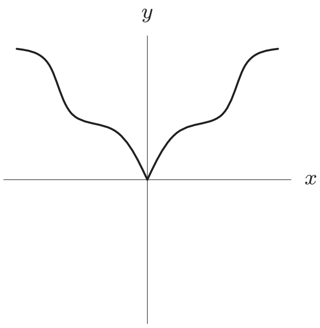

Figure 5.50: The graph of $f(x)$ that is symmetric across the y-axis

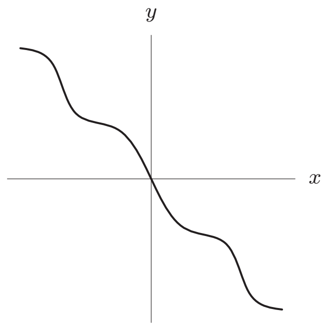

Figure 5.51: The graph of $f(x)$ that is symmetric across the origin

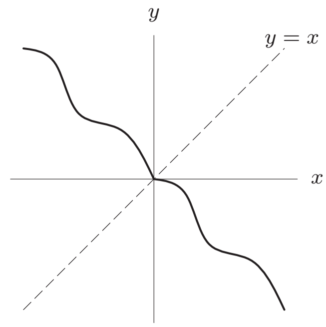

Figure 5.52: The graph of $f(x)$ that is symmetric across the line $y = x$

(c) Figure 5.52 shows the graph of function f that is symmetric across the line $y = x$.

32. **(a)** Since the values of $f(x)$ and $f(-x)$ are the same, $f(x)$ appears to be symmetric across the y-axis. Thus, $f(x)$ could be an even function.

(b) Since the value of $g(-x)$ is the opposite of $g(x)$, we know that $g(x)$ could be symmetric about the origin. Thus, $g(x)$ could be an odd function.

(c) Let $h(x) = f(x) + g(x)$. The value of $h(-x) = f(-x) + g(-x)$ is not the same as either $h(x) = f(x) + g(x)$ or $-h(x) = -(f(x) + g(x))$, so $h(x)$ is not symmetric.

(d) Let $j(x) = f(x+1)$. Note that $j(1) = f(1+1) = f(2) = 1$ and $j(-1) = f(-1+1) = f(0) = -3$. Thus, $j(-x)$ does not equal either $j(x)$ or $-j(x)$, so $j(x)$ is not symmetric.

33. See Figure 5.53.

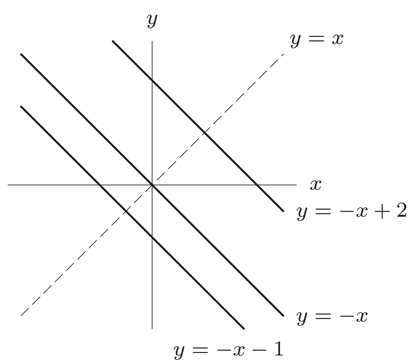

Figure 5.53: The graphs of $y = -x + 2$, $y = -x$, and $y = -x - 1$ are all symmetric across the line $y = x$.

Any straight line perpendicular to $y = x$ is symmetric across $y = x$. Its slope must be -1, so $y = -x + b$, for an arbitrary constant b, is symmetric across $y = x$.

Also, the line $y = x$ is symmetric about itself.

34. One way to do this is to sketch a graph of $y = h(x)$ to see that it appears to be symmetric across the origin. In other words, we can visually check to see that flipping the graph of $y = h(x)$ about the y-axis and then the x-axis (or vice-versa) does not change the appearance of the function's graph. See Figure 5.54.

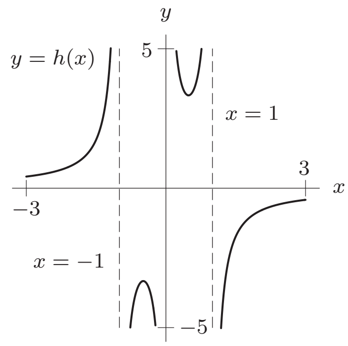

Figure 5.54: The graph of $y = h(x) = \frac{1+x^2}{x-x^3}$ is symmetric across the origin

To confirm that $h(x)$ is symmetric across the origin, we use algebra. We need to show that $h(-x) = -h(x)$ for any x. Finding the formula for $h(-x)$, we have

$$\begin{aligned} h(-x) &= \frac{1+(-x)^2}{(-x)-(-x)^3} = \frac{1+x^2}{-x+x^3} = \frac{1+x^2}{-(x-x^3)} \\ &= \frac{1+x^2}{x-x^3} = -h(x). \end{aligned}$$

Thus, the formula for $h(-x)$ is the same as the formula for $-h(x)$, and so the graph of $y = h(x)$ is symmetric across the origin.

35. The argument that $f(x)$ is not odd is correct. However, the statement "something is either even or odd" is false. This function is neither an odd function nor an even function.

36. No, it is not possible for an odd function to be strictly concave up. If it were concave up in the first or second quadrants, then the fact that it is odd would mean it would have to be symmetric across the origin, and so would be concave down in the third or fourth quadrants.

37. Because $f(x)$ is an odd function, $f(x) = -f(-x)$. Setting $x = 0$ gives $f(0) = -f(0)$, so $f(0) = 0$. Since $c(0) = 1$, $c(x)$ is not odd. Since $d(0) = 1$, $d(x)$ is not odd.

38. Because $f(x)$ is an even function, it is symmetric across the y-axis. Thus, in the second quadrant it must be decreasing and concave down.

39. To show that $f(x) = x^{1/3}$ is an odd function, we must show that $f(x) = -f(-x)$:

$$-f(-x) = -(-x)^{1/3} = x^{1/3} = f(x).$$

However, not all power functions are odd. The function $f(x) = x^2$ is an even function because $f(x) = f(-x)$ for all x. Another counter-example is $f(x) = \sqrt{x} = x^{1/2}$. This function is not odd because it is not defined for negative values of x.

40. Figure 5.55 shows the graphs of $s(x)$, $c(x)$, and $n(x)$. Based on the graphs, it appears that $s(x)$ is an even function (symmetric across the y-axis), $c(x)$ is an odd function (symmetric across the origin), and $n(x)$ is neither.

$s(-x) = 2^{-x} + (\frac{1}{2})^{-x} = (\frac{1}{2})^x + 2^x = 2^x + (\frac{1}{2})^x = s(x)$, so $s(x)$ is an even function.

$c(-x) = 2^{-x} - (\frac{1}{2})^{-x} = (\frac{1}{2})^x - 2^x = -2^x + (\frac{1}{2})^x = -c(x)$, so $c(x)$ is an odd function.

$n(-x) = 2^{-x} - (\frac{1}{2})^{-x-1} = (\frac{1}{2})^x + 2^{x+1} = 2 \cdot 2^x + (\frac{1}{2})^x$. Since $n(-x) \neq n(x)$ and $n(-x) \neq -n(x)$, $n(x)$ is neither even nor odd.

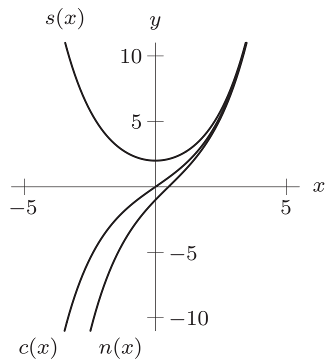

Figure 5.55

41. Suppose $f(x)$ is both even and odd. If $f(x)$ is even, then

$$f(-x) = f(x).$$

If $f(x)$ is odd, then

$$f(-x) = -f(x).$$

Since $f(-x)$ equals both $f(x)$ and $-f(x)$, we have

$$f(x) = -f(x).$$

Add $f(x)$ to both sides of the equation to get

$$2f(x) = 0$$

or

$$f(x) = 0.$$

Thus, the function $f(x) = 0$ is the only function which is both even and odd. There are no *nontrivial* functions that have both symmetries.

42. There is only one such function, and a rather unexciting one at that. Any function with symmetry about the x-axis would look unchanged if you flipped its graph about the x-axis. For any function f, the graph of $y = -f(x)$ is the graph of $y = f(x)$ flipped about the x-axis. Assuming this does not change the appearance of its graph, we have the equation

$$f(x) = -f(x).$$

Adding $f(x)$ to both sides gives

$$2f(x) = 0,$$

or simply

$$f(x) = 0.$$

Thus the only function that is symmetrical about the x-axis is the x-axis itself – that is, the line $y = 0$. If you think about it, you will see that any other curve that is symmetrical about the x-axis would necessarily fail the vertical line test, and would thus not represent the graph of a function.

Solutions for Section 5.3

Exercises

1. To increase by a factor of 10, multiply by 10. The right shift of 2 is made by substituting $x - 2$ for x in the function formula. Together they give $y = 10f(x - 2)$.

2. The transformation affects only the y-coordinate, multiplying it by 3 and then raising it by 1. The new y-coordinate is $3 \cdot 1/3 + 1 = 2$. The point on the new graph is $(5, 2)$.

3. All of the output values are smaller by a factor of 0.25, so the compressed range is $-0.25 \leq 0.25C(x) \leq 0.25$.

4. See Figure 5.56.

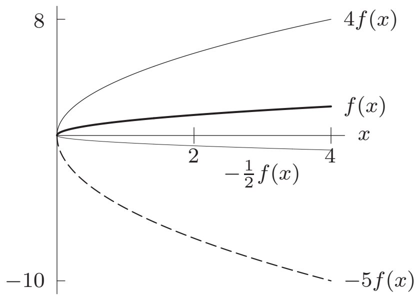

Figure 5.56

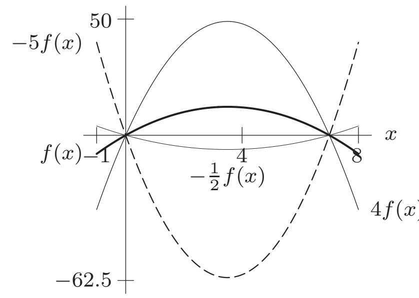

Figure 5.57

5. See Figure 5.57.

6. See Figure 5.58.

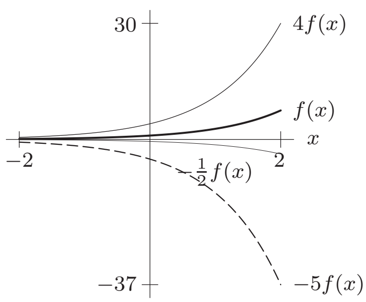

Figure 5.58

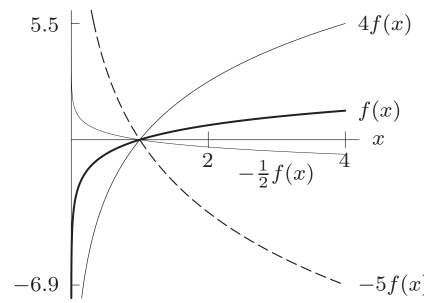

Figure 5.59

7. See Figure 5.59.

8. **(a)** To get the table for $f(x)/2$, you need to divide each entry for $f(x)$ by 2. See Table 5.6.

Table 5.6

x	-3	-2	-1	0	1	2	3
$f(x)/2$	1	1.5	3.5	-0.5	-1.5	2	4

(b) In order to get the table for $-2f(x+1)$, first get the table for $f(x+1)$. To do this, note that, if $x = 0$, then $f(x+1) = f(0+1) = f(1) = -3$ and if $x = -4$, then $f(x+1) = f(-4+1) = f(-3) = 2$. Since $f(x)$ is defined for $-3 \leq x \leq 3$, where x is an integer, then $f(x+1)$ is defined for $-4 \leq x \leq 2$.

Table 5.7

x	-4	-3	-2	-1	0	1	2
$f(x+1)$	2	3	7	-1	-3	4	8

Next, multiply each value of $f(x+1)$ entry by -2.

Table 5.8

x	-4	-3	-2	-1	0	1	2
$-2f(x+1)$	-4	-6	-14	2	6	-8	-16

(c) To get the table for $f(x)+5$, you need to add 5 to each entry for $f(x)$ in in the table given in the problem.

Table 5.9

x	-3	-2	-1	0	1	2	3
$f(x)+5$	7	8	12	4	2	9	13

(d) If $x=3$, then $f(x-2)=f(3-2)=f(1)=-3$. Similarly if $x=2$ then $f(x-2)=f(0)=-1$, since $f(x)$ is defined for integral values of x from -3 to 3, $f(x-2)$ is defined for integral values of x, which are two units higher, that is from -1 to 5.

Table 5.10

x	-1	0	1	2	3	4	5
$f(x-2)$	2	3	7	-1	-3	4	8

(e) If $x=3$, then $f(-x)=f(-3)=2$, whereas if $x=-3$, then $f(-x)=f(3)=8$. So, to complete the table for $f(-x)$, flip the values of $f(x)$ given in the problem about the origin.

Table 5.11

x	-3	-2	-1	0	1	2	3
$f(-x)$	8	4	-3	-1	7	3	2

(f) To get the table for $-f(x)$, take the negative of each value of $f(x)$ from the table given in the problem.

Table 5.12

x	-3	-2	-1	0	1	2	3
$-f(x)$	-2	-3	-7	1	3	-4	-8

9. (a)

Table 5.13

x	-4	-3	-2	-1	0	1	2	3	4
$f(-x)$	13	6	1	-2	-3	-2	1	6	13

(b)

Table 5.14

x	-4	-3	-2	-1	0	1	2	3	4
$-f(x)$	-13	-6	-1	2	3	2	-1	-6	-13

(c)

Table 5.15

x	-4	-3	-2	-1	0	1	2	3	4
$3f(x)$	39	18	3	-6	-9	-6	3	18	39

(d) All three functions are even.

10. **(a)** II
(b) III
(c) IV
(d) I

11. The function is $y = f(x+3)$. Since $f(x) = |x|$, we want $y = |x+3|$. The transformation shifts the graph of $f(x)$ by 3 units to the left. See Figure 5.60.

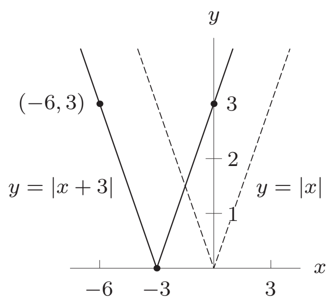

Figure 5.60

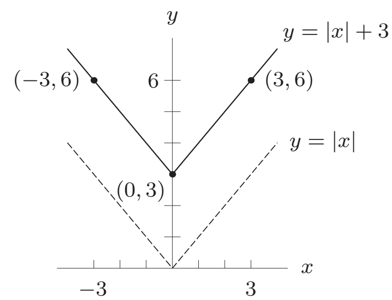

Figure 5.61

12. Again, $f(x) = |x|$. Therefore, $y = f(x) + 3$ means that we would shift the graph of $y = |x|$ upward 3 units. See Figure 5.61.

13. Since $g(x) = x^2$, $-g(x) = -x^2$. The graph of $g(x)$ is flipped over the x-axis. See Figure 5.62.

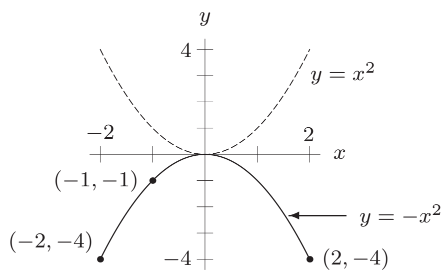

Figure 5.62

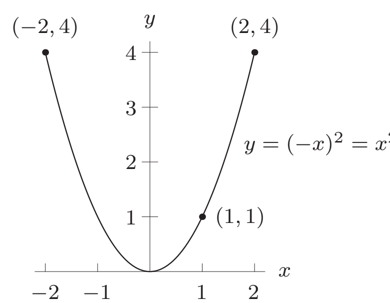

Figure 5.63

14. As before, $g(x) = x^2$. Thus $y = g(-x) = (-x)^2$, but $(-x)^2 = x^2$, so $g(-x) = x^2 = g(x)$. Since $g(x)$ is an even function, reflecting its graph across the y-axis leaves the graph unchanged. See Figure 5.63.

15. Since $h(x) = 2^x$, $3h(x) = 3 \cdot 2^x$. The graph of $h(x)$ is stretched vertically by a factor of 3. See Figure 5.64.

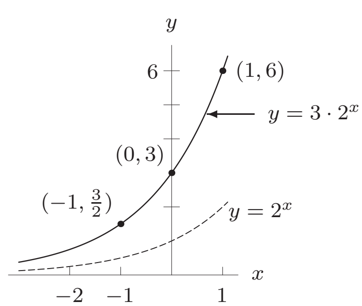

Figure 5.64

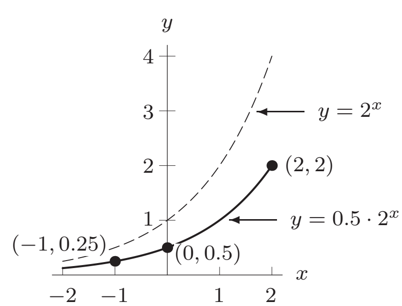

Figure 5.65

16. Since $h(x) = 2^x$, $0.5h(x) = 0.5 \cdot 2^x$. The graph of $h(x)$ is compressed vertically by a factor of 2. See Figure 5.65.

17. (i) i: The graph of $y = f(x)$ has been stretched vertically by a factor of 2.
(ii) c: The graph of $y = f(x)$ has been stretched vertically by $1/3$, or compressed.
(iii) b: The graph of $y = f(x)$ has been reflected over the x-axis and raised by 1.
(iv) g: The graph of $y = f(x)$ has been shifted left by 2, and raised by 1.
(v) d: The graph of $y = f(x)$ has been reflected over the y-axis.

Problems

18. The input $(x + 1)$ moves the graph 1 unit to the left. Then the factor of 2 doubles the resulting y-values (stretching the graph vertically), and finally the graph is shifted down 3 units.

19. The new average rate of change is:

$$\frac{\frac{1}{2}s(4) - \frac{1}{2}s(0)}{4 - 0} = \frac{1}{2} \cdot \frac{s(4) - s(0)}{4 - 0} = \frac{1}{2}(70) = 35 \text{ mph},$$

which is half the original average rate of change.

20. The graph of $f(t) = 1/(1 + x^2)$ resembles a bell-shaped curve. (It is not, however, a true "bell curve.") See Figure 5.66.

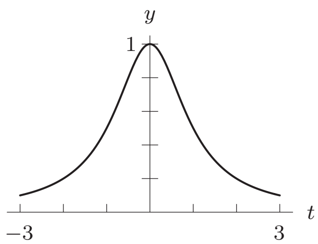

Figure 5.66

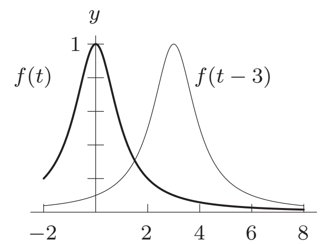

Figure 5.67

21. The graph of $f(t - 3)$ is the graph of $f(t)$ shifted to the right by 3 units. See Figure 5.67.

22. The graph of $0.5f(t)$ resembles a flattened version of the graph of $f(t)$. Each point on the new graph is half as far from the t-axis as the same point on the graph of f. See Figure 5.68.

23. The graph of $-f(t)$ is a vertically flipped version of the graph of $f(t)$. See Figure 5.69.

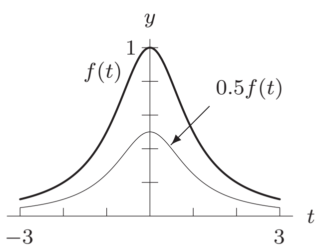

Figure 5.68

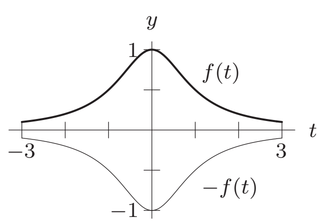

Figure 5.69

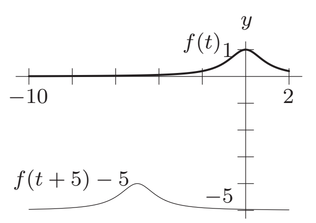

Figure 5.70

24. The graph of $f(t + 5) - 5$ is the graph of $f(t)$ shifted to the left by 5 units and then down by 5 units. See Figure 5.70.

25. **(a)** (iii) The number of gallons needed to cover the house is $f(A)$; two more gallons will be $f(A) + 2$.
(b) (i) To cover the house twice, you need $f(A) + f(A) = 2f(A)$.
(c) (ii) The sign is an extra 2 ft^2 so we need to cover the area $A + 2$. Since $f(A)$ is the number of gallons needed to cover A square feet, $f(A + 2)$ is the number of gallons needed to cover $A + 2$ square feet.

26. I is (b)
II is (d)
III is (c)
IV is (h)

27. **(a)** If t represents the number of the months, then $t+1$ represents one month later than month t. So $P(t+1)$ represents the number of rabbits one month later.

For instance, if $t=3$, then $P(3)$ stands for the number of rabbits on April 1. Thus, $P(t+1)=P(3+1)=P(4)$ stands for the number of rabbits a month later (May 1).

(b) $2P(t)$ stands for twice the number of rabbits in the park in month t.

For instance, if $t=3$ and $P(3)=500$, then $2P(t)=2P(3)=2(500)=1000$, which is twice the number of rabbits in the park on April 1.

28. **(a)** Since $y=f(x)$ and $y=3\cdot 2^x$ are both increasing functions whose values approach zero as $x\to-\infty$, $f(x)$ is a possible match for $3\cdot 2^x$.

(b) Since $5^{-x}\to\infty$ as $x\to-\infty$, there is no graph given that could represent the function as $y=5^{-x}$.

(c) Since $y=-5^x=-(5^x)$ is an exponential growth function flipped about the x-axis, $g(x)$ is a possible match for $y=-5^x$.

(d) The graph of $y=2-2^{-x}$ is the graph of an exponential function, $y=2^x$, flipped over both axes and shifted up by 2 units. The graph of $j(x)$ could satisfy these conditions except that it also passes through $(0,0)$, while the graph of $y=2-2^{-x}$ passes through $(0,1)$ $[y=2-2^{-0}=2-1=1]$. So, none of these graphs could represent the function $y=2-2^{-x}$.

(e) The graph of $y=1-(\frac{1}{2})^x$ and $y=j(x)$ could represent the same function because:

i. they are both increasing functions;
ii. they both pass through $(0,0)$ $[y=1-(\frac{1}{2})^0=1-1=0]$;
iii. they both approach an asymptote as x gets very large (as x gets bigger and bigger, $(\frac{1}{2})^x$ gets closer and closer to zero, so $y=1-(\frac{1}{2})^x$ gets closer and closer to one).

29. See Figure 5.71. The graph is shifted to the right by 3 units.

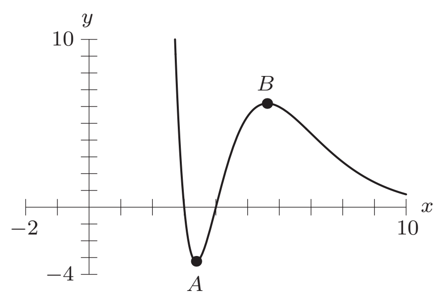

Figure 5.71

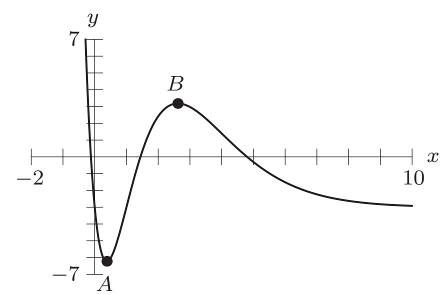

Figure 5.72

30. See Figure 5.72. The graph is shifted down by 3 units.

31. See Figure 5.73. The graph is horizontally flipped and vertically compressed by a factor of 3.

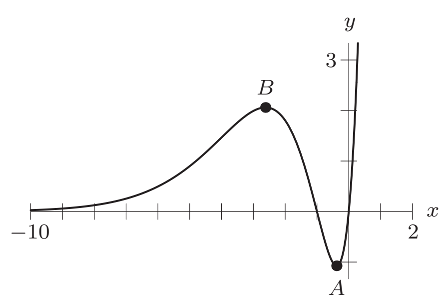

Figure 5.73

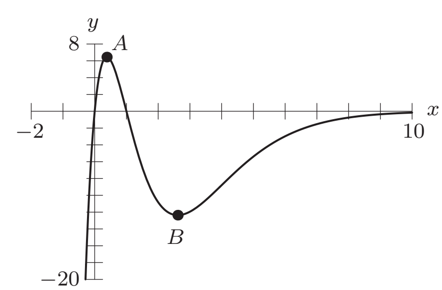

Figure 5.74

32. See Figure 5.74. The graph is vertically flipped and vertically stretched by a factor of 2.

33. See Figure 5.75. The graph is vertically flipped, horizontally shifted left by 5 units, and vertically shifted up by 5 units.

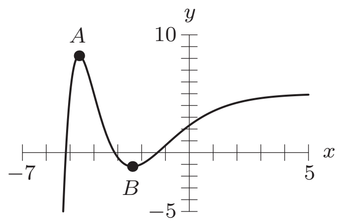

Figure 5.75

34. **(a)** This figure is the graph of $f(t)$ shifted upward by two units. Thus its formula is $y = f(t) + 2$. Since on the graph of $f(t)$ the asymptote occurs at $y = 5$ on this graph the asymptote must occur at $y = 7$.

(b) This figure is the graph of $f(t)$ shifted to the left by one unit. Thus its formula is $y = f(t + 1)$. Since on the graph of $f(t)$ the asymptote occurs at $y = 5$, on this graph the asymptote also occurs at $y = 5$. Note that a horizontal shift does not affect the horizontal asymptotes.

(c) This figure is the graph of $f(t)$ shifted downward by three units and to the right by two units. Thus its formula is $y = f(t - 2) - 3$. Since on the graph of $f(t)$ the asymptote occurs at $y = 5$, on this graph the asymptote must occur at $y = 2$. Again, the horizontal shift does not affect the horizontal asymptote. However, outside changes (vertical shifts) do change the horizontal asymptote.

35. **(a)** $y = -2f(x)$. The function has been reflected over the x-axis, and stretched vertically by a factor of 2.

(b) $y = f(x) + 2$. This function has been shifted upward 2 units.

(c) $y = 3f(x - 2)$. This function has been shifted 2 units to the right and stretched vertically by a factor of 3.

36.

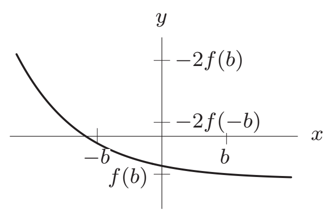

Figure 5.76

37. Figure 5.77 gives a graph of a function $y = f(x)$ together with graphs of $y = \frac{1}{2}f(x)$ and $y = 2f(x)$. All three graphs cross the x-axis at $x = -2, x = -1$, and $x = 1$. Likewise, all three functions are increasing and decreasing on the same intervals. Specifically, all three functions are increasing for $x < -1.55$ and for $x > 0.21$ and decreasing for $-1.55 < x < 0.21$.

Even though the stretched and compressed versions of f shown by Figure 5.77 are increasing and decreasing on the same intervals, they are doing so at different rates. You can see this by noticing that, on every interval of x, the graph of $y = \frac{1}{2}f(x)$ is less steep than the graph of $y = f(x)$. Similarly, the graph of $y = 2f(x)$ is steeper than the graph of $y = f(x)$. This indicates that the magnitude of the average rate of change of $y = \frac{1}{2}f(x)$ is less than that of $y = f(x)$, and that the magnitude of the average rate of change of $y = 2f(x)$ is greater than that of $y = f(x)$.

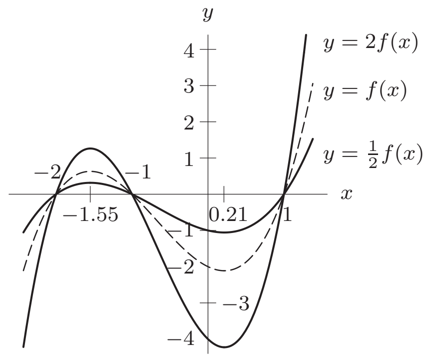

Figure 5.77: The graph of $y = 2f(x)$ and $y = \frac{1}{2}f(x)$ compared to the graph of $f(x)$

38. Since $g(x) = 5e^{x-2}$ and $g(x) = kf(x) = ke^x$, we have

$$5e^{x-2} = ke^x.$$

Divide both sides by e^x to isolate k,

$$5\frac{e^{x-2}}{e^x} = k,$$

and simplify to find $k = \dfrac{5}{e^2}$.

Solutions for Section 5.4

Exercises

1. The graph is compressed horizontally by a factor of $1/2$ so the transformed function gives an output of 3 when the input is 1. Thus the point $(1, 3)$ lies on the graph of $g(2x)$.

2. The inside change stretches the graph horizontally by a factor of 10, while the outside change stretches it vertically by a factor of 10.

3. If $x = -2$, then $f(\frac{1}{2}x) = f(\frac{1}{2}(-2)) = f(-1) = 7$, and if $x = 6$, then $f(\frac{1}{2}x) = f(\frac{1}{2} \cdot 6) = f(3) = 8$. In general, $f(\frac{1}{2}x)$ is defined for values of x which are twice the values for which $f(x)$ is defined.

Table 5.16

x	-6	-4	-2	0	2	4	6
$f(\frac{1}{2}x)$	2	3	7	-1	-3	4	8

4.

Table 5.17

x	-3	-2	-1	0	1	2	3
$f(x)$	-4	-1	2	3	0	-3	-6
$f(\frac{1}{2}x)$	–	2	–	3	–	0	–
$f(2x)$	–	–	-1	3	-3	–	–

5. The graph in Figure 5.78 of $n(x) = e^{2x}$ is a horizontal compression of the graph of $m(x) = e^x$. The graph of $p(x) = 2e^x$ is a vertical stretch of the graph of $m(x) = e^x$. All three graphs have a horizontal asymptote at $y = 0$. The y-intercept of $n(x) = e^{2x}$ is the same as for $m(x)$, but the graph of $p(x) = 2e^x$ has a y-intercept of $(0, 2)$.

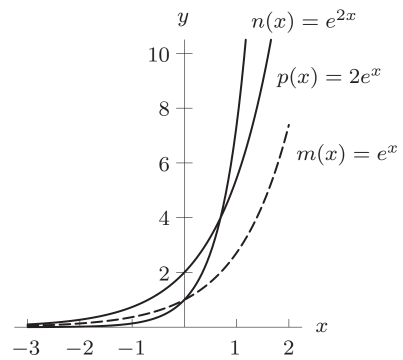

Figure 5.78

6. Since $h(x) = 2^x$, we know that $h(3x) = 2^{(3x)}$. Since we are multiplying x by a factor of 3, the graph of $2^{(3x)}$ is going to be 1/3 as wide as the graph of 2^x.

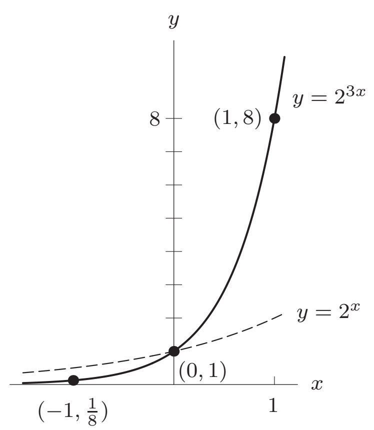

Figure 5.79

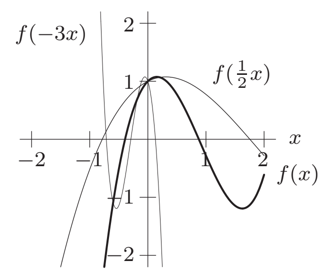

Figure 5.80

7. See Figure 5.80.

8. See Figure 5.81.

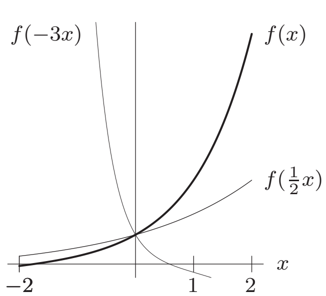

Figure 5.81

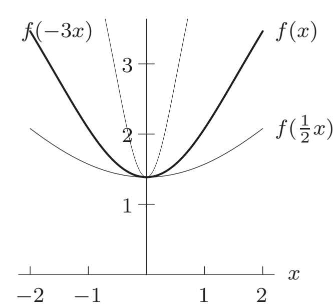

Figure 5.82

9. See Figure 5.82.

10. (i) e: The graph of $y = f(x)$ has been compressed horizontally by a factor of 2.
(ii) i: The graph of $y = f(x)$ has been compressed horizontally by a factor of 2, and stretched vertically by a factor of 2.
(iii) No match. None of these figures show $y = f(x)$ stretched horizontally by a factor of 2.

Problems

11. To get an output of 4 in the transformed function, we need the input to f to be 2. So $3p = 2$ and thus $p = \frac{2}{3}$.

12. **(a)** The graph is compressed horizontally by a factor of $\frac{1}{2}$, so the new domain is $-6 \le x \le 6$. There is no outside change so the range is still $0 \le l(2x) \le 3$.
(b) The graph is stretched horizontally by a factor of 2, so the new domain is $-24 \le x \le 24$. The change is only an inside change, so the range is still $0 \le l(\frac{1}{2}x) \le 3$.

13. The stretch away from the y-axis by a factor of d requires multiplying the x-coordinate by $1/d$, and the upward translation requires adding c to the y-coordinate. This gives $(a/d, b + c)$.

14. This graph is the graph of f shifted to the right by one unit, flipped vertically, and stretched vertically by a factor of 2. See Figure 5.83.

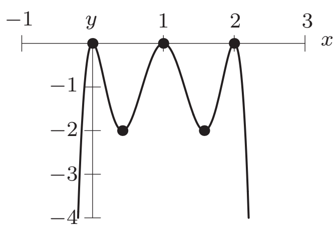

Figure 5.83

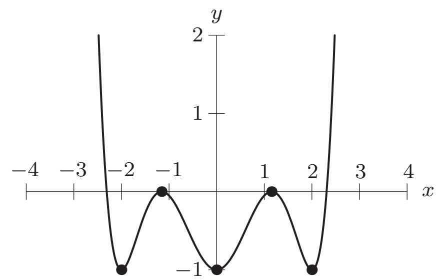

Figure 5.84

15. This graph is the graph of f stretched horizontally by a factor of 2 and then shifted down by 1 unit. See Figure 5.84.

16. **(a)** (ii) The \$5 tip is added to the fare $f(x)$, so the total is $f(x) + 5$.
(b) (iv) There were 5 extra miles so the trip was $x + 5$. I paid $f(x + 5)$.
(c) (i) Each trip cost $f(x)$ and I paid for 5 of them, or $5f(x)$.
(d) (iii) The miles were 5 times the usual so $5x$ is the distance, and the cost is $f(5x)$.

17. If profits are $r(t) = 0.5P(t)$ instead of $P(t)$, then profits are half the dollar level expected. If profits are $s(t) = P(0.5t)$ instead of $P(t)$, then profits are accruing half as fast as the projected rate.

18. **(a)** The formula is $A = f(r) = \pi r^2$.
(b) If the radius is increased by 10%, then the new radius is $r + (10\%)r = (110\%)r = 1.1r$. We want to know the output when our input is $1.1r$, so the appropriate expression is $f(1.1r)$.
(c) Since $f(1.1r) = \pi(1.1r)^2 = 1.21\pi r^2$, the new area is the old area multiplied by 1.21, or 121% of the old area. In other words, the area of a circle is increased by 21% when its radius is increased by 10%.

19. **(a)** Since III is horizontally stretched compared to one graph and compressed compared to another, it should be $f(x)$.
(b) The most horizontally compressed of the graphs are II and IV, so they should be $f(-2x)$ and $f(2x)$. Since II appears to be III reflected over the y-axis and compressed, it should be $f(-2x)$.
(c) The most horizontally stretched of the graphs should be $f(-\frac{1}{2}x)$, which is I.
(d) The most horizontally compressed of the graphs are II and IV, so they should be $f(-2x)$ and $f(2x)$. Since IV appears to be III compressed, it should be $f(2x)$.

20. **(a)** Since I is horizontally stretched compared to one graph and compressed compared to another, it should be $f(x)$.
(b) The most horizontally compressed of the graphs are III and II, so they should be $f(-2x)$ and $f(2x)$. Since III appears to be a compressed version of I reflected across the y-axis, it should be $f(-2x)$.
(c) The most horizontally stretched of the graphs should be $f(-\frac{1}{2}x)$, which is IV.
(d) The most horizontally compressed of the graphs are III and II, so they should be $f(-2x)$ and $f(2x)$. Since II appears to be a compressed version of I, it should be $f(2x)$.

21. The function f has been reflected over the x-axis and the y-axis and stretched horizontally by a factor of 2. Thus, $y = -f(-\frac{1}{2}x)$.

22. **(a)** See Figure 5.85. The zeros of $f(x)$ are at $x = \pm 2$.
(b) $g(x) = f(0.5x) = 4 - (0.5x)^2 = 4 - (0.25x)^2$. From the graph we see the zeros at ± 4.
(c) $h(x) = f(2x) = 4 - (2x)^2 = 4 - 4x^2$. From the graph we see the zeros at ± 1.
(d) The zeros are compressed toward the origin by a factor of $1/10$, so the zeros are ± 0.2.

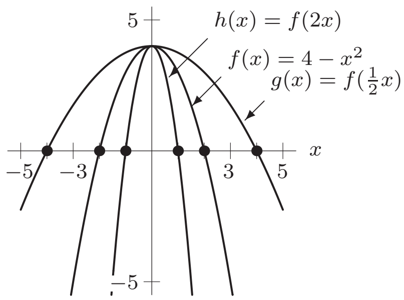

Figure 5.85

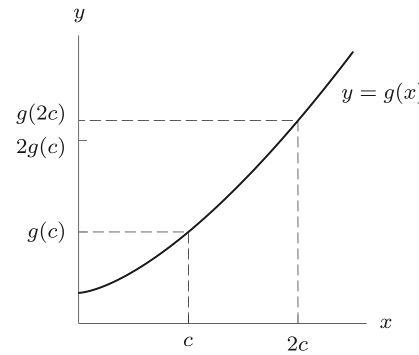

Figure 5.86

23. **(a)** To find $g(c)$, locate the point on $y = g(x)$ whose x-coordinate is c. The corresponding y-coordinate is $g(c)$.
(b) On the y-axis, go up twice the length of $g(c)$ to locate $2g(c)$.
(c) To find $g(2c)$, you must first find the location of $2c$ on the x-axis. This occurs twice as far from the origin as c. Then find the point on $y = g(x)$ whose x-coordinate is $2c$. The corresponding y-coordinate is $g(2c)$. See Figure 5.86.

24. Temperatures in this borehole are 3°C lower than at the same depth in the Belleterre borehole. See Table 5.18.

Table 5.18

d	25	50	75	100	125	150	175	200
$g(d)$	2.5	2.2	2.1	2.1	2.3	2.5	2.75	3

25. Temperatures in this borehole are the same as temperatures 5 meters deeper in the Belleterre borehole. See Table 5.19.

Table 5.19

d	20	45	70	95	120	145	170	195
$h(d)$	5.5	5.2	5.1	5.1	5.3	5.5	5.75	6

26. Temperatures in this borehole are the same as temperatures 10 meters less deep in the Belleterre borehole. See Table 5.20.

Table 5.20

d	35	60	85	110	135	160	185	210
$m(d)$	5.5	5.2	5.1	5.1	5.3	5.5	5.75	6

27. Temperatures in this borehole are 50% higher than temperatures at the same depth in the Belleterre borehole. See Table 5.21.

Table 5.21

d	25	50	75	100	125	150	175	200
$n(d)$	8.25	7.8	7.65	7.65	7.95	8.25	8.63	9

28. Temperatures in this borehole are the same as temperatures 20% less deep in the Belleterre borehole. See Table 5.22.

Table 5.22

d	31.25	62.5	93.75	125	156.25	187.5	218.75	250
$p(d)$	5.5	5.2	5.1	5.1	5.3	5.5	5.75	6

29. Temperatures in this borehole are 2°C warmer than temperatures 50% higher than temperatures at the same depth in the Belleterre borehole. See Table 5.23.

Table 5.23

d	25	50	75	100	125	150	175	200
$q(d)$	10.25	9.8	9.65	9.65	9.95	10.25	10.63	11

Solutions for Section 5.5

Exercises

1. By comparing $f(x)$ to the vertex form, $y = a(x-h)^2 + k$, we see the vertex is $(h,k) = (1,2)$. The axis of symmetry is the vertical line through the vertex, so the equation is $x = 1$. The parabola opens upward because the value of a is positive 3.

2. To compare $g(x)$ with the vertex form, rewrite it as $g(x) = -1(x-(-3))^2 + (-4)$. We then see the vertex is $(h,k) = (-3,-4)$. The axis of symmetry is the vertical line through the vertex, so the equation is $x = -3$. The parabola opens downward because the value of a is negative 1.

3. **(a)** See Figure 5.87. For g, we have $a = 1$, $b = 0$, and $c = 3$. Its vertex is at $(0,3)$, and its axis of symmetry is the y-axis, or the line $x = 0$. This function has no zeros.

(b) See Figure 5.88. For f, we have $a = -2$, $b = 4$, and $c = 16$. The axis of symmetry is the line $x = 1$ and the vertex is at $(1,18)$. The zeros, or x-intercepts, are at $x = -2$ and $x = 4$. The y-intercept is at $y = 16$.

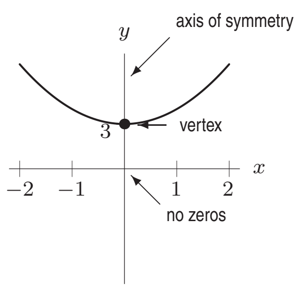

Figure 5.87: $g(x) = x^2 + 3$

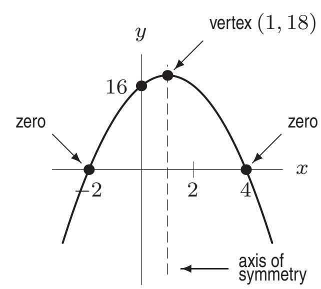

Figure 5.88: $f(x) = -2x^2 + 4x + 16$

4. To complete the square, we take $\frac{1}{2}$ of the coefficient of t and square the result. This gives $(\frac{1}{2}\cdot 11)^2 = (\frac{11}{2})^2 = \frac{121}{4}$. Using this number, we can rewrite the formula for $v(t)$:

$$v(t) = \underbrace{t^2 + 11t + \left(\frac{11}{2}\right)^2}_{\text{completing the square}} - \underbrace{\frac{121}{4}}_{\text{compensating term}} - 4$$

$$= \left(t + \frac{11}{2}\right)^2 - \frac{137}{4}.$$

Thus, the vertex of v is $(-\frac{11}{2}, -\frac{137}{4})$ and the axis of symmetry is $t = -\frac{11}{2}$.

5. Since the coefficient of x^2 is not 1, we first factor out the coefficient of x^2 from the formula. This gives

$$w(x) = -3\left(x^2 + 10x - \frac{31}{3}\right).$$

We next complete the square of the expression in parentheses. To do this, we add $(\frac{1}{2} \cdot 10)^2 = 25$ inside the parentheses:

$$w(x) = -3(\underbrace{x^2 + 10x + 25}_{\text{completing the square}} - \underbrace{25}_{\text{compensating term}} - 31/3).$$

Thus,

$$w(x) = -3((x+5)^2 - 106/3)$$
$$w(x) = -3(x+5)^2 + 106$$

so the vertex of the graph of this function is $(-5, 106)$, and the axis of symmetry is $x = -5$. Also, since $a = -3$ is negative, the graph is a downward opening parabola.

6. Factoring out negative one (to make the coefficient of x^2 equal 1) and completing the square gives

$$\begin{aligned} y &= -1 \cdot \left(x^2 - 7x + \left(-\frac{7}{2}\right)^2 - \left(-\frac{7}{2}\right)^2 + 13\right) \\ &= -\left(x - \frac{7}{2}\right)^2 + \frac{49}{4} - 13 \\ &= -\left(x - \frac{7}{2}\right)^2 - \frac{3}{4}. \end{aligned}$$

Thus, the graph of this function is a downward-opening parabola with a vertex below the x-axis. Since the graph is below the x-axis and opens down, it does not intersect the x-axis. We conclude that this function has no zeros which are real numbers.

To see this algebraically, notice that the equation $y = 0$ has no real-valued solution, because solving

$$-\left(x - \frac{7}{2}\right)^2 - \frac{3}{4} = 0$$

gives

$$x = \frac{7}{2} \pm \sqrt{-\frac{3}{4}}$$

and $\sqrt{-\frac{3}{4}}$ is not a real number.

7. The coordinates of the point $(6, 13)$ must satisfy the equation, so

$$13 = (6-3)^2 + k.$$

Solving for k gives $k = 4$. The formula is: $y = (x-3)^2 + 4$.

8. Substituting the coordinates of the vertex gives

$$-2 = a(0)^2 + k.$$

Solving for k gives $k = -2$. The formula now is $y = ax^2 - 2$. Substituting the coordinates of the point now gives

$$4 = a(3)^2 - 2.$$

Solving for a gives $a = \frac{2}{3}$. The formula is:

$$y = \frac{2}{3}x^2 - 2.$$

9. Since the vertex is $(4,7)$, we use the form $y = a(x-h)^2 + k$, with $h = 4$ and $k = 7$. We solve for a, substituting in the second point, $(0,4)$.

$$\begin{aligned} y &= a(x-4)^2 + 7 \\ 4 &= a(0-4)^2 + 7 \\ -3 &= 16a \\ -\frac{3}{16} &= a. \end{aligned}$$

Thus, an equation for the parabola is

$$y = -\frac{3}{16}(x-4)^2 + 7.$$

10. Since the vertex is $(3,3)$, we use the form $y = a(x-h)^2 + k$, with $h = 3$ and $k = 3$. We solve for a, substituting in the second point, $(5,5)$.

$$\begin{aligned} y &= a(x-3)^2 + 3 \\ 5 &= a(5-3)^2 + 3 \\ 2 &= 4a \\ \frac{1}{2} &= a. \end{aligned}$$

Thus, an equation for the parabola is

$$y = \frac{1}{2}(x-3)^2 + 3.$$

11. We know there are zeros at $x = -1$ and $x = 3$, so we use the factored form

$$y = a(x+1)(x-3).$$

We solve for a by substituting $x = 0$, $y = -1$ giving

$$\begin{aligned} -1 &= a(1)(-3) \\ a &= \frac{1}{3}. \end{aligned}$$

Thus, the parabola is

$$y = \frac{1}{3}(x+1)(x-3)$$

or

$$y = \frac{1}{3}x^2 - \frac{2}{3}x - 1.$$

12. The vertex is the point $(h,k) = (3,-5)$. Thus, a possible formula for this function is of the form

$$y = a(x-3)^2 - 5.$$

To find the value of a, we use the fact that the y-intercept of this function is $(0,2)$. Thus, we have $x = 0, y = 2$, so

$$\begin{aligned} a(0-3)^2 - 5 &= 2 \\ 9a &= 7 \\ a &= \frac{7}{9}. \end{aligned}$$

The formula for this quadratic function is $y = \frac{7}{9}(x-3)^2 - 5$. Since $|a| < 1$, this graph is wider than the graph of $y = x^2$.

13. We know there are zeros at $x = -6$ and $x = 2$, so we use the factored form:

$$y = a(x+6)(x-2)$$

and solve for a. At $x = 0$, we have

$$\begin{aligned} 5 &= a(0+6)(0-2) \\ 5 &= -12a \\ -\frac{5}{12} &= a. \end{aligned}$$

Thus,

$$y = -\frac{5}{12}(x+6)(x-2)$$

or

$$y = -\frac{5}{12}x^2 - \frac{5}{3}x + 5.$$

14. The function has zeros at $x = -4$ and $x = 5$, and appears quadratic, so it could be of the form $y = a(x+4)(x-5)$. Since $y = 36$ when $x = 2$, we know that $y = a(2+4)(2-5) = -18a = 36$, so $a = -2$. Therefore, $y = -2(x+4)(x-5)$ is a possible formula..

15. The square of half the coefficient of the x-term is $(\frac{8}{2})^2 = 16$. Adding and subtracting this number after the x-term gives

$$f(x) = x^2 + 8x + 16 - 16 + 3.$$

This can be simplified to $f(x) = (x+4)^2 - 13$. The vertex is $(-4, -13)$ and the axis of symmetry is $x = -4$.

16. Factoring out the coefficient of x^2 gives

$$g(x) = -2(x^2 - 6x - 2).$$

Inside the parenthesis, we add and subtract the square of half the coefficient of the x-term, $(-6/2)^2 = 9$, to get:

$$\begin{aligned} g(x) &= -2(x^2 - 6x + 9 - 9 - 2) \\ g(x) &= -2((x-3)^2 - 11) \\ g(x) &= -2(x-3)^2 + 22. \end{aligned}$$

The vertex is $(3, 22)$ and the axis of symmetry is $x = 3$.

17. Using the vertex form $y = a(x-h)^2 + k$, where $(h, k) = (2, 5)$, we have

$$y = a(x-2)^2 + 5.$$

Since the parabola passes through $(1, 2)$, these coordinates must satisfy the equation, so

$$2 = a(1-2)^2 + 5.$$

Solving for a gives $a = -3$. The formula is:

$$y = -3(x-2)^2 + 5.$$

18. Since the parabola has x-intercepts at $x = -1$ and $x = 5$, its formula is:

$$y = a(x+1)(x-5).$$

The coordinates $(-2, 6)$ must satisfy the equation, so

$$6 = a(-2+1)(-2-5).$$

Solving for a gives $a = \frac{6}{7}$. The formula is:

$$y = \frac{6}{7}(x+1)(x-5).$$

Problems

19. We have $(h,k)=(4,2)$, so $y=a(x-4)^2+2$. Solving for a, we have

$$\begin{aligned} a(0-4)^2+2 &= 6 \\ 16a &= 4 \\ a &= \frac{1}{4}, \end{aligned}$$

so $y=(1/4)(x-4)^2+2$.

20. We have $(h,k)=(4,2)$, so $y=a(x-4)^2+2$. Solving for a, we have

$$\begin{aligned} a(0-4)^2+2 &= -4 \\ 16a &= -6 \\ a &= -\frac{6}{16} = -\frac{3}{8}, \end{aligned}$$

so $y=(-3/8)(x-4)^2+2$.

21. We have $(h,k)=(4,2)$, so $y=a(x-4)^2+2$. Solving for a, we have

$$\begin{aligned} a(11-4)^2+2 &= 0 \\ 49a &= -2 \\ a &= -\frac{2}{49}, \end{aligned}$$

so $y=(-2/49)(x-4)^2+2$. We can verify this formula using the other zero:

$$(-2/49)(-3-4)^2+2=(-2/49)(-7)^2+2=-2+2=0,$$

as required.

22. The zeros are $x=1,4$, so $y=a(x-1)(x-4)$. Solving for a, we have

$$\begin{aligned} a(0-1)(0-4) &= 7 \\ 4a &= 7 \\ a &= \frac{7}{4}, \end{aligned}$$

so $y=(7/4)(x-1)(x-4)$.

23. There is one zero at $x=-2$, so by symmetry the vertex is $(-2,0)$. We have $y=a(x+2)^2$. Solving for a, we have

$$\begin{aligned} a(0+2)^2 &= 7 \\ 4a &= 7 \\ a &= \frac{7}{4}, \end{aligned}$$

so $y=(7/4)(x+2)^2$.

24. We have $(h,k)=(-7,-3)$, so $y=a(x+7)^2-3$. Solving for a, we have

$$\begin{aligned} a(-3+7)^2-3 &= -7 \\ 16a &= -4 \\ a &= -1/4, \end{aligned}$$

so $y=(-1/4)(x+7)^2-3$.

25. The graph of $y = x^2 - 10x + 25$ appears to be the graph of $y = x^2$ moved to the right by 5 units. See Figure 5.89. If this were so, then its formula would be $y = (x-5)^2$. Since $(x-5)^2 = x^2 - 10x + 25$, $y = x^2 - 10x + 25$ is, indeed, a horizontal shift of $y = x^2$.

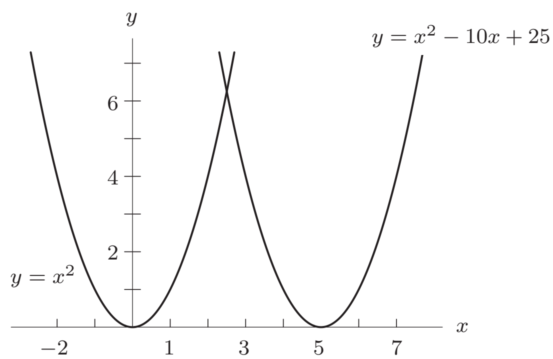

Figure 5.89

26. (a) See Figure 5.90.

(b) From Figure 5.90, it appears as though the graph of h might be found by flipping the graph of $f(x) = x^2$ over the x-axis and then shifting it to the left. In other words, we might guess that the graph of h is given by $y = -f(x+2)$. This graph is shown in Figure 5.91. Notice, though, that $y = -f(x+2)$ is not as steep as the graph of $y = h(x)$. However, we can look at the y–intercepts of both graphs and notice that -4 must be stretched to -8. Thus, we can try applying a stretch factor of 2 to our guess. The resulting graph, given by

$$y = -2f(x+2),$$

does indeed match h.

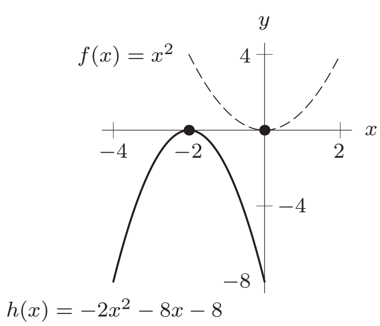

Figure 5.90: The graphs of $f(x) = x^2$ and $h(x) = -2x^2 - 8x - 8$

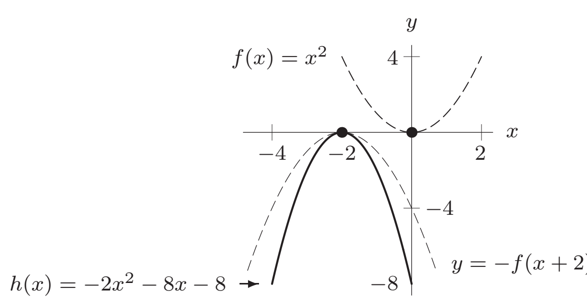

Figure 5.91: The graph of $y = -f(x+2)$ is a compressed version of the graph of h. The graph of $y = -2f(x+2)$ is the same as $y = h(x)$

We can verify the last result algebraically. If the graph of h is given by $y = -2f(x+2)$, then

$$\begin{aligned} y &= -2f(x+2) \\ &= -2(x+2)^2 \qquad \text{(because } f(x) = x^2\text{)} \\ &= -2(x^2 + 4x + 4) \\ &= -2x^2 - 8x - 8. \end{aligned}$$

This is the formula given for h.

27. **(a)**

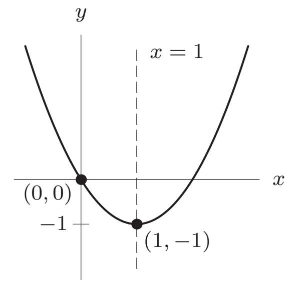

Figure 5.92

(b) Since the vertex is at $(1, -1)$, the parabola could be described by

$$f(x) = a(x-1)^2 - 1.$$

Since the parabola passes through $(0, 0)$

$$\begin{aligned} 0 &= a(0-1)^2 - 1 \\ 0 &= a - 1. \end{aligned}$$

Therefore

$$a = 1.$$

So, the equation is

$$f(x) = (x-1)^2 - 1$$

or

$$f(x) = x^2 - 2x.$$

(c) Since the vertex is at $(1, -1)$ and the parabola is concave up, the range of this function is all real numbers greater than or equal to -1.

(d) Since one zero is at $x = 0$, which is one unit to the left of the axis of symmetry at $x = 1$, the other zero will occur at one unit to the right of the axis of the symmetry at $x = 2$.

28. **(a)** The graph of g can be found by shifting the graph of f to the right 3 units and then up 2 units; $g(x) = f(x-3) + 2$.

(b) Yes, g is a quadratic function. To see this, notice that

$$\begin{aligned} g(x) &= (x-3)^2 + 2 \\ &= x^2 - 6x + 11. \end{aligned}$$

Thus, g is a quadratic function with parameters $a = 1$, $b = -6$, and $c = 11$.

(c) Figure 5.93 gives graphs of $f(x) = x^2$ and $g(x) = (x-3)^2 + 2$. Notice that g's axis of symmetry can be found by shifting f's axis of symmetry to the right 3 units. The vertex of g can be found by shifting f's vertex to the right 3 units and then up 2 units.

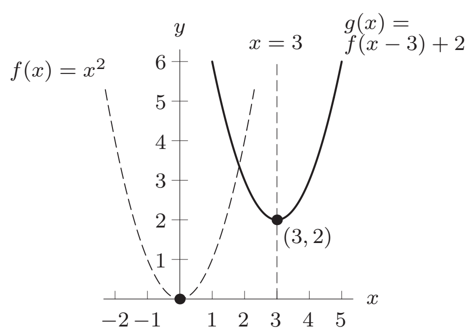

Figure 5.93: The graphs of $f(x) = x^2$ and $g(x) = f(x-3) + 2$.

29. Yes, we can find the function. Because the vertex is $(1, 4)$, $f(x) = a(x-1)^2 + 4$ for some a. To find a, we use the fact that $x = -1$ is a zero, that is, the fact that $f(-1) = 0$. We can write $f(-1) = a(-1-1)^2 + 4 = 0$, so $4a + 4 = 0$ and $a = -1$. Thus $f(x) = -(x-1)^2 + 4$.

30. We will reverse Gwendolyn's actions. First, we can shift the parabola back two units to the right by replacing x in $y = (x-1)^2 + 3$ with $(x-2)$. This gives

$$\begin{aligned} y &= ((x-2)-1)^2 + 3 \\ &= (x-3)^2 + 3. \end{aligned}$$

We subtract 3 from this function to move the parabola down three units, so

$$\begin{aligned} y &= (x-3)^2 + 3 - 3 \\ &= (x-3)^2. \end{aligned}$$

Finally, to flip the parabola back across the horizontal axis, we multiply the function by -1. Thus, Gwendolyn's original equation was

$$y = -(x-3)^2.$$

31. (a)

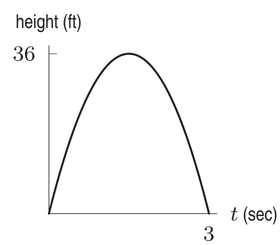

Figure 5.94

(b) To find t when $d(t) = 0$, either use the graph or factor $-16t^2 + 48t$ and set it equal to zero. Factoring yields $-16t^2 + 48t = -16t(t-3)$, so $d(t) = 0$ when $t = 0$ or $t = 3$. The first time $d(t) = 0$ is at the moment the tomato is being thrown up into the air. The second time is when the tomato hits the ground.

(c) The maximum height occurs on the axis of symmetry, which is halfway between the zeros, at $t = 1.5$. So, the tomato is highest 1.5 seconds after it is thrown.

(d) The maximum height reached is $d(1.5) = 36$ feet.

32. (a) Use a graphing calculator or computer to sketch a graph of the function over its relevant domain, $x \geq 0$, then find its vertex. (See Figure 5.95).

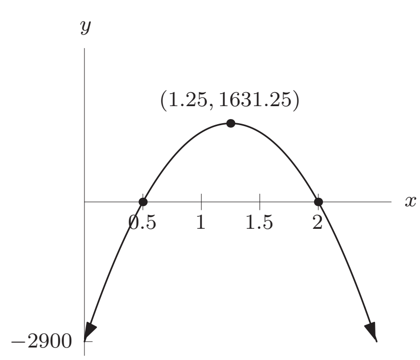

Figure 5.95

The maximum profit is \$ 1631.25 per week, when the price per cup is \$ 1.25.

(b) $P(x-2)$ is a horizontal shift of $P(x)$, while $P(x)-2$ is a vertical shift down ($2 less profit). Since the maximum value of the function will not change under a horizontal shift, $P(x-2)$ will also have a maximum profit of $1631.25 per week. Since $P(x-2)$ is $P(x)$ shifted two units to the right, the maximum profit will occur at $1.25+2$ or at the price of $ 3.25 per cup (very expensive!)

(c) Since $P(x)+50$ is a vertical shift of $P(x)$, the maximum value will still occur when $x = 1.25$ but the maximum profit is now 1631.25+$50=$1681.25.

33. The distance around any rectangle with a height of h units and a base of b units is $2b + 2h$. See Figure 5.96. Since the string forming the rectangle is 50 cm long, we know that $2b + 2h = 50$ or $b + h = 25$. Therefore, $b = 25 - h$. The area, A, of such a rectangle is

$$A = bh$$
$$A = (25 - h)(h).$$

The zeros of this quadratic function are $h = 0$ and $h = 25$, so the axis of symmetry, which is halfway between the zeros, is $h = 12.5$. Since the maximum value of A occurs on the axis of symmetry, the area will be the greatest when the height is 12.5 and the base is also 12.5 ($b = 25 - h = 25 - 12.5 = 12.5$).

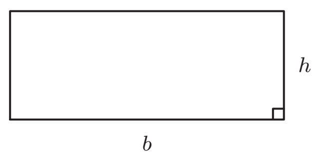

Figure 5.96

If the string were k cm long, $2b+2h = k$ or $b+h = \frac{k}{2}$, so $b = \frac{k}{2} - h$. $A = bh = \left(\frac{k}{2} - h\right) h$. The zeros in this case are $h = 0$ and $h = \frac{k}{2}$, so the axis of symmetry is $h = \frac{k}{4}$. If $h = \frac{k}{4}$, then $b = \frac{k}{2} - h = \frac{k}{2} - \frac{k}{4} = \frac{k}{4}$. So the dimensions for maximum area are $\frac{k}{4}$ by $\frac{k}{4}$; in other words, the rectangle with the maximum area is a square whose side measures $\frac{1}{4}$ of the length of the string.

34. **(a)** See Figure 5.97

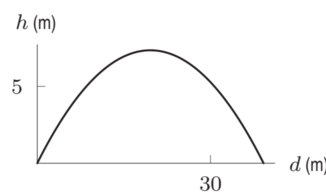

Figure 5.97

(b) When the ball hits the ground $h = 0$, so $h = 0.75d - 0.0192d^2 = d(0.75 - 0.0192d) = 0$ and we get $d = 0$ or $d \approx 39.063$ m. Since $d = 0$ is the position where the kicker is standing, the ball must hit the ground about 39.063 meters from the point where it is kicked.

(c) The path is parabolic and the maximum height occurs at the vertex, which lies on the axis of symmetry, mid-way between the zeros at $d \approx 19.531$ m. Since $h = 0.75(19.531) - 0.0192(19.531)^2 \approx 7.324$, we know that the ball reaches 7.324 meters above the ground before it begins to fall.

(d) From part (c), the horizontal distance traveled when the ball reaches its maximum height is ≈ 19.531 m.

35. **(a)** Factoring gives $h(t) = -16t^2 + 16Tt = 16t(T - t)$. Since $h(t) \geq 0$ only for $0 \leq t \leq T$, the model makes sense only for these values of t.

(b) The times $t = 0$ and $t = T$ give the start and end of the jump. The maximum height occurs halfway in between, at $t = T/2$.

(c) Since $h(t) = 16t(T - t)$, we have

$$h\left(\frac{T}{2}\right) = 16\left(\frac{T}{2}\right)\left(T - \frac{T}{2}\right) = 4T^2.$$

Solutions for Chapter 5 Review

Exercises

1. **(a)** The input is $2x = 2 \cdot 2 = 4$.
(b) The input is $\frac{1}{2}x = \frac{1}{2} \cdot 2 = 1$.
(c) The input is $x + 3 = 2 + 3 = 5$.
(d) The input is $-x = -2$.

2. **(a)** The input is $2x$, and so $2x = 2$, which means $x = 1$.
(b) The input is $\frac{1}{2}x$, and so $\frac{1}{2}x = 2$, which means $x = 4$.
(c) The input is $x + 3$, and so $x + 3 = 2$, which means $x = -1$.
(d) The input is $-x$, and so $-x = 2$, which means $x = -2$.

3. **(a)** $(6, 5)$ **(b)** $(2, 1)$ **(c)** $(\frac{1}{2}, 5)$ **(d)** $(2, 20)$

4. **(a)** $(-9, 4)$ **(b)** $(-3, \frac{4}{3})$ **(c)** $(1, 4)$ **(d)** $(-1, -4)$

5. A function is odd if $a(-x) = -a(x)$.

$$\begin{aligned} a(x) &= \frac{1}{x} \\ a(-x) &= \frac{1}{-x} = -\frac{1}{x} \\ -a(x) &= -\frac{1}{x} \end{aligned}$$

Since $a(-x) = -a(x)$, we know that $a(x)$ is an odd function.

6. $m(-x) = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2} = m(x)$, so $m(x)$ is an even function.

7. In this case, $e(x) = x + 3$, $e(-x) = -x + 3$ and $-e(x) = -(x + 3) = -x - 3$. Since $e(-x)$ equals neither $e(x)$ nor $-e(x)$, the function $e(x)$ is neither even nor odd.

8. $p(-x) = (-x)^2 + 2(-x) = x^2 - 2x$, and $-p(x) = -x^2 - 2x$. Since $p(-x) \neq p(x)$ and $p(-x) \neq -p(x)$, the function p is neither even nor odd.

9. A function is even if $b(-x) = b(x)$.

$$\begin{aligned} b(x) &= |x| \\ b(-x) &= |-x| = |x| \end{aligned}$$

Since $b(-x) = b(x)$, we know that $b(x)$ is an even function.

10. $q(-x) = 2^{-x+1}$, and $-q(x) = -2^{x+1}$. Since $q(-x) \neq q(x)$ and $q(-x) \neq -q(x)$, the function q is neither even nor odd.

11. **(a)** $f(2x) = 1 - (2x) = 1 - 2x$
(b) $f(x + 1) = 1 - (x + 1) = -x$
(c) $f(1 - x) = 1 - (1 - x) = x$
(d) $f(x^2) = 1 - x^2$
(e) $f(1/x) = 1 - (1/x) = (x/x) - (1/x) = (x - 1)/x$
(f) $f(\sqrt{x}) = 1 - \sqrt{x}$

12.

Table 5.24

x	-3	-2	-1	0	1	2	3
$f(x)$	-4	-1	2	3	0	-3	-6
$f(-x)$	-6	-3	0	3	2	-1	-4
$-f(x)$	4	1	-2	-3	0	3	6
$f(x)-2$	-6	-3	0	1	-2	-5	-8
$f(x-2)$	–	–	-4	-1	2	3	0
$f(x)+2$	-2	1	4	5	2	-1	-4
$f(x+2)$	2	3	0	-3	-6	–	–
$2f(x)$	-8	-2	4	6	0	-6	-12
$-f(x)/3$	$4/3$	$1/3$	$-2/3$	-1	0	1	2

13. The function has zeros at $x=-1$ and $x=3$, and appears quadratic, so it could be of the form $y=a(x+1)(x-3)$. Since $y=-3$ when $x=0$, we know that $y=a(0+1)(0-3)=-3a=-3$, so $a=1$. Thus $y=(x+1)(x-3)$ is a possible formula.

14. The vertex is the point $(h,k)=(-6,9)$, so a possible equation is

$$y=a(x+6)^2+9.$$

Solving for a, we use the fact that the graph has an x-intercept of 15, so $y=0$ when $x=15$. This gives

$$\begin{aligned} a(-15+6)^2+9&=0\\ 81a&=-9\\ a&=-\frac{1}{9}, \end{aligned}$$

so $y=-\frac{1}{9}(x+6)^2+9$. Since $a<0$, this graph is a flipped upside down and much wider than $y=x^2$.

15. The function appears quadratic with vertex at $(2,0)$, so it could be of the form $y=a(x-2)^2$. For $x=0$, $y=-4$, so $y=a(0-2)^2=4a=-4$ and $a=-1$. Thus $y=-(x-2)^2$ is a possible formula.

16. Since the vertex is $(6,5)$, we use the form $y=a(x-h)^2+k$, with $h=6$ and $k=5$. We solve for a, substituting in the second point, $(10,8)$.

$$\begin{aligned} y&=a(x-6)^2+5\\ 8&=a(10-6)^2+5\\ 3&=16a\\ \frac{3}{16}&=a. \end{aligned}$$

Thus, an equation for the parabola is

$$y=\frac{3}{16}(x-6)^2+5.$$

Problems

17. The graph is the graph of f shifted to the left by 2 and up by 2. See Figure 5.98.

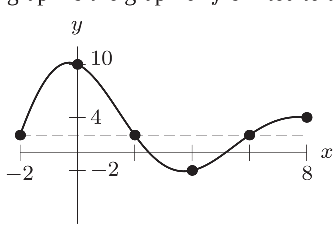

Figure 5.98

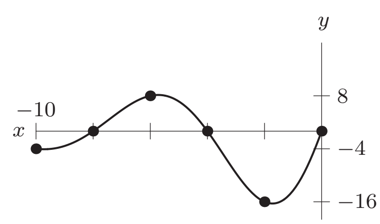

Figure 5.99

18. The graph is the graph of f flipped across the y-axis, stretched vertically by a factor of 2, and flipped across the x-axis. See Figure 5.99.

19. The graph appears to be the graph of f shifted to the left by 4 and down by 8, so $y = f(t+4) - 8$.

20. The graph appears to be the graph of f flattened vertically by a factor of 0.5, flipped across the x-axis, and shifted vertically by 4 units, so $y = -0.5f(x) + 4$.

21. **(a)** $D(225)$ represents the number of iced cappuccinos sold at a price of \$2.25.
(b) $D(p)$ is likely to be a decreasing function. The coffeehouse will probably sell fewer iced cappuccinos if they charge a higher price for them.
(c) p is the price the coffeehouse should charge if they want to sell 180 iced cappuccinos per week.
(d) $D(1.5t)$ represents the number of iced cappuccinos the coffeehouse will sell if they charge one and a half times the average price. $1.5D(t)$ represents 1.5 times the number of cappuccinos sold at the average price. $D(t+50)$ is the number of iced cappuccinos they will sell if they charge 50 cents more than the average price. $D(t)+50$ represents 50 more cappuccinos than the number they can sell at the average price.

22. **(a)** $j(25)$ is the average amount of water (in gallons) required daily by a 25-foot oak. However, $j^{-1}(25)$ is the height of an oak requiring an average of 25 gallons of water per day.
(b) $j(v) = 50$ means that an oak of height v requires 50 gallons of water daily. This statement can be rewritten $j^{-1}(50) = v$.
(c) This statement can be written $j(z) = p$, or as $j^{-1}(p) = z$.
(d)
- $j(2z)$ is the amount of water required by a tree that is twice average height.
- $2j(z)$ is enough water for two oak trees of average height. This expression equals $2p$.
- $j(z+10)$ is enough water for an oak tree ten feet taller than average.
- $j(z)+10$ is the amount of water required by an oak of average height, plus 10 gallons. Thus, this expression equals $p+10$.
- $j^{-1}(2p)$ is the height of an oak requiring $2p$ gallons of water.
- $j^{-1}(p+10)$ is the height of an oak requiring $p+10$ gallons of water.
- $j^{-1}(p)+10$ is the height of an oak that is 10 feet taller than average. Thus, this expression equals $z+10$.

23. **(a)** (VI) We know that $y = e^x$ is an increasing function with a y-intercept of 1. When $x = 1$, $y = e$, or, a little less than 3.
(b) (V) The graph of $y = e^{5x}$ is similar to that of $y = e^x$, but it is compressed horizontally by a factor of 5. It is a more rapidly increasing function with a y-intercept of 1.
(c) (III) The graph of $y = 5e^x$ is a vertical stretch, by a factor of 5, of $y = e^x$. It is an increasing function with a y-intercept of 5.
(d) (IV) The graph of $y = e^{x+5}$ is the graph of $y = e^x$ shifted to the left by 5 units.
(e) (I) The graph of $y = e^{-x}$ is the graph of $y = e^x$ transposed across the y-axis. It is a decreasing function with a y-intercept of 1.
(f) (II) The graph of $y = e^x + 5$ is the graph of $y = e^x$ shifted vertically upward by 5 units. The y-intercept will be $1 + 5 = 6$.

24. In the southern hemisphere the seasons are reversed, that is, they come a half year earlier (or later) than in the northern hemisphere. So we need to shift the graph horizontally by half a year. Whether we shift the graph left or right makes no difference. Therefore, possible formulas for the shifted curve include $L(d + \frac{365}{2})$ and $L(d - \frac{365}{2})$. In Figure 5.100, we have shifted $L(d)$ one-half year forward (to the left) giving $L(d + \frac{365}{2})$.

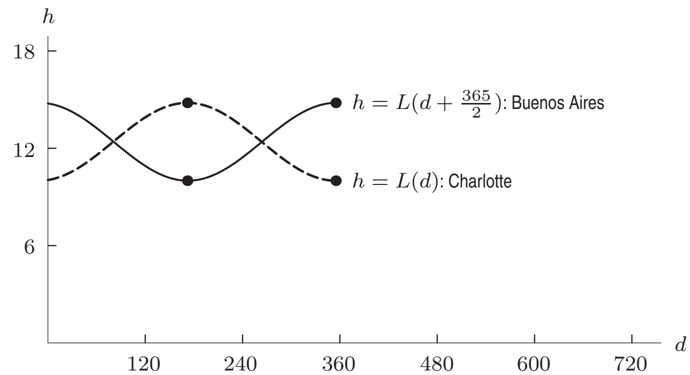

Figure 5.100

25. **(a)** Using the formula for $d(t)$, we have

$$\begin{aligned} d(t) - 15 &= (-16t^2 + 38) - 15 \\ &= -16t^2 + 23. \end{aligned}$$

$$\begin{aligned} d(t - 1.5) &= -16(t - 1.5)^2 + 38 \\ &= -16(t^2 - 3t + 2.25) + 38 \\ &= -16t^2 + 48t + 2. \end{aligned}$$

(b)

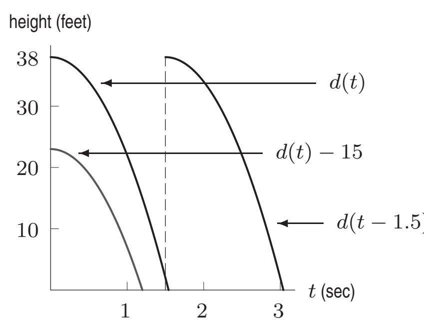

Figure 5.101

(c) $d(t) - 15$ represents the height of a brick which falls from $38 - 15 = 23$ feet above the ground. On the other hand, $d(t-1.5)$ represents the height of a brick which began to fall from 38 feet above the ground at one and a half seconds after noon.

(d) (i) The brick hits the ground when its height is 0. Thus, if we represent the brick's height above the ground by $d(t)$, we get

$$\begin{aligned} 0 &= d(t) \\ 0 &= -16t^2 + 38 \\ -38 &= -16t^2 \\ t^2 &= \frac{38}{16} \\ t^2 &= 2.375 \\ t &= \pm\sqrt{2.375} \approx \pm 1.541. \end{aligned}$$

We are only interested in positive values of t, so the brick must hit the ground 1.541 seconds after noon.

(ii) If we represent the brick's height above the ground by $d(t) - 15$ we get

$$\begin{aligned} 0 &= d(t) - 15 \\ 0 &= -16t^2 + 23 \\ -23 &= -16t^2 \\ t^2 &= \frac{23}{16} \\ t^2 &= 1.4375 \\ t &= \pm\sqrt{1.4375} \approx \pm 1.199. \end{aligned}$$

Again, we are only interested in positive values of t, so the the brick hits the ground 1.199 seconds after noon.

(e) Since the brick, whose height is $d(t - 1.5)$, begins falling 1.5 seconds after the brick whose height is $d(t)$, we expect the brick whose height is $d(t-1.5)$ to hit the ground 1.5 seconds after the brick whose height is $d(t)$. Thus, the brick should hit the ground $1.5 + 1.541 = 3.041$ seconds after noon.

26. The graph could be $y = |x|$ shifted right by one unit and up two units. Thus, let

$$y = |x - 1| + 2.$$

27. The graph is the cubic function that has been flipped about the x-axis and shifted left and up one unit. Thus, we could try

$$y = -(x + 1)^3 + 1.$$

28. The graph appears to have been compressed horizontally by a factor of 2, flipped vertically, and shifted vertically by 2 units, so

$$y = -h(2x) + 2.$$

29. The graph appears to have been shifted to the left 6 units, compressed vertically by a factor of 2, and shifted vertically by 1 unit, so

$$y = \frac{1}{2}h(x + 6) + 1.$$

30. (a) $f(10) = 6000$. The total cost for a carpenter to build 10 wooden chairs is $\$6000$.

(b) $f(30) = 7450$. The total cost for a carpenter to build 30 wooden chairs is $\$7450$.

(c) $z = 40$. A carpenter can build 40 wooden chairs for $\$8000$.

(d) $f(0) = 5000$. This is the fixed cost of production, or how much it costs the carpenter to set up before building any chairs.

31. Assuming f is linear between 10 and 20, we get

$$\begin{aligned} \frac{6400 - f(10)}{p - 10} &= \frac{f(20) - f(10)}{20 - 10}, \\ \text{or} \quad \frac{400}{p - 10} &= \frac{800}{10}. \end{aligned}$$

Solving for p yields $p = 15$. Assuming f is linear between 20 and 30, we get

$$\begin{aligned} \frac{q - f(20)}{26 - 20} &= \frac{f(30) - f(20)}{30 - 20}, \\ \text{or} \quad \frac{q - 6800}{6} &= \frac{650}{10}. \end{aligned}$$

Solving for q yields $q = 7190$.

32. (a) We have

$$\begin{aligned} d_1 &= f(30) - f(20) = 650 \\ d_2 &= f(40) - f(30) = 550 \\ d_3 &= f(50) - f(40) = 500 \end{aligned}$$

(b) d_1, d_2, and d_3 tell us how much building an additional 10 chairs will cost if the carpenter has already built 20, 30, and 40 chairs respectively.

33.

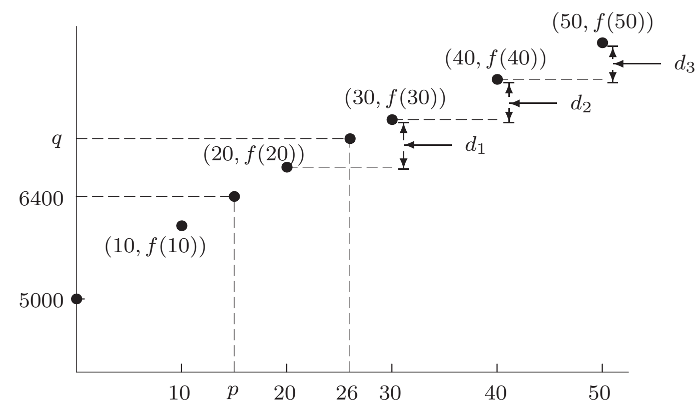

Figure 5.102

34. (a) (i) $f(k+10)$ is how much it costs to produce 10 more than the normal weekly number of chairs.
(ii) $f(k)+10$ is 10 dollars more than the cost of a normal week's production.
(iii) $f(2k)$ is the normal cost of two week's production.
(iv) $2f(k)$ is twice the normal cost of one week's production (which may be greater than $f(2k)$ since the fixed costs are included twice in $2f(k)$).

(b) The total amount the carpenter gets will be $1.8f(k)$ plus a five percent sales tax: that is, $1.05(1.8f(k)) = 1.89f(k)$.

35. See Figure 5.103.

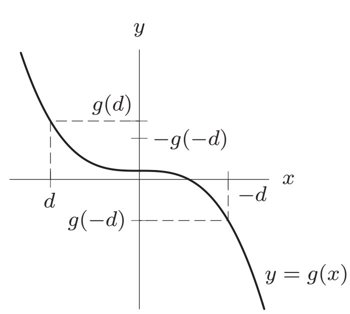

Figure 5.103

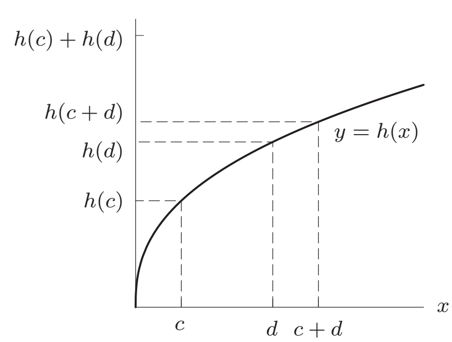

Figure 5.104

36. See Figure 5.104.

37. There is a vertical stretch of 3 so
$$y = 3h(x).$$

38. We have a reflection through the x-axis and a horizontal shift to the right by 1.
$$y = -h(x-1)$$

39. There is a reflection through the y-axis, a horizontal compression by a factor of 2, a horizontal shift to the right by 1 unit, and a reflection through the x-axis. Combining these transformations we get
$$y = -h(-2(x-1)) \qquad \text{or} \qquad y = -h(2-2x).$$

40. **(a)** Sketch varies – should show appropriate seasonal increases/decreases, such as in Figure 5.105.

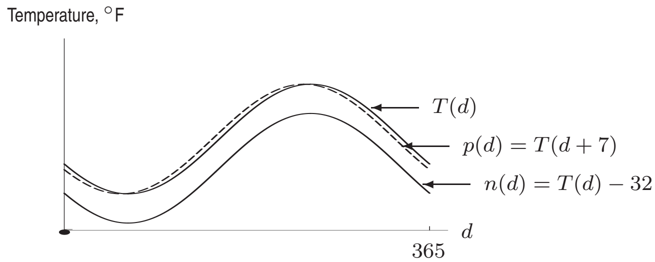

Figure 5.105: Possible graphs of T and n and p

(b) Freezing is 32°F. If $T(d)$ is the temperature for a particular day, you can determine how far above (or below) freezing $T(d)$ is by subtracting 32 from it. So $n(d) = T(d) - 32$. The sketch of n is the sketch of T shifted 32 units (32°F) downward. See Figure 5.105.

(c) Since low temperatures this year are a week ahead of those of last year, the low temperature on the 100th day of this year, $p(100)$, is the same as the low temperature on the 107th day of last year, $T(107)$. More generally, $p(d) = T(d+7)$. The graph of p is 7 units (7 days) to the left of the graph of T because all low temperatures are occurring seven days earlier. See Figure 5.105.

41. Figure 5.106 shows a graph of the basketball player's trajectory for $T = 1$ second. Since this is the graph of a parabola, the maximum height occurs at the t-value which is halfway between the zeros, 0 and 1. Thus, the maximum occurs at $t = 1/2$ second. The maximum height is $h(1/2) = 4$ feet, and 75% of 4 is 3. Thus, when the basketball player is above 3 feet from the ground, he is in the top 25% of his trajectory. To find when he reaches a height of 3 feet, set $h(t) = 3$. Solving for t gives $t = 0.25$ or $t = 0.75$ seconds. Thus, from $t = 0.25$ to $t = 0.75$ seconds, the basketball player is in the top 25% of his jump, as indicated in Figure 5.106. We see that he spends half of the time at the top quarter of the height of this jump, giving the impression that he hangs in the air.

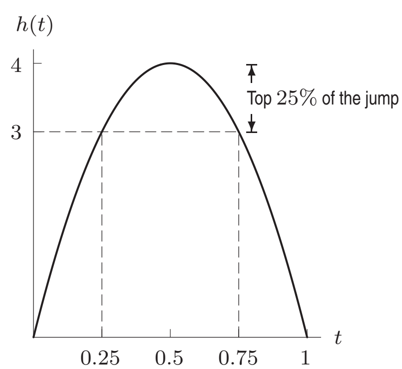

Figure 5.106: A graph of $h(t) = -16t^2 + 16Tt$ for $T = 1$

42.

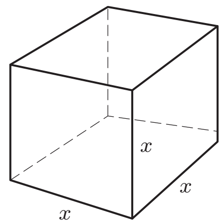

Figure 5.107

(a) $L(x) = 12x$
(b) $L(x) - 6$ cm of tape would be short by 6 cm of doing the job. $L(x - 6)$ cm of tape would be short by 6 cm for each edge (a total of 72 cm short all together).
(c) Each edge would require 2 extra cm of tape, so the required amount would be $L(x + 2)$.
(d) $S(x) = 6x^2$.
(e) $V(x) = x^3$.

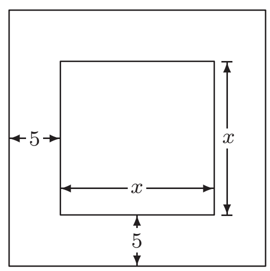

Figure 5.108: cross sectioned view

(f) Surface area of larger box $= S(x + 10)$.
(g) Volume of larger box $= V(x + 10)$.
(h) Tape on edge of larger box $= L(x + 10)$.
(i) Double taping the outer box would require $2L(x + 10)$, or $L(2(x + 10))$. ($L(2x)$ will give the same function values as $2L(x)$.)
(j) A box with edge length 20% longer than x has length $x + 0.20x = 1.2x$; the "taping function" should input that edge length, hence $L(1.2x)$ does the job. (But note that $1.2L(x)$ will also work in this case.)

CHECK YOUR UNDERSTANDING

1. True. The graph of $g(x)$ is a copy of the graph of f shifted vertically up by three units.
2. False. The horizontal shift is two units to the right.
3. True. The graph is shifted down by $|k|$ units.
4. True.
5. True. The reflection across the x-axis of $y = f(x)$ is $y = -f(x)$.
6. False. For an odd function $f(x) = -f(-x)$.
7. False. The graphs of odd functions are symmetric about the origin.
8. True. Any point (x, y) on the graph of $y = f(x)$ reflects across the x-axis to the point $(x, -y)$, which lies on the graph of $y = -f(x)$.

9. True. Any point (x, y) on the graph of $y = f(x)$ reflects across the y-axis to the point $(-x, y)$, which lies on the graph of $y = f(-x)$.

10. True. Symmetry about the y-axis means that if any point (x, y) is on the graph of the function then $(-x, y)$ must also be on the graph of the function. If (x, y) is on the graph, then $y = f(x)$. Thus, if the graph is symmetric, we also know that $y = f(-x)$, so $f(x) = f(-x)$.

11. False. In the figure in the problem, it appears that $g(x) = f(x-2) + 1$ because the graph is two units to the right and one unit up from the graph of f.

12. False. Substituting $(x-2)$ in to the formula for g gives $g(x-2) = (x-2)^2 + 4 = x^2 - 4x + 4 + 4 = x^2 - 4x + 8$.

13. False. If $f(x) = x^2$, then $f(x+1) = x^2 + 2x + 1 \neq x^2 + 1 = f(x) + 1$.

14. True. This looks like the absolute value function shifted right 1 unit and down 2 units.

15. True. The reflection across the x-axis of the graph of $f(x)$ has equation $y = -f(x)$, and a four unit upward shift of that graph has equation $y = -f(x) + 4 = -3^x + 4$.

16. False. If $q(p) = p^2 + 2p + 4$ then $-q(-p) = -((-p)^2 + 2(-p) + 4) = -(p^2 - 2p + 4) = -p^2 + 2p - 4$.

17. True.

18. True. Since

$$\text{Rate of change } = \frac{f(b) - f(a)}{b - a},$$

multiplying f by k multiplies the rate of change by k.

19. False. The graph of g has the same shape as f so it has not been stretched. It appears that $g(x) = -f(x+1) + 3$.

20. False. From the table, we have $g(-2) = -\frac{1}{2}f(-2+1) - 3 = -\frac{1}{2}f(-1) - 3 = -\frac{1}{2}(4) - 3 = -5$.

21. False. Consider $f(x) = x^2$. Shifting up first and then compressing vertically gives the graph of $g(x) = \frac{1}{2}(x^2 + 1) = \frac{1}{2}x^2 + \frac{1}{2}$. Compressing first and then shifting gives the graph of $h(x) = \frac{1}{2}x^2 + 1$.

22. False. In the graph, it appears that $g(x) = 3f(2x)$.

23. True. For $3f(2x) + 1$, at $x = -2$, we have $3f(2(-2)) + 1 = 3f(-4) + 1 = 3(1) + 1 = 4$. For $-2f(\frac{1}{2}x)$, at $x = 2$, we have $-2f(\frac{1}{2}(-2)) = -2f(-1) = -2(-2) = 4$.

24. True.

25. True. This is the definition of a vertex.

26. False. If a parabola is concave up its vertex is a minimum point.

27. False. The vertex is located at the point (h, k).

28. True. The vertex is (h, k), and the axis of symmetry is the vertical line $x = h$ through the vertex.

29. True. Transform $y = ax^2 + bx + c$ to the form $y = a(x-h)^2 + k$ where it can be seen that if $a < 0$, then the value of y has a maximum at the vertex (h, k), and the parabola opens downward.

Solutions to Tools for Chapter 5

1. $x^2 + 8x = x^2 + 8x + 16 - 16 = (x+4)^2 - 16$

2. $y^2 - 12y = y^2 - 12y + 36 - 36 = (y-6)^2 - 36$

3. $w^2 + 7w = w^2 + 7w + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 = \left(w + \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2$

4. $2r^2 + 20r = 2(r^2 + 10r) = 2(r^2 + 10r + 25 - 25) = 2((r+5)^2 - 25) = 2(r+5)^2 - 50$.

5. $s^2 + 6s - 8 = s^2 + 6s + 9 - 9 - 8 = (s+3)^2 - 17$

6. $3t^2 + 24t - 13 = 3(t^2 + 8t) - 13 = 3(t^2 + 8t + 16) - 48 - 13 = 3(t+4)^2 - 61$

7. We add and subtract the square of half the coefficient of the a-term, $(\frac{-2}{2})^2 = 1$, to get

$$a^2 - 2a - 4 = a^2 - 2a + 1 - 1 - 4$$

$$\begin{aligned} &= (a^2 - 2a + 1) - 1 - 4 \\ &= (a-1)^2 - 5. \end{aligned}$$

8. We add and subtract the square of half the coefficient of the n-term, $(\frac{4}{2})^2 = 4$, to get

$$\begin{aligned} n^2 + 4n - 5 &= n^2 + 4n + 4 - 4 - 5 \\ &= (n^2 + 4n + 4) - 4 - 5 \\ &= (n+2)^2 - 9. \end{aligned}$$

9. We add and subtract the square of half the coefficient of the c-term, $(\frac{3}{2})^2 = \frac{9}{4}$, to get

$$\begin{aligned} c^2 + 3c - 7 &= c^2 + 3c + \frac{9}{4} - \frac{9}{4} - 7 \\ &= \left(c^2 + 3c + \frac{9}{4}\right) - \frac{9}{4} - 7 \\ &= \left(c + \frac{3}{2}\right)^2 - \frac{37}{4}. \end{aligned}$$

10. Factoring out the coefficient of r^2 gives

$$3r^2 + 9r - 4 = 3\left(r^2 + 3r - \frac{4}{3}\right).$$

Inside the parenthesis, we add and subtract the square of half the coefficient of the r-term, $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$, to get

$$\begin{aligned} 3\left(r^2 + 3r - \frac{4}{3}\right) &= 3\left(r^2 + 3r + \frac{9}{4} - \frac{9}{4} - \frac{4}{3}\right) \\ &= 3\left(\left(r + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{4}{3}\right) \\ &= 3\left(r + \frac{3}{2}\right)^2 - \frac{43}{4}. \end{aligned}$$

11. Factoring out the coefficient of s^2 gives

$$4\left(s^2 + \frac{1}{4}s + \frac{1}{2}\right).$$

Inside the parenthesis, we add and subtract the square of half the coefficient of the s-term, $\left(\frac{1/4}{2}\right)^2 = \frac{1}{64}$, to get

$$\begin{aligned} 4\left(s^2 + \frac{1}{4}s + \frac{1}{2}\right) &= 4\left(s^2 + \frac{1}{4}s + \frac{1}{64} - \frac{1}{64} + \frac{1}{2}\right) \\ &= 4\left(\left(s + \frac{1}{8}\right)^2 - \frac{1}{64} + \frac{1}{2}\right) \\ &= 4\left(s + \frac{1}{8}\right)^2 + \frac{31}{16}. \end{aligned}$$

12. Factoring out the coefficient of g^2 gives

$$12g^2 + 8g + 5 = 12\left(g^2 + \frac{2}{3}g + \frac{5}{12}\right).$$

Inside the parenthesis, we add and subtract the square of half the coefficient of the g-term, $\left(\frac{2/3}{2}\right)^2 = \frac{1}{9}$, to get

$$\begin{aligned}
12\left(g^2 + \frac{2}{3}g + \frac{5}{12}\right) &= 12\left(g^2 + \frac{2}{3}g + \frac{1}{9} - \frac{1}{9} + \frac{5}{12}\right) \\
&= 12\left(\left(g + \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{5}{12}\right) \\
&= 12\left(g + \frac{1}{3}\right)^2 + \frac{11}{3}.
\end{aligned}$$

13. Completing the square yields

$$x^2 - 2x - 3 = (x^2 - 2x + 1) - 1 - 3 = (x-1)^2 - 4.$$

14. First we rewrite $10 - 6x + x^2$ as $x^2 - 6x + 10$ and then complete the square. So

$$10 - 6x + x^2 = x^2 - 6x + 10 = (x^2 - 6x + 9) - 9 + 10 = (x-3)^2 + 1.$$

15. First we factor out -1. Then

$$\begin{aligned}
-x^2 + 6x - 2 &= -(x^2 - 6x + 2) = -(x^2 - 6x + 9 - 9 + 2) \\
&= -(x^2 - 6x + 9 - 7) = -(x^2 - 6x + 9) + 7 \\
&= -(x-3)^2 + 7.
\end{aligned}$$

16. First we factor 3 from the first two terms. Then

$$\begin{aligned}
3x^2 - 12x + 13 &= 3(x^2 - 4x) + 13 = 3(x^2 - 4x + 4 - 4) + 13 \\
&= 3(x^2 - 4x + 4) - 12 + 13 = 3(x-2)^2 + 1.
\end{aligned}$$

17. Complete the square and write in vertex form.

$$\begin{aligned}
y &= x^2 + 6x + 3 \\
&= x^2 + 6x + 9 - 9 + 3 \\
&= (x+3)^2 - 6.
\end{aligned}$$

The vertex is $(-3, -6)$.

18. Complete the square and write in vertex form.

$$\begin{aligned}
y &= x^2 - x + 4 \\
&= x^2 - x + \frac{1}{4} - \frac{1}{4} + 4 \\
&= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 4 \\
&= \left(x - \frac{1}{2}\right)^2 + \frac{15}{4}.
\end{aligned}$$

The vertex is $(1/2, 15/4)$.

19. Complete the square and write in vertex form.

$$\begin{aligned} y &= -x^2 - 8x + 2 \\ &= -(x^2 + 8x - 2) \\ &= -(x^2 + 8x + 16 - 16 - 2) \\ &= -((x+4)^2 - 18) \\ &= -(x+4)^2 + 18. \end{aligned}$$

The vertex is $(-4, 18)$

20. Complete the square and write in vertex form.

$$\begin{aligned} y &= x^2 - 3x - 3 \\ &= x^2 - 3x + \frac{9}{4} - \frac{9}{4} - 3 \\ &= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 3 \\ &= \left(x - \frac{3}{2}\right)^2 - \frac{21}{4}. \end{aligned}$$

The vertex is $(3/2, -21/4)$.

21. Complete the square and write in vertex form.

$$\begin{aligned} y &= -x^2 + x - 6 \\ &= -(x^2 - x + 6) \\ &= -\left(x^2 - x + \frac{1}{4} - \frac{1}{4} + 6\right) \\ &= -\left(\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 6\right) \\ &= -\left(x - \frac{1}{2}\right)^2 - \frac{23}{4}. \end{aligned}$$

The vertex is $(1/2, -23/4)$.

22. Complete the square and write in vertex form.

$$\begin{aligned} y &= 3x^2 + 12x \\ &= 3(x^2 + 4x) \\ &= 3(x^2 + 4x + 4 - 4) \\ &= 3((x+2)^2 - 4) \\ &= 3(x+2)^2 - 12. \end{aligned}$$

The vertex is $(-2, -12)$.

23. Complete the square and write in vertex form.

$$\begin{aligned} y &= -4x^2 + 8x - 6 \\ &= -4\left(x^2 - 2x + \frac{6}{4}\right) \\ &= -4\left(x^2 - 2x + 1 - 1 + \frac{6}{4}\right) \\ &= -4\left((x-1)^2 - 1 + \frac{6}{4}\right) \\ &= -4(x-1)^2 - 2. \end{aligned}$$

The vertex is $(1, -2)$.

24. Complete the square and write in vertex form.

$$\begin{aligned} y &= 5x^2 - 5x + 7 \\ &= 5\left(x^2 - x + \frac{7}{5}\right) \\ &= 5\left(x^2 - x + \left(\frac{1}{4} - \frac{1}{4} + \frac{7}{5}\right)\right) \\ &= 5\left(\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{7}{5}\right) \\ &= 5\left(x - \frac{1}{2}\right)^2 + \frac{23}{4}. \end{aligned}$$

The vertex is $(1/2, 23/4)$.

25. Complete the square and write in vertex form.

$$\begin{aligned} y &= 2x^2 - 7x + 3 \\ &= 2\left(x^2 - \frac{7}{2}x + \frac{3}{2}\right) \\ &= 2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} + \frac{3}{2}\right) \\ &= 2\left(\left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2}\right) \\ &= 2\left(x - \frac{7}{4}\right)^2 - \frac{25}{8}. \end{aligned}$$

The vertex is $(7/4, -25/8)$.

26. Complete the square and write in vertex form.

$$\begin{aligned} -3x^2 - x - 2 &= -3\left(x^2 + \frac{1}{3}x + \frac{2}{3}\right) \\ &= -3\left(x^2 + \frac{1}{3}x + \frac{1}{36} - \frac{1}{36} + \frac{2}{3}\right) \\ &= -3\left(\left(x + \frac{1}{6}\right)^2 - \frac{1}{36} + \frac{2}{3}\right) \\ &= -3\left(x + \frac{1}{6}\right)^2 - \frac{23}{12}. \end{aligned}$$

The vertex is $(-1/6, -23/12)$.

27. Get the variables on the left side, the constants on the right side and complete the square using $(\frac{-6}{2})^2 = 9$.

$$\begin{aligned} r^2 - 6r &= -8 \\ r^2 - 6r + 9 &= 9 - 8 \\ (r-3)^2 &= 1. \end{aligned}$$

Take the square root of both sides and solve for r.

$$\begin{aligned} r - 3 &= \pm 1 \\ r &= 3 \pm 1. \end{aligned}$$

So, $r = 4$ or $r = 2$.

28. Get the variables on the left side, the constants on the right side and complete the square using $(\frac{-2}{2})^2 = 1$.

$$\begin{aligned} g^2 - 2g &= 24 \\ g^2 - 2g + 1 &= 24 + 1 \\ (g-1)^2 &= 25. \end{aligned}$$

Take the square root of both sides and solve for g.

$$\begin{aligned} g - 1 &= \pm 5 \\ g &= 1 \pm 5. \end{aligned}$$

So, $g = 6$ or $g = -4$.

29. Complete the square using $(-2/2)^2 = 1$, take the square root of both sides and solve for p.

$$\begin{aligned} p^2 - 2p &= 6 \\ p^2 - 2p + 1 &= 6 + 1 \\ (p-1)^2 &= 7 \\ p - 1 &= \pm\sqrt{7} \\ p &= 1 \pm \sqrt{7}. \end{aligned}$$

30. Get the variables on the left side, the constants on the right side and complete the square using $\left(\frac{-3}{2}\right)^2 = \frac{9}{4}$.

$$\begin{aligned} n^2 - 3n &= 18 \\ n^2 - 3n + \frac{9}{4} &= 18 + \frac{9}{4} \\ \left(n - \frac{3}{2}\right)^2 &= 18 + \frac{9}{4} \\ \left(n - \frac{3}{2}\right)^2 &= \frac{81}{4}. \end{aligned}$$

Take the square root of both sides and solve for n.

$$\begin{aligned} n - \frac{3}{2} &= \pm\frac{9}{2} \\ n &= \frac{3}{2} \pm \frac{9}{2}. \end{aligned}$$

So, $n = 6$ or $n = -3$.

31. Complete the square with $(\frac{1}{2})^2 = \frac{1}{4}$ and take the square root of both sides to solve for d.

$$\begin{aligned} d^2 - d + \frac{1}{4} &= \frac{1}{4} + 2 \\ \left(d - \frac{1}{2}\right)^2 &= \frac{9}{4} \\ d - \frac{1}{2} &= \pm\frac{3}{2} \\ d &= \frac{1}{2} \pm \frac{3}{2}. \end{aligned}$$

So $d = 2$ or $d = -1$.

32. Get the variables on the left side, the constants on the right side and complete the square using $(\frac{2}{2})^2 = 1$.

$$\begin{aligned} 2r^2 + 4r &= 5 \\ 2(r^2 + 2r) &= 5 \\ 2(r^2 + 2r + 1) &= 5 + 2 \cdot 1 \\ 2(r+1)^2 &= 7. \end{aligned}$$

Divide by 2, take the square root of both sides and solve for r.

$$\begin{aligned} (r+1)^2 &= \frac{7}{2} \\ r + 1 &= \pm\sqrt{\frac{7}{2}} \\ r &= -1 \pm \sqrt{\frac{7}{2}}. \end{aligned}$$

33. Get the variables on the left side, the constants on the right side and complete the square using $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$.

$$\begin{aligned} 2s^2 + 10s &= 1 \\ 2\left(s^2 + 5s\right) &= 1 \\ 2\left(s^2 + 5s + \frac{25}{4}\right) &= 2\left(\frac{25}{4}\right) + 1 \\ 2\left(s + \frac{5}{2}\right)^2 &= \frac{25}{2} + 1 \\ 2\left(s + \frac{5}{2}\right)^2 &= \frac{27}{2}. \end{aligned}$$

Divide by 2, take the square root of both sides and solve for s.

$$\begin{aligned} \left(s + \frac{5}{2}\right)^2 &= \frac{27}{4} \\ s + \frac{5}{2} &= \pm\sqrt{\frac{27}{4}} \\ s + \frac{5}{2} &= \pm\frac{\sqrt{27}}{2} \\ s &= -\frac{5}{2} \pm \frac{\sqrt{27}}{2}. \end{aligned}$$

34. Get the variables on the left side, the constants on the right side and complete the square using $(\frac{2/5}{2})^2 = \frac{1}{25}$.

$$\begin{aligned} 5q^2 - 2q &= 8 \\ 5\left(q^2 - \frac{2}{5}q\right) &= 8 \\ 5\left(q^2 - \frac{2}{5}q + \frac{1}{25}\right) &= 5\left(\frac{1}{25}\right) + 8 \\ 5\left(q - \frac{1}{5}\right)^2 &= \frac{1}{5} + 8 \\ 5\left(q - \frac{1}{5}\right)^2 &= \frac{41}{5}. \end{aligned}$$

Divide by 5, take the square root of both sides and solve for q.

$$\begin{aligned}\left(q-\frac{1}{5}\right)^2 &= \frac{41}{25}\\ q-\frac{1}{5} &= \pm\sqrt{\frac{41}{25}}\\ q-\frac{1}{5} &= \pm\frac{\sqrt{41}}{5}\\ q &= \frac{1}{5}\pm\frac{\sqrt{41}}{5}.\end{aligned}$$

35. Get the variables on the left side and the constants on the right side, and complete the square.

$$\begin{aligned}7r^2-3r &= 6\\ 7\left(r^2-\frac{3}{7}r\right) &= 6\\ 7\left(r^2-\frac{3}{7}r+\left(\frac{3}{14}\right)^2\right) &= 7\left(\frac{3}{14}\right)^2+6\\ 7\left(r-\frac{3}{14}\right)^2 &= \frac{9}{28}+6\\ 7\left(r-\frac{3}{14}\right)^2 &= \frac{177}{28}.\end{aligned}$$

Divide by 7, take the square root of both sides and solve for r.

$$\begin{aligned}\left(r-\frac{3}{14}\right)^2 &= \frac{177}{196}\\ r-\frac{3}{14} &= \pm\sqrt{\frac{177}{196}}\\ r-\frac{3}{14} &= \pm\frac{\sqrt{177}}{14}\\ r &= \frac{3}{14}\pm\frac{\sqrt{177}}{14}.\end{aligned}$$

36. Complete the square on the left side.

$$\begin{aligned}5\left(p^2+\frac{9}{5}p\right) &= 1\\ 5\left(p^2+\frac{9}{5}p+\frac{81}{100}\right) &= 5\left(\frac{81}{100}\right)+1\\ 5\left(p+\frac{9}{10}\right)^2 &= \frac{81}{20}+1\\ 5\left(p+\frac{9}{10}\right)^2 &= \frac{101}{20}.\end{aligned}$$

Divide by 5 and take the square root of both sides to solve for p.

$$\begin{aligned}\left(p+\frac{9}{10}\right)^2 &= \frac{101}{100}\\ p+\frac{9}{10} &= \pm\sqrt{\frac{101}{100}}\end{aligned}$$

$$\begin{aligned} p+\frac{9}{10} &= \pm\frac{\sqrt{101}}{10} \\ p &= -\frac{9}{10} \pm \frac{\sqrt{101}}{10}. \end{aligned}$$

37. With $a = 1$, $b = -4$, and $c = -12$, we use the quadratic formula,

$$\begin{aligned} n &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\ &= \frac{4 \pm \sqrt{(-4)^2 - 4\cdot 1\cdot(-12)}}{2\cdot 1} \\ &= \frac{4 \pm \sqrt{16+48}}{2} \\ &= \frac{4 \pm \sqrt{64}}{2} \\ &= \frac{4 \pm 8}{2}. \end{aligned}$$

So, $n = 6$ or $n = -2$.

38. Rewrite the equation so the left side is zero and use the quadratic formula with $a = 2$, $b = 5$, and $c = 2$.

$$\begin{aligned} y &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{5^2 - 4\cdot 2\cdot 2}}{2\cdot 2} \\ &= \frac{-5 \pm \sqrt{25-16}}{4} \\ &= \frac{-5 \pm \sqrt{9}}{4} \\ &= \frac{-5 \pm 3}{4}. \end{aligned}$$

So, $y = -\frac{1}{2}$ or $y = -2$.

39. Set the equation equal to zero, $6k^2 + 11k + 3 = 0$. With $a = 6$, $b = 11$, and $c = 3$, we use the quadratic formula,

$$\begin{aligned} k &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\ &= \frac{-11 \pm \sqrt{11^2 - 4\cdot 6\cdot 3}}{2\cdot 6} \\ &= \frac{-11 \pm \sqrt{121-72}}{12} \\ &= \frac{-11 \pm \sqrt{49}}{12} \\ &= \frac{-11 \pm 7}{12}. \end{aligned}$$

So, $k = -\frac{1}{3}$ or $k = -\frac{3}{2}$.

40. Set the equation equal to zero, $w^2 + w - 4 = 0$. With $a = 1$, $b = 1$, and $c = -4$, we use the quadratic formula,

$$\begin{aligned} w &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4\cdot 1\cdot(-4)}}{2\cdot 1} \end{aligned}$$

$$
\begin{aligned}
&= \frac{-1 \pm \sqrt{1+16}}{2} \\
&= \frac{-1 \pm \sqrt{17}}{2}.
\end{aligned}
$$

41. Set the equation equal to zero, $z^2 + 4z - 6 = 0$. With $a = 1$, $b = 4$, and $c = -6$, we use the quadratic formula,

$$
\begin{aligned}
z &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} \\
&= \frac{-4 \pm \sqrt{16+24}}{2} \\
&= \frac{-4 \pm \sqrt{40}}{2} \\
&= \frac{-4 \pm 2\sqrt{10}}{2} \\
&= -2 \pm \sqrt{10}.
\end{aligned}
$$

42. With $a = 2$, $b = 6$, and $c = -3$, we use the quadratic formula

$$
\begin{aligned}
q &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-6 \pm \sqrt{6^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} \\
&= \frac{-6 \pm \sqrt{36+24}}{4} \\
&= \frac{-6 \pm \sqrt{60}}{4} \\
&= \frac{-6 \pm 2\sqrt{15}}{4} \\
&= \frac{-3 \pm \sqrt{15}}{2}
\end{aligned}
$$

43. Rewrite the equation to equal zero, and factor.

$$
\begin{aligned}
r^2 - 2r - 8 &= 0 \\
(r-4)(r+2) &= 0.
\end{aligned}
$$

So, $r - 4 = 0$ or $r + 2 = 0$ and $r = 4$ or $r = -2$.

44. Use the quadratic formula with $a = -3$, $b = 4$, and $c = 9$, to get

$$
\begin{aligned}
t &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-3) \cdot 9}}{2 \cdot (-3)} \\
&= \frac{-4 \pm \sqrt{16+108}}{-6} \\
&= \frac{-4 \pm \sqrt{124}}{-6}
\end{aligned}
$$

$$\begin{aligned} &= \frac{-4 \pm 2\sqrt{31}}{-6} \\ &= \frac{2 \pm \sqrt{31}}{3}. \end{aligned}$$

45. Rewrite the equation to equal zero, and factor.

$$\begin{aligned} n^2 + 4n - 5 &= 0 \\ (n+5)(n-1) &= 0. \end{aligned}$$

So, $n + 5 = 0$ or $n - 1 = 0$, thus $n = -5$ or $n = 1$.

46. Solve by completing the square using $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

$$\begin{aligned} s^2 + 3s + \frac{9}{4} &= 1 + \frac{9}{4} \\ \left(s + \frac{3}{2}\right)^2 &= 1 + \frac{9}{4} \\ \left(s + \frac{3}{2}\right)^2 &= \frac{13}{4}. \end{aligned}$$

Taking the square root of both sides and solving for s,

$$\begin{aligned} s + \frac{3}{2} &= \pm\sqrt{\frac{13}{4}} \\ s + \frac{3}{2} &= \pm\frac{\sqrt{13}}{2} \\ s &= -\frac{3}{2} \pm \frac{\sqrt{13}}{2} \\ s &= \frac{-3 \pm \sqrt{13}}{2}. \end{aligned}$$

47. Rewrite the equation to equal zero, and factor by grouping.

$$\begin{aligned} z^3 + 2z^2 - 3z - 6 &= 0 \\ z^2(z+2) - 3(z+2) &= 0 \\ (z+2)(z^2-3) &= 0. \end{aligned}$$

So, $z + 2 = 0$ or $z^2 - 3 = 0$, thus $z = -2$ or $z = \pm\sqrt{3}$.

48. Rewrite the equation as $2q^2 + 4q = 13$ and solve by completing the square.

$$\begin{aligned} 2(q^2 + 2q) &= 13 \\ 2(q^2 + 2q + 1) &= 2 \cdot 1 + 13 \\ 2(q+1)^2 &= 2 + 13 \\ 2(q+1)^2 &= 15. \end{aligned}$$

Dividing by 2, taking the square root of both sides and solving for q, we get

$$\begin{aligned} (q+1)^2 &= \frac{15}{2} \\ q + 1 &= \pm\sqrt{\frac{15}{2}} \\ q &= -1 \pm \sqrt{\frac{15}{2}}. \end{aligned}$$

49. Set the equation equal to zero, and use the quadratic formula with $a = 25$, $b = -30$, and $c = 4$.

$$\begin{aligned} u &= \frac{30 \pm \sqrt{(-30)^2 - 4 \cdot 25 \cdot 4}}{2 \cdot 25} \\ &= \frac{30 \pm \sqrt{900 - 400}}{50} \\ &= \frac{30 \pm \sqrt{500}}{50} \\ &= \frac{30 \pm 10\sqrt{5}}{50} \\ &= \frac{3 \pm \sqrt{5}}{5}. \end{aligned}$$

50. This equation can be solved by completing the square.

$$\begin{aligned} v^2 - 4v &= 9 \\ v^2 - 4v + 4 &= 9 + 4 \\ (v-2)^2 &= 13. \end{aligned}$$

Take the square root of both sides and solve for v to get $v = 2 \pm \sqrt{13}$.

51. Simplify by dividing by 3 and solve by completing the square.

$$\begin{aligned} y^2 &= 2y + 6 \\ y^2 - 2y &= 6 \\ y^2 - 2y + 1 &= 1 + 6 \\ (y-1)^2 &= 7. \end{aligned}$$

Take the square root of both sides and solve for y to get $y = 1 \pm \sqrt{7}$.

52. Set the equation equal to zero and use the quadratic formula with $a = 2$, $b = -14$, and $c = 23$.

$$\begin{aligned} p &= \frac{14 \pm \sqrt{(-14)^2 - 4 \cdot 2 \cdot 23}}{2 \cdot 2} \\ &= \frac{14 \pm \sqrt{196 - 184}}{4} \\ &= \frac{14 \pm \sqrt{12}}{4} \\ &= \frac{14 \pm 2\sqrt{3}}{4} \\ &= \frac{7 \pm \sqrt{3}}{2}. \end{aligned}$$

53. Set the equation equal to zero and use factoring.

$$\begin{aligned} 2w^3 - 6w^2 - 8w + 24 &= 0 \\ w^3 - 3w^2 - 4w + 12 &= 0 \\ w^2(w-3) - 4(w-3) &= 0 \\ (w-3)(w^2-4) &= 0 \\ (w-3)(w-2)(w+2) &= 0. \end{aligned}$$

So, $w - 3 = 0$ or $w - 2 = 0$ or $w + 2 = 0$ thus, $w = 3$ or $w = 2$ or $w = -2$.

54. Solve by completing the square.

$$\begin{aligned}
4x^2 + 16x &= 5 \\
4(x^2 + 4x) &= 5 \\
4(x^2 + 4x + 4) &= 4 \cdot 4 + 5 \\
4(x+2)^2 &= 21 \\
(x+2)^2 &= \frac{21}{4}.
\end{aligned}$$

Take the square root of both sides and solve for x.

$$\begin{aligned}
x + 2 &= \pm\sqrt{\frac{21}{4}} \\
x + 2 &= \pm\frac{\sqrt{21}}{2} \\
x &= -2 \pm \frac{\sqrt{21}}{2}.
\end{aligned}$$

55. Use the quadratic formula with $a = 49$, $b = 70$, $c = 22$, to solve this equation.

$$\begin{aligned}
m &= \frac{-70 \pm \sqrt{70^2 - 4 \cdot 49 \cdot 22}}{2 \cdot 49} \\
&= \frac{-70 \pm \sqrt{4900 - 4312}}{98} \\
&= \frac{-70 \pm \sqrt{588}}{98} \\
&= \frac{-70 \pm 14\sqrt{3}}{98} \\
&= \frac{-5 \pm \sqrt{3}}{7}.
\end{aligned}$$

56. Before completing the square, get all terms on the left, and divide by the coefficient of x^2:

$$\begin{aligned}
8x^2 - 1 &= 2x \\
8x^2 - 2x - 1 &= 0 \\
x^2 - \frac{1}{4}x - \frac{1}{8} &= 0.
\end{aligned}$$

Now complete the square

$$\begin{aligned}
x^2 - \frac{1}{4}x + \frac{1}{64} - \frac{1}{64} - \frac{1}{8} &= 0 \\
(x - \frac{1}{8})^2 - \frac{9}{64} &= 0 \\
x - \frac{1}{8} &= \pm\sqrt{\frac{9}{64}} \\
x &= \frac{1}{8} \pm \frac{3}{8}
\end{aligned}$$

So the solutions are $x = 1/2$ and $x = -1/4$.