FIND SOLUTIONS ON NEXT PAGE

CHAPTER SIX

Solutions for Section 6.1

Exercises

1. This function appears to be periodic because it repeats regularly.

2. This appears to be a periodic function, because it repeats regularly.

3. This does not appear to be a periodic function, because, while it rises and falls, it does not rise and fall to the same level regularly.

4. This appears to be a periodic function, because it repeats regularly.

5. This function does not appear to be periodic. Though it does rise and fall, it does not do so regularly.

6. This is not the graph of a periodic function. Although the y-values repeat, they do not repeat at regular intervals. The intervals get progressively shorter.

7. This could be a periodic function with a period of 3. The values of $f(t)$ repeat each time t increases by 3.

8. This function does not appear to be periodic. Though it does rise and fall, it seems to do so irregularly.

9. The period is approximately 4.

10. The graph appears to have a period of b. Every change in the x value of b brings us back to the same y value.

11. The period appears to be 3.

12. The period appears to be $41 - 1 = 40$.

Problems

13. The wheel will complete two full revolutions after 20 minutes, so the function is graphed on the interval $0 \leq t \leq 20$. See Figure 6.1.

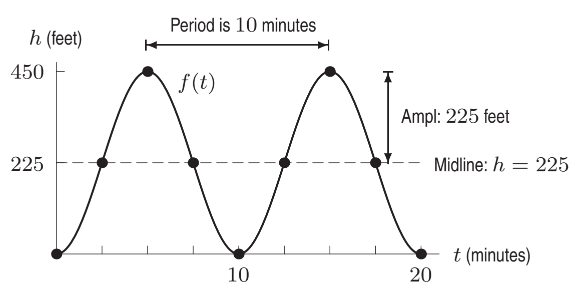


Figure 6.1: Graph of $h = f(t)$, $0 \leq t \leq 20$

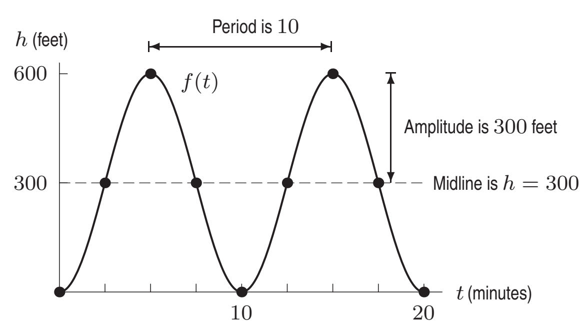


Figure 6.2: Graph of $h = f(t)$, $0 \leq t \leq 20$

14. The wheel will complete two full revolutions after 20 minutes, and the height ranges from $h = 0$ to $h = 600$. So the function is graphed on the interval $0 \leq t \leq 20$. See Figure 6.2.

15. The wheel will complete two full revolutions after 10 minutes. See Figure 6.3.

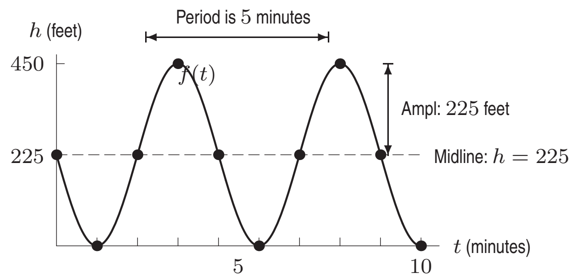


Figure 6.3: Graph of $h = f(t)$, $0 \le t \le 10$

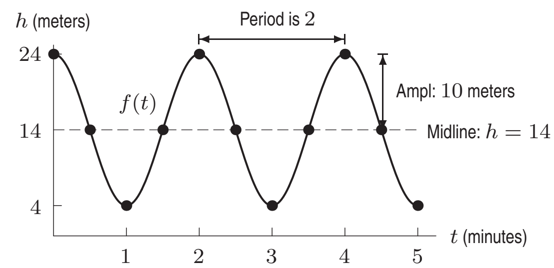


Figure 6.4: Graph of $h = f(t)$, $0 \le t \le 5$

16. See Figure 6.4.

17. See Figure 6.5.

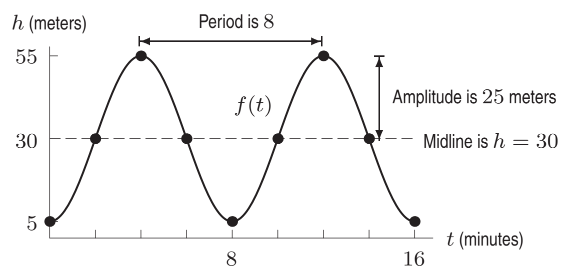


Figure 6.5: Graph of $h = f(t)$, $0 \le t \le 16$

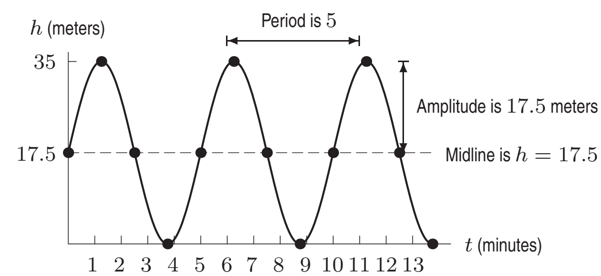


Figure 6.6: Graph of $h = f(t)$, $0 \le t \le 13.75$

18. See Figure 6.6.

19. At $t = 0$, we see $h = 20$, so you are level with the center of the wheel. Your initial position is at three o'clock (or nine o'clock) and initially you are rising. On the interval $0 \le t \le 7$ the wheel completes seven fourths of a revolution. Therefore, if p is the period, we know that

$$\frac{7}{4}p = 7$$

which gives $p = 4$. This means that the ferris wheel takes 4 minutes to complete one full revolution. The minimum value of the function is $h = 5$, which means that you get on and get off of the wheel from a 5 meter platform. The maximum height above the midline is 15 meters, so the wheel's diameter is 30 meters. Notice that the wheel completes a total 2.75 cycles. Since each period is 4 minutes long, you ride the wheel for $4(2.75) = 11$ minutes.

20. Your initial position is twelve o'clock, since at $t = 0$, the value of h is at its maximum of 35. The period is 4 because the wheel completes one cycle in 4 minutes. The diameter is 30 meters and the boarding platform is 5 meters above ground. Because you go through 2.5 cycles, the length of time spent on the wheel is 10 minutes.

21. At $t = 0$, we see $h = 20$ m, and you are at the midline, so your initial height is level with the center of the wheel. Your initial position is at the three o'clock (or nine o'clock) position, and you are moving upward at $t = 0$. The amplitude of this function is 20, which means that the wheel's diameter is 40 meters. The minimum value of the function is $h = 0$, which means you board and get off the wheel at ground level. The period of this function is 5, which means that it takes 5 minutes for the wheel to complete one full revolution. Notice that the function completes 2.25 periods. Since each period is 5 minutes long, this means you ride the wheel for $5(2.25) = 11.25$ minutes.

22. At $t = 0$, we see $h = 20$ m, and you are at the midline, so your initial height is level with the center of the wheel. Your initial position is at the three o'clock (or the nine o'clock) position and at first your height is decreasing, so you are descending. The amplitude of this function is 20, which means that the wheel's diameter is 40 meters. The minimum value of the function is $h = 0$, which means you board and get off the wheel at ground level. The period of this function is 5, which means that it takes 5 minutes for the wheel to complete one full revolution. Notice that the function completes 2.25 periods. Since each period is 5 minutes long, this means you ride the wheel for $5(2.25) = 11.25$ minutes.

23. The midline of f is $d = 10$. The period of f is 1, the amplitude 4 cm, and its minimum and maximum values are 6 cm and 14 cm, respectively. The fact that $f(t)$ is wave-shaped means that the spring is bobbing up and down, or *oscillating*. The fact that the period of f is 1 means that it takes the weight one second to complete one oscillation and return to its original position. Studying the graph, we see that it takes the weight 0.25 seconds to move from its initial position at the midline to its maximum at $d = 14$, where it is farthest from the ceiling (and the spring is at its maximum extension). It takes another 0.25 seconds to return to its initial position at $d = 10$ cm. It takes another 0.25 seconds to rise up to its closest distance from the ceiling at $d = 6$ (the minimum extension of the spring). In 0.25 seconds more it moves back down to its initial position at $d = 10$. (This sequence of motions by the weight, completed in one second, represents one full oscillation.) Since Figure 6.9 of the text gives 3 full periods of $f(t)$, it represents the 3 complete oscillations made by the weight in 3 seconds.

24. The amplitude, period, and midline are the same for Figures 6.9 and 6.10 in the text. In Figure 6.10, the weight is initially moving upward toward the ceiling, since d, the distance from the ceiling, begins to decrease at $t = 0$, whereas in Figure 6.9, d begins to increase at $t = 0$. Thus, the motion described in Figure 6.10 must have resulted from pulling the weight away from the ceiling at $t = -0.25$, whereas the motion described by Figure 6.9 must have resulted from pushing the weight toward the ceiling at $t = -0.25$.

25.

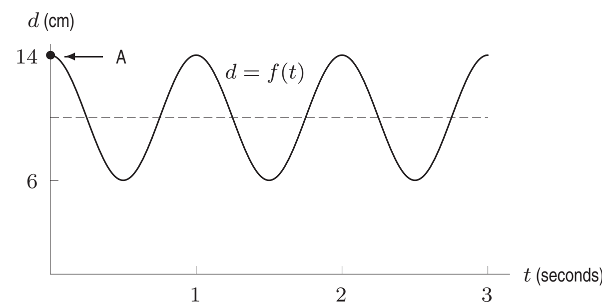


Figure 6.7: Graph of $d = f(t)$ for $0 \leq t \leq 3$

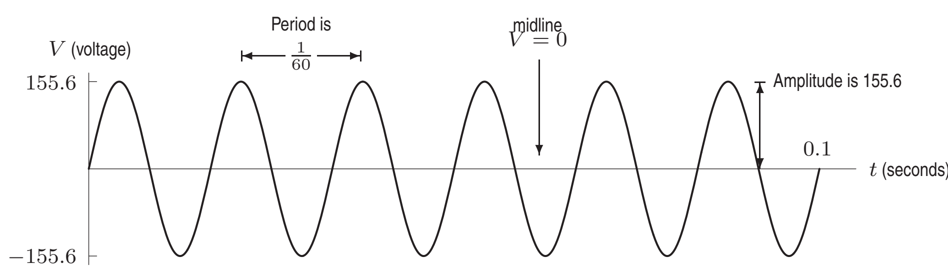
Since the weight is released at $d = 14$ cm when $t = 0$, it is initially at the point in Figure 6.7 labeled A. The weight will begin to oscillate in the same fashion as described by Figures 6.9 and 6.10. Thus, the period, amplitude, and midline for Figure 6.7 are the same as for Figures 6.9 and 6.10 in the text.

26. **(a)** Weight B, because the midline is $d = 10$, compared to $d = 20$ for weight A. This means that when the spring is not oscillating, weight B is 10 cm from the ceiling, while weight A is 20 cm from the ceiling.

(b) Weight A, because its amplitude is 10 cm, compared to the amplitude of 5 cm for weight B.

(c) Weight A, because its period is 0.5, compared to the period of 2 for weight B. This means that it takes weight A only half a second to complete one oscillation, whereas weight B completes one oscillation in 2 seconds.

27. **(a)**



(b) The period is $\frac{1}{60}$ of a second as there are 60 cycles each second. The midline value, 0 volts, is the average voltage over one whole period. The amplitude, 155.6 volts, is the maximum amount by which the voltage can vary, either above or below the midline.

28. **(a)** Two possible answers are shown in Figures 6.8 and 6.9.

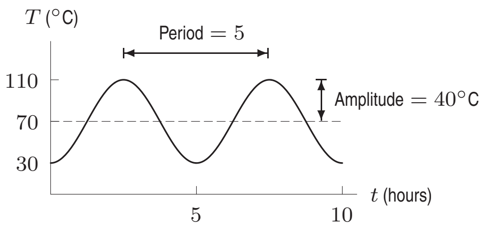


Figure 6.8

or

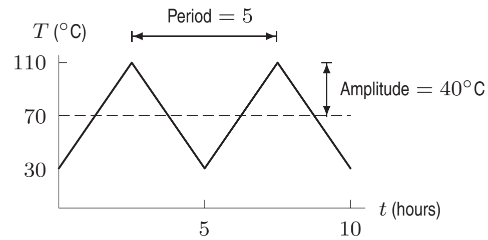


Figure 6.9

(b) The period is 5 hours. This is the time required for the temperature to cycle from $30°$ to $110°$ and back to $30°$. The midline, or average temperature, is $T = (110 + 30)/2 = 70°$. The amplitude is $40°$ since this is the amount of temperature variation (up or down) from the average.

29. **(a)** Periodic
(b) Not periodic
(c) Not periodic
(d) Not periodic
(e) Periodic
(f) Not periodic
(g) Periodic

30. Notice that the function is only approximately periodic. See Figure 6.10.

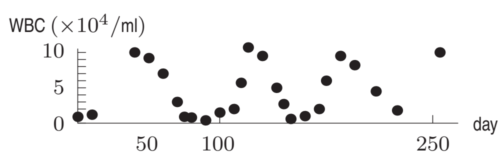


Figure 6.10

The midline is half way between the maximum and minimum WBC values.

$$y = \frac{(10.7 + 0.4)}{2} = 5.55.$$

The amplitude is the difference between the maximum and midline, so $A = 5.15$. The period is the length of time from peak to peak. Measuring between successive peaks gives $p_1 = 120 - 40 = 80$ days; $p_2 = 185 - 120 = 65$ days; $p_3 = 255 - 185 = 70$ days. Using the average of the three periods we get $p \approx 72$ days.

31. By plotting the data in Figure 6.11, we can see that the midline is at $h = 2$ (approximately). Since the maximum value is 3 and the minimum value is 1, we have

$$\text{Amplitude} = 2 - 1 = 1.$$

Finally, we can see from the graph that one cycle has been completed from time $t = 0$ to time $t = 1$, so the period is 1 second.

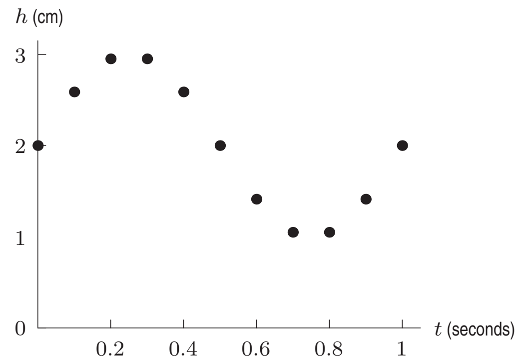


Figure 6.11

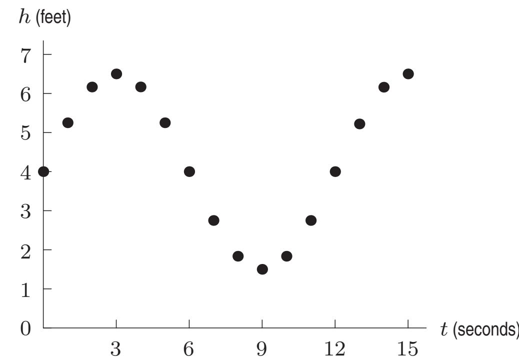


Figure 6.12

32. By plotting the data in Figure 6.12, we can see that the midline is at $h = 4$ (approximately). Since the maximum value is 6.5 and the minimum value is 1.5, we have

$$\text{Amplitude } = 4 - 1.5 = 6.5 - 4 = 2.5.$$

Finally, we can see from the graph that one cycle has been completed from time $t = 0$ to $t = 12$, so the period is 12 seconds.

Solutions for Section 6.2

Exercises

1.

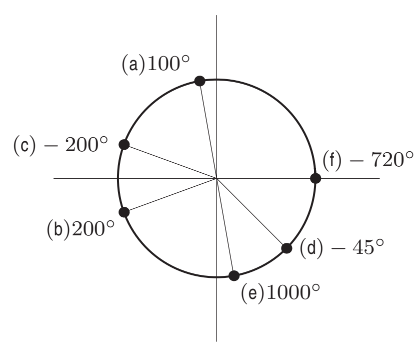


Figure 6.13

(a) $(\cos 100°, \sin 100°) = (-0.174, 0.985)$
(b) $(\cos 200°, \sin 200°) = (-0.940, -0.342)$
(c) $(\cos(-200°), \sin(-200°)) = (-0.940, 0.342)$
(d) $(\cos(-45°), \sin(-45°)) = (0.707, -0.707)$
(e) $(\cos 1000°, \sin 1000°) = (0.174, -0.985)$
(f) $(\cos 720°, \sin 720°) = (1, 0)$

2. If we go around four times, we make four full circles, which is $360° \cdot 4 = 1440$ degrees.

3. If we go around two times, we make two full circles, which is $360° \cdot 2 = 720$ degrees. Since we're going around in the negative (clockwise) direction, we have -720 degrees.

4. If we go around 16.4 times, we make 16.4 full circles, which is $360^\circ \cdot 16.4 = 5904$ degrees.

5.

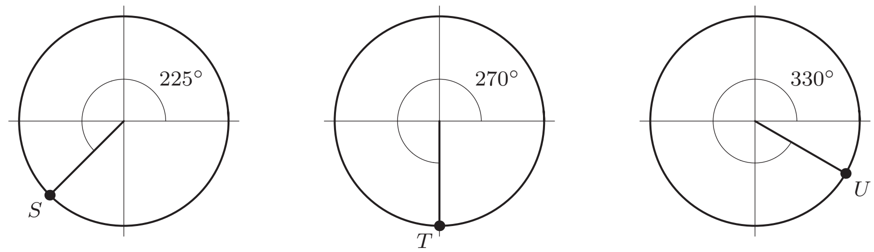


Figure 6.14

$S = (-0.707, -0.707), T = (0, -1), U = (0.866, -0.5)$

6. To locate the points D, E, and F, we mark off their respective angles, -90°, -135°, and -225°, by measuring these angles from the positive x-axis in the clockwise direction. See Figure 6.15.

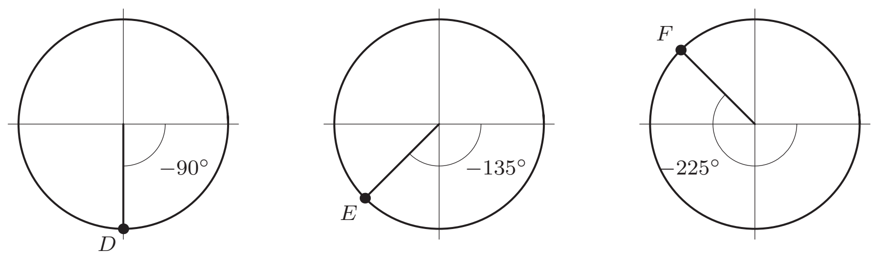


Figure 6.15

$D = (0, -1), E = (-0.707, -0.707), F = (-0.707, 0.707)$

7. Point A is at 390°. The angle 390° is located by first wrapping around the circle, which accounts for 360°, and then continuing for an additional 30°. Similarly, B, which is at 495°, is located by first wrapping around the circle, and then continuing for an additional $495^\circ - 360^\circ = 135^\circ$. Finally, C, which as at 690°, is located by first wrapping around the circle, giving 360°, and then continuing for an additional $690^\circ - 360^\circ = 330^\circ$.

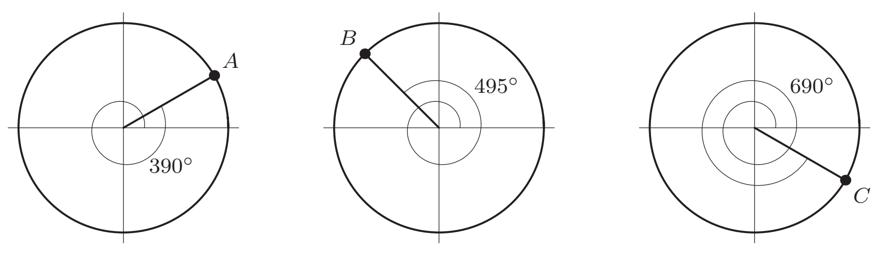


Figure 6.16

$A = (0.866, 0.5), B = (-0.707, 0.707), C = (0.866, -0.5)$

8. To locate the points P, Q, and R, we mark off their respective angles, 540°, -180°, and 450°, by measuring these angles from the positive x-axis in the counterclockwise direction if the angle is positive and in the clockwise direction if the angle is negative. See Figure 6.17.

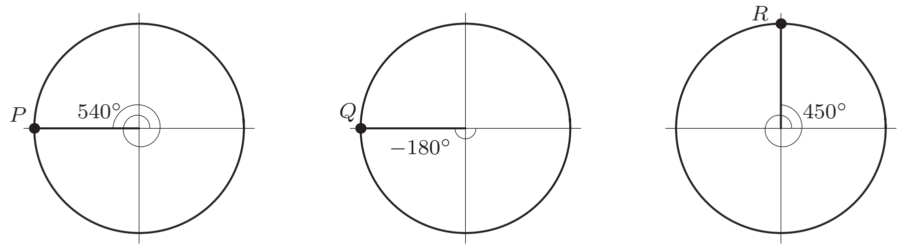


Figure 6.17

$P = (-1, 0), Q = (-1, 0), R = (0, 1)$

9. Since $x = r\cos\theta$ and $y = r\sin\theta$ we have

$$\begin{aligned} S &= (5\cos 225^\circ, 5\sin 225^\circ) = (-3.536, -3.536) \\ T &= (5\cos 270^\circ, 5\sin 270^\circ) = (0, -5) \\ U &= (5\cos 330^\circ, 5\sin 330^\circ) = (4.330, -2.5) \end{aligned}$$

10. Since $x = r\cos\theta$ and $y = r\sin\theta$ we have

$$\begin{aligned} A &= (3\cos 390^\circ, 3\sin 390^\circ) = (2.598, 1.5) \\ B &= (3\cos 495^\circ, 3\sin 495^\circ) = (-2.121, 2.121) \\ C &= (3\cos 690^\circ, 3\sin 690^\circ) = (2.598, -1.5) \end{aligned}$$

11. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = 90^\circ$, the point is $(3.8\cos 90, 3.8\sin 90) = (0, 3.8)$.

12. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = 180^\circ$, the point is $(3.8\cos 180, 3.8\sin 180) = (-3.8, 0)$.

13. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = -180^\circ$, the point is $(3.8\cos(-180), 3.8\sin(-180)) = (-3.8, 0)$.

14. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = -90^\circ$, the point is $(3.8\cos(-90), 3.8\sin(-90)) = (0, -3.8)$.

15. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = -270^\circ$, the point is $(3.8\cos(-270), 3.8\sin(-270)) = (0, 3.8)$.

16. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = -540^\circ$, the point is $(3.8\cos(-540), 3.8\sin(-540)) = (-3.8, 0)$.

17. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = 1426^\circ$, the point is $(3.8\cos 1426, 3.8\sin 1426) = (3.687, -0.919)$.

18. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$ and $\theta = 1786^\circ$, the point is $(3.8\cos 1786, 3.8\sin 1786) = (3.687, -0.919)$.

19. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$, the point is $(3.8\cos 45, 3.8\sin 45) = (3.8\sqrt{2}/2, 3.8\sqrt{2}/2) = (2.687, 2.687)$.

20. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$, the point is $(3.8\cos 135, 3.8\sin 135) = (-3.8\sqrt{2}/2, 3.8\sqrt{2}/2) = (-2.687, 2.687)$.

21. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$, the point is $(3.8\cos 225, 3.8\sin 225) = (-3.8\sqrt{2}/2, -3.8\sqrt{2}/2) = (-2.687, -2.687)$.

22. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$, the point is $(3.8\cos 315, 3.8\sin 315) = (3.8\sqrt{2}/2, -3.8\sqrt{2}/2) = (2.687, -2.687)$.

23. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$, the point is $(3.8\cos(-10), 3.8\sin(-10)) = (3.742, -0.660)$.

24. Since the x-coordinate is $r\cos\theta$ and the y-coordinate is $r\sin\theta$ and $r = 3.8$, the point is $(3.8\cos(-20), 3.8\sin(-20)) = (3.571, -1.300)$.

Problems

25. The car on the ferris wheel starts at the 3 o'clock position. Let's suppose that you see the wheel rotating counterclockwise. (If not, move to the other side of the wheel.)

The angle $\phi = 420^\circ$ indicates a counterclockwise rotation of the ferris wheel from the 3 o'clock position all the way around once (360°), and then two-thirds of the way back up to the top (an additional 60°). This leaves you in the 1 o'clock position, or at the angle 60°.

A negative angle represents a rotation in the opposite direction, that is clockwise. The angle $\theta = -150^\circ$ indicates a rotation from the 3 o'clock position in the clockwise direction, past the 6 o'clock position and two-thirds of the way up to the 9 o'clock position. This leaves you in the 8 o'clock position, or at the angle 210°. (See Figure 6.18.)

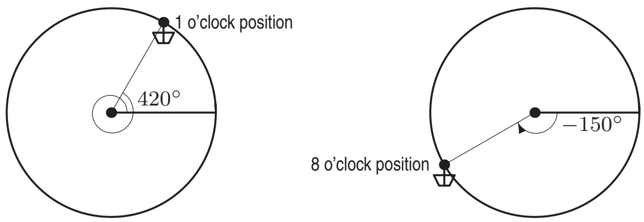


Figure 6.18: The positions and displacements on the ferris wheel described by 420° and -150°

26. See Figure 6.19.

(a) $\cos 240^\circ$ is negative, so we need an angle in the second quadrant with the same x-coordinate since $240^\circ = 180^\circ + 60^\circ$. This angle is $180^\circ - 60^\circ = 120^\circ$.

(b) $\sin 240^\circ$ is negative, so we need an angle in the fourth quadrant with the same y-coordinate. This angle is $360^\circ - 60^\circ = 300^\circ$.

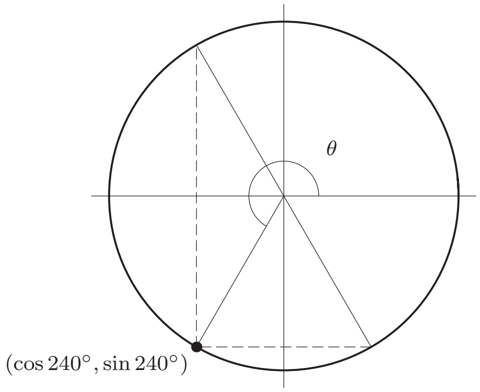


Figure 6.19

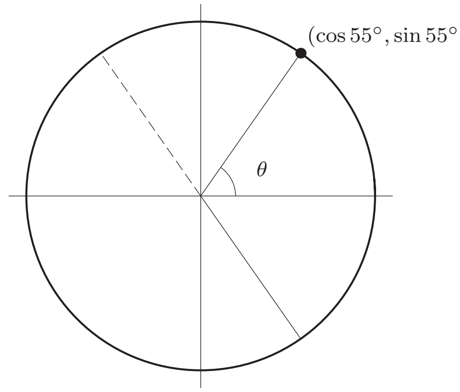


Figure 6.20

27. See Figure 6.20.

(a) $\cos 53^\circ$ is positive, so we need an angle in the fourth quadrant with the same x-coordinate. This angle is $360^\circ - 53^\circ = 307^\circ$.

(b) $\sin 53^\circ$ is positive, so we need an angle in the second quadrant with the same y-coordinate. This angle is $180^\circ - 53^\circ = 127^\circ$.

28. **(a)** As we see from Figure 6.21, the angle $135°$ specifies a point P' on the unit circle directly across the y-axis from the point P. Thus, P' has the same y-coordinate as P, but its x-coordinate is opposite in sign to the x-coordinate of P. Therefore, $\sin 135° = 0.707$, and $\cos 135° = -0.707$.

(b) As we see from Figure 6.22, the angle $285°$ specifies a point Q' on the unit circle directly across the x-axis from the point Q. Thus, Q' has the same x-coordinate as Q, but its y-coordinate is opposite in sign to the y-coordinate of Q. Therefore, $\sin 285° = -0.966$, and $\cos 285° = 0.259$.

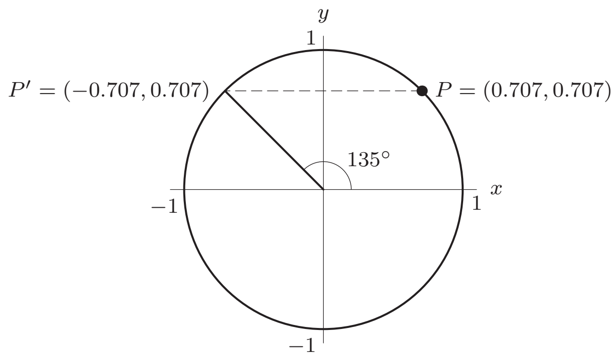


Figure 6.21: The sine and cosine of $135°$ can be found by referring to the sine and cosine of $45°$

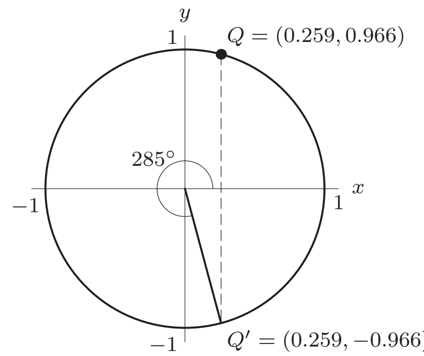
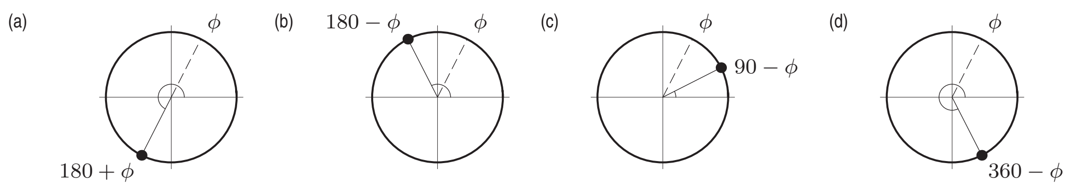


Figure 6.22: The sine and cosine of $285°$ can be found by referring to the sine and cosine of $75°$

29. The graphs follow.



30. **(a)** $\sin(\theta + 360°) = \sin\theta = a$, since the sine function is periodic with a period of $360°$.

(b) $\sin(\theta + 180°) = -a$. (A point on the unit circle given by the angle $\theta + 180°$ diametrically opposite the point given by the angle θ. So the y-coordinates of these two points are opposite in sign, but equal in magnitude.)

(c) $\cos(90° - \theta) = \sin\theta = a$. This is most easily seen from the right triangles in Figure 6.23.

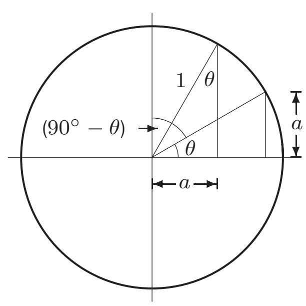


Figure 6.23

(d) $\sin(180° - \theta) = a$. (A point on the unit circle given by the angle $180° - \theta$ has a y-coordinate equal to the y-coordinate of the point on the unit circle given by θ.)

(e) $\sin(360° - \theta) = -a$. (A point on the unit circle given the the angle $360° - \theta$ has a y-coordinate of the same magnitude as the y-coordinate of the point on the unit circle given by θ, but is of opposite sign.)

(f) $\cos(270° - \theta) = -\sin\theta = -a$.

31. Given the angle θ, draw a line l through the origin making an angle θ with the x-axis. Go counterclockwise if $\theta > 0$ and clockwise if $\theta < 0$, wrapping around the unit circle more than once if necessary. Let $P = (x, y)$ be the point where l intercepts the unit circle. Then the definition of sine is that $\sin\theta = y$.

32. The period of the ferris wheel is 30 minutes, so in 17.5 minutes you will travel $17.5/30$, or $7/12$, of a complete revolution, which is $7/12(360^\circ) = 210^\circ$ from your starting position (the 6 o'clock position). This location is 120° from the horizontal line through the center of the wheel. Thus, your height is $225 + 225\sin 120^\circ = 225 + 225(0.866) \approx 419.856$ feet above the ground.

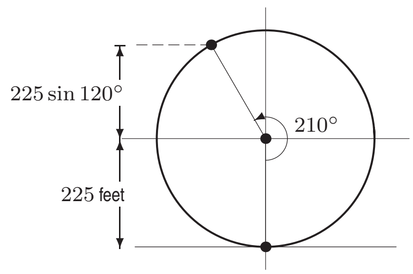


Figure 6.24

33. The radius is 10 meters. So when the seat height is 15 meters, the seat will be 5 meters above the horizontal line through the center of the wheel. This produces an angle whose sine is $5/10 = 1/2$, which we know is the angle 30°, or the 2 o'clock position. This situation is shown in Figure 6.25:

The seat is above 15 meters when it is between the 2 o'clock and 10 o'clock positions. This happens $4/12 = 1/3$ of the time, or for $4/3$ minutes each revolution.

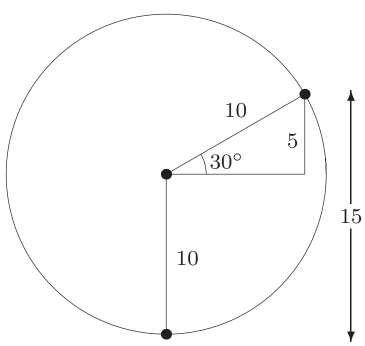


Figure 6.25

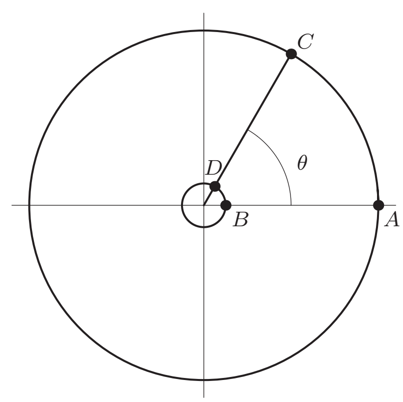


Figure 6.26

34. See Figure 6.26. Since the diameter is 120 mm, the radius is 60 mm. The coordinates of the outer edge point, A, on the x-axis is $(60, 0)$. Similarly the inner edge at point B has coordinates $(7.5, 0)$.

Points C and D are at an angle θ from the x-axis and have coordinates of the form $(r\cos\theta, r\sin\theta)$. For the outer edge, $r = 60$ so $C = (60\cos\theta, 60\sin\theta)$. The inner edge has $r = 7.5$, so $D = (7.5\cos\theta, 7.5\sin\theta)$.

35. **(a)** Since the four panels divide a full rotation or 360° into four equal spaces, the angle between two adjacent panels is

$$\frac{360^\circ}{4} = 90^\circ.$$

(b) The angle created by rotating a panel from B to A is equal to the angle between each panel, or 90°.
(c) Point B is directly across from point D. So the angle between the two is 180°.
(d) If the door moves from B to D, the angle of rotation is 180°.
(e) Each person, whether entering or leaving, must rotate the door by 180°. Thus the total rotation is $(3+5)(180^\circ) = 8(180^\circ) = 1440^\circ$. Since $1440^\circ = 4(360^\circ)$ the rotation is equivalent to 0°. Thus, the panel at point A ends up at point A.

36. **(a)** The five panels split the circle into five equal parts, so the angle between each panel is $360^\circ/5 = 72^\circ$.
(b) Point B is directly across the circle from D, so 180°.
(c) The angle from A to D is the same as the angle from B to C, and the BC angle is the angle between panels, which is 72°. So moving the panel between A and D gives an angle of $(72^\circ)/2 = 36^\circ$. The panel then goes from point D to point B spanning another 180°. Thus in total the panel traveled $36^\circ + 180^\circ = 216^\circ$.

Solutions for Section 6.3

Exercises

1. To convert 60° to radians, multiply by $\pi/180^\circ$:

$$60^\circ\left(\frac{\pi}{180^\circ}\right) = \left(\frac{60^\circ}{180^\circ}\right)\pi = \frac{\pi}{3}.$$

We say that the radian measure of a 60° angle is $\pi/3$.

2. To convert 45° to radians, multiply by $\pi/180^\circ$:

$$45^\circ\left(\frac{\pi}{180^\circ}\right) = \left(\frac{45^\circ}{180^\circ}\right)\pi = \frac{\pi}{4}.$$

Thus we say that the radian measure of a 45° angle is $\pi/4$.

3. If ϕ is the radian measure of 100°, then

$$\phi = \left(\frac{\pi}{180^\circ}\right)100^\circ \approx 1.7453 \text{ radians.}$$

4. If θ is the radian measure of 17°, then

$$\theta = \left(\frac{\pi}{180^\circ}\right)17^\circ \approx 0.297 \text{ radians.}$$

5. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $150\cdot\pi/180$, giving $\frac{5}{6}\pi$ radians.

6. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $120\cdot\pi/180$, giving $\frac{2}{3}\pi$ radians.

7. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $-270\cdot\pi/180$, giving $-\frac{3}{2}\pi$ radians.

8. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $\pi\cdot\pi/180$, giving $\pi^2/180 \approx 0.0548$ radians.

9. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $\frac{7}{2}\pi\cdot 180/\pi$, giving 630 degrees.

10. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $5\pi\cdot 180/\pi$, giving 900 degrees.

11. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $90 \cdot 180/\pi$, giving $16{,}200/\pi \approx 5156.620$ degrees.

12. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $2 \cdot 180/\pi$, giving $360/\pi \approx 114.592$ degrees.

13. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $45 \cdot 180/\pi$, giving $8100/\pi \approx 2578.310$ degrees.

14. **(a)** $30 \cdot \dfrac{\pi}{180} = \dfrac{\pi}{6}$ or 0.52

(b) $120 \cdot \dfrac{\pi}{180} = \dfrac{2\pi}{3}$ or 2.09

(c) $200 \cdot \dfrac{\pi}{180} = \dfrac{10\pi}{9}$ or 3.49

(d) $315 \cdot \dfrac{\pi}{180} = \dfrac{7\pi}{4}$ or 5.50

15. **(a)** I
(b) II
(c) II
(d) III
(e) IV
(f) IV
(g) I
(h) II
(i) II
(j) III

16. If we go around once, we make one full circle, which is 2π radians.

17. If we go around twice, we make two full circles, which is $2\pi \cdot 2 = 4\pi$ radians. Since we're going around in the negative direction, we have -4π radians.

18. If we go around 0.75 times, we make three-fourths of a full circle, which is $2\pi \cdot \frac{3}{4} = 3\pi/2$ radians.

19. If we go around 4.27 times, we make 4.27 full circles, which is $2\pi \cdot 4.27 = 8.54\pi$ radians.

20. The arc length, s, corresponding to an angle of θ radians in a circle of radius r is $s = r\theta$. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $-180 \cdot \pi/180$, giving $-\pi$ radians. The negative sign indicates rotation in a clockwise, rather than counterclockwise, direction. Since length cannot be negative, we find the arc length corresponding to π radians. Thus, our arc length is $6.2\pi \approx 19.478$.

21. The arc length, s, corresponding to an angle of θ radians in a circle of radius r is $s = r\theta$. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $45 \cdot \pi/180$, giving $\frac{\pi}{4}$ radians. Thus, our arc length is $6.2\pi/4 \approx 4.869$.

22. The arc length, s, corresponding to an angle of θ radians in a circle of radius r is $s = r\theta$. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $180/\pi \cdot \pi/180$, giving 1 radian. Thus, our arc length is $6.2 \cdot 1 = 6.2$.

23. The arc length, s, corresponding to an angle of θ radians in a circle of radius r is $s = r\theta$. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $a \cdot \pi/180$ radians. Thus, our arc length is $6.2a\pi/180$.

Problems

24. Using $s = r\theta$ gives $30 = 3r$. Solving for r we have $r = 10$ cm.

25. First 225° has to be converted to radian measure:

$$225 \cdot \frac{\pi}{180} = \frac{5\pi}{4}.$$

Using $s = r\theta$ gives

$$s = 4 \cdot \frac{5\pi}{4} = 5\pi \text{ feet}.$$

26. Using $s = r\theta$, we have $s = 8(2) = 16$ inches.

27.

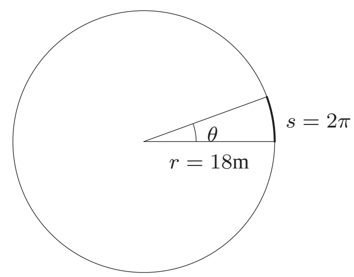


Figure 6.27

In Figure 6.27, we have $s = 2\pi$ and $r = 18$. Therefore,

$$\theta = \frac{s}{r} = \frac{2\pi}{18} = \frac{\pi}{9}.$$

Now,

$$\frac{\pi}{9} \text{ radians} = \frac{\pi}{9}\left(\frac{180^\circ}{\pi}\right) = 20^\circ.$$

Therefore, an arc of length 2π m on a circle of radius 18 m determines an angle of $\pi/9$ radians or 20°.

28. We have $s = 3$ and $r = 5$. Using the formula $s = r\theta$, we have $\theta = 3/5$ radians or

$$\theta = \frac{3}{5} \cdot \frac{180^\circ}{\pi} = 34.3775^\circ.$$

The coordinates of P are $(r\cos\theta, r\sin\theta) = (4.1267, 2.8232)$.

29. From the figure, we see that $P = (7,4)$. This means $r = \sqrt{7^2 + 4^2} = \sqrt{65}$. We know that $\sin\theta = 4/\sqrt{65}$, and we can use a graphing calculator to estimate that $\theta = 0.5191$ radians or 29.7449°. This gives $s = 0.5191\sqrt{65} = 4.185$.

30. We have $\theta = 22^\circ$ or, in radians,

$$22^\circ \cdot \frac{\pi}{180^\circ} = 0.3840.$$

We also know that $r = 0.05$, so

$$s = 0.05(0.3840) = 0.01920.$$

The coordinates of P are $(r\cos\theta, r\sin\theta) = (0.0464, 0.0187)$.

31. We have $\theta = 1.3$ rad or, in degrees,

$$1.3\left(\frac{180^\circ}{\pi}\right) = 74.4845^\circ.$$

We also have $r = 12$, so

$$s = 12(1.3) = 15.6,$$

and $P = (r\cos\theta, r\sin\theta) = (3.2100, 11.5627)$.

32. We have $\theta = 3\pi/7$ or, in degrees,

$$\theta = \frac{3\pi}{7} \cdot \frac{180^\circ}{\pi} = 77.1429^\circ.$$

We also have $r = 80$, so

$$s = 80 \cdot \frac{3\pi}{7} = \frac{240\pi}{7} = 107.7117,$$

and $P = (r\cos\theta, r\sin\theta) = (17.8017, 77.9942)$.

33. We do not know the value of r or s, but we know that $s = r\theta$, so

$$\theta = \frac{s}{r} = 0.4,$$

or, in degrees,

$$\theta = 0.4\left(\frac{180^\circ}{\pi}\right) = 22.918^\circ.$$

This means that $P = (r\cos\theta, r\sin\theta) = (0.9211r, 0.3894r)$.

34. **(a)** Yes, in both it seems to be roughly 60°.
(b) Just over 6 arcs fit into the circumference, since the circumference is $2\pi r = 2\pi(2) = 12.566$.

35. **(a)** Negative
(b) Negative
(c) Positive
(d) Positive

36. **(a)** $-2\pi/3 < 2/3 < 2\pi/3 < 2.3$
(b) $\cos 2.3 < \cos(-2\pi/3) = \cos(2\pi/3) < \cos(2/3)$

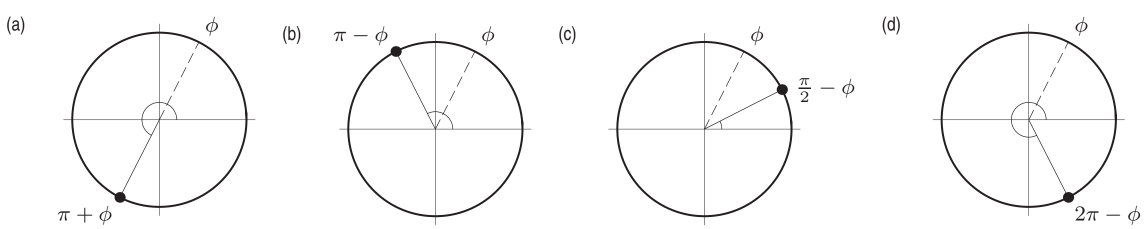
37. $\sin\theta = 0.6$, $\cos\theta = -0.8$.

38. $\sin\theta = 0.8$, $\cos\theta = -0.6$.

39. Since the ant traveled three units on the unit circle, the traversed arc must be spanned by an angle of three radians. Thus the ant's coordinates must be

$$(\cos 3, \sin 3) \approx (-0.99, 0.14).$$

40. The graphs are found below.



41. As the bob moves from one side to the other, as in Figure 6.28, the string moves through an angle on 10°. We are therefore looking for the arc length on a circle of radius 3 feet cut off by an angle of 10°. First we convert 10° to radians

$$10^\circ = 10 \cdot \frac{\pi}{180} = \frac{\pi}{18} \text{ radians.}$$

Then we find

$$\begin{aligned}\text{arc length} &= \text{radius} \cdot \text{angle spanned in radians}\\ &= 3\left(\frac{\pi}{18}\right)\\ &= \frac{\pi}{6} \text{ feet.}\end{aligned}$$

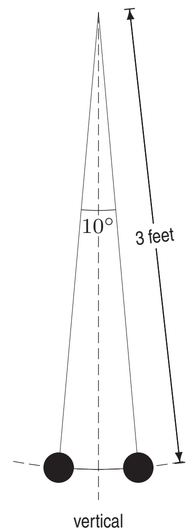


Figure 6.28

42. A complete revolution is an angle of 2π radians and this takes 60 minutes. In 35 minutes, the angle of movement in radians is $35/60 \cdot 2\pi = 7\pi/6$. The arc length is equal to the radius times the radian measure, which is $6(7\pi/6) = 7\pi \approx 21.991$ inches.

43. Converting the angle into radians, we get

$$1.4333^\circ = 1.4333\left(\frac{2\pi}{360}\right) \approx 0.0250 \text{ radians.}$$

Now we know that

$$\text{Arc length} = \text{Radius} \cdot \text{Angle in radians.}$$

Thus the radius is

$$\text{Radius} = \frac{\text{Arc length}}{\text{Angle in radians}} \approx \frac{100 \text{ miles}}{1.4333(2\pi/360)} = 3998.310 \text{ miles.}$$

44. The value of t is bigger than the value of $\sin t$ on $0 < t < \pi/2$. On a unit circle, the vertical segment, $\sin t$, is shorter than the arc, t. See Figure 6.29.

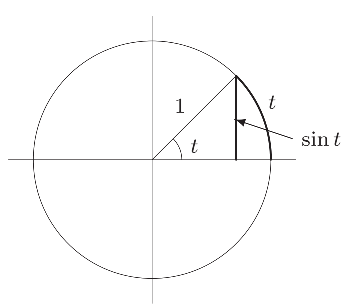


Figure 6.29

45. Make a table, such as Table 6.1, using your calculator to see that $\cos t$ is decreasing and the values of t are increasing.

Table 6.1

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\cos t$	1	0.995	0.980	0.953	0.921	0.878	0.825	0.765	0.697	0.622

Use a more refined table to see $t \approx 0.74$. (See Table 6.2.) Further refinements lead to $t \approx 0.739$.

Table 6.2

t	0.70	0.71	0.72	0.73	0.74	0.75	0.76
$\cos t$	0.765	0.758	0.752	0.745	0.738	0.732	0.725

Alternatively, consider the graphs of $y = t$ and $y = \cos t$ in Figure 6.30. They intersect at a point in the first quadrant, so for the t-coordinate of this point, $t = \cos t$. Trace with a calculator to find $t \approx 0.739$.

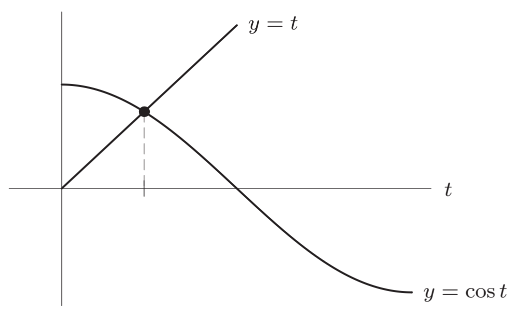


Figure 6.30

Solutions for Section 6.4

Exercises

1. Since the maximum value of the sine function is 1 and the minimum is -1, this function varies between -1 and 1 in value, giving it a midline of $y = 0$ and an amplitude of 1.

2. Since the maximum value of the sine function is 1 and the minimum is -1, the $6\sin(4x + 3)$ varies between -6 and 6 in value. The addition of 5 means that it varies between -1 and 11, giving it a midline of $y = 5$ and an amplitude of 6.

3. Since the maximum value of the cosine function is 1 and the minimum is -1, the $-7\cos(2x - 8)$ varies between -7 and 7 in value. The subtraction of 4 means that it varies between -11 and 3, giving it a midline of $y = -4$ and an amplitude of 7.

4. Since the maximum value of the function is 3 and the minimum is 1, its midline is $y = 2$, and its amplitude is 1.

5. Since the maximum value of the function is 1 and the midline appears to be $y = -2$, the amplitude is 3.

6. Since the minimum value of the function is -10 and the midline appears to be $y = -3$, the amplitude is 7.

7. Since the middle of the clock's face is at 185 cm, the midline is at 185 cm, and since the hand is 15 cm long, the amplitude is 15 cm.

8. Since the middle of the clock's face is at 223 cm, the midline is at 223 cm, and since the hand is 20 cm long, the amplitude is 20 cm.

9.

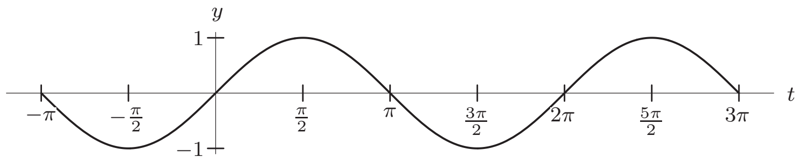


Figure 6.31

(a) (i) For $0 < t < \pi$ and $2\pi < t < 3\pi$ the function $\sin t$ is positive.
(ii) It is increasing for $-\frac{\pi}{2} < t < \frac{\pi}{2}$ and $\frac{3\pi}{2} < t < \frac{5\pi}{2}$.
(iii) For $-\pi < t < 0$ and $\pi < t < 2\pi$ it is concave up.

(b) The function appears to have the maximum rate of increase at $t = 0, 2\pi$.

10. The calculator gives the value $0.707107\ldots$ for both expressions $\sqrt{\dfrac{1}{2}}$ and $\dfrac{\sqrt{2}}{2}$. In fact, $\sqrt{\dfrac{1}{2}} = \dfrac{\sqrt{2}}{2}$. This is because

$$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

The value 0.7071 is a good approximation of $\sqrt{2}/2$.

11. The calculator gives the value $0.866025\ldots$ for both $\sqrt{\frac{3}{4}}$ and $\frac{\sqrt{3}}{2}$. In fact, $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$. The value 0.8660 is a good approximation of $\frac{\sqrt{3}}{2}$.

12. Since we know that the x-coordinate on the unit circle at $7\pi/6$ is the negative of the x-coordinate at $\pi/6$, and since $\pi/6$ radians is the same as 30°, we know that $\cos(7\pi/6) = -\cos(\pi/6) = -\cos 30^\circ = -\sqrt{3}/2$.

13. Since we know that the y-coordinate on the unit circle at $2\pi/3$ is the same as the y-coordinate at $\pi/3$, and since $\pi/3$ radians is the same as 60°, we know that $\sin(2\pi/3) = \sin(\pi/3) = \sin 60^\circ = \sqrt{3}/2$.

14. Since we know that the y-coordinate on the unit circle at $-\pi/3$ is the negative of the y-coordinate at $\pi/3$, and since $\pi/3$ radians is the same as 60°, we know that $\sin(-\pi/3) = -\sin(\pi/3) = -\sin 60^\circ = -\sqrt{3}/2$.

15. Since we know that the x-coordinate on the unit circle at $-\pi/6$ is the same as the x-coordinate at $\pi/6$, and since $\pi/6$ radians is the same as 30°, we know that $\cos(-\pi/6) = \cos(\pi/6) = \cos 30^\circ = \sqrt{3}/2$.

Problems

16. $f(x) = (\sin x) + 1$
$g(x) = (\sin x) - 1$

17. $g(x) = \cos x$, $a = \pi/2$ and $b = 1$.

18. **(a)** $7\pi/8$
(b) $\pi - 1$

19. $f(x) = \sin(x + \frac{\pi}{2})$
$g(x) = \sin(x - \frac{\pi}{2})$

20. $A = 1, B = \frac{\pi}{2}, C = 2, D = \pi, E = 4, F = \frac{3\pi}{2}, G = 5$

21. Since $\sin\theta$ is the y-coordinate of a point on the unit circle, its height above the x-axis can never be greater than 1. Otherwise the point would be outside the circle. See Figure 6.32.

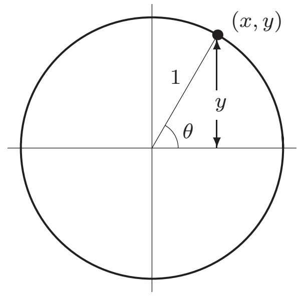


Figure 6.32

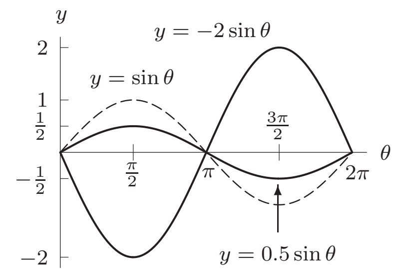


Figure 6.33

22. We can sketch these graphs using a calculator or computer. Figure 6.33 gives a graph of $y = \sin\theta$, together with the graphs of $y = 0.5\sin\theta$ and $y = -2\sin\theta$, where θ is in radians and $0 \le \theta \le 2\pi$. These graphs are similar but not the same. The amplitude of $y = 0.5\sin\theta$ is 0.5 and the amplitude of $y = -2\sin\theta$ is 2. The graph of $y = -2\sin\theta$ is vertically reflected relative to the other two graphs. These observations are consistent with the fact that the constant A in the equation

$$y = A\sin\theta$$

may result in a vertical stretching or shrinking and/or a reflection over the x-axis. Note that all three graphs have a period of 2π.

23. **(a)** Since $3\pi/4$ is in the second quadrant,

$$\sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

(b) Since $5\pi/3$ is in the fourth quadrant,

$$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}.$$

(c) Since $7\pi/6$ is in the third quadrant,

$$\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}.$$

(d) Since $11\pi/6$ is in the fourth quadrant,

$$\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}.$$

(e) Since $9\pi/4$ is in the first quadrant,

$$\sin\left(\frac{9\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

24. Since 45° is half 90°, the point P in Figure 6.34 lies on the line $y = x$. Substituting $y = x$ into the equation of the circle, $x^2 + y^2 = 1$, gives $x^2 + x^2 = 1$. Solving for x, we get

$$\begin{aligned} 2x^2 &= 1 \\ x^2 &= \frac{1}{2} \\ x &= \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}. \end{aligned}$$

Since P is in the first quadrant, x and y are positive, so

$$x = \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \text{and} \qquad y = \sin 45^\circ = \frac{1}{\sqrt{2}}.$$

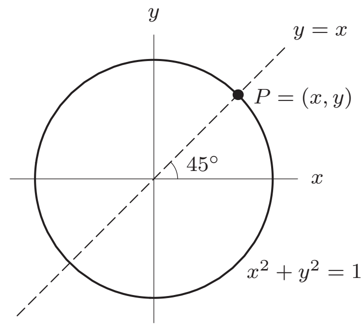


Figure 6.34: Calculating $\cos 45°$ and $\sin 45°$

25.

$$x = r\cos\theta = 10\cos 210^\circ = 10(-\sqrt{3}/2) = -5\sqrt{3}$$

and

$$y = r\sin\theta = 10\sin 210^\circ = 10(-1/2) = -5,$$

so the coordinates of W are $(-5\sqrt{3}, -5)$.

26. (a)

(b)

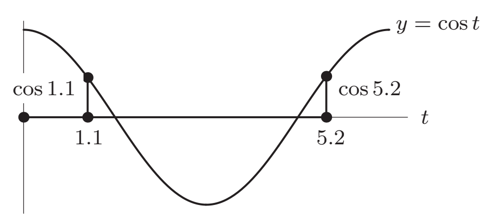


Figure 6.35

27. (a) (i) p
(ii) s
(iii) q
(iv) r

(b)

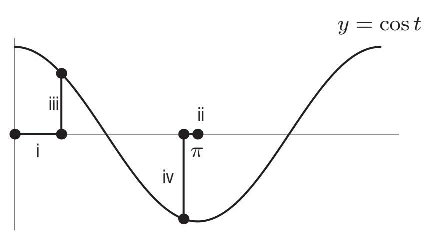


Figure 6.36

28. If the circle were centered at $(0,0)$ we would see immediately that $x = 5\cos\theta$. The shift up 7 units has no effect on the x-value but the shift 6 units left means $x = 5\cos\theta - 6$. Thus $f(\theta) = 5\cos\theta - 6$. We can check this by plugging in convenient θ-values. For instance, $f(\pi/2) = -6$ makes sense because it is the 12 o'clock position on the circle.

29. **(a)**

$$m = \frac{\cos b - \cos a}{b - a}$$

(b)

$$\frac{\cos\frac{4\pi}{3} - \cos\frac{\pi}{4}}{\frac{4\pi}{3} - \frac{\pi}{4}} = \frac{\frac{-1}{2} - \frac{\sqrt{2}}{2}}{\frac{13\pi}{12}} = \frac{-1-\sqrt{2}}{2} \cdot \frac{12}{13\pi} = \frac{-6(1+\sqrt{2})}{13\pi}$$

30. **(a)** Slope, m, of segment joining S and T:

$$m = \frac{\sin(a+h) - \sin a}{(a+h) - a} = \frac{\sin(a+h) - \sin a}{h}$$

(b) If $a = 1.7$ and $h = 0.05$,

$$m = \frac{\sin 1.75 - \sin 1.7}{1.75 - 1.7} \approx -0.15$$

Solutions for Section 6.5

Exercises

1. The midline is $y = 0$. The amplitude is 6. The period is 2π.

2. The midline is $y = -8$. The amplitude is 7. The period is $2\pi/4 = \pi/2$.

3. We first divide both sides by 2, giving

$$y = \frac{1}{2}\cos(8(t-6)) + 1.$$

The midline is $y = 1$. The amplitude is $\frac{1}{2}$. The period is $2\pi/8 = \pi/4$.

4. The midline is $y = -1$. The amplitude is π. The period is $2\pi/2 = \pi$.

5. We see that the phase shift is -4, since the function is in a form that shows it. To find the horizontal shift, we factor out a 3 within the cosine function, giving us

$$y = 2\cos\left(3\left(t + \frac{4}{3}\right)\right) - 5.$$

Thus, the horizontal shift is $-4/3$.

6. We see that the phase shift is -13, since the function is in a form that shows it. To find the horizontal shift, we factor out a 7 within the cosine function, giving us

$$y = -4\cos\left(7\left(t + \frac{13}{7}\right)\right) - 5.$$

Thus, the horizontal shift is $-13/7$.

7. Both f and g have periods of 1, amplitudes of 1, and midlines $y = 0$.

8. **(a)** The function $y = \sin(-t)$ is periodic, and its period is 2π. The function begins repeating every 2π units, as is clear from its graph. Recall that $f(-x)$ is a reflection about the y-axis of the graph of $f(x)$, so the periods for $\sin(t)$ and $\sin(-t)$ are the same. See Figure 6.37.

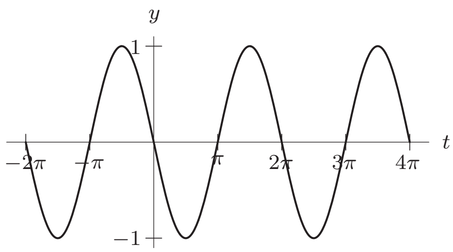


Figure 6.37: $y = \sin(-t)$

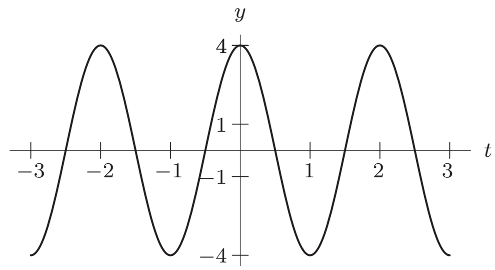


Figure 6.38: $y = 4\cos(\pi t)$

(b) The function $y = 4\cos(\pi t)$ is periodic, and its period is 2. This is because when $0 \le t \le 2$, we have $0 \le \pi t \le 2\pi$ and the cosine function has period 2π. Note the amplitude of $4\cos(\pi t)$ is 4, but changing the amplitude does not affect the period. See Figure 6.38.

(c) The function $y = \sin(t) + t$ is not periodic, because as t gets large, $\sin(t) + t$ gets large as well. In fact, since $\sin(t)$ varies from -1 to 1, y is always between $t - 1$ and $t + 1$. So the values of y cannot repeat. See Figure 6.39.

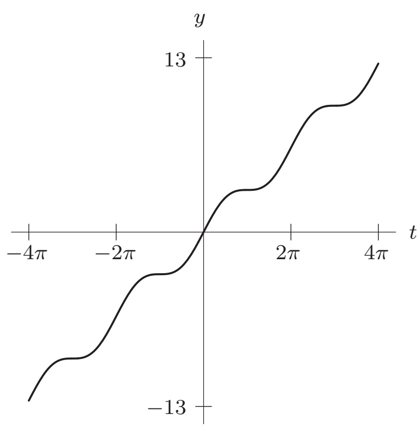


Figure 6.39: $y = \sin(t) + t$

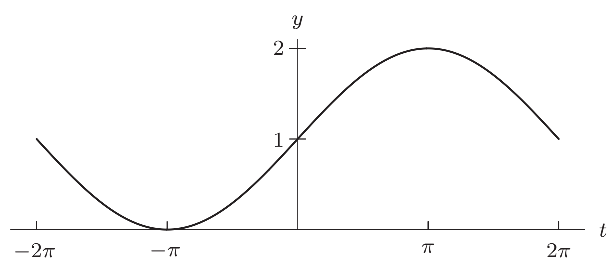


Figure 6.40: $y = \sin(t/2) + 1$

(d) In general $f(x)$ and $f(x) + c$ will have the same period if they are periodic. The function $y = \sin(\frac{t}{2}) + 1$ is periodic, because $\sin(\frac{t}{2})$ is periodic. Since $\sin(t/2)$ completes one cycle for $0 \le t/2 \le 2\pi$, or $0 \le t \le 4\pi$, we see the period of $y = \sin(t/2) + 1$ is 4π. See Figure 6.40.

9. The function completes one and a half oscillations in 9 units of t, so the period is $9/1.5 = 6$, the amplitude is 5, and the midline is 0.

10. The period is 4, the amplitude 3, and the midline -3.

11. This function resembles a sine curve in that it passes through the origin and then proceeds to grow from there. We know that the smallest value it attains is -4 and the largest it attains is 4, thus its amplitude is 4. It has a period of 1. Thus in the equation

$$g(t) = A\sin(Bt)$$

we know that $A =$ and

$$1 = \text{period} = \frac{2\pi}{B}.$$

So $B = 2\pi$, and then

$$h(t) = 4\sin(2\pi t).$$

12. This function resembles a cosine curve in that it attains its maximum value when $t = 0$. We know that the smallest value it attains is -3 and that its midline is $y = 0$. Thus its amplitude is 3. It has a period of 4. Thus in the equation

$$f(t) = A\cos(Bt)$$

we know that $A = 3$ and

$$4 = \text{period} = \frac{2\pi}{B}.$$

So $B = \pi/2$, and then

$$f(t) = 3\cos\left(\frac{\pi}{2}t\right).$$

13. This function resembles an inverted cosine curve in that it attains its minimum value when $t = 0$. We know that the smallest value it attains is 0 and that its midline is $y = 2$. Thus its amplitude is 2 and it is shifted upward by two units. It has a period of 4π. Thus in the equation

$$g(t) = -A\cos(Bt) + D$$

we know that $A = -2$, $D = 2$, and

$$4\pi = \text{period} = \frac{2\pi}{B}.$$

So $B = 1/2$, and then

$$g(t) = -2\cos\left(\frac{t}{2}\right) + 2.$$

14. The graph is a horizontally and vertically compressed sine function. The midline is $y = 0$. The amplitude is 0.8. We see that $\pi/7 =$ two periods, so the period is $\pi/14$. Hence $B = 2\pi/(\text{period}) = 28$, and so

$$y = 0.8\sin(28\theta).$$

15. The midline is $y = 4000$. The amplitude is $8000 - 4000 = 4000$. The period is 60, so the angular frequency is $2\pi/60$. The graph at $x = 0$ rises from its midline, so we use the sine. Thus,

$$y = 4000 + 4000\sin\left(\frac{2\pi}{60}x\right).$$

16. The midline is $y = 20$. The amplitude is $30 - 20 = 10$. The period is 12, so the angular frequency is $2\pi/12$. The graph at $x = 0$ decreases from its maximum, like the cosine function. Thus,

$$y = 10\cos\left(\frac{2\pi}{12}x\right) + 20.$$

17. The graph resembles a sine function that is vertically reflected, horizontally and vertically stretched, and vertically shifted. There is no horizontal shift since the function hits its midline at $\theta = 0$. The midline is halfway between 0 and 4, so it has the equation $y = 2$. The amplitude is 2. Since we see 9 is $\frac{3}{4}$ of the length of a cycle, the period is 12. Hence $B = 2\pi/(\text{period}) = \pi/6$, and so

$$y = -2\sin\left(\frac{\pi}{6}\theta\right) + 2.$$

18. We see the interval from 0 to 2 is half a period, so the period $P = 4$. Hence $B = 2\pi/P = \pi/2$. The midline is shown at $y = 3$, so $D = 3$. We see the amplitude $|A| = 3$. Since g has a minimum at $\theta = 0$ like $-\cos\theta$, A is negative. Hence $A = -3$. Thus,

$$g(\theta) = -3\cos\left(\frac{\pi}{2}\theta\right) + 3.$$

Problems

19. See Figure 6.41.

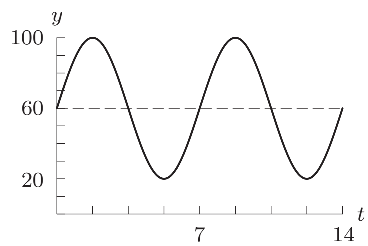


Figure 6.41

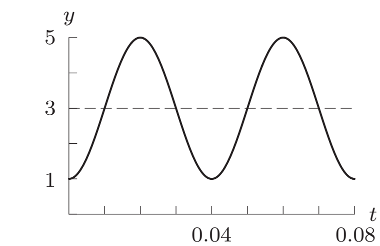


Figure 6.42

20. See Figure 6.42.

21. See Figure 6.43.

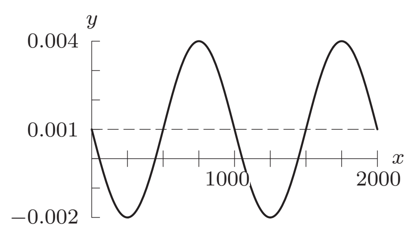


Figure 6.43

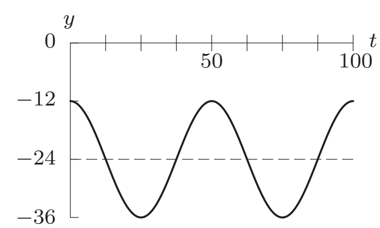


Figure 6.44

22. See Figure 6.44.

23. First, we note that the graph indicates that f completes one cycle in 8 units. Therefore, the period of f equals 8, and we can also see that by shifting the graph of f 2 units to the right, we obtain the graph of g. Thus, f must be shifted by $1/4$ of a period to the right to obtain g, meaning that the phase shift of g is $2\pi \cdot (1/4) = \pi/2$. Therefore, we have $g(x) = 3\sin((\pi/4)x - \pi/2)$.

24. First, we note that the graph indicates that f completes one cycle in 10 units. Therefore, the period of f equals 10, and we can also see that by shifting the graph of f 3 units to the right, we obtain the graph of g. Thus, f must be shifted by $3/10$ of a period to the right to obtain g, meaning that the phase shift of g is $2\pi \cdot (3/10) = 3\pi/5$. Therefore, we have $g(x) = 10\sin((\pi/5)x - 3\pi/5)$.

25. Because the period of $\sin x$ is 2π, and the period of $\sin 2x$ is π, so from the figure in the problem we see that

$$f(x) = \sin x.$$

The points on the graph are $a = \pi/2$, $b = \pi$, $c = 3\pi/2$, $d = 2\pi$, and $e = 1$.

26. This graph of the function $y = \cos(5t + \pi/4)$ is the graph of $y = \cos 5t$ shifted horizontally to the left. (In general, the graph of $f(x+k)$ is the graph of $f(x)$ shifted left if k is positive.) Since

$$5t + \frac{\pi}{4} = 5\left(t + \frac{\pi}{20}\right),$$

the graph shifts left a distance of $\pi/20$ units. The phase shift is $\pi/4$. Since $\pi/4$ is one-eighth of the period of $\cos t$, the graph of this function is $y = \cos 5t$ shifted left by one-eighth of its period.

27. From Figure 6.45, we see the amplitude A of this function is 20. We see 4 cycles in 3 seconds so the period is $\frac{3}{4}$.

The amplitude tells us the maximum amount by which the blood pressure can vary from its average value of 100 mm Hg. The period of $\frac{3}{4}$ seconds tells us the duration of one cycle of blood pressure change.

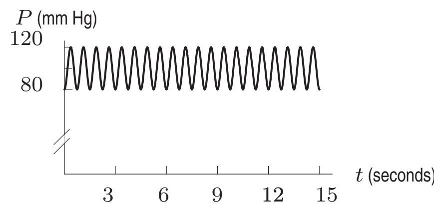


Figure 6.45

28. $f(t) = 17.5 + 17.5 \sin\left(\frac{2\pi}{5}t\right)$

29. $f(t) = 14 + 10 \sin\left(\pi t + \frac{\pi}{2}\right)$

30. The amplitude and midline are 20 and the period is 5. The graph is a sine curve shifted half a period to the right (or left), so the phase shift is $-\pi$. Thus

$$h = 20 + 20 \sin\left(\frac{2\pi}{5}t - \pi\right).$$

31. $f(t) = 20 + 15 \sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right)$

32. From Figure 6.5, we see that the midline and amplitude are both 225 and the period is 30. Since the graph looks like a cosine reflected about the midline, we take $k = 225$ and $A = -225$. In addition

$$\frac{2\pi}{B} = 30 \qquad \text{so} \qquad B = \frac{\pi}{15}.$$

The cosine function does not need to be shifted horizontally, so

$$f(t) = -225 \cos\left(\frac{\pi}{15}t\right) + 225.$$

33. (a) The ferris wheel makes one full revolution in 30 minutes. Since one revolution is 360°, the wheel turns

$$\frac{360}{30} = 12^\circ \text{ per minute.}$$

(b) The angle representing your position, measured from the 6 o'clock position, is $12t^\circ$. However, the angle shown in Figure 6.46 is measured from the 3 o'clock position, so

$$\theta = (12t - 90)^\circ.$$

(c) With y as shown in Figure 6.46, we have

$$\text{Height} = 225 + y = 225 + 225 \sin\theta,$$

so

$$f(t) = 225 + 225 \sin(12t - 90)^\circ.$$

Note that the expression $\sin(12t - 90)^\circ$ means $\sin\left((12t - 90)^\circ\right)$.

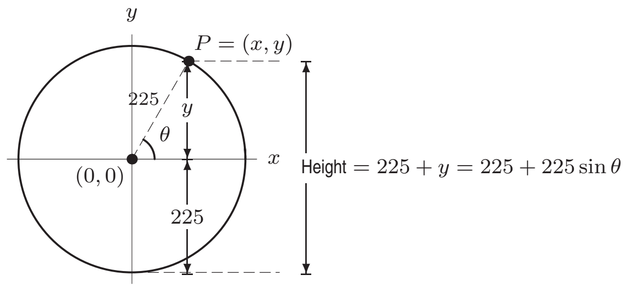


Figure 6.46

(d) Using a calculator, we obtain the graph of $h = f(t) = 225 + 225\sin(12t - 90)^\circ$ in Figure 6.47. The period is 30 minutes, the midline and amplitude are 225 feet.

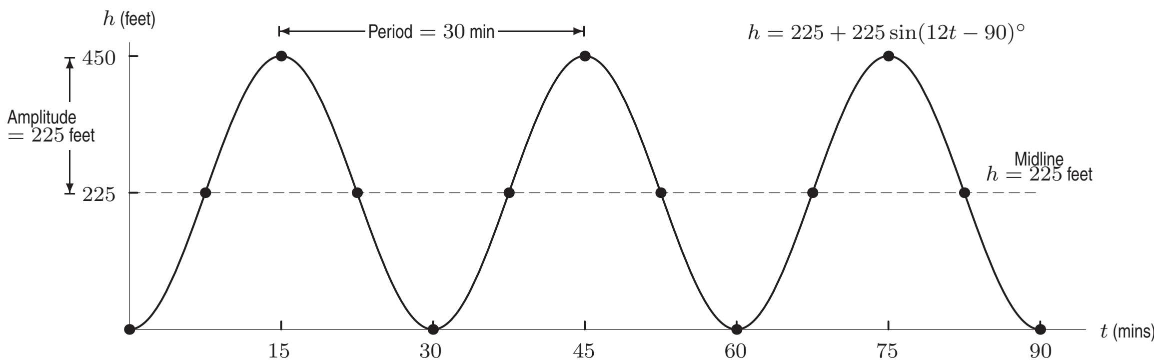


Figure 6.47: On the ferris wheel: Height, h, above ground as function of time, t

34. (a) The population has initial value 1500 and grows at a constant rate of 200 animals per year.
(b) The population has initial value 2700 and decreases at a constant rate of 80 animals per year.
(c) The population has initial value 1800 and increases at the constant percent rate of 3% per year.
(d) The population has initial value 800 and decreases at the *continuous* percent rate of 4% per year.
(e) The population has initial value 3800, climbs to $3800 + 230 = 4030$, drops to $3800 - 230 = 3570$, and climbs back to 3800 over a 7 year period. This pattern keeps repeating itself.

35. (a) The midline is at $P = (2200 + 1300)/2 = 1750$. The amplitude is $|A| = 2200 - 1750 = 450$. The population starts at its minimum so it is modeled by vertically reflected cosine curve. This means $A = -450$, and that there is no phase shift. Since the period is 12, we have $B = 2\pi/12 = \pi/6$. This means the formula is

$$P = f(t) = -450\cos\left(\frac{\pi}{6}t\right) + 1750.$$

(b) The midline, $P = 1750$, is the average population value over one year. The period is 12 months (or 1 year), which means the cycle repeats annually. The amplitude is the amount that the population varies above and below the average annual population.
(c) Figure 6.48 is a graph of $f(t) = -450\cos(\frac{\pi}{6}t) + 1750$ and $P = 1500$.

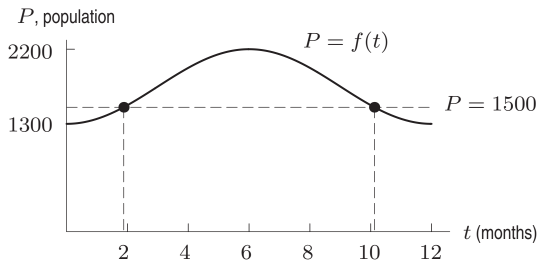


Figure 6.48

From a graph we get approximations of $t_1 \approx 1.9$ and $t_2 \approx 10.1$. This means that the population is 1500 sometime in late February and again sometime in early November.

36. The data given describe a trigonometric function shifted vertically because all the $g(x)$ values are greater than 1. Since the maximum is approximately 3 and the minimum approximately 1, the midline value is 2. We choose the sine function over the cosine function because the data tell us that at $x = 0$ the function takes on its midline value, and then increases. Thus our function will be of the form

$$g(x) = A\sin(Bx) + k.$$

We know that A represents the amplitude, k represents the vertical shift, and the period is $2\pi/B$.

We've already noted the midline value is $k \approx 2$. This means $A = \max - k = 1$. We also note that the function completes a full cycle after 1 unit. Thus

$$1 = \frac{2\pi}{B}$$

so

$$B = 2\pi.$$

Thus

$$g(x) = \sin(2\pi x) + 2.$$

37. This function has an amplitude of 3 and a period 1, and resembles a sine graph. Thus $y = 3f(x)$.

38. This function has an amplitude of 2 and a period of 3, and resembles vertically reflected cosine graph. Thus $y = -2g(x/3)$.

39. This function has an amplitude of 1 and a period of 0.5, and resembles an inverted sine graph. Thus $y = -f(2x)$.

40. This function has an amplitude of 2 and a period of 1 and a midline of $y = -3$, and resembles a cosine graph. Thus $y = 2g(x) - 3$.

41.

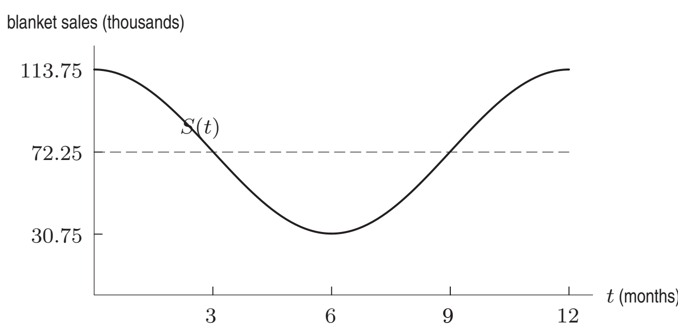


Figure 6.49

The amplitude of this graph is 41.5. The period is $P = 2\pi/B = (2\pi)/(\pi/6) = 12$ months. The amplitude of 41.5 tells us that during winter months sales of electric blankets are 41,500 above the average. Similarly, sales reach a minimum of 41,500 below average in the summer months. The period of one year indicates that this seasonal sales pattern repeats annually.

42. **(a)**

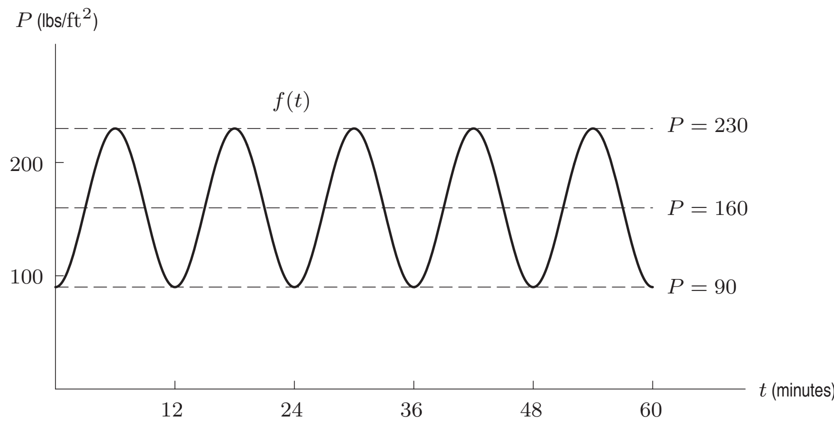


Figure 6.50

This function is a vertically reflected cosine function which has been vertically shifted. Thus the function for this equation will be of the form

$$P = f(t) = -A\cos(Bt) + k.$$

(b) The midline value is $k = (90 + 230)/2 = 160$.
The amplitude is $|A| = 230 - 160 = 70$.
A complete oscillation is made each 12 minutes, so the period is 12. This means $B = 2\pi/12 = \pi/6$. Thus $P = f(t) = -70\cos(\pi t/6) + 160$.

(c) Graphing $P = f(t)$ on a calculator for $0 \leq t \leq 2$ and $90 \leq P \leq 230$, we see that $P = f(t)$ first equals 115 when $t \approx 1.67$ minutes.

43. Figure 6.51 highlights the two parts of the graph. In the first hour, the plane is approaching Boston. In the second hour, the plane is circling Boston.

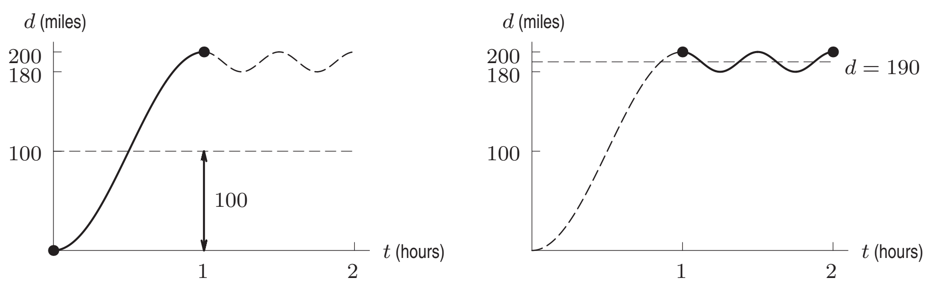


Figure 6.51: We can split the function $d = f(t)$ into two pieces, both of which are cosine curves

From Figure 6.51, we see that both parts of $f(t)$ look like cosine curves. The first part has the equation

$$f(t) = -100\cos(\pi t) + 100, \quad \text{for } 0 \leq t \leq 1.$$

In the second part, the period is 1/2, the midline is 190, and the amplitude is $200 - 190 = 10$, so

$$f(t) = 10\cos(4\pi t) + 190, \quad \text{for } 1 \leq t \leq 2.$$

Thus, a piecewise formula for $f(t)$ could be

$$f(t) = \begin{cases} -100\cos(\pi t) + 100 & \text{for } 0 \leq t \leq 1 \\ 10\cos(4\pi t) + 190 & \text{for } 1 < t \leq 2. \end{cases}$$

44. (a) See Figure 6.52.

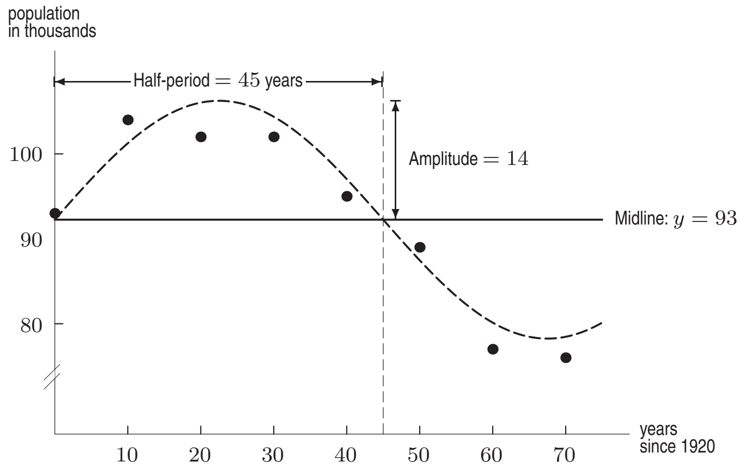


Figure 6.52: Population (in thousands) versus time, together with sine curve.

(b) The variation is possibly sinusoidal, but not necessarily. The population rises first, then falls. From the graph, it appears it could soon rise again, but this need not be the case.

(c) See Figure 6.52. There are many possible answers.

(d) For the graph drawn, the amplitude of the population function is $107-93 = 14$. The average value of P from the data is 93. The graph of $P = f(t)$ behaves as the graph of $\sin t$, for $3/4$ of a period. Therefore, we look for a reasonable approximation to the data of the form $P = f(t) = 14\sin(Bt)+93$. To determine B, we assume that 45 years is half the period of f. Thus, the period equals 90 years and so $B = 2\pi/90 = \pi/45$. Hence an approximation to the data is

$$P = f(t) = 14\sin\left(\frac{\pi}{45}t\right) + 93.$$

(e) $P = f(-10) \approx 84.001$, which means that our formula predicts a population of about 84,000. This is not too far off the mark, but not all that close, either.

45. (a) See Figure 6.53, where January is represented by $t = 0$.

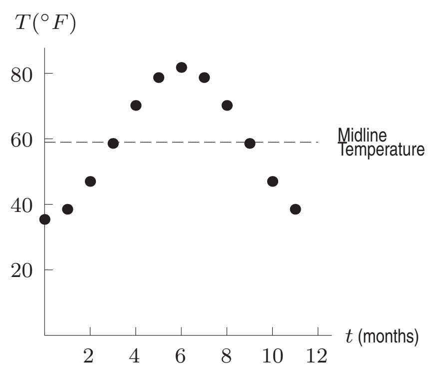


Figure 6.53

(b) The midline temperature is approximately $(81.8 + 35.4)/2 = 58.6$ degrees. The amplitude of the temperature function is then $81.8 - 58.6 = 23.2$ degrees. The period equals 12 months.

(c) We choose the approximating function $T = f(t) = -A\cos(Bt) + D$. Since the graph resembles an inverted cosine curve, we know $A = 23.2$ and $D = 58.6$. Since the period is 12, $B = 2\pi/12 = \pi/6$. Thus

$$T = f(t) = -23.2\cos\left(\frac{\pi}{6}t\right) + 58.6$$

is a good approximation, though it does not exactly agree with all the data.

(d) In October, $T = f(9) = -23.2\cos((\pi/6)9) + 58.6 \approx 58.6$ degrees, while the table shows an October value of 62.5.

46. (a) Although the graph has a rough wavelike pattern, the wave is not perfectly regular in each 7-day interval. A true periodic function has a graph which is absolutely regular, with values that repeat exactly every period.

(b) Usage spikes every 7 days or so, usually about midweek (8/7, 8/14, 8/21, etc.). It drops to a low point every 7 days or so, usually on Saturday or Sunday (8/10, 8/17, 8/25, etc.). This indicates that scientists use the site less frequently on weekends and more frequently during the week.

(c) See Figure 6.54 for one possible approximation. The function shown here is given by $n = a\cos(B(t-h))+k$ where t is the number of days from Monday, August 5, and $a = 45{,}000$, $B = 2\pi/7$, $h = 2$, and $k = 100{,}000$. The midline $k = 100{,}000$ tells us that usage rises and falls around an approximate average of 100,000 connections per day. The amplitude $a = 45{,}000$ tells us that usage tends to rise or fall by about 45,000 from the average over the course of the week. The period is 7 days, or one week, giving $B = 2\pi/7$, and the curve resembles a cosine function shifted to the right by about $h = 2$ days. Thus,

$$n = 45{,}000\cos\left(\frac{2\pi}{7}(t-2)\right) + 100{,}000.$$

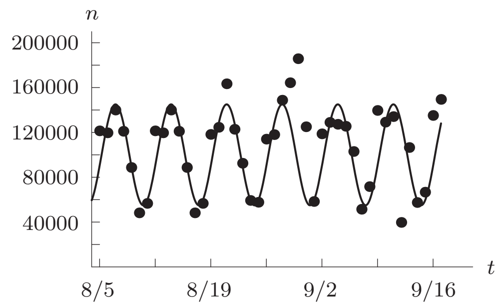


Figure 6.54: Fitting a trigonometric function to the arXiv.org usage data

47. The petroleum import data is graphed in Figure 6.55.

We start with a sine function of the form

$$f(t) = A\sin(B(t-h)) + k.$$

Since the maximum here is 18 and the minimum is 12, the midline value is $k = (18+12)/2 = 15$. The amplitude is then $A = 18 - 15 = 3$. The period, measured peak to peak, is 12. So $B = 2\pi/12 = \pi/6$. Lastly, we calculate h, the horizontal shift to the right. Our data are close to the midline value for $t \approx 74$, whereas $\sin t$ is at its midline value for $t = 0$. So $h = 74$ and the equation is

$$f(t) = 3\sin\frac{\pi}{6}(t-74) + 15.$$

We can check our formula by graphing it and seeing how close it comes to the data points. See Figure 6.56.

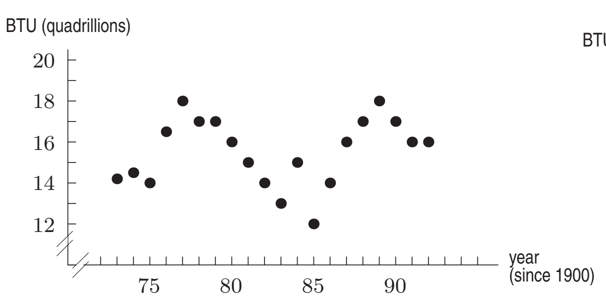


Figure 6.55: US Imports of Petroleum

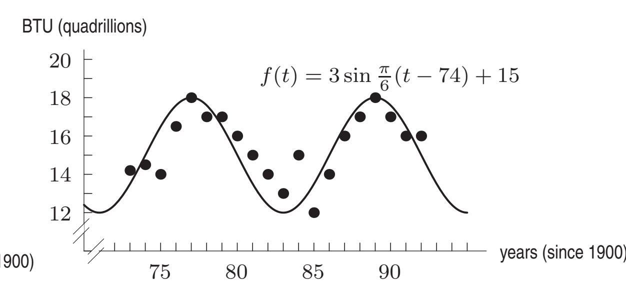


Figure 6.56

Solutions for Section 6.6

Exercises

1. $\sin 0^\circ = 0$, $\cos 0^\circ = 1$, $\tan 0^\circ = \sin 0^\circ / \cos 0^\circ = 0/1 = 0$.

2. $\cos 90^\circ = 0$

3. $\sin 90^\circ = 1$

4. $\tan 90^\circ$ is undefined, because at $\theta = 90^\circ$, the x-coordinate is 0. So in order to evaluate $\tan 90^\circ$, we would have to divide by 0.

5. $\sin 270^\circ = -1$

6. Since 225° is in the third quadrant,

$$\tan 225^\circ = \tan 45^\circ = 1.$$

7. Since 135° is in the second quadrant,

$$\tan 135^\circ = -\tan 45^\circ = -1.$$

8. $\tan 540^\circ = \tan 180^\circ = \dfrac{\sin 180^\circ}{\cos 180^\circ} = \dfrac{0}{-1} = 0.$

9. 1

10. $\sqrt{3}$

11. $-\sqrt{3}$

12. $\dfrac{-1}{\sqrt{3}}$

13. Since $\csc(5\pi/4) = 1/\sin(5\pi/4)$, we know that $\csc(5\pi/4) = 1/(-1/\sqrt{2}) = -\sqrt{2}$.

14. Since $\cot(5\pi/3) = 1/(\tan(5\pi/3) = 1/(\sin(5\pi/3)/\cos(5\pi/3)) = \cos(5\pi/3)/\sin(5\pi/3)$, we know that $\cot(5\pi/3) = (1/2)/(-\sqrt{3}/2) = -1/\sqrt{3}$.

15. Since $\sec(-\pi/6) = 1/\cos(-\pi/6)$, we know that $\sec(-\pi/6) = 1/(\sqrt{3}/2) = 2/\sqrt{3}$.

16. Since $\sec(11\pi/6) = 1/\cos(11\pi/6)$, we know that $\sec(11\pi/6) = 1/(\sqrt{3}/2) = 2/\sqrt{3}$.

Problems

17. Since $\sec\theta = 1/\cos\theta$, we have $\sec\theta = 1/(1/2) = 2$. Since $1 + \tan^2\theta = \sec^2\theta$,

$$\begin{aligned} 1 + \tan^2\theta &= 2^2 \\ \tan^2\theta &= 4 - 1 \\ \tan\theta &= \pm\sqrt{3}. \end{aligned}$$

Since $0 \le \theta \le \pi/2$, we know that $\tan\theta \ge 0$, so $\tan\theta = \sqrt{3}$.

18. Since $\csc\theta = 1/\sin\theta$, we begin by using the Pythagorean Identity to find $\sin\theta$. Since $\cos^2\theta + \sin^2\theta = 1$, we have

$$\begin{aligned} \left(\frac{1}{2}\right)^2 + \sin^2\theta &= 1 \\ \sin^2\theta &= 1 - \frac{1}{4} \\ \sin\theta &= \pm\sqrt{\frac{3}{4}}. \end{aligned}$$

Since $0 \le \theta \le \pi/2$, $\sin\theta \ge 0$, so $\sin\theta = \sqrt{\frac{3}{4}}$. Thus, $\csc\theta = 1/\sqrt{3/4} = \sqrt{4/3}$.

Since $\cot\theta = \cos\theta/\sin\theta$, we have $\cot\theta = (1/2)/\sqrt{3/4} = (1/2)(2\sqrt{3}) = 1/\sqrt{3}$.

19. Since $\sec\theta = 1/\cos\theta$, we begin with the Pythagorean Identity, $\cos^2\theta + \sin^2\theta = 1$. We have

$$\begin{aligned}\cos^2\theta + \left(\frac{1}{3}\right)^2 &= 1\\ \cos^2\theta &= 1 - \frac{1}{9}\\ \cos\theta &= \pm\sqrt{\frac{8}{9}} = \pm\frac{\sqrt{8}}{3}.\end{aligned}$$

Since $0 \le \theta \le \pi/2$, we know that $\cos\theta \ge 0$, so $\cos\theta = \sqrt{8}/3$. Therefore, $\sec\theta = 1/(\sqrt{8}/3) = 3/\sqrt{8}$.

Since $\tan\theta = \sin\theta/\cos\theta$, we have $\tan\theta = (1/3)/(\sqrt{8}/3) = 1/\sqrt{8}$.

20. Since $\cos\theta = 1/\sec\theta$, $\cos\theta = 1/17$. Using the Pythagorean Identity, $\cos^2\theta + \sin^2\theta = 1$, we have

$$\begin{aligned}\sin^2\theta + \left(\frac{1}{17}\right)^2 &= 1\\ \sin^2\theta &= 1 - \frac{1}{17^2}\\ \sin\theta &= \pm\sqrt{\frac{288}{289}} = \pm\frac{\sqrt{288}}{17}.\end{aligned}$$

Since $0 \le \theta \le \pi/2$, we know that $\sin\theta \ge 0$, so $\sin\theta = \sqrt{288}/17$.

Using the identity $\tan\theta = \sin\theta/\cos\theta$, we see that $\tan\theta = (\sqrt{288}/17)/(1/17) = \sqrt{288}$.

21. This looks like a tangent graph. At $\pi/4$, $\tan\theta = 1$. Since on this graph, $f(\pi/4) = 1/2$, and since it appears to have the same period as $\tan\theta$ without a horizontal or vertical shift, a possible formula is $f(\theta) = \frac{1}{2}\tan\theta$.

22. This looks like a tangent graph. At $\pi/4$, $\tan\theta = 1$. Since on this graph, $f(\pi/2) = 1$, and since it appears to have the same period as $\tan\theta$ without a vertical shift, but shifted $\pi/4$ to the right, a possible formula is $f(\theta) = \tan(\theta - \pi/4)$.

23. **(a)** $\sin^2\alpha = 1 - \cos^2\alpha = 1 - \left(\dfrac{-\sqrt{3}}{5}\right)^2 = 1 - \dfrac{3}{25} = \dfrac{22}{25}$. Since α is in the third quadrant,

$$\sin\alpha = -\sqrt{\frac{22}{25}} = \frac{-\sqrt{22}}{5}.$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\sqrt{22}/5}{-\sqrt{3}/5} = \sqrt{\frac{22}{3}}.$$

(b) $\tan\beta = \dfrac{\sin\beta}{\cos\beta} = \dfrac{4}{3}$, so $\sin\beta = \dfrac{4}{3}\cos\beta$. Then

$$1 = \sin^2\beta + \cos^2\beta = \left(\frac{4}{3}\cos\beta\right)^2 + \cos^2\beta = \frac{25}{9}\cos^2\beta.$$

Since β is in the third quadrant, $\cos\beta = -\sqrt{\dfrac{9}{25}} = \dfrac{-3}{5}$ and $\sin\beta = \dfrac{4}{3}\left(\dfrac{-3}{5}\right) = \dfrac{-4}{5}$.

24. **(a)** $\sin^2\phi = 1 - \cos^2\phi = 1 - (0.4626)^2$ and $\sin\phi$ is negative, so $\sin\phi = -\sqrt{1-(0.4626)^2} = -0.8866$. Thus $\tan\phi = (\sin\phi)/(\cos\phi) = (-0.8866)/(0.4626) = -1.9166$.

(b) $\cos^2\theta = 1 - \sin^2\theta = 1 - (-0.5917)^2$ and $\cos\theta$ is negative, so $\cos\theta = -\sqrt{1-(-0.5917)^2} = -0.8062$. Thus $\tan\theta = (\sin\theta)/(\cos\theta) = (-0.5917)/(-0.8062) = 0.7339$.

25. Since $y = \sin\theta$, we can construct the following triangle:

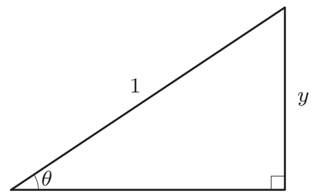


Figure 6.57

The adjacent side, using the Pythagorean theorem, has length $\sqrt{1-y^2}$. So, $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{1-y^2}}{1} = \sqrt{1-y^2}$.

26. For all t where the functions are defined

$$\tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin t}{\cos t} = -\tan t.$$

This shows that the tangent function satisfies the definition of an odd function.

27. Divide both sides of $\cos^2\theta + \sin^2\theta = 1$ by $\sin^2\theta$. For $\sin\theta \neq 0$,

$$\begin{aligned}\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} &= \frac{1}{\sin^2\theta}\\ \left(\frac{\cos\theta}{\sin\theta}\right)^2 + 1 &= \left(\frac{1}{\sin\theta}\right)^2\\ \cot^2\theta + 1 &= \csc^2\theta.\end{aligned}$$

28.

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{x}{3}\right)^2 = 1 - \frac{x^2}{9} = \frac{9-x^2}{9},$$

so

$$\cos\theta = \sqrt{\frac{9-x^2}{9}} = \frac{\sqrt{9-x^2}}{3}.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{3}\cdot\frac{3}{\sqrt{9-x^2}} = \frac{x}{\sqrt{9-x^2}}$$

29. $\sin^2\theta = 1 - \cos^2\theta = 1 - (4/x)^2 = 1 - 16/x^2 = (x^2-16)/x^2$, so $\sin\theta = \sqrt{(x^2-16)/x^2} = \sqrt{(x^2-16)}/x$. Thus $\tan\theta = \sin\theta/\cos\theta = \sqrt{x^2-16}/x \cdot x/4 = \sqrt{x^2-16}/4$.

30. First notice that $\cos\theta = \frac{x}{2}$, then $\sin^2\theta = 1-\cos^2\theta = 1-(x/2)^2 = 1-x^2/4 = (4-x^2)/4$, so $\sin\theta = \sqrt{(4-x^2)/4} = \sqrt{4-x^2}/2$. Thus $\tan\theta = \sin\theta/\cos\theta = \sqrt{4-x^2}/2 \cdot 2/x = \sqrt{4-x^2}/x$.

31. First notice that $\tan\theta = \frac{x}{9}$ so $\tan\theta = \sin\theta/\cos\theta = x/9$, so $\sin\theta = x/9\cdot\cos\theta$. Now to find $\cos\theta$ by using $1 = \sin^2\theta + \cos^2\theta = (x^2/81)\cos^2\theta + \cos^2\theta = \cos^2\theta(x^2/81+1)$, so $\cos^2\theta = 81/(x^2+81)$ and $\cos\theta = 9/\sqrt{x^2+81}$. Thus, $\sin\theta = (x/9)\cdot(9/\sqrt{x^2+81}) = x/\sqrt{x^2+81}$.

32. **(a)** The slope of the line is $m = \tan(5\pi/6) = -1/\sqrt{3}$, and the y-intercept is $b = 2$, so $y = (-1/\sqrt{3})x + 2$.
(b) Solve $0 = (-1/\sqrt{3})x + 2$ to get $x = 2\sqrt{3}$.

33. The point-slope formula for a line is $y = y_0 + m(x - x_0)$, where m is the slope and (x_0, y_0) is a point on the line. Here the slope of line l is $(\sin\theta)/(\cos\theta) = \tan\theta$. Thus, $y = y_0 + (\tan\theta)(x - x_0)$, where (x_0, y_0) is a point on the line.

34. **(a)** The graph of $y = \tan t$ has vertical asymptotes at odd multiples of $\pi/2$, that is, at $\pi/2$, $3\pi/2$, $5\pi/2$, etc., and their negatives. The graph of $y = \cos t$ has t-intercepts at the same values.
(b) The graph of $y = \tan t$ has t-intercepts at multiples of π, that is, at 0, $\pm\pi$, $\pm2\pi$, $\pm3\pi$, etc. The graph of $y = \sin t$ has t-intercepts at the same values.

35.

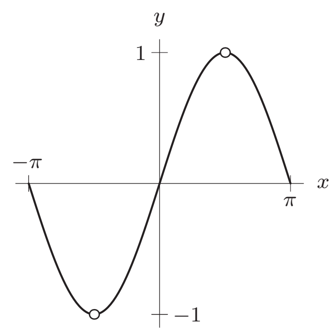


Figure 6.58

Though the function $y = f(x) = \cos x \cdot \tan x$ can be simplified by

$$\cos x \cdot \tan x = \cos x \cdot \frac{\sin x}{\cos x} = \sin x,$$

it is important to notice that $f(x)$ is not defined at the points where $\cos x = 0$. There would be division by zero at such points. Note the holes in the graph which denote undefined values of the function.

36. The angle spanned by the arc shown is $\theta = s/r = 4/5$ radians, so $m = r\cos\theta = 5\cos(4/5)$ and $n = r\sin\theta = 5\sin(4/5)$. By the Pythagorean theorem,

$$\begin{aligned} p^2 &= n^2 + (5-m)^2 \\ &= n^2 + m^2 - 10m + 25 \\ &= 25\sin^2(4/5) + 25\cos^2(4/5) - 10m + 25 \\ &= 50 - 10m \end{aligned}$$

so

$$\begin{aligned} p &= \sqrt{50 - 10m} \\ &= \sqrt{50 - 50\cos(4/5)} \\ &= 5\sqrt{2(1-\cos(4/5))}. \end{aligned}$$

37. The angle spanned by the arc shown is $\theta = s/r = 10/5 = 2$ radians, so $v = r\sin\theta = 5\sin 2$. Since u is positive as it is a length, $u = -r\cos\theta = -5\cos 2$, because $\cos 2$ is negative. By the Pythagorean theorem,

$$\begin{aligned} w^2 &= v^2 + (5+u)^2 \\ &= v^2 + u^2 + 10u + 25 \\ &= 25\sin^2 2 + 25\cos^2 2 + 10u + 25 \\ &= 50 + 10u, \end{aligned}$$

and

$$w = \sqrt{50 + 10u} = \sqrt{50 - 50\cos 2} = 5\sqrt{2(1-\cos 2)}.$$

38.

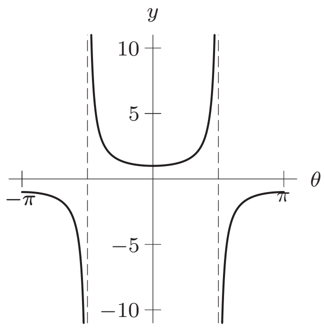


Figure 6.59: The graph of $\sec x$

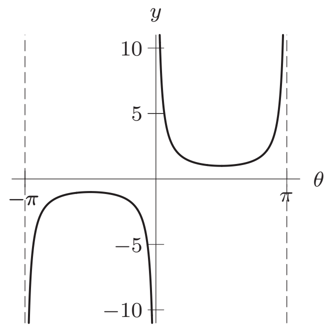


Figure 6.60: The graph of $\csc x$

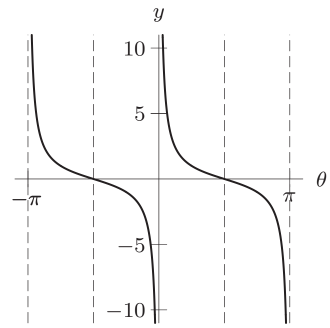


Figure 6.61: The graph of $\cot x$

The functions $\sec x$ and $\csc x$, shown in Figures 6.59 and 6.60, have period 2π, while the function $\cot x$, shown in Figure 6.61, has period π. Each of these functions tends toward infinity where the reciprocal function approaches zero. The function $y = \sec x = 1/\cos x$ is positive on the intervals where $\cos x$ is positive and negative where $\cos x$ is negative. Similarly, the function $y = \csc x = 1/\sin x$ is positive on the intervals where $\sin x$ is positive and negative where $\sin x$ is negative. The function $y = \cot x = 1/\tan x$ is positive on the intervals where $\tan x$ is positive and negative where $\tan x$ is negative. The function $\cot x$ has zeros where $\tan x$ values approach infinity.

Solutions for Section 6.7

Exercises

1. We use the inverse tangent function on a calculator to get $\theta = 1.570$.

2. We use the inverse sine function on a calculator to get $\theta = 0.608$.

3. We divide both sides by 3 to get $\cos\theta = 0.238$. We use the inverse cosine function on a calculator to get $\theta = 1.330$.

4. Since the cosine function always has a value between -1 and 1, there are no solutions.

5. We use the inverse tangent function on a calculator to get $5\theta + 7 = -0.236$. Solving for θ, we get $\theta = -1.447$.

6. We divide both sides by 2, giving us $\sin(4\theta) = 0.3335$. We then use the inverse sine function on a calculator to get $4\theta = 0.340$, so $\theta = 0.085$.

7. Since $\sin t = -1$, we have $t = 3\pi/2$.

8. Since $\sin t = 1/2$, we have $t = \pi/6$ and $t = 5\pi/6$.

9. Since $\cos t = -1$, we have $t = \pi$.

10. Since $\cos t = 1/2$, we have $t = \pi/3$ and $t = 5\pi/3$.

11. Since $\tan t = 1$, we have $t = \pi/4$ and $t = 5\pi/4$.

12. Since $\tan t = -1$, we have $t = 3\pi/4$ and $t = 7\pi/4$.

13. Since $\tan t = \sqrt{3}$, we have $t = \pi/3$ and $t = 4\pi/3$.

14. Since $\tan t = 0$, we have $t = 0$, $t = \pi$, and $t = 2\pi$.

15. **(a)** Tracing along the graph in Figure 6.62, we see that the approximations for the two solutions are

$$t_1 \approx 1.88 \qquad \text{and} \qquad t_2 \approx 4.41.$$

Note that the first solution, $t_1 \approx 1.88$, is in the second quadrant and the second solution, $t_2 \approx 4.41$, is in the third quadrant. We know that the cosine function is negative in those two quadrants. You can check the two solutions by substituting them into the equation:

$$\cos 1.88 \approx -0.304 \qquad \text{and} \qquad \cos 4.41 \approx -0.298,$$

both of which are close to -0.3.

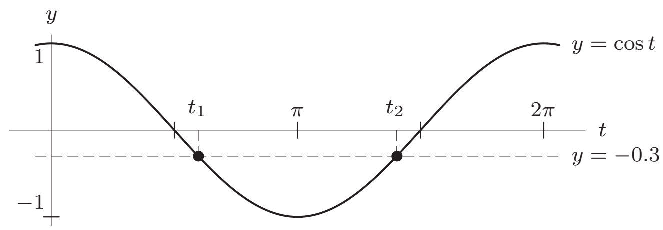


Figure 6.62: The angles t_1 and t_2 are the two solutions to $\cos t = -0.3$ for $0 \le t \le 2\pi$

(b) If your calculator is in radian mode, you should find

$$\cos^{-1}(-0.3) \approx 1.875,$$

which is one of the values we found in part (a) by using a graph. Using the $\boxed{\cos^{-1}}$ key gives only one of the solutions to a trigonometric equation. We find the other solutions by using the symmetry of the unit circle. Figure 6.63 shows that if $t_1 \approx 1.875$ is the first solution, then the second solution is

$$\begin{aligned} t_2 &= 2\pi - t_1 \\ &\approx 2\pi - 1.875 \approx 4.408. \end{aligned}$$

Thus, the two solutions are $t \approx 1.88$ and $t \approx 4.41$.

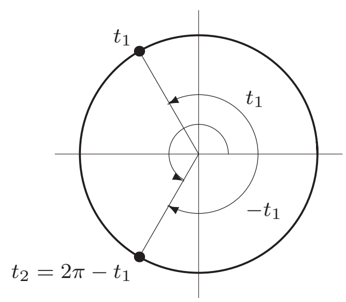


Figure 6.63: By the symmetry of the unit circle, $t_2 = 2\pi - t_1$

16. (a) We know that $\cos(\pi/3) = 1/2$. From the graph of $y = \cos t$ in Figure 6.64, we see that $t = \pi/3$, $t = 5\pi/3$, $t = -\pi/3$, and $t = -5\pi/3$ are all solutions, as are any values of t obtained by adding or subtracting multiples of 2π to these values.

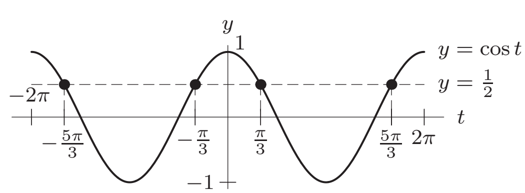


Figure 6.64: $y = \cos t$ has an infinite number of t-values for $y = 1/2$

(b) If we restrict our attention to the interval $0 \le t \le 2\pi$, we find two solutions, $t = \pi/3$ and $t = 5\pi/3$. To see why this is so, look at the unit circle in Figure 6.65. During one revolution around the circle, there are always two angles with the same cosine (or sine, or tangent), unless the cosine is 1 or -1. Therefore, we expect the equation $\cos t = 1/2$ to have two solutions in the interval $0 \le t \le 2\pi$.

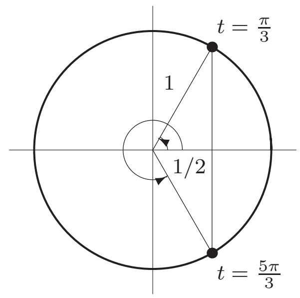


Figure 6.65: During one revolution around the unit circle, the two angles $\pi/3$ and $5\pi/3$ have the cosine value of $1/2$

17. Since every angle that is a multiple of 2π different from our original angle gives the same point on the unit circle, we first find the angle closest to zero after subtracting as many multiples of 2π as necessary. In this case, since $13\pi/6 = 2\pi + \pi/6$, we subtract 2π once, to give us $\pi/6$.

18. Since every angle that is a multiple of 2π different from our original angle gives the same point on the unit circle, we first find the angle closest to zero after adding as many multiples of 2π as necessary. In this case, since $-7\pi/3 = -2pi - \pi/3$, we add 2π once, to give us $-\pi/3$. Since this angle is in the fourth quadrant, the reference angle is $\pi/3$.

19. Since every angle that is a multiple of 2π different from our original angle gives the same point on the unit circle, we first find the angle closest to zero after subtracting as many multiples of 2π as necessary. In this case, since $10\pi/3 = 2\pi + 4\pi/3$, we subtract 2π once, to give us $4\pi/3$. Since this angle is in the third quadrant, the reference angle is $\pi/3$.

20. Since every angle that is a multiple of 2π different from our original angle gives the same point on the unit circle, we first find the angle closest to zero after subtracting as many multiples of 2π as necessary. In this case, since $11\pi/4 = 2\pi + 3\pi/4$, we subtract 2π once, to give us $3\pi/4$. Since this angle is in the second quadrant, we the reference angle is $\pi/4$.

21. Since every angle that is a multiple of 2π different from our original angle give the same point on the unit circle, we first find the angle closest to zero after subtracting as many multiples of 2π as necessary. In this case, since $73\pi/3 = 24\pi + \pi/3 = 12 \cdot 2\pi + \pi/3$, we subtract 2π twelve times, to give us $\pi/3$, which, since it's in the first quadrant, is our reference angle.

22. Since every angle that is a multiple of 2π different from our original angle gives the same point on the unit circle, we first find the angle closest to zero after adding as many multiples of 2π as necessary. In this case, since $-46\pi/7 = -6\pi - 4\pi/7 = -3 \cdot 2\pi - 4\pi/7$, we add 2π three times, to give us $-4\pi/7$. Since this angle is in the third quadrant, the reference angle is $3\pi/7$.

23. Since every angle that is a multiple of 2π different from our original angle gives the same point on the unit circle, we first find the angle closest to zero after subtracting as many multiples of 2π as necessary. In this case, we subtract 2π three times, to give us $18 - 6\pi \approx -0.850$, which is in the fourth quadrant, between 0 and $-\pi/2$. Therefore, the reference angle is 0.850.

24. Since every angle that is a multiple of 2π different from our original angle gives the same point on the unit circle, we first find the angle closest to zero after adding as many multiples of 2π as necessary. In this case, we add 2π four times, to give us $-22 + 8\pi \approx 3.133$, which is in the second quadrant, between $\pi/2$ and π. Therefore, our reference angle is 0.009.

25. **(a)** The reference angle for 120° is $180^\circ - 120^\circ = 60^\circ$, so $\cos 120^\circ = -\cos 60^\circ = -1/2$.
(b) The reference angle for 135° is $180^\circ - 135^\circ = 45^\circ$, so $\sin 135^\circ = \sin 45^\circ = \sqrt{2}/2$.
(c) The reference angle for 225° is $225^\circ - 180^\circ = 45^\circ$, so $\cos 225^\circ = -\cos 45^\circ = -\sqrt{2}/2$.
(d) The reference angle for 300° is $360^\circ - 300^\circ = 60^\circ$, so $\sin 300^\circ = -\sin 60^\circ = -\sqrt{3}/2$.

26. **(a)** The reference angle for $2\pi/3$ is $\pi - 2\pi/3 = \pi/3$, so $\sin(2\pi/3) = \sin(\pi/3) = \sqrt{3}/2$.
(b) The reference angle for $3\pi/4$ is $\pi - 3\pi/4 = \pi/4$, so $\cos(3\pi/4) = -\cos(\pi/4) = -1/\sqrt{2}$.
(c) The reference angle for $-3\pi/4$ is $\pi - 3\pi/4 = \pi/4$, so $\tan(-3\pi/4) = \tan(\pi/4) = 1$.
(d) The reference angle for $11\pi/6$ is $2\pi - 11\pi/6 = \pi/6$, so $\cos(11\pi/6) = \cos(\pi/6) = \sqrt{3}/2$.

27. Graph $y = \sin\theta$ on $0 \le \theta \le 2\pi$ and locate the two points with y-coordinate 0.65. The θ-coordinates of these points are approximately $\theta = 0.708$ and $\theta = 2.434$. See Figure 6.66.

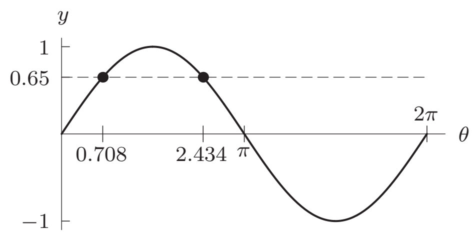


Figure 6.66

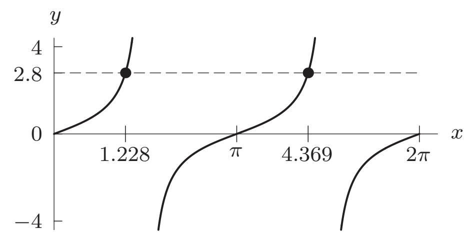


Figure 6.67

28. Graph $y = \tan x$ on $0 \le x \le 2\pi$ and locate the two points with y-coordinate 2.8. The x-coordinates of these points are approximately $x = 1.228$ and $x = 4.369$. See Figure 6.67.

29. Graph $y = \cos t$ on $0 \le t \le 2\pi$ and locate the two points with y-coordinate -0.24. The t-coordinates of these points are approximately $t = 1.813$ and $t = 4.473$. See Figure 6.68.

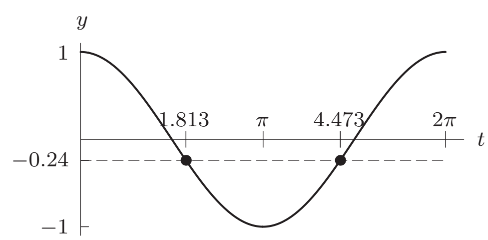


Figure 6.68

Problems

30. We have $x = \cos^{-1}(0.6) = 0.927$. A graph of $\cos x$ shows that the second solution is $x = 2\pi - 0.927 = 5.356$.

31. Collecting the $\sin x$ terms on the left gives

$$\begin{aligned}
2\sin x &= 1 - \sin x \\
3\sin x &= 1 \\
\sin x &= 1/3 \\
x &= \sin^{-1}(1/3) = 0.340.
\end{aligned}$$

A graph of $\sin x$ shows the second solutions is $x = \pi - 0.340 = 2.802$.

32. Multiplying both sides by $\cos x$ gives

$$\begin{aligned}
5\cos x &= 1/\cos x \\
5\cos^2 x &= 1 \\
\cos^2 x &= 1/5 \\
\cos x &= \pm\sqrt{\frac{1}{5}}.
\end{aligned}$$

Thus $x = \cos^{-1}\sqrt{1/5} = 1.107$ or $x = \cos^{-1}\left(-\sqrt{1/5}\right) = 2.034$. A graph of $\cos x$ shows that there are other solutions with $\cos x = \sqrt{1/5}$ given by $x = 2\pi - 1.107 = 5.176$ and with $\cos x = -\sqrt{1/5}$ given by $x = 2\pi - 2.034 = 4.249$. Thus the solutions are

$$1.107, 2.034, 4.249, 5.176.$$

33. Since $\sin 2x = 0.3$ we know

$$2x = \sin^{-1}(0.3) = 0.3047.$$

Because we must divide by 2 to find x, we start by finding all solutions to $\sin 2x = 0.3$ between $0 \le x \le 4\pi$. These are

$$\begin{aligned} 2x &= 0.3047 \\ 2x &= 2\pi - 0.3047 = 5.9785 \\ 2x &= 2\pi + 0.3047 = 6.5879 \\ 2x &= 4\pi - 0.3047 = 12.2617. \end{aligned}$$

Thus,

$$x = 0.152, 2.989, 3.294, 6.131.$$

34. Since $\sin(x-1) = 0.25$, we know

$$\begin{aligned} x - 1 &= \sin^{-1}(0.25) = 0.253. \\ x &= 1.253. \end{aligned}$$

Another solution for $x - 1$ is given by

$$\begin{aligned} x - 1 &= \pi - 0.253 = 2.889 \\ x &= 3.889. \end{aligned}$$

35. Since $5\cos(x+3) = 1$, we have

$$\begin{aligned} \cos(x+3) &= \frac{1}{5} = 0.2 \\ x + 3 &= \cos^{-1}(0.2) = 1.3694 \\ x &= 1.3694 - 3 = -1.6306. \end{aligned}$$

This value of x is not in the interval $0 \le x \le 2\pi$. To obtain values of x in this interval, we find values of $x + 3$ in the interval between 3 and $3 + 2\pi$, that is between 3 and 9.283. These values of $x + 3$ are

$$\begin{aligned} x + 3 &= 2\pi - 1.369 = 4.914 \\ x + 3 &= 2\pi + 1.369 = 7.653. \end{aligned}$$

Thus

$$x = 1.914, 4.653.$$

36. By sketching a graph, we see that there are four solutions (see Figure 6.69). The first solution is given by $x = \cos^{-1}(0.6) = 0.927$, which is equivalent to the length labeled "b" in Figure 6.69. Next, note that, by the symmetry of the graph of the cosine function, we can obtain a second solution by subtracting the length b from 2π. Therefore, a second solution to the equation is given by $x = 2\pi - 0.927 = 5.356$. Similarly, our final two solutions are given by $x = 2\pi + 0.927 = 7.210$ and $x = 4\pi - 0.927 = 11.639$.

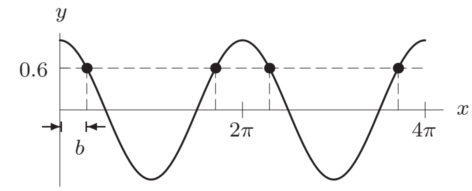


Figure 6.69

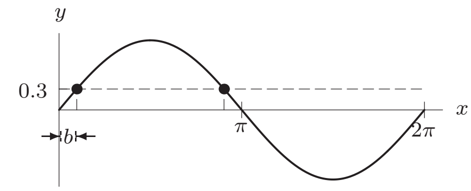


Figure 6.70

37. By sketching a graph, we see that there are two solutions (see Figure 6.70). The first solution is given by $x = \sin^{-1}(0.3) = 0.305$, which is equivalent to the length labeled "b" in Figure 6.70. Next, note that, by the symmetry of the graph of the sine function, we can obtain the second solution by subtracting the length b from π. Therefore, the other solution is given by $x = \pi - 0.305 = 2.837$.

38. By sketching a graph, we see that there are two solutions (see Figure 6.71). The first solution is given by $x = \cos^{-1}(-0.7) = 2.346$, which corresponds to the leftmost point in Figure 6.71. To find the second solution, we first calculate the distance labeled "b" in Figure 6.71 to obtain $b = \pi - \cos^{-1}(-0.7) = 0.795$. Therefore, by the symmetry of the cosine function, the second solution is given by $x = \pi + b = 3.937$.

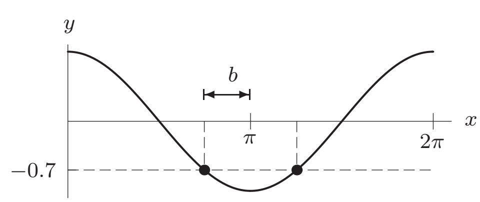


Figure 6.71

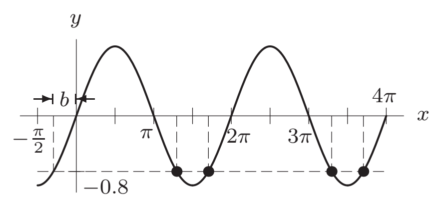


Figure 6.72

39. By sketching a graph, we see that there are four solutions (see Figure 6.72). To find the four solutions, we begin by calculating $\sin^{-1}(-0.8) = -0.927$. Therefore, the length labeled "b" in Figure 6.72 is given by $b = -\sin^{-1}(-0.8) = 0.927$. Now, using the symmetry of the graph of the sine function, we can see that the four solutions are given by $x = \pi + b = 4.069$, $x = 2\pi - b = 5.356$, $x = 3\pi + b = 10.352$, and $x = 4\pi - b = 11.639$.

40. One solution is $\theta = \sin^{-1}(-\sqrt{2}/2) = -\pi/4$, and a second solution is $5\pi/4$, since $\sin(5\pi/4) = -\sqrt{2}/2$. All other solutions are found by adding integer multiples of 2π to these two solutions. See Figure 6.73.

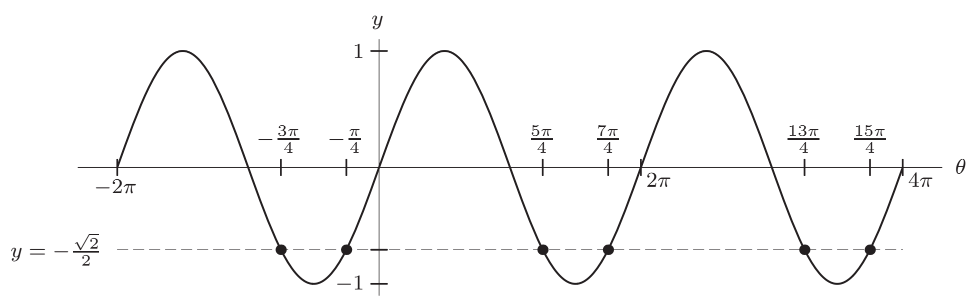


Figure 6.73

41. One solution is $\theta = \cos^{-1}(\sqrt{3}/2) = \pi/6$, and a second solution is $11\pi/6$, since $\cos(11\pi/6) = \sqrt{3}/2$. All other solutions are found by adding integer multiples of 2π to these two solutions. See Figure 6.74.

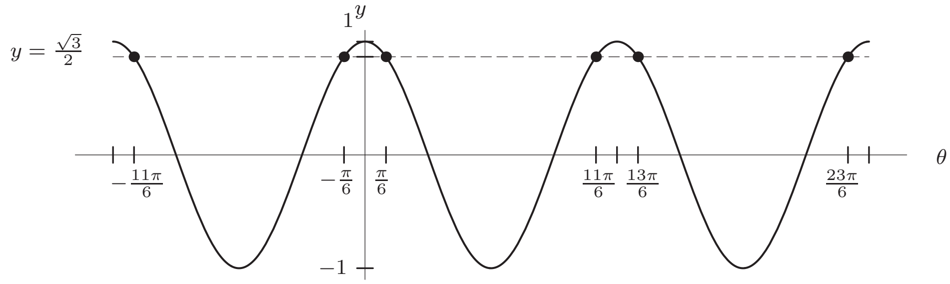


Figure 6.74

42. One solution is $\theta = \tan^{-1}(-\sqrt{3}/3) = -\pi/6$, All other solutions are found by adding integer multiples of π to $-\pi/6$. See Figure 6.75.

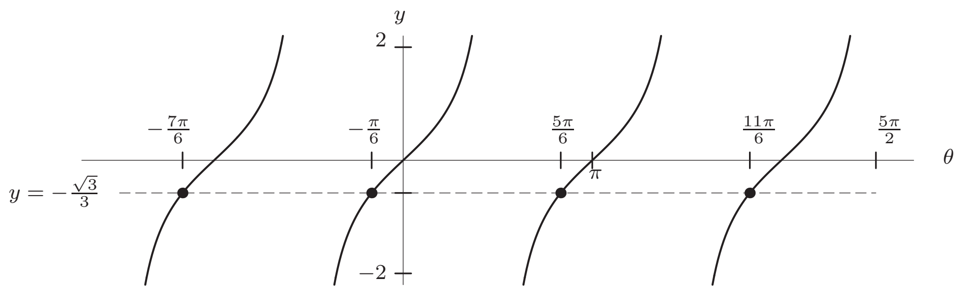


Figure 6.75

43. The angle between $-\pi/2$ and 0 whose tangent is -3 is $\tan^{-1}(-3)$. Thus, the angle in the second quadrant is

$$\theta = \pi + \tan^{-1}(-3) \approx 1.893$$

44.

$$\begin{aligned}
\sec^2\alpha + 3\tan\alpha &= \tan\alpha \\
1 + \tan^2\alpha + 3\tan\alpha &= \tan\alpha \\
\tan^2\alpha + 2\tan\alpha + 1 &= 0 \\
(\tan\alpha + 1)^2 &= 0 \\
\tan\alpha &= -1 \\
\alpha &= \frac{3\pi}{4},\ \frac{7\pi}{4}
\end{aligned}$$

45. From Figure 6.76 we can see that the solutions lie on the intervals $\frac{\pi}{8} < t < \frac{\pi}{4}$, $\frac{3\pi}{4} < t < \frac{7\pi}{8}$, $\frac{9\pi}{8} < t < \frac{5\pi}{4}$ and $\frac{7\pi}{4} < t < \frac{15\pi}{8}$. Using the trace mode on a calculator, we can find approximate solutions $t = 0.52$, $t = 2.62$, $t = 3.67$ and $t = 5.76$.

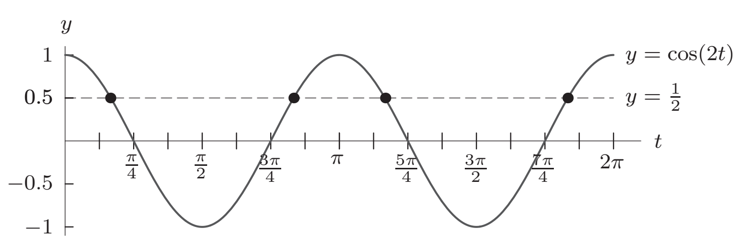


Figure 6.76

For a more precise answer we solve $\cos(2t) = \frac{1}{2}$ algebraically. To find $2t = \arccos(1/2)$. One solution is $2t = \pi/3$. But $2t = 5\pi/3$, $7\pi/3$, and $11\pi/3$ are also angles that have a cosine of $1/2$. Thus $t = \pi/6$, $5\pi/6$, $7\pi/6$, and $11\pi/6$ are the solutions between 0 and 2π.

46. To solve

$$\tan t = \frac{1}{\tan t}$$

we multiply both sides of the equation by $\tan t$. Multiplication gives us

$$\tan^2 t = 1 \qquad \text{or} \qquad \tan t = \pm 1.$$

From Figure 6.77, we see that there are two solutions for $\tan t = 1$, and two solutions for $\tan t = -1$, they are approximately $t = 0.79$, $t = 3.93$, and $t = 2.36$, $t = 5.50$.

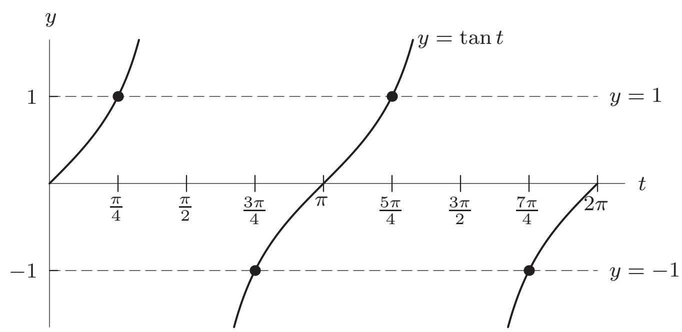


Figure 6.77

To find exact solutions, we have $t = \arctan(\pm 1) = \pm\pi/4$. There are other angles that have a $\tan$ of ± 1, namely $\pm 3\pi/4$. So $t = \pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$ are the solutions in the interval from 0 to 2π.

47. From Figure 6.78, we see that $2\sin t\cos t - \cos t = 0$ has four roots between 0 and 2π. They are approximately $t = 0.52$, $t = 1.57$, $t = 2.62$, and $t = 4.71$.

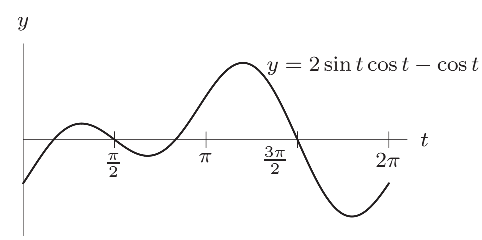


Figure 6.78

To solve the problem symbolically, we factor out $\cos t$:

$$2\sin t\cos t - \cos t = \cos t(2\sin t - 1) = 0.$$

So solutions occur either when $\cos t = 0$ or when $2\sin t - 1 = 0$. The equation $\cos t = 0$ has solutions $\pi/2$ and $3\pi/2$. The equation $2\sin t - 1 = 0$ has solution $t = \arcsin(1/2) = \pi/6$, and also $t = \pi - \pi/6 = 5\pi/6$. Thus the solutions to the original problem are

$$t = \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{\pi}{6} \text{ and } \frac{5\pi}{6}.$$

48. The solutions to the equation are at points where the graphs of $y = 3\cos^2 t$ and $y = \sin^2 t$ cross. From Figure 6.79, we see that $3\cos^2 t = \sin^2 t$ has four solutions between 0 and 2π, they are approximately $t = 1.05$, $t = 2.09$, $t = 4.19$, and $t = 5.24$.

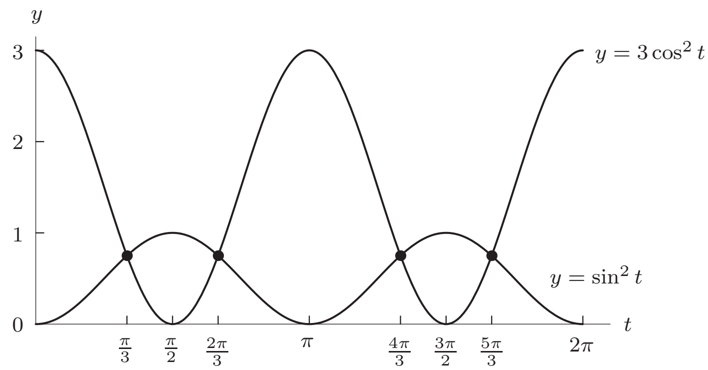


Figure 6.79

To solve $3\cos^2 t = \sin^2 t$ we divide both sides by $\cos^2 t$ and rewrite the equation as

$$3 = \tan^2 t \qquad \text{or} \qquad \tan t = \pm\sqrt{3}.$$

(Dividing by $\cos^2 t$ is valid only if $\cos t \neq 0$. Since $t = \pi/2$ and $t = 3\pi/2$ are not solutions, $\cos t \neq 0$.)

Using the inverse tangent and reference angles we find that the solutions occur at the points

$$t = \frac{\pi}{3},\ \frac{4\pi}{3},\ \frac{2\pi}{3} \text{ and } \frac{5\pi}{3}.$$

49. See Figure 6.80. The angle θ is the sun's angle of elevation. Here, $\tan\theta = \dfrac{50}{60} = \dfrac{5}{6}$. So, $\theta = \tan^{-1}\left(\dfrac{5}{6}\right) \approx 39.806°$.

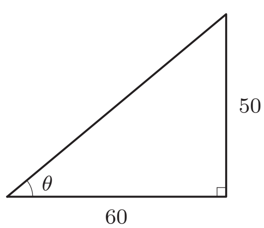


Figure 6.80

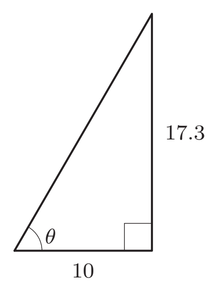


Figure 6.81

50. Draw a picture as in Figure 6.81. The angle that we want is labeled θ in this picture. We see that $\tan\theta = \dfrac{17.3}{10} = 1.73$. Evaluating $\tan^{-1}(1.73)$ on a calculator, we get $\theta \approx 59.971°$.

51. (a) The maximum is $\$100{,}000$ and the minimum is $\$20{,}000$. Thus in the function

$$f(t) = A\cos(B(t-h)) + k,$$

the midline is

$$k = \frac{100,000 + 20,000}{2} = \$60{,}000$$

and the amplitude is

$$A = \frac{100,000 - 20,000}{2} = \$40{,}000.$$

The period of this function is 12 since the sales are seasonal. Since

$$\text{period } = 12 = \frac{2\pi}{B},$$

we have

$$B = \frac{\pi}{6}.$$

The company makes its peak sales in mid-December, which is month -1 or month 11. Since the regular cosine curve hits its peak at $t = 0$ while ours does this at $t = -1$, we find that our curve is shifted horizontally 1 unit to the left. So we have

$$h = -1.$$

So the sales function is

$$f(t) = 40{,}000 \cos\left(\frac{\pi}{6}(t+1)\right) + 60{,}000 = 40{,}000 \cos\left(\frac{\pi}{6}t + \frac{\pi}{6}\right) + 60{,}000.$$

(b) Mid-April is month $t = 3$. Substituting this value into our function, we get

$$f(3) = \$40{,}000.$$

(c) To solve $f(t) = 60{,}000$ for t, we write

$$\begin{aligned} 60{,}000 &= 40{,}000 \cos\left(\frac{\pi}{6}t + \frac{\pi}{6}\right) + 60{,}000 \\ 0 &= 40{,}000 \cos\left(\frac{\pi}{6}t + \frac{\pi}{6}\right) \\ 0 &= \cos\left(\frac{\pi}{6}t + \frac{\pi}{6}\right). \end{aligned}$$

Therefore, $(\frac{\pi}{6}t + \frac{\pi}{6})$ equals $\frac{\pi}{2}$ or $\frac{3\pi}{2}$. Solving for t, we get $t = 2$ or $t = 8$. So in mid-March and mid-September the company has sales of $\$60{,}000$ (which is the average or midline sales value.)

52. (a) Let t be the time in hours since 12 noon. Let $d = f(t)$ be the depth in feet in Figure 6.82.

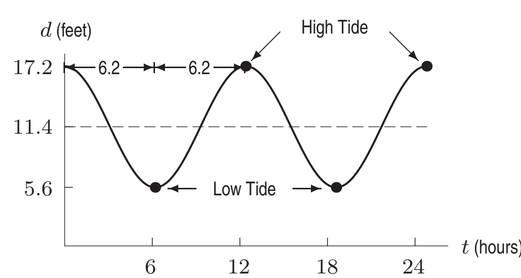


Figure 6.82

(b) The midline is $d = \dfrac{17.2 + 5.6}{2} = 11.4$ and the amplitude is $17.2 - 11.4 = 5.8$. The period is 12.4. Thus we get $d = f(t) = 11.4 + 5.8 \cos\left(\dfrac{\pi}{6.2}t\right)$.

(c) We find the first t value when $d = f(t) = 8$:

$$8 = 11.4 + 5.8 \cos\left(\frac{\pi}{6.2}t\right)$$

Using the $\cos^{-1}$ function

$$\begin{aligned}\frac{-3.4}{5.8} &= \cos\left(\frac{\pi}{6.2}t\right)\\ \cos^{-1}\left(\frac{-3.4}{5.8}\right) &= \frac{\pi}{6.2}t\\ t &= \frac{6.2}{\pi}\cos^{-1}\left(\frac{-3.4}{5.8}\right) \approx 4.336 \text{ hours.}\end{aligned}$$

Since $0.336(60) \approx 20$ minutes, the latest time the boat can set sail is 4:20 pm.

53. The curve is a sine curve with an amplitude of 5, a period of 8 and a vertical shift of -3. Thus the equation for the curve is $y = 5\sin\left(\frac{\pi}{4}x\right) - 3$. Solving for $y = 0$, we have

$$\begin{aligned}5\sin\left(\frac{\pi}{4}x\right) &= 3\\ \sin\left(\frac{\pi}{4}x\right) &= \frac{3}{5}\\ \frac{\pi}{4}x &= \sin^{-1}\left(\frac{3}{5}\right)\\ x &= \frac{4}{\pi}\sin^{-1}\left(\frac{3}{5}\right) \approx 0.819.\end{aligned}$$

This is the x-coordinate of P. The x-coordinate of Q is to the left of 4 by the same distance P is to the right of O, by the symmetry of the sine curve. Therefore,

$$x \approx 4 - 0.819 = 3.181$$

is the x-coordinate of Q.

54. **(a)** $\sin^{-1} x$ is the angle whose sine is x. When $x = 0.5$, $\sin^{-1}(0.5) = \pi/6$.
(b) $\sin(x^{-1})$ is the sine of $1/x$. When $x = 0.5$, $\sin(0.5^{-1}) = \sin(2) \approx 0.909$.
(c) $(\sin x)^{-1} = \dfrac{1}{\sin x}$. When $x = 0.5$, $(\sin 0.5)^{-1} = \dfrac{1}{\sin(0.5)} \approx 2.086$.

55. **(a)** $\arccos(0.5) = \pi/3$
(b) $\arccos(-1) = \pi$
(c) $\arcsin(0.1) \approx 0.1$

56. Graph $y = 12 - 4\cos(3t)$ on $0 \le t \le 2\pi/3$ and locate the two points with y-coordinate 14. (See Figure 6.83.) These points have t-coordinates of approximately $t = 0.698$ and $t = 1.396$. There are six solutions in three cycles of the graph between 0 and 2π.

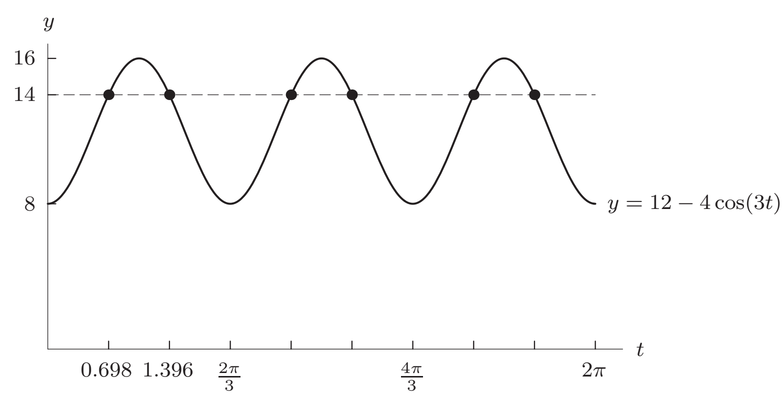


Figure 6.83

57. (a) Graph $y = 3 - 5\sin 4t$ on the interval $0 \le t \le \pi/2$, and locate values where the function crosses the t-axis. Alternatively, we can find the points where the graph $5\sin 4t$ and the line $y = 3$ intersect. By looking at the graphs of these two functions on the interval $0 \le t \le \pi/2$, we find that they intersect twice. By zooming in we can identify these points of intersection as roughly $t_1 \approx 0.16$ and $t_2 \approx 0.625$. See Figure 6.84.

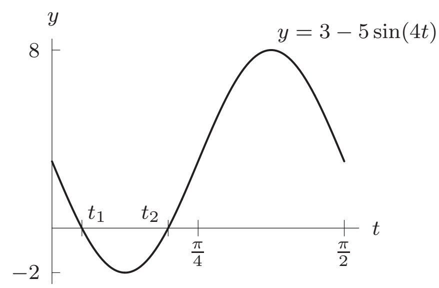


Figure 6.84

(b) Solve for $\sin(4t)$ and then use arcsine:

$$\begin{aligned} 5\sin(4t) &= 3 \\ \sin(4t) &= \frac{3}{5} \\ 4t &= \arcsin\left(\frac{3}{5}\right). \end{aligned}$$

So $t_1 = \dfrac{\arcsin(3/5)}{4} \approx 0.161$ is a solution. But the angle $\pi - \arcsin(3/5)$ has the same sine as $\arcsin(3/5)$. Solving $4t = \pi - \arcsin(3/5)$ gives $t_2 = \dfrac{\pi}{4} - \dfrac{\arcsin(3/5)}{4} \approx 0.625$ as a second solution.

58. (a) The domain of f is $-1 \le x \le 1$, so $\sin^{-1}(x)$ is defined only if x is between -1 and 1 inclusive. The range is between $-\pi/2$ and $\pi/2$ inclusive.

(b) The domain of g is $-1 \le x \le 1$, so $\cos^{-1}(x)$ is defined only if x is between -1 and 1 inclusive. The range is between 0 and π inclusive.

(c) The domain of h is all real numbers, so $\tan^{-1}(x)$ is defined for all values of x. The range is between but not including $-\pi/2$ and $\pi/2$.

59. Statement II is always true, because $\arcsin x$ is an angle whose sine is x, and thus the sine of $\arcsin x$ will necessarily equal x. Statement I could be true or false. For example,

$$\arcsin\left(\sin\frac{\pi}{4}\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$$

On the other hand,

$$\arcsin(\sin\pi) = \arcsin(0) = 0,$$

which is not equal to π.

60. (a) The value of $\sin t$ will be between -1 and 1. This means that $k\sin t$ will be between $-k$ and k. Thus, $t^2 = k\sin t$ will be between 0 and k. So

$$-\sqrt{k} \le t \le \sqrt{k}.$$

(b) Plotting $2\sin t$ and t^2 on a calculator, we see that $t^2 = 2\sin t$ for $t = 0$ and $t \approx 1.40$.

(c) Compare the graphs of $k\sin t$, a sine wave, and t^2, a parabola. As k increases, the amplitude of the sine wave increases, and so the sine wave intersects the parabola in more points.

(d) Plotting $k\sin t$ and t^2 on a calculator for different values of k, we see that if $k \approx 20$, this equation will have a negative solution at $t \approx -4.3$, but that if k is any smaller, there will be no negative solution.

61. (a) In Figure 6.85, the earth's center is labeled O and two radii are extended, one through S, your ship's position, and one through H, the point on the horizon. Your line of sight to the horizon is tangent to the surface of the earth. A line tangent to a circle at a given point is perpendicular to the circle's radius at that point. Thus, since your line of sight is tangent to the earth's surface at H, it is also perpendicular to the earth's radius at H. This means that triangle OCH is a right triangle. Its hypotenuse is $r + x$ and its legs are r and d. From the Pythagorean theorem, we have

$$\begin{aligned} r^2 + d^2 &= (r+x)^2 \\ d^2 &= (r+x)^2 - r^2 \\ &= r^2 + 2rx + x^2 - r^2 = 2rx + x^2. \end{aligned}$$

Since d is positive, we have $d = \sqrt{2rx + x^2}$.

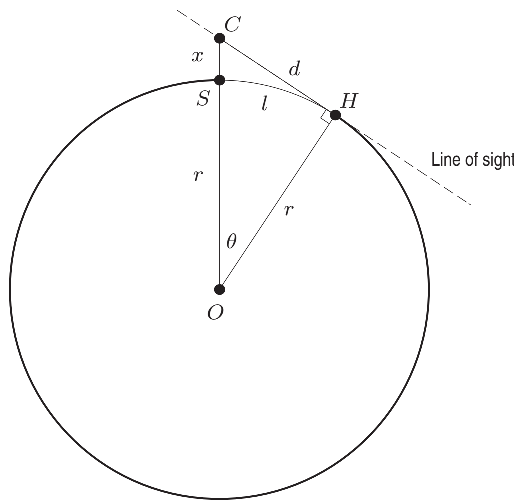


Figure 6.85

(b) We begin by using the formula obtained in part (a).

$$\begin{aligned} d &= \sqrt{2rx + x^2} \\ &= \sqrt{2(6{,}370{,}000)(50) + 50^2} \\ &\approx 25{,}238.908. \end{aligned}$$

Thus, you would be able to see a little over 25 kilometers from the crow's nest C.

Having found a formula for d, we will now try to find a formula for l, the distance along the earth's surface from the ship to the horizon H. In Figure 6.85, l is the arc length specified by the angle θ (in radians). The formula for arc length is

$$l = r\theta.$$

In this case, we must determine θ. From Figure 6.85 we see that

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{r}{r+x}.$$

Thus,

$$\theta = \cos^{-1}\left(\frac{r}{r+x}\right)$$

since $0 \le \theta \le \pi/2$. This means that

$$l = r\theta = r\cos^{-1}\left(\frac{r}{r+x}\right)$$
$$= 6{,}370{,}000\cos^{-1}\left(\frac{6{,}370{,}000}{6{,}370{,}050}\right) \approx 25{,}238.776 \text{ meters.}$$

There is very little difference—about 0.13 m or 13 cm—between the distance d that you can see and the distance l that the ship must travel to reach the horizon. If this is surprising, keep in mind that Figure 6.85 has not been drawn to scale. In reality, the mast height x is significantly smaller than the earth's radius r so that the point C in the crow's nest is very close to the ship's position at point S. Thus, the line segment d and the arc l are almost indistinguishable.

62. The angle between a line $y = b + mx$ and the x-axis is $\arctan m$. Thus, ℓ_1 makes an angle of $\arctan m_1$ with the x-axis, and ℓ_2 makes an angle of $\arctan m_2$ with the x-axis. The angle between the two lines is thus

$$\theta = \arctan(m_1) - \arctan(m_2).$$

Solutions for Chapter 6 Review

Exercises

1. In graph A, the average speed is relatively high with little variation, which corresponds to (iii). In graph B, the average speed is lower and there are significant speed-ups and slow-downs, which corresponds to (i). In graph C, the average speed is low and frequently drops to 0, which this corresponds to (ii).

2.

$$x = r\cos\theta = 16\cos(-72^\circ) \approx 4.944$$

and

$$y = r\sin\theta = 16\sin(-72^\circ) \approx -15.217,$$

so the approximate coordinates of Z are $(4.944, -15.217)$.

3. **(a)** $\sin\theta$ and $\cos\theta$ are both positive in quadrant I only.
(b) $\tan\theta > 0$ in quadrants I and III.
(c) $\tan\theta < 0$ in quadrants II and IV.
(d) $\sin\theta < 0$ in quadrants III and IV. $\cos\theta > 0$ in quadrants I and IV. Thus both are true only in quadrant IV.
(e) $\cos\theta < 0$ in quadrants II and III, and $\tan\theta > 0$ in quadrants I and III. Both are true only in quadrant III.

4. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $330 \cdot \pi/180$, giving $\frac{11}{6}\pi$ radians.

5. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $315 \cdot \pi/180$, giving $\frac{7}{4}\pi$ radians.

6. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $-225 \cdot \pi/180$, giving $-\frac{5}{4}\pi$ radians.

7. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $6\pi \cdot \pi/180$, giving $\pi^2/30 \approx 0.329$ radians.

8. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $\frac{3}{2}\pi \cdot 180/\pi$, giving 270 degrees.

9. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $180 \cdot 180/\pi$, giving $32{,}400/\pi \approx 10{,}313.240$ degrees.

10. In order to change from radians to degrees, we multiply the number of radians by $180/\pi$, so we have $(5\pi/\pi)(180/\pi)$, giving $900/\pi = 286.479$ degrees.

11. If we go around four times, we make four full circles, which is $2\pi \cdot 4 = 8\pi$ radians.

12. If we go around six times, we make six full circles, which is $2\pi \cdot 6 = 12\pi$ radians. Since we're going in the negative direction, we have -12π radians.

13. If we go around 16.4 times, we make 16.4 full circles, which is $2\pi \cdot 16.4 = 32.8\pi$ radians.

14. **(a)** Since $1.57 < 2 < 3.14$, you will be in the quadrant II.
(b) Since $3.14 < 4 < 4.71$, you will be in the quadrant III.
(c) Since $4.71 < 6 < 6.28$, you will be in the quadrant IV.
(d) Since $0 < 1.5 < 1.57$, you will be in the quadrant I.
(e) Since $3.14 < 3.2 < 4.71$, you will be in the quadrant III.

15. The arc length, s, corresponding to an angle of θ radians in a circle of radius r is $s = r\theta$. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $17 \cdot \pi/180$, giving $\frac{17}{180}\pi$ radians. Thus, our arc length is $6.2 \cdot 17\pi/180 \approx 1.840$.

16. The arc length, s, corresponding to an angle of θ radians in a circle of radius r is $s = r\theta$. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $-585 \cdot \pi/180$, giving $-\frac{13}{4}\pi$ radians. Since length cannot be negative, we take the absolute value, giving us $13\pi/4$ radians. Thus, our arc length is $6.2 \cdot 13\pi/4 \approx 63.303$.

17. The arc length, s, corresponding to an angle of θ radians in a circle of radius r is $s = r\theta$. In order to change from degrees to radians, we multiply the number of degrees by $\pi/180$, so we have $-360/\pi \cdot \pi/180$, giving -2 radians. The negative sign indicates rotation in a clockwise, rather than counterclockwise, direction. Since length cannot be negative, we find the arc length corresponding to 2 radians. Thus, our arc length is $6.2 \cdot 2 = 12.4$.

18. **(a)** We are looking for the graph of a function with amplitude one but a period of π; only $C(t)$ qualifies.
(b) We are looking for the graph of a function with amplitude one and period 2π but which is shifted up by two units; only $D(t)$ qualifies.
(c) We are looking for the graph of a function with amplitude 2 and period 2π; only $A(t)$ qualifies.
(d) Only $B(t)$ is left and we are looking for the graph of a function with amplitude one and period 2π but which has been shifted to the left by two units. This checks with $B(t)$.

19. The midline is 3. The amplitude is 1. The period is 2π.

20. The midline is 7. The amplitude is 1. The period is 2π.

21. We first divide both sides of the equation by 6, giving

$$y = 2\sin(\pi t - 7) + 7.$$

The midline is 7. The amplitude is 2. The period is $2\pi/\pi = 2$.

22. The amplitude is 4, the period is $\frac{2\pi}{1} = 2\pi$, the phase shift and horizontal shift are 0. See Figure 6.86.

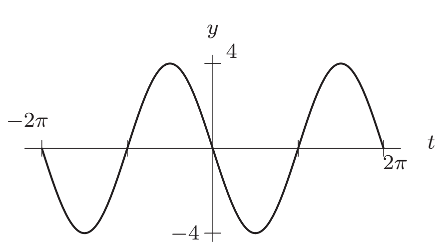


Figure 6.86: $y = -4\sin t$

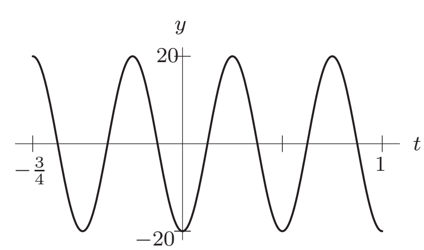


Figure 6.87: $y = -20\cos(4\pi t)$

23. The amplitude is 20, the period is $\frac{2\pi}{4\pi} = \frac{1}{2}$, the phase shift and horizontal shift are 0. See Figure 6.87.

24. The amplitude is 1, the period is $2\pi/2 = \pi$, the phase shift is $-\pi/2$, and

$$\text{Horizontal shift } = -\frac{\pi/2}{2} = -\frac{\pi}{4}.$$

Since the horizontal shift is negative, the graph of $y = \cos(2t)$ is shifted $\pi/4$ units to the left to give the graph in Figure 6.88.

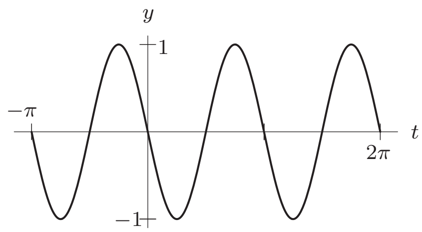


Figure 6.88: $y = \cos(2t + \pi/2)$

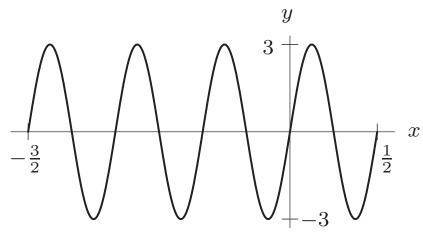


Figure 6.89: $y = 3\sin(4\pi t + 6\pi)$

25. The amplitude is 3, the period is $\frac{2\pi}{4\pi} = \frac{1}{2}$, the phase shift is -6π, and

$$\text{Horizontal shift } = -\frac{6\pi}{4\pi} = -\frac{3}{2}.$$

Since the horizontal shift is negative, the graph of $y = 3\sin(4\pi t)$ is shifted $\frac{3}{2}$ units to the left to give the graph in Figure 6.89. Note that a shift of $\frac{3}{2}$ units produces the same graph as the unshifted graph. We expect this since $y = 3\sin(4\pi t + 6\pi) = 3\sin(4\pi t)$. See Figure 6.89.

26. The amplitude is 30, the midline is $y = 60$, and the period is 4.

27. The amplitude is 30; the midline is $y = 60$, and the period is 20.

28. Amplitude is 40; midline is $y = 50$; period is 16.

29. Amplitude is 50; midline is $y = 50$; period is 64.

30. We would use $y = \sin x$ for $f(x)$ and $k(x)$, as both these graphs cross the midline at $x = 0$. We would use $y = \cos x$ for $g(x)$ and $h(x)$, as both these graphs are as far as possible from the midline at $x = 0$.

31. See Figure 6.90.

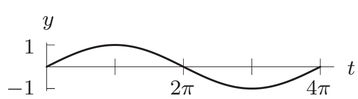


Figure 6.90: $y = \sin(t/2)$

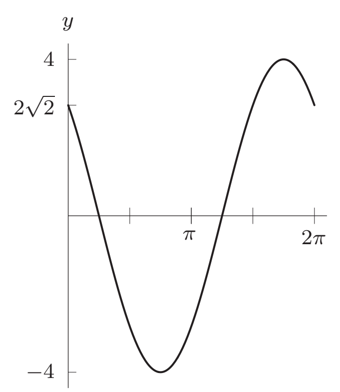


Figure 6.91: $y = 4\cos(t + \pi/4)$

32. See Figure 6.91.

33. See Figure 6.92.

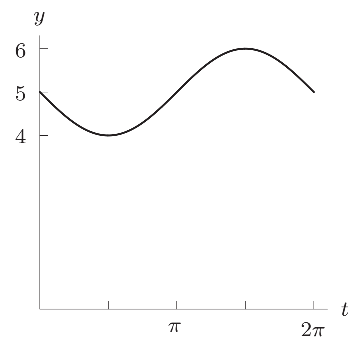


Figure 6.92: $y = 5 - \sin t$

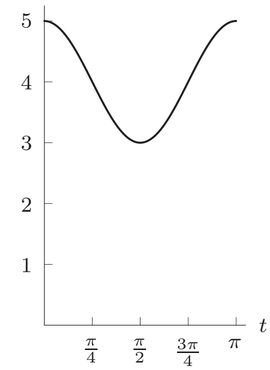


Figure 6.93: $y = \cos(2t) + 4$

34. See Figure 6.93.

Problems

35. $\cos 540° = \cos(360° + 180°) = \cos 180° = -1$.

36. Since we know that the y-coordinate on the unit circle at $7\pi/6$ is the y-coordinate at $\pi/6$ multiplied by -1, and since $\pi/6$ radians is the same as $30°$, we know that $\sin(7\pi/6) = -\sin(\pi/6) = -\sin 30° = -1/2$.

37. Since $\tan(-2\pi/3) = \sin(-2\pi/3)/\cos(-2\pi/3)$, we know that $\tan(-2\pi/3) = (-\sqrt{3}/2)/(-1/2) = \sqrt{3}$.

38. If we consider a triangle with opposite side of length 3 and hypotenuse 5, we can use the Pythagorean theorem to find the length of the adjacent side as

$$\sqrt{5^2 - 3^2} = 4.$$

This gives a triangle with sides 3 and 4, and hypotenuse 5. Since tangent is negative in the fourth quadrant, $\tan\theta = -\frac{3}{4}$. See Figure 6.94.

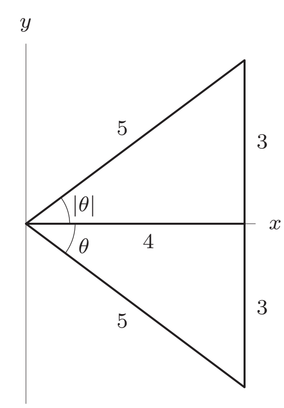


Figure 6.94

39. We use the inverse sine function on a calculator to get $\theta = 0.412$.

40. We use the inverse tangent function on a calculator to get $\theta - 1 = 0.168$, which gives us $\theta = 1.168$.

41. From the figure, we see that

$$\sin x = \frac{0.83}{1},$$

so

$$x = \sin^{-1}(0.83).$$

Using a calculator, we find that $x = \sin^{-1}(0.83) \approx 0.979$.

42. **(a)** The graph resembles a cosine function with midline $k = 8$, amplitude $A = 10$, and period $p = 60$, so

$$y = 10\cos\left(\frac{2\pi}{60}x\right) + 8.$$

(b) We find the zeros at x_1 and x_2 by setting $y = 0$. Solving gives

$$\begin{aligned} 10\cos\left(\frac{2\pi}{60}x\right) + 8 &= 0 \\ \cos\left(\frac{2\pi}{60}x\right) &= -\frac{8}{10} \\ \frac{2\pi}{60}x &= \cos^{-1}(-0.8) \\ x &= \frac{60}{2\pi}\cos^{-1}(-0.8) \\ &= 23.8550. \end{aligned}$$

Judging from the graph, this is the value of x_1. By symmetry, $x_2 = 60 - x_1 = 36.1445$.

43. **(a)** The period is $p = 80$, so $b = 2\pi/80$. The amplitude is $|a| = 300$, and the curve resembles an upside-down cosine, so $a = -300$. The midline is $k = 600$.

Putting this information together gives $y = 600 - 300\cos(2\pi x/80)$.

(b) These points fall on the line $y = 475$.

The period is $p = 80$, so $b = 2\pi/80$. The amplitude is $|a| = 300$, and the curve resembles an upside-down cosine, so $a = -300$. The midline is $k = 600$. Thus, a formula for the sinusoidal function is $y = 600 - 300\cos(2\pi x/80)$, so we obtain one possible solution as follows:

$$\begin{aligned} 600 - 300\cos\left(\frac{2\pi}{80}x\right) &= 475 \\ -300\cos\left(\frac{2\pi}{80}x\right) &= 475 - 600 = -125 \\ \cos\left(\frac{2\pi}{80}x\right) &= \frac{-125}{-300} = \frac{5}{12} \\ \frac{2\pi}{80}x &= \cos^{-1}\left(\frac{5}{12}\right) \\ x &= \frac{80}{2\pi}\cos^{-1}\left(\frac{5}{12}\right) \\ &= 14.5279. \end{aligned}$$

We denote this solution by x_0, and it gives the x-coordinate of the left-most point. By symmetry, we see that

$$\begin{aligned} x_1 &= 80 - x_0 = 65.4721 \\ x_2 &= 80 + x_0 = 94.5279. \end{aligned}$$

44. First, by looking at the graph of f, we note that its amplitude is 5, and its midline is given by $y = 5$. Therefore, we have $A = \pm 5$ and $k = 5$. Also, since the graph of f completes one full cycle in 8 units, we see that the period of f is 8, so we have $B = 2\pi/8 = \pi/4$. Combining these observations, we see that we can take the four formulas to be horizontal translations of the functions $y_1 = 5\cos((\pi/4)x) + 5$, $y_2 = -5\cos((\pi/4)x) + 5$, $y_3 = 5\sin((\pi/4)x) + 5$, and $y_4 = -5\sin((\pi/4)x) + 5$, all of which have the same amplitude, period, and midline as f. After sketching y_1, we see that we can obtain the graph of f by shifting the graph of y_1 4 units to the right; therefore, $f_1(x) = 5\cos((\pi/4)(x-4))+5$. A sketch of the graph of y_2 reveals that the graphs of f and y_2 are identical, so $f_2(x) = -5\cos((\pi/4)x)+5$. A sketch of the graph of y_3 reveals that we can obtain the graph of f by shifting y_3 to the right 2 units, so $f_3(x) = 5\sin((\pi/4)(x-2))+5$. Similarly, a sketch of y_4 allows us to conclude that $f_4(x) = -5\sin((\pi/4)(x-6))+5$. Answers may vary.

45. First, by looking at the graph of f, we note that its amplitude is 6, and its midline is given by $y = 2$. Therefore, we have $A = \pm 6$ and $k = 2$. Also, since the graph of f completes one full cycle in 4π units, we see that the period of f is 4π, so we have $B = (2\pi)/(4\pi) = 1/2$. Combining these observations, we see that we can take the four formulas to be horizontal translations of the functions $y_1 = 6\cos((1/2)x)+2, y_2 = -6\cos((1/2)x)+2, y_3 = 6\sin((1/2)x)+2$, and $y_4 = -6\sin((1/2)x)+2$, all of which have the same amplitude, period, and midline as f. After sketching y_1, we see that we can obtain the graph of f by shifting the graph of y_1 to the right 3π units; therefore, $f_1(x) = 6\cos((1/2)(x-3\pi))+2$. Similarly, we can obtain the graph of f by shifting the graph of y_2 to the right π units, so $f_2(x) = -6\cos((1/2)(x-\pi))+2$, or by shifting the graph of y_3 to the right 2π units, so $f_3(x) = 6\sin((1/2)(x-2\pi))+2$. Finally, a sketch of y_4 reveals that y_4 and f describe identical functions, so $f_4(x) = -6\sin((1/2)x)+2$. Answers may vary.

46. First, by looking at the graph of f, we note that its amplitude is 3, and its midline is given by $y = -3$. Therefore, we have $A = \pm 3$ and $k = -3$. Also, since the graph of f completes one full cycle between $x = -3$ and $x = 5$, we see that the period of f is 8, so we have $B = 2\pi/8 = \pi/4$. Combining these observations, we see that we can take the four formulas to be horizontal translations of the functions $y_1 = 3\cos((\pi/4)x)-3, y_2 = -3\cos((\pi/4)x)-3, y_3 = 3\sin((\pi/4)x)-3$, and $y_4 = -3\sin((\pi/4)x)-3$, all of which have the same amplitude, period, and midline as f. After sketching y_1, we see that we can obtain the graph of f by shifting the graph of y_1 3 units to the right; therefore, $f_1(x) = 3\cos((\pi/4)(x-3))-3$. Similarly, we can obtain the graph of f by shifting the graph of y_2 to the left 1 unit, so $f_2(x) = -3\cos((\pi/4)(x+1))-3$, or by shifting the graph of y_3 to the right 1 unit, so $f_3(x) = 3\sin((\pi/4)(x-1))-3$. Finally, a sketch of y_4 reveals that we can obtain the graph of f by shifting the graph of y_4 to the right 5 units, so $f_4(x) = -3\sin((\pi/4)(x-5))-3$. Answers may vary.

47. First, by looking at the graph of f, we see that the difference between the maximum and minimum output values is $8-(-2) = 10$, so the amplitude is 5. Therefore, the midline of the graph is given by the line $y = -2+5 = 3$, so we have $A = \pm 5$ and $k = 3$. We also note that the graph of f completes 1.5 cycles between $x = -2$ and $x = 16$, so the period of the graph is 12, from which it follows that $B = 2\pi/12 = \pi/6$.

Combining these observations, we see that we can take the four formulas to be horizontal translations of the functions $y_1 = 5\cos((\pi/6)x)+3, y_2 = -5\cos((\pi/6)x)+3, y_3 = 5\sin((\pi/6)x)+3$, and $y_4 = -5\sin((\pi/6)x)+3$, all of which have the same amplitude, period, and midline as f. After sketching y_1, we see that we can obtain the graph of f by shifting the graph of y_1 2 units to the left; therefore, $f_1(x) = 5\cos((\pi/6)(x+2))+3$. Similarly, we can obtain the graph of f by shifting the graph of y_2 to the right 4 units, so $f_2(x) = -5\cos((\pi/6)(x-4))+3$, or by shifting the graph of y_3 to the right 7 units, so $f_3(x) = 5\sin((\pi/6)(x-7))+3$. Finally, a sketch of y_4 reveals that we can obtain the graph of f by shifting the graph of y_4 to the right 1 unit, so $f_4(x) = -5\sin((\pi/6)(x-1))+3$. Answers may vary.

48. We first solve for $\cos\alpha$,

$$
\begin{aligned}
2\cos\alpha &= 1 \\
\cos\alpha &= \frac{1}{2} \\
\alpha &= \frac{\pi}{3},\ \frac{5\pi}{3}.
\end{aligned}
$$

49. We first solve for $\tan\alpha$,

$$
\begin{aligned}
\tan\alpha &= \sqrt{3} - 2\tan\alpha \\
3\tan\alpha &= \sqrt{3} \\
\tan\alpha &= \frac{\sqrt{3}}{3} \\
\alpha &= \frac{\pi}{6},\ \frac{7\pi}{6}
\end{aligned}
$$

50. We first solve for $\sin(2\alpha)$,

$$\begin{aligned}\sin(2\alpha)+3&=4\\ \sin(2\alpha)&=1\\ 2\alpha&=\frac{\pi}{2},\ \frac{5\pi}{2}\\ \alpha&=\frac{\pi}{4},\ \frac{5\pi}{4}.\end{aligned}$$

51. We first solve for $\tan\alpha$,

$$\begin{aligned}4\tan\alpha+3&=2\\ 4\tan\alpha&=-1\\ \tan\alpha&=-\frac{1}{4}\\ \alpha&=2.897,\ 6.038.\end{aligned}$$

52. We first solve for $\sin\alpha$,

$$\begin{aligned}3\sin^2\alpha+4&=5\\ 3\sin^2\alpha&=1\\ \sin^2\alpha&=\frac{1}{3}\\ \sin\alpha&=\pm\sqrt{\frac{1}{3}}\end{aligned}$$

$$\begin{aligned}\sin\alpha&=\sqrt{\tfrac{1}{3}} & \sin\alpha&=-\sqrt{\tfrac{1}{3}}\\ \alpha&=0.616,\ 2.526 & \alpha&=3.757,\ 5.668\end{aligned}$$

53. We first solve for $\tan\alpha$,

$$\begin{aligned}\tan^2\alpha&=2\tan\alpha\\ \tan^2\alpha-2\tan\alpha&=0\\ \tan\alpha(\tan\alpha-2)&=0\end{aligned}$$

$$\begin{aligned}\tan\alpha&=0 & \tan\alpha-2&=0\\ \alpha&=0,\ \pi & \tan\alpha&=2\\ & & \alpha&=1.107,\ 4.249\end{aligned}$$

54. The arc length is equal to the radius times the radian measure, so

$$d=(2)\left(\frac{87}{60}\right)(2\pi)=5.8\pi\approx 18.221 \text{ inches.}$$

55. We know $r=3960$ and $\theta=1^\circ$. Change θ to radian measure and use $s=r\theta$.

$$s=3960(1)\left(\frac{\pi}{180}\right)\approx 69.115 \text{ miles.}$$

56. We can approximate this angle by using $s = r\theta$. The arc length is approximated by the moon diameter; and the radius is the distance to the moon. Therefore $\theta = s/r = 2160/238,860 \approx 0.009$ radians. Change this to degrees to get $\theta = 0.009(180/\pi) \approx 0.516°$. Note that we could also consider the radius to cut across the moon's center, in which case the radius would be $r = 238,860 + 2160/2 = 239,940$. The difference in the two answers is negligible.

57. The circumference of the outer edge is

$$6(2\pi) = 12\pi \text{ cm.}$$

A point on the outer edge travels 100 times this distance in one minute. Thus, a point on the outer edge must travel at the speed of 1200π cm/minute or roughly 3770 cm/min.

The circumference of the inner edge is

$$0.75(2\pi) = 1.5\pi \text{ cm.}$$

A point on the inner edge travels 100 times this distance in one minute. Thus, a point on the inner edge must travel at the speed of 150π cm/minute or roughly 471 cm/min.

58. The function $\sin^2\theta$ means $(\sin\theta)^2$. Thus, you first evaluate the sine of θ and then square the result.
The function $\sin\theta^2$ means $\sin(\theta^2)$. Thus, you first square θ and then evaluate the sine of this result.
For example:

$$\sin^2\frac{\pi}{4} = \left(\sin\frac{\pi}{4}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2},$$

while

$$\sin\left(\left(\frac{\pi}{4}\right)^2\right) = \sin\left(\frac{\pi^2}{16}\right) \approx 0.578.$$

59. Using $s = r\theta$, we know the arc length $s = 600$ and $r = 3960 + 500$. Therefore $\theta = 600/4460 \approx 0.1345$ radians.

60. **(a)** The midline, $D = 10$, the amplitude $A = 4$, and the period 1, so

$$1 = \frac{2\pi}{B} \qquad \text{and} \qquad B = 2\pi.$$

Therefore the formula is

$$f(t) = 4\sin(2\pi t) + 10.$$

(b) Solving $f(t) = 12$, we have

$$\begin{aligned} 12 &= 4\sin(2\pi t) + 10 \\ 2 &= 4\sin(2\pi t) \\ \sin 2\pi t &= \frac{1}{2} \\ 2\pi t &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \\ t &= \frac{1}{12}, \frac{5}{12}, \frac{13}{12}, \frac{17}{12} \end{aligned}$$

The spring is 12 centimeters from the ceiling at $1/12$ sec, $5/12$ sec, $13/12$ sec, $17/12$ sec.

61. The function has a maximum of 3000, a minimum of 1200 which means the upward shift is $\frac{3000+1200}{2} = 2100$. A period of eight years means the angular frequency is $\frac{\pi}{4}$. The amplitude is $|A| = 3000 - 2100 = 900$. Thus a function for the population would be an inverted cosine and $f(t) = -900\cos((\pi/4)t) + 2100$.

62. **(a)** The average monthly temperature in Fairbanks is shown in Figure 6.95.
(b) These data are best modeled by a vertically reflected cosine curve, which is reasonable for something like temperature that oscillates with a 12 month period.
(c) The midline temperature is $(61.3 + (-11.5))/2 = 24.9$. The amplitude is $61.3 - 24.9 = 36.4$ Since the period is 12, we have $B = 2\pi/12 = \pi/6$. There is no horizontal shift, so $C = 0$. Hence our function is

$$f(t) = 24.9 - 36.4\cos\left(\frac{\pi}{6}t\right).$$

(d) To solve $f(t) = 32$, we must solve the equation

$$\begin{aligned} 32 &= 24.9 - 36.4\cos\left(\frac{\pi}{6}t\right) \\ 7.1 &= -36.4\cos\left(\frac{\pi}{6}t\right) \\ \cos\left(\frac{\pi}{6}t\right) &= -0.195 \end{aligned}$$

$$\frac{\pi}{6}t = \cos^{-1}(-0.195) = 1.767 \qquad \frac{\pi}{6}t = 2\pi - 1.767 = 4.516$$

$$t = 3.375 \qquad\qquad t = 8.625$$

The temperature in Fairbanks reaches the freezing point in the middle of April and the middle of September.

(e) See Figure 6.96.

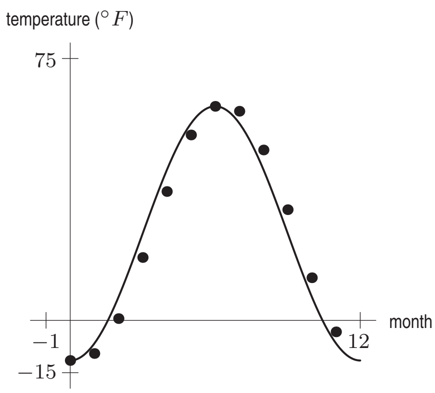


Figure 6.95

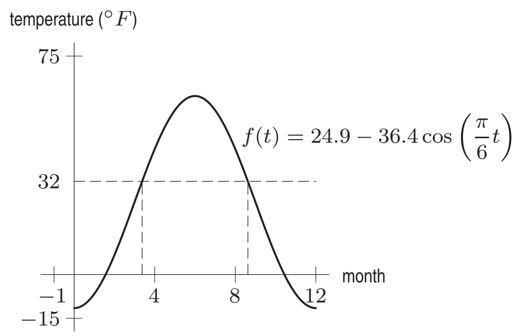


Figure 6.96

(f) The amplitude, period, and midline would be the same, but the function would be vertically reflected:

$$y = 24.9 + 36.4\cos\left(\frac{\pi}{6}t\right).$$

63. The data shows the period is about 0.6 sec. The angular frequency is $B = 2\pi/0.6 \approx 10.472$. The amplitude is $(\text{high} - \text{low})/2 = (180 - 120)/2 = 60/2 = 30$ cm. The midline is $(\text{Minimum}) + (\text{Amplitude}) = 120 + 30 = 150$ cm. The weight starts in a low position so it is a quarter cycle behind the sine; that is, the phase shift is $\pi/2$:

$$y = 30\sin\left(105t - \frac{\pi}{2}\right) + 150.$$

64. **(a)** The setting must be roughly the average temperature or 70°.

(b) At $t = 0$, the furnace has been running for a while and the house has begun to warm. At $t = 0.25$, the house is at 70° and the furnace turns off. At $t = 0.5$, the house stops getting warmer and begins to cool. At $t = 0.75$, the furnace turns back on, but the house continues to cool. At $t = 1.0$, the house stops cooling and begins to warm.

(c) The graph resembles an inverted cosine curve with period 1, amplitude 2 and midline $T = 70$ so a function would be $T = f(t) = 2\cos(2\pi t) + 70$

(d) The period is the length of one cooling/heating cycle. The amplitude is the temperature variation from the average. The midline value is the thermostat setting.

(e) Figure 6.97 shows one possible graph. This is a graph of a piecewise defined trigonometric function with one formula for $0 \le t \le 0.25$, and another for $0.25 < t \le 1$. At time $t = 0.125$ the furnace turns off, and at $t = 0.625$ the furnace turns back on. At time $t = 0.25$ the room reaches its maximum temperature.

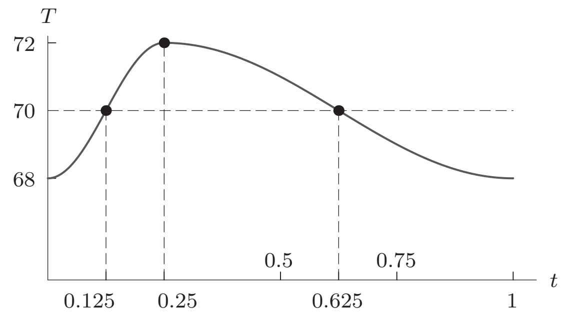


Figure 6.97

CHECK YOUR UNDERSTANDING

1. True, because $\sin x$ is an odd function.

2. False, because $\cos x$ is not an odd function.

3. False, because $\sin x$ is not an even function.

4. True, because $\cos x$ is an even function.

5. True, by the sum of angles identity.

6. False. Not true for $x = 0$. The graphs of $\cos(x + \pi)$ and $\sin x$ do not look identical.

7. True, since the period of $\cos x$ is 2π and 4π is two periods.

8. False, since $2\cos(x)$ has amplitude 2 and $\cos(2x)$ has amplitude 1.

9. False, since $\cos(1/x)$ is undefined at $x = 0$, whereas $(\cos 1)/(\cos x)$ is 1 at $x = 0$.

10. False. $\sec^2 x - 1 = \tan^2 x$.

11. False. Not true for $x = 0$. The two sides have different midlines.

12. True. The graph of a periodic function is obtained by starting with the graph on one period and horizontally shifting it by all multiples of the period. The entire graph is determined by the graph of a single period. Since the graphs of the periodic functions $f(t)$ and $g(t)$ are the same on the period $0 \le t < A$, their graphs are the same everywhere.

13. True, since the values of $\sin x$ start to repeat after one rotation through 2π.

14. False, since $\sin(\pi x)$ has period $2\pi/\pi = 2$.

15. False. A parabola is a quadratic function and can never repeat its values more than twice.

16. True. This is the definition of a periodic function.

17. True. This is the definition of the period of f.

18. False. The amplitude is half the difference between its maximum and minimum values.

19. True.

20. False. A unit circle must have a radius of 1.

21. False. The point is $(-1, 0)$ on the unit circle.

22. True. The point $(1, 0)$ on the unit circle is the starting point to measure angles so $\theta = 0^\circ$.

23. True. This is the definition of $\sin\theta$.

24. False. The point is $(2\cos 240^\circ, 2\sin 240^\circ) = (-1, -\sqrt{3})$.

25. True. Since $\cos\theta = x/r$ we have $x = r\cos\theta$.

26. True. Because P and Q have the same x-coordinates.

27. True. Because P and R have the same y-coordinates.

28. True. Since the angle is greater than 180° but less than 270°, the point is in the third quadrant.

29. False. Since both angles are negative they are measured in the clockwise direction. Going clockwise beyond 180° takes the point into the second quadrant.

30. False. One radian is about 57 degrees.

31. True. This is the definition of radian measure.

32. False. One radian is about 57 degrees, so three radians is more than 90° but less than 180°. Thus the point is in the second quadrant.

33. False. Multiply the angle by $\pi/180^\circ$.

34. False. Use $s = r\theta$ to get, $s = 3 \cdot (\pi/3) = \pi$.

35. True. In radians, points in the second quadrant correspond to angles from $\pi/2$ to π.

36. True. Since $s = r\theta$, one complete revolution is 2π and $r = 1$, we have $s = 1 \cdot 2\pi \approx 6.28$.

37. True. We know $\cos 30^\circ = \sqrt{3}/2 = \sin \frac{\pi}{3}$.

38. False. $\sin \frac{\pi}{6} = \frac{1}{2}$.

39. True. Since $\sin \frac{\pi}{4} = \sqrt{2}/2 = \cos \frac{\pi}{4}$.

40. True. Since sine is an odd function.

41. False. The angle 315° is in the fourth quadrant, so the cosine value is positive. The correct value is $\sqrt{2}/2$.

42. True. The period appears to be about 4.

43. False. The amplitude is (maximum $-$ minimum)$/2 = (6-2)/2 = 2$.

44. True. Since $f(x+4) = f(x)$ and since $f(x+c) = f(c)$ is true for no smaller positive value of c, the period is 4.

45. True. The midline is $y =$ (maximum $+$ minimum)$/2 = (6+2)/2 = 4$.

46. False. The period of g is only half as long as the period of f.

47. False. Amplitude is always positive. In this case it is equal to three.

48. False. The amplitude is 10.

49. True. The function is just a vertical stretch and an upward shift of the cosine function, so the period remains unchanged and is equal to 2π.

50. False. The maximum y-value is when $\cos x = 1$. It is 35.

51. True. The minimum y-value is when $\cos x = -1$. It is 15.

52. False. The midline equation is $y = 25$

53. True, since $\cos x = A \cos B(x-h) + k$ with $A = 1, B = 1, h = 0$, and $k = 0$.

54. False. The amplitude is positive 2.

55. False. The cosine function has been vertically stretched and then shifted downward by 4. It has not been reflected.

56. True. The function $\sin(2t)$ is a sine function horizontally compressed by $\frac{1}{2}$ so it has half the period of $\sin t$.

57. False. The period is $\frac{1}{3}$ that of $y = \cos x$.

58. True. The period is $2\pi/B$.

59. False. A shift to the left by h units is given by $y = A\sin(2(x+h)) + k$.

60. True. The conditions for A, B and C in $y = A\cos(Bx) + C$ are perfectly met.

61. True. The amplitude is $\frac{1}{2}$, the period is π and the midline is $y = 1$ and its horizontal shift is correct.

62. False. Numerically, we could check the equation at $x = 0$ to find $y = -0.5\cos(2 \cdot 0 + \frac{\pi}{3}) + 1 = 0.75$, but the graph at $x = 0$ shows $y = 1$.

63. True. On the unit circle we define $x = \cos\theta$ and $y = \sin\theta$, so we have $\tan\theta = y/x = \sin\theta/\cos\theta$.

64. False. It is undefined at $\pi/2$ and $3\pi/2$.

65. True. Because the angles θ and $\theta + \pi$ determine the same line through the origin and hence have the same slope, which is the tangent.

66. False. The tangent is undefined at $\frac{\pi}{2}$. Note that $\tan x$ approaches $+\infty$ as x approaches $\pi/2$ from the left and approaches $-\infty$ as x approaches $\pi/2$ from the right.

67. True. Let $\theta = 5x$ in the identity $\sin^2\theta + \cos^2\theta = 1$.

68. False. In the second quadrant the tangent is negative.

69. True. We have $\sec\pi = 1/\cos\pi = 1/(-1) = -1$.

70. True. The value of $\csc\pi = 1/\sin\pi = 1/0$ is undefined.

71. False. The reciprocal of the sine function is the cosecant function.

72. True. This is true since $\cos\frac{\pi}{3} = 0.5$ and $0 < \pi/3 < \pi$.

73. True. Since $y = \arctan(-1) = -\pi/4$, we have $\sin(-\pi/4) = -\sqrt{2}/2$.

74. True. $\sin(\pi/3) = \sqrt{3}/2$.

75. True. When defined, $\cos(\cos^{-1}(x)) = x$.

76. False. $y = \sin^{-1}x = \arcsin x \neq 1/\sin x$.

77. False. The domain is the range of the cosine function, $-1 \leq x \leq 1$.

78. True. If $\cos t = 1$, then $\sin t = 0$ so $\tan t = 0/1 = 0$.

79. False. The reference angle for 120° is 60°, since it takes 60° to reach the x-axis from 120°.

80. True. It takes 60° to reach the x-axis from 300°.

81. False. Find $x = \sin(\arcsin x) = \sin(0.5) \neq \pi/6$. The following statement is true. If $\arcsin 0.5 = x$ then $x = \pi/6$.

82. False. Any angle $\theta = \frac{\pi}{4} + n\pi$, or $\theta = -\frac{\pi}{4} + n\pi$ with n an integer, will have the same cosine value.

83. False; for instance, $\cos^{-1}(\cos(2\pi)) = 0$.

84. True

85. True; because $\tan A = \tan B$ means $A = B + k\pi$, for some integer k. Thus $\frac{A-B}{\pi} = k$ is an integer.

86. False; for example, $\cos\left(\dfrac{\pi}{4}\right) = \cos\left(\frac{7\pi}{4}\right)$ but $\sin\left(\dfrac{\pi}{4}\right) = -\sin\left(\dfrac{7\pi}{4}\right)$.

87. False. For example, $\cos(\pi/4) = \cos(-\pi/4)$.

Solutions to Tools for Chapter 6

1. By the Pythagorean theorem, the hypotenuse has length $\sqrt{1^2+2^2}=\sqrt{5}$.

(a) $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{2}{1} = 2.$

(b) $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{2}{\sqrt{5}}.$

(c) $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{1}{\sqrt{5}}.$

2. Since we know all the sides of this right triangle, we have

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}, \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}, \quad \text{and} \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}.$$

Similarly,

$$\sin\phi = \frac{4}{5}, \quad \cos\phi = \frac{3}{5}, \quad \text{and} \quad \tan\phi = \frac{4}{3}.$$

3. **(a)** $\dfrac{5}{\sqrt{125}}$

(b) $\dfrac{10}{\sqrt{125}}$
(c) $\dfrac{10}{\sqrt{125}}$
(d) $\dfrac{5}{\sqrt{125}}$
(e) $1/2$
(f) 2

4. We know one of the legs and all three angles of this triangle. We need to find the other leg and the hypotenuse. Since x is opposite and 4 is adjacent to the 28° angle, we have

$$\tan 28^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{4}.$$

This means

$$x = 4\tan 28^\circ \approx 4(0.5317) = 2.1268.$$

Since h is the length of the hypotenuse, we have

$$\cos 28^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{h},$$

which gives

$$h = \frac{4}{\cos 28^\circ} \approx 4.5303.$$

5. By the Pythagorean Theorem, we know that the third side must be $\sqrt{7^2 - 2^2} = \sqrt{45}$.

(a) Since $\sin\theta$ is opposite side over hypotenuse, we have $\sin\theta = \sqrt{45}/7$.
(b) Since $\cos\theta$ is adjacent side over hypotenuse, we have $\cos\theta = 2/7$.
(c) Since $\tan\theta$ is opposite side over adjacent side, we have $\tan\theta = \sqrt{45}/2$.

6. By the Pythagorean Theorem, we know that the third side must be $\sqrt{9^2 + 5^2} = \sqrt{106}$.

(a) Since $\sin\theta$ is opposite side over hypotenuse, we have $\sin\theta = 5/\sqrt{106}$.
(b) Since $\cos\theta$ is adjacent side over hypotenuse, we have $\cos\theta = 9/\sqrt{106}$.
(c) Since $\tan\theta$ is opposite side over adjacent side, we have $\tan\theta = 5/9$.

7. By the Pythagorean Theorem, we know that the third side must be $\sqrt{12^2 - 8^2} = \sqrt{80}$.

(a) Since $\sin\theta$ is opposite side over hypotenuse, we have $\sin\theta = 8/12$.
(b) Since $\cos\theta$ is adjacent side over hypotenuse, we have $\cos\theta = \sqrt{80}/12$.
(c) Since $\tan\theta$ is opposite side over adjacent side, we have $\tan\theta = 8/\sqrt{80}$.

8. Because the two angles are the same, we know that the two missing sides must be equal. Therefore, by the Pythagorean Theorem, we know that the missing sides are (which we call s) are given by:

$$\begin{aligned} 17^2 &= s^2 + s^2 \\ 289 &= 2s^2 \\ \sqrt{\frac{289}{2}} &= s. \end{aligned}$$

(a) Since $\sin\theta$ is opposite side over hypotenuse, we have $\sin\theta = \sqrt{289/2}/17 = 1/\sqrt{2}$.
(b) Since $\cos\theta$ is adjacent side over hypotenuse, we have $\cos\theta = \sqrt{289/2}/17 = 1/\sqrt{2}$.
(c) Since $\tan\theta$ is opposite side over adjacent side, we have $\tan\theta = 1$.

9. By the Pythagorean Theorem, we know that the third side must be $\sqrt{11^2 - 2^2} = \sqrt{117}$.

(a) Since $\sin\theta$ is opposite side over hypotenuse, we have $\sin\theta = \sqrt{117}/11$.
(b) Since $\cos\theta$ is adjacent side over hypotenuse, we have $\cos\theta = 2/11$.
(c) Since $\tan\theta$ is opposite side over adjacent side, we have $\tan\theta = \sqrt{117}/2$.

10. By the Pythagorean Theorem, we know that the third side must be $\sqrt{a^2+b^2}$.

(a) Since $\sin\theta$ is opposite side over hypotenuse, we have $\sin\theta = a/\sqrt{a^2+b^2}$.
(b) Since $\cos\theta$ is adjacent side over hypotenuse, we have $\cos\theta = b/\sqrt{a^2+b^2}$.
(c) Since $\tan\theta$ is opposite side over adjacent side, we have $\tan\theta = a/b$.

11. Since $\sin 17^\circ = r/7$, we have $r = 7\sin 17^\circ$. Similarly, since $\cos 17^\circ = q/7$, we have $q = 7\cos 17^\circ$.

12. Since $\sin 12^\circ = 4/r$, we have $r = 4/\sin 12^\circ$. Similarly, since $\tan 12^\circ = 4/q$, we have $q = 4/\tan 12^\circ$.

13. Since $\cos 37^\circ = 6/r$, we have $r = 6/\cos 37^\circ$. Similarly, since $\tan 37^\circ = q/6$, we have $q = 6\tan 37^\circ$.

14. Since $\sin 40^\circ = r/15$, we have $r = 15\sin 40^\circ$. Similarly, since $\cos 40^\circ = q/15$, we have $q = 15\cos 40^\circ$.

15. Since $\tan 77^\circ = 9/r$, we have $r = 9/\tan 77^\circ$. Similarly, since $\sin 77^\circ = 9/q$, we have $q = 9/\sin 77^\circ$.

16. Since $\sin 22^\circ = \lambda/r$, we have $r = \lambda/\sin 22^\circ$. Similarly, since $\tan 22^\circ = \lambda/q$, we have $q = \lambda/\tan 22^\circ$.

17. We have

$$\begin{aligned} c &= \sqrt{a^2+b^2} = \sqrt{1184} \approx 34.409 \\ \sin A &= \frac{a}{c} \\ A &= \sin^{-1}\frac{a}{c} = 35.538^\circ \\ B &= 90^\circ - A = 54.462^\circ. \end{aligned}$$

18. We have

$$\begin{aligned} b &= \sqrt{c^2-a^2} = \sqrt{384} \approx 19.596 \\ \sin A &= \frac{a}{c} \\ A &= \sin^{-1}\frac{a}{c} = 45.585^\circ \\ B &= 90^\circ - A = 44.415^\circ. \end{aligned}$$

19. We have

$$\begin{aligned} B &= 90^\circ - A = 62^\circ \\ a &= c\cdot\sin A = 20\sin 28^\circ = 9.389 \\ b &= c\cdot\sin B = 20\sin 62^\circ = 17.659. \end{aligned}$$

20. We have

$$\begin{aligned} A &= 90^\circ - B = 62^\circ \\ a &= c\cdot\sin A \\ c &= \frac{20}{\sin 62^\circ} = 22.651 \\ \tan B &= \frac{b}{20} \\ b &= 20\tan 62^\circ = 10.634. \end{aligned}$$

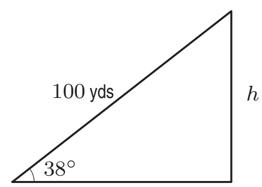
21. Using $\tan 13^\circ = \dfrac{\text{height}}{200}$ to find the height we get

$$\text{height} = 200\tan 13^\circ \approx 46.174\text{feet}.$$

Using $\cos 13^\circ = \dfrac{200}{\text{incline}}$ to find the incline we get

$$\text{incline} = 200/\cos 13^\circ \approx 205.261\text{feet}.$$

22.



$$\sin(38^\circ) = \frac{h}{100},$$

which implies that

$$h = 100\sin(38^\circ) \approx 61.566 \text{ yards} \quad \text{or} \quad 184.698 \text{ feet.}$$

23. Figure 6.98 illustrates this situation.

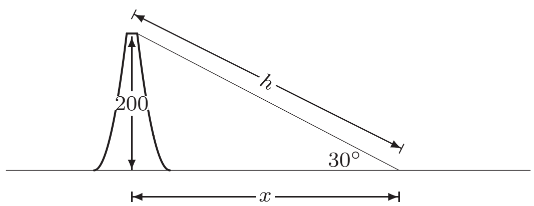


Figure 6.98

We have a right triangle with legs x and 200 and hypotenuse h. Thus,

$$\sin 30^\circ = \frac{200}{h}$$
$$h = \frac{200}{\sin 30^\circ} = \frac{200}{0.5} = 400 \text{ feet.}$$

To find the distance x, we can relate the angle and its opposite and adjacent legs by writing

$$\tan 30^\circ = \frac{200}{x}$$
$$x = \frac{200}{\tan 30^\circ} \approx 346.410 \text{ feet.}$$

We could also write the equation $x^2 + 200^2 = h^2$ and substitute $h = 400$ ft to solve for x.

24.

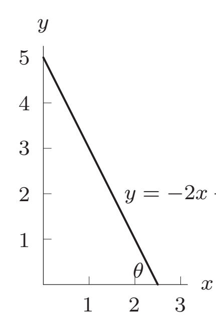


Figure 6.99

The y-intercept of this line is 5; the x-intercept is 2.5. These are the lengths of the legs of the right triangle in Figure 6.99 so $\tan\theta = \dfrac{5}{2.5} = 2$, or $\theta = \tan^{-1}(2) \approx 63.435^\circ$.

25. If the horizontal distance is d, then

$$\frac{20}{d} = \tan 15^\circ,$$

so

$$d = \frac{20}{\tan 15^\circ} \approx 74.641 \text{ feet.}$$

26. Let d be the distance from the base of the ladder to the wall; see Figure 6.100. Then, $d/3 = \cos\alpha$, so $d = 3\cos\alpha$ meters.

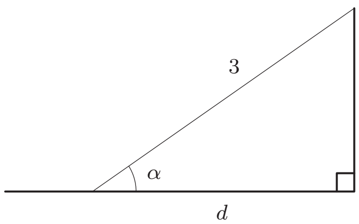


Figure 6.100

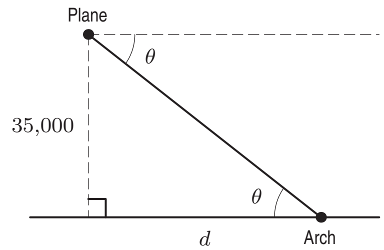


Figure 6.101

27. Let d be the horizontal distance from the airplane to the arch. See Figure 6.101. Then, $\tan\theta = 35000/d$, or $d = 35000/\tan\theta$ feet.

28. **(a)** The relationship between the radius d, the arc length, $s = 70$, and the angle $\theta = 8^\circ = 8\pi/180$ radians is $s = d\theta$, so

$$70 = d\frac{8\pi}{180},$$

$$d = \frac{70 \cdot 180}{8\pi} \approx 501 \text{ inches.}$$

Your friend is roughly 41.8 feet away.

(b) Using $\tan 8^\circ = 70/d$, we find that $d = 70/\tan 8^\circ \approx 498.076$ inches $= 41.506$ ft.

(c) The difference would decrease as the angle decreases.

29. Let d be the distance from Hampton to the point where the beam strikes the shore. Then, $\tan\phi = d/3$, so $d = 3\tan\phi$ miles.

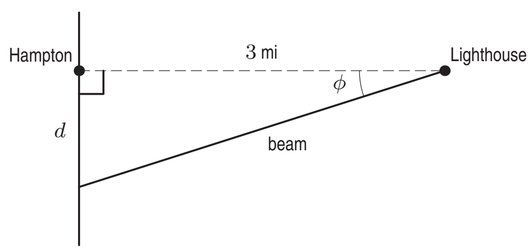


Figure 6.102

30. **(a)** Since $\sin 45^\circ = h/125$, we have $h = 125\sin 45^\circ \approx 88.388$ feet.

(b) Since $\sin 30^\circ = h/125$, we have $h = 125\sin 30^\circ = 62.5$ feet.

(c) Since $\cos 45^\circ = c/125$, we have $c = 125\cos(45^\circ) \approx 88.388$ feet.
Since $\cos 30^\circ = d/125$, we have $d \approx 108.253$ feet.

31. Since the distance from P to A is $\dfrac{50}{\tan 42^\circ}$ and the distance from P to B is $\dfrac{50}{\tan 35^\circ}$,

$$d = \frac{50}{\tan 35^\circ} - \frac{50}{\tan 42^\circ} \approx 15.877 \text{ feet.}$$