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FIND SOLUTIONS ON NEXT PAGE

CHAPTER TWO

Solutions for Section 2.1

Exercises

1. To evaluate when $x = -7$, we substitute -7 for x in the function, giving $f(-7) = -\frac{7}{2} - 1 = -\frac{9}{2}$.

2. To evaluate when $x = -7$, we substitute -7 for x in the function, giving $f(-7) = (-7)^2 - 3 = 49 - 3 = 46$.

3. We have

$$y = f(4) = \frac{6}{2 - 4^3} = \frac{6}{2 - 64} = \frac{6}{-62} = -\frac{3}{31}.$$

Solve for x:

$$\begin{aligned}
\frac{6}{2 - x^3} &= 6 \\
6 &= 6(2 - x^3) \\
1 &= 2 - x^3 \\
x^3 &= 1 \\
x &= 1.
\end{aligned}$$

4. We have

$$y = f(4) = \sqrt{20 + 2 \cdot 4^2} = \sqrt{52}.$$

Solve for x:

$$\begin{aligned}
\sqrt{20 + 2x^2} &= 6 \\
20 + 2x^2 &= 36 \\
2x^2 &= 16 \\
x^2 &= 8 \\
x &= \pm\sqrt{8}.
\end{aligned}$$

5. We have

$$y = f(4) = 4 \cdot 4^{3/2} = 4 \cdot 2^3 = 4 \cdot 8 = 32.$$

Solve for x:

$$\begin{aligned}
4x^{3/2} &= 6 \\
x^{3/2} &= 6/4 \\
x^3 &= 36/16 = 9/4 \\
x &= \sqrt[3]{9/4}.
\end{aligned}$$

6. We find:

$$y = f(4) = (4)^{-3/4} - 2 = \frac{1}{(\sqrt[4]{4})^3} - 2 = \frac{1}{(\sqrt{2})^3} - 2 = \frac{1}{\sqrt{2}\sqrt{2}\sqrt{2}} - 2 = \frac{1}{2\sqrt{2}} - 2.$$

Solve for x:

$$\begin{aligned}
x^{-3/4} - 2 &= 6 \\
x^{-3/4} &= 8 \\
x &= 8^{-4/3} = \frac{1}{(\sqrt[3]{8})^4} = \frac{1}{2^4} \\
&= \frac{1}{16}.
\end{aligned}$$

7. **(a)** Substituting $x = 0$ gives $f(0) = 2(0) + 1 = 1$.
(b) Setting $f(x) = 0$ and solving gives $2x + 1 = 0$, so $x = -1/2$.

8. **(a)** Substituting $t = 0$ gives $f(0) = 0^2 - 4 = -4$.
(b) Setting $f(t) = 0$ and solving gives $t^2 - 4 = 0$, so $t^2 = 4$, so $t = \pm 2$.

9. **(a)** Substituting $x = 0$ gives $g(0) = 0^2 - 5(0) + 6 = 6$.
(b) Setting $g(x) = 0$ and solving gives $x^2 - 5x + 6 = 0$.
Factoring gives $(x-2)(x-3) = 0$, so $x = 2, 3$.

10. **(a)** Substituting $t = 0$ gives

$$g(0) = \frac{1}{0+2} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}.$$

(b) Setting $g(t) = 0$ and solving gives

$$\begin{aligned} \frac{1}{t+2} - 1 &= 0 \\ \frac{1}{t+2} &= 1 \\ 1 &= t + 2 \\ t &= -1. \end{aligned}$$

11. Substituting -2 for x gives

$$f(-2) = \frac{-2}{1-(-2)^2} = \frac{-2}{1-4} = \frac{2}{3}.$$

12. Substituting zero for x gives

$$h(0) = a \cdot 0^2 + b \cdot 0 + c = c.$$

13. Substituting 4 for t gives

$$P(4) = 170 - 4 \cdot 4 = 154.$$

Similarly, with $t = 2$,

$$P(2) = 170 - 4 \cdot 2 = 162,$$

so

$$P(4) - P(2) = 154 - 162 = -8.$$

14. Substituting -27 for x gives

$$g(-27) = -\frac{1}{2}(-27)^{1/3} = -\frac{1}{2}(-3) = \frac{3}{2}.$$

15. **(a)** Substituting, $h(x+3) = \dfrac{1}{x+3}$.
(b) Substituting and adding, $h(x) + h(3) = \dfrac{1}{x} + \dfrac{1}{3}$.

16. We need to solve for x in the equation:

$$0.3 = \frac{2x+1}{x+1}.$$

Multiplying both sides by $x + 1$ gives:

$$\begin{aligned} 0.3(x+1) &= 2x + 1 \\ 0.3x + 0.3 &= 2x + 1 \\ -1.7x &= 0.7 \\ x &= -0.412. \end{aligned}$$

17. **(a)** Reading from the table, we have $f(1) = 2$, $f(-1) = 0$, and $-f(1) = -2$.
(b) When $x = -1$, $f(x) = 0$.

18. **(a)** $f(0)$ is the value of the function when $x = 0$, $f(0) = 3$.
(b) $f(x) = 0$ for $x = -1$ and $x = 3$.
(c) $f(x)$ is positive for $-1 < x < 3$.

19. The input, t, is the number of months since January 1, and the output, F, is the number of foxes. The expression $g(9)$ represents the number of foxes in the park on October 1. Table 1.3 on page 5 of the text gives $F = 100$ when $t = 9$. Thus, $g(9) = 100$. On October 1, there were 100 foxes in the park.

20. The output $g(t)$ stands for a number of foxes. We want to know in what month there are 75 foxes. Table 1.3 on page 5 of the text tells us that this occurs when $t = 4$ and $t = 8$; that is, in May and in September.

Problems

21. Substituting $r = 3$ and $h = 2$ gives

$$V = \frac{1}{3}\pi 3^2 \cdot 2 = 6\pi \text{ cubic inches.}$$

22. **(a)** Substituting, $q(5) = 3 - (5)^2 = -22$.
(b) Substituting, $q(a) = 3 - a^2$.
(c) Substituting, $q(a - 5) = 3 - (a - 5)^2 = 3 - (a^2 - 10a + 25) = -a^2 + 10a - 22$.
(d) Using the answer to part (b), $q(a) - 5 = 3 - a^2 - 5 = -a^2 - 2$.
(e) Using the answer to part (b) and (a), $q(a) - q(5) = (3 - a^2) - (-22) = -a^2 + 25$.

23. Substituting -1 gives $p(-1) = (-1)^2 + (-1) + 1 = 1$. Substituting 1 and taking the negative gives, $-p(1) = -((1)^2 + (1) + 1) = -3$. Thus, $p(-1) \neq -p(1)$. They are not equal.

24. Substituting $\frac{1}{3}$ for x gives

$$f\left(\frac{1}{3}\right) = 3 + 2\left(\frac{1}{3}\right)^2 = 3 + \frac{2}{9} = 3.222.$$

On the other hand

$$f(1) = 3 + 2(1)^2 = 5$$
$$f(3) = 3 + 2(3)^2 = 21.$$

So $\dfrac{f(1)}{f(3)} = \dfrac{5}{21} = 0.238$, and we see that

$$f\left(\frac{1}{3}\right) \neq \frac{f(1)}{f(3)}.$$

They are not equal.

25. **(a)** The table shows $f(6) = 3.7$, so $t = 6$. In a typical June, Chicago has 3.7 inches of rain.
(b) First evaluate $f(2) = 1.8$. Solving $f(t) = 1.8$ gives $t = 1$ or $t = 2$. Chicago has 1.8 inches of rain in January and in February.

26. **(a)** $g(x) = x^2 + x$
$g(-3x) = (-3x)^2 + (-3x)$
$g(-3x) = 9x^2 - 3x$
(b) $g(1 - x) = (1 - x)^2 + (1 - x) = (1 - 2x + x^2) + (1 - x) = x^2 - 3x + 2$
(c) $g(x + \pi) = (x + \pi)^2 + (x + \pi) = (x^2 + 2\pi x + \pi^2) + (x + \pi) = x^2 + (2\pi + 1)x + \pi^2 + \pi$
(d) $g(\sqrt{x}) = (\sqrt{x})^2 + \sqrt{x} = x + \sqrt{x}$
(e) $g\left(\dfrac{1}{x+1}\right) = \left(\dfrac{1}{(x+1)^2}\right) + \dfrac{1}{x+1} = \dfrac{1}{(x+1)^2} + \dfrac{x+1}{(x+1)^2} = \dfrac{x+2}{(x+1)^2}$
(f) $g(x^2) = (x^2)^2 + x^2 = x^4 + x^2$

27. **(a)** (i) $\dfrac{\frac{1}{t}}{\frac{1}{t} - 1} = \dfrac{\frac{1}{t}}{\frac{1-t}{t}} = \dfrac{1}{t} \cdot \dfrac{t}{1-t} = \dfrac{1}{1-t}.$

(ii) $\dfrac{\frac{1}{t+1}}{\frac{1}{t+1} - 1} = \dfrac{1}{t+1} \cdot \dfrac{t+1}{1-t-1} = -\dfrac{1}{t}.$

(b) Solve $f(x) = \dfrac{x}{(x-1)} = 3$, so

$$\begin{aligned} x &= 3x - 3 \\ 3 &= 2x \\ x &= \frac{3}{2}. \end{aligned}$$

28. **(a)** To find a point on the graph of $h(x)$ whose x-coordinate is 5, we substitute 5 for x in the formula for $h(x)$. $h(5) = \sqrt{5+4} = \sqrt{9} = 3$. Thus, the point $(5, 3)$ is on the graph of $h(x)$.

(b) Here we want to find a value of x such that $h(x) = 5$. We set $h(x) = 5$ to obtain

$$\begin{aligned} \sqrt{x+4} &= 5 \\ x + 4 &= 25 \\ x &= 21. \end{aligned}$$

Thus, $h(21) = 5$, and the point $(21, 5)$ is on the graph of $h(x)$.

(c) Figure 2.1 shows the desired graph. The point in part (a) is $(5, h(5))$, or $(5, 3)$. This point is labeled A in Figure 2.1. The point in part (b) is $(21, 5)$. This point is labeled B in Figure 2.1.

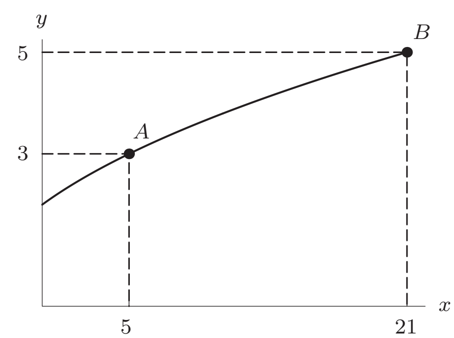


Figure 2.1

(d) If $p = 2$, then $h(p+1) - h(p) = h(2+1) - h(2) = h(3) - h(2)$. But $h(3) = \sqrt{3+4}$, while $h(2) = \sqrt{2+4}$, thus, $h(p+1) - h(p)$ for $p = 2$ equals $h(3) - h(2) = \sqrt{7} - \sqrt{6} \approx 0.1963$.

29. **(a)** In order to find $f(0)$, we need to find the value which corresponds to $x = 0$. The point $(0, 24)$ seems to lie on the graph, so $f(0) = 24$.

(b) Since $(1, 10)$ seems to lie on this graph, we can say that $f(1) = 10$.

(c) The point that corresponds to $x = b$ seems to be about $(b, -7)$, so $f(b) = -7$.

(d) When $x = c$, we see that $y = 0$, so $f(c) = 0$.

(e) When your input is d, the output is about 20, so $f(d) = 20$.

30. **(a)**

x	-2	-1	0	1	2	3
$h(x)$	0	9	8	3	0	6

(b) From the table, we see that $h(3) = 6$, while $h(1) = 3$. Thus, $h(3) - h(1) = 6 - 3 = 3$.

(c) From the table, we see that $h(2) = 0$, and $h(0) = 8$. Thus, $h(2) - h(0) = 0 - 8 = -8$.

(d) From the table, we see that $h(0) = 8$. Thus, $2h(0) = 2(8) = 16$.

(e) From the table, we see that $h(1) = 3$. Thus, $h(1) + 3 = 3 + 3 = 6$.

31. **(a)** Substituting $x = 0$ gives $f(0) = \sqrt{0^2 + 16} - 5 = \sqrt{16} - 5 = 4 - 5 = -1$.

(b) We want to find x such that $f(x) = \sqrt{x^2+16} - 5 = 0$. Thus, we have

$$\begin{aligned} \sqrt{x^2+16} - 5 &= 0 \\ \sqrt{x^2+16} &= 5 \\ x^2 + 16 &= 25 \\ x^2 &= 9 \\ x &= \pm 3. \end{aligned}$$

Thus, $f(x) = 0$ for $x = 3$ or $x = -3$.

(c) In part (b), we saw that $f(3) = 0$. You can verify this by substituting $x = 3$ into the formula for $f(x)$:

$$f(3) = \sqrt{3^2+16} - 5 = \sqrt{25} - 5 = 5 - 5 = 0.$$

(d) The vertical intercept is the value of the function when $x = 0$. We found this to be -1 in part (a). Thus the vertical intercept is -1.

(e) The graph touches the x-axis when $f(x) = 0$. We saw in part (b) that this occurs at $x = 3$ and $x = -3$.

32. (a) From Figure 2.2, we see that $P = (b, a)$ and $Q = (d, e)$.

(b) To evaluate $f(b)$, we want to find the y-value when the x-value is b. Since (b, a) lies on this graph, we know that the y-value is a, so $f(b) = a$.

(c) To solve $f(x) = e$, we want to find the x-value for a y-value of e. Since (d, e) lies on this curve, $x = d$ is our solution.

(d) To solve $z = f(x)$, we need to first find a value for z; in other words, we need to first solve for $f(z) = c$. Since $(0, c)$ lies on this graph, we know that $z = 0$. Now we need to solve $0 = f(x)$ by finding the point whose y-value is 0. That point is $(h, 0)$, so $x = h$ is our solution.

(e) We know that $f(b) = a$ and $f(d) = e$. Thus, if $f(b) = -f(d)$, we know that $a = -e$.

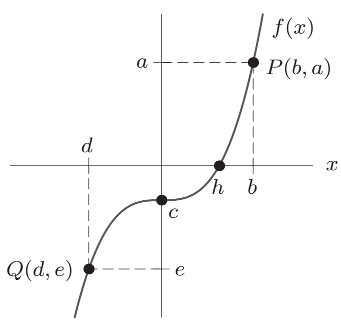


Figure 2.2

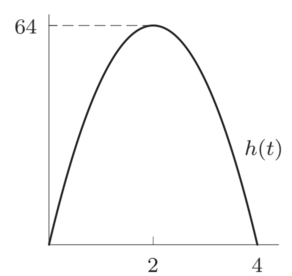


Figure 2.3

33. (a) Substituting into $h(t) = -16t^2 + 64t$, we get

$$\begin{aligned} h(1) &= -16(1)^2 + 64(1) = 48 \\ h(3) &= -16(3)^2 + 64(3) = 48 \end{aligned}$$

Thus the height of the ball is 48 feet after 1 second and after 3 seconds.

(b) The graph of $h(t)$ is in Figure 2.3. The ball is on the ground when $h(t) = 0$. From the graph we see that this occurs at $t = 0$ and $t = 4$. The ball leaves the ground when $t = 0$ and hits the ground at $t = 4$ or after 4 seconds. From the graph we see that the maximum height is 64 ft.

34. (a) Substituting $t = 0$ gives $v(0) = 0^2 - 2(0) = 0 - 0 = 0$.

(b) To find when the object has velocity equal to zero, we solve the equation

$$
\begin{aligned}
t^2 - 2t &= 0 \\
t(t-2) &= 0 \\
t &= 0 \quad \text{or} \quad t = 2.
\end{aligned}
$$

Thus the object has velocity zero at $t = 0$ and at $t = 2$.

(c) The quantity $v(3)$ represents the velocity of the object at time $t = 3$. Its units are ft/sec.

35. (a) The car's position after 2 hours is denoted by the expression $s(2)$. The position after 2 hours is

$$s(2) = 11(2)^2 + 2 + 100 = 44 + 2 + 100 = 146.$$

(b) This is the same as asking the following question: "For what t is $v(t) = 65$?"

(c) To find out when the car is going 67 mph, we set $v(t) = 67$. We have

$$
\begin{aligned}
22t + 1 &= 67 \\
22t &= 66 \\
t &= 3.
\end{aligned}
$$

The car is going 67 mph at $t = 3$, that is, 3 hours after starting. Thus, when $t = 3$, $S(3) = 11(3^2) + 3 + 100 = 202$, so the car's position when it is going 67 mph is 202 miles.

36. (a) Her tax is \$973 on the first \$20,000 plus 6.85% of the remaining \$48,000:

$$\text{Tax owed } = \$973 + 0.0685(\$48{,}000) = \$973 + \$3288 = \$4261.$$

(b) Her taxable income, $T(x)$, is 80% of her total income, or 80% of x. So $T(x) = 0.8x$.

(c) Her tax owed is \$973 plus 6.85% of her taxable income over \$20,000. Since her taxable income is $0.8x$, her taxable income over \$20,000 is $0.8x - 20{,}000$. Therefore,

$$L(x) = 973 + 0.0685(0.8x - 20{,}000),$$

so multiplying out and simplifying, we obtain

$$L(x) = 0.0548x - 397.$$

(d) Evaluating for $x = \$85{,}000$, we have

$$
\begin{aligned}
L(85{,}000) &= 973 + 0.0685(0.8(85{,}000) - 20{,}000) \\
&= \$4261.
\end{aligned}
$$

The values are the same.

37. (a)

n	1	2	3	4	5	6	7	8	9	10	11	12
$f(n)$	1	1	2	3	5	8	13	21	34	55	89	144

(b) We note that for every value of n, we can find a unique value for $f(n)$ (by adding the two previous values of the function). This satisfies the definition of function, so $f(n)$ is a function.

(c) Using the pattern, we can figure out $f(0)$ from the fact that we must have

$$f(2) = f(1) + f(0).$$

Since $f(2) = f(1) = 1$, we have

$$1 = 1 + f(0),$$

so

$$f(0) = 0.$$

Likewise, using the fact that $f(1) = 1$ and $f(0) = 0$, we have

$$\begin{aligned} f(1) &= f(0) + f(-1) \\ 1 &= 0 + f(-1) \\ f(-1) &= 1. \end{aligned}$$

Similarly, using $f(0) = 0$ and $f(-1) = 1$ gives

$$\begin{aligned} f(0) &= f(-1) + f(-2) \\ 0 &= 1 + f(-2) \\ f(-2) &= -1. \end{aligned}$$

However, there is no obvious way to extend the definition of $f(n)$ to non-integers, such as $n = 0.5$. Thus we cannot easily evaluate $f(0.5)$, and we say that $f(0.5)$ is undefined.

38. (a) We calculate the values of $f(x)$ and $g(x)$ using the formulas given in Table 2.1.

Table 2.1

x	-2	-1	0	1	2
$f(x)$	6	2	0	0	2
$g(x)$	6	2	0	0	2

The pattern is that $f(x) = g(x)$ for $x = -2, -1, 0, 1, 2$. Based on this, we might speculate that f and g are really the same function. This is, in fact, the case, as can be verified algebraically:

$$\begin{aligned} f(x) &= 2x(x-3) - x(x-5) \\ &= 2x^2 - 6x - x^2 + 5x \\ &= x^2 - x \\ &= g(x). \end{aligned}$$

Their graphs are the same, and are shown in Figure 2.4.

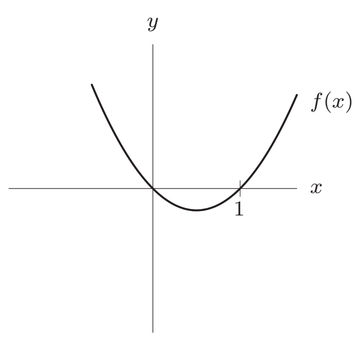


Figure 2.4

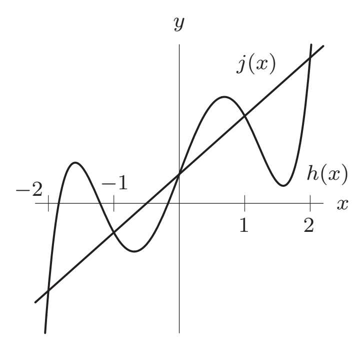


Figure 2.5

(b) Using the formulas for $h(x)$ and $j(x)$, we obtain Table 2.2.

Table 2.2

x	-2	-1	0	1	2
$h(x)$	-3	-1	1	3	5
$j(x)$	-3	-1	1	3	5

The pattern is that $h(x) = j(x)$ for $x = -2, -1, 0, 1, 2$. Based on this, we might speculate that h and j are really the same function. The graphs of these functions are shown in Figure 2.5. We see that the graphs share only the points of the table and are thus two different functions.

39. **(a)** This tells us that a person who loses 30 minutes of sleep takes 5 minutes longer to complete the task than a person who loses no sleep.
(b) This tells us that a person who loses t_1 minutes of sleep takes twice as long to complete the task as a person who loses no sleep.
(c) This tells us that a person who loses $2t_1$ minutes of sleep takes 50% longer to complete the task as a person who loses t_1 minutes of sleep.
(d) This tells us that a person who loses $t_2 + 60$ minutes of sleep takes 10 minutes longer to complete the task than a person who loses only $t_2 + 30$ minutes of sleep.

40. $r(0.5s_0)$ is the wind speed at a half the height above ground of maximum wind speed.

41. In $r(s) = 0.75v_0$, the variable s is the height (or heights) at which the wind speed is 75% of the maximum wind speed.

Solutions for Section 2.2

Exercises

1. The graph of $f(x) = 1/x$ for $-2 \le x \le 2$ is shown in Figure 2.6. From the graph, we see that $f(x) = -(1/2)$ at $x = -2$. As we approach zero from the left, $f(x)$ gets more and more negative. On the other side of the y-axis, $f(x) = (1/2)$ at $x = 2$. As x approaches zero from the right, $f(x)$ grows larger and larger. Thus, on the domain $-2 \le x \le 2$, the range is $f(x) \le -(1/2)$ or $f(x) \ge (1/2)$.

2. The graph of $f(x) = 1/x^2$ for $-1 \le x \le 1$ is shown in Figure 2.7. From the graph, we see that $f(x) = 1$ at $x = -1$ and $x = 1$. As we approach 0 from 1 or from -1, the graph increases without bound. The lower limit of the range is 1, while there is no upper limit. Thus, on the domain $-1 \le x \le 1$, the range is $f(x) \ge 1$.

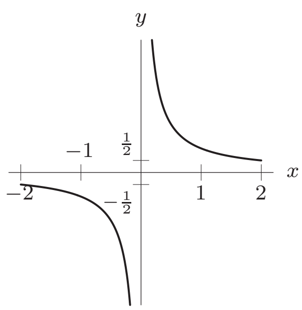


Figure 2.6

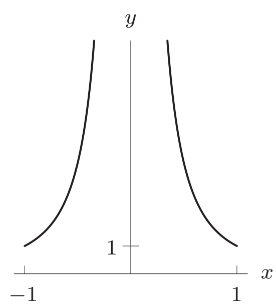


Figure 2.7

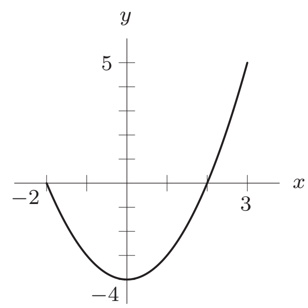


Figure 2.8

3. The graph of $f(x) = x^2 - 4$ for $-2 \le x \le 3$ is shown in Figure 2.8. From the graph, we see that $f(x) = 0$ at $x = -2$, that $f(x)$ decreases down to -4 at $x = 0$, and then increases to $f(x) = 3^2 - 4 = 5$ at $x = 3$. The minimum value of $f(x)$ is -4, while the maximum value is 5. Thus, on the domain $-2 \le x \le 3$, the range is $-4 \le f(x) \le 5$.

4. The graph of $f(x) = \sqrt{9 - x^2}$ for $-3 \le x \le 1$ is shown in Figure 2.9. From the graph, we see that $f(x) = 0$ at $x = -3$, and that $f(x)$ increases to a maximum value of 3 at $x = 0$, and then decreases to a value of $f(x) = \sqrt{9 - 1^2} \approx 2.83$ or $= 2\sqrt{2}$ at $x = 1$. Thus, on the domain $-3 \le x \le 1$, the range is $0 \le f(x) \le 3$.

5. The graph of $f(x) = (x - 4)^3$ is given in Figure 2.10.
The domain is all real x; the range is all real $f(x)$.

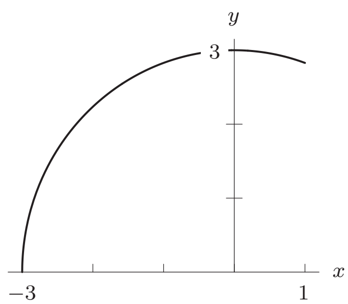


Figure 2.9

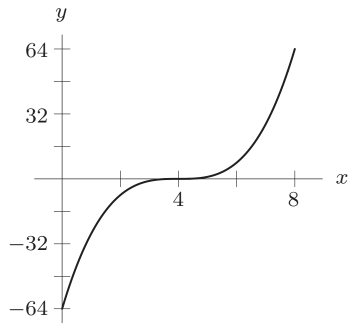


Figure 2.10

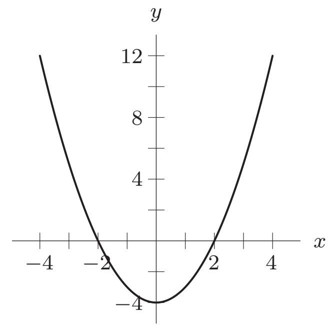


Figure 2.11

6. The graph of $f(x) = x^2 - 4$ is given in Figure 2.11.
The domain is all real x; the range is all $f(x) \geq -4$.

7. The graph of $f(x) = 9 - x^2$ is given in Figure 2.12. The domain is all real x; the range is all $f(x) \leq 9$.

8. The graph of $f(x) = x^3 + 2$ is given in Figure 2.13. The domain is all real x; the range is all real $f(x)$.

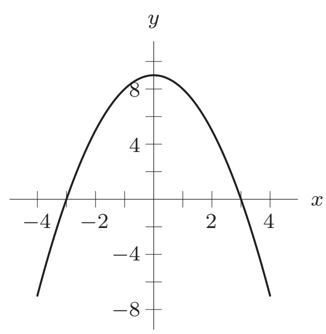


Figure 2.12

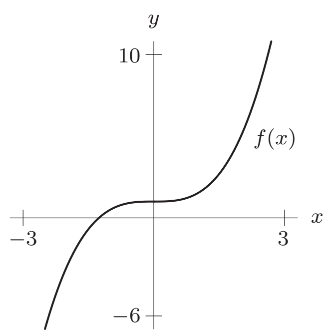


Figure 2.13

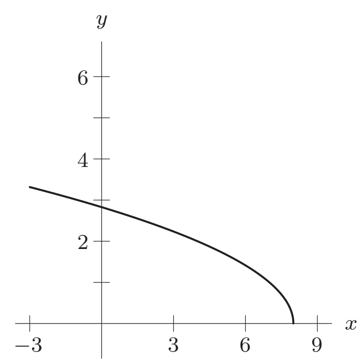


Figure 2.14

9. The graph of $f(x) = \sqrt{8 - x}$ is given in Figure 2.14. The domain is all real $x \leq 8$; the range is all $f(x) \geq 0$.

10. The graph of $f(x) = \sqrt{x - 3}$ is given in Figure 2.15. The domain is all real $x \geq 3$; the range is all $f(x) \geq 0$.

11. The graph of $f(x) = -1/(x + 1)^2$ is given in Figure 2.16. The domain is all real x, $x \neq -1$; the range is all $f(x) < 0$.

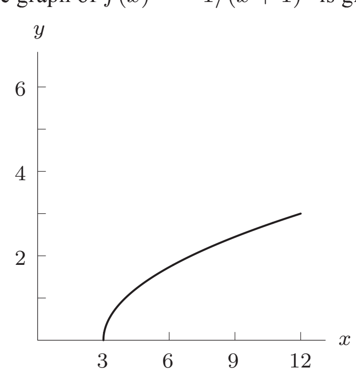


Figure 2.15

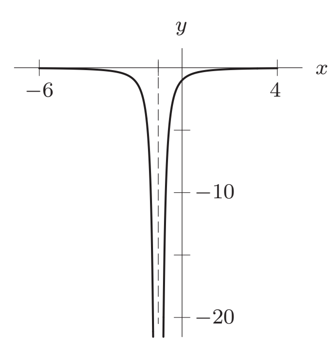


Figure 2.16

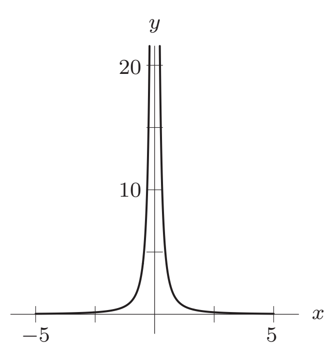


Figure 2.17

12. The graph of $f(x) = 1/x^2$ is given in Figure 2.17.
The domain is all real numbers x, $x \neq 0$; the range is all $f(x) > 0$.

13. The domain is $1 \leq x \leq 7$. The range is $2 \leq f(x) \leq 18$.

14. The domain is $2 \leq x \leq 6$. The range is $1 \leq f(x) \leq 3$.

Problems

15. The expression $x^2 - 9$, found inside the square root sign, must always be non-negative. This happens when $x \geq 3$ or $x \leq -3$, so our domain is $x \geq 3$ or $x \leq -3$.
For the range, the smallest value $\sqrt{x^2 - 9}$ can have is zero. There is no largest value, so the range is $q(x) \geq 0$.

16. To evaluate $f(x)$, we must have $x - 4 > 0$. Thus

$$\text{Domain: } x > 4.$$

To find the range, we want to know all possible output values. We solve the equation $y = f(x)$ for x in terms of y. Since

$$y = \frac{1}{\sqrt{x-4}},$$

squaring gives

$$y^2 = \frac{1}{x-4},$$

and multiplying by $x - 4$ gives

$$\begin{aligned} y^2(x-4) &= 1 \\ y^2x - 4y^2 &= 1 \\ y^2x &= 1 + 4y^2 \\ x &= \frac{1+4y^2}{y^2}. \end{aligned}$$

This formula tells us how to find the x-value which corresponds to a given y-value. The formula works for any y except $y = 0$ (which puts a 0 in the denominator). We know that y must be positive, since $\sqrt{x-4}$ is positive, so we have

$$\text{Range: } y > 0.$$

17. Since for any value of x that you might choose you can find a corresponding value of $m(x)$, we can say that the domain of $m(x) = 9 - x$ is all real numbers.
For any value of $m(x)$ there is a corresponding value of x. So the range is also all real numbers.

18. Since you can choose any value of x and find an associated value for $n(x)$, we know that the domain of this function is all real numbers.
However, there are some restrictions on the range. Since x^4 is always positive for any value of x, $9 - x^4$ will have a largest value of 9 when $x = 0$. So the range is $n(x) \leq 9$.

19. Any number can be squared, so the domain is all real numbers. Since x^2 is always greater than or equal to zero, we see that $-x^2 \leq 0$. Thus, $f(x) = -x^2 + 7 \leq 7$. Thus, the range is all real numbers ≤ 7.

20. We can take the cube root of any number, so the domain is all real numbers. By using appropriate input values we can get any real number as a result, so the range is all real numbers.

21. Any number can be squared, so the domain is all real numbers. Since x^2 is always greater than or equal to zero, we see that $f(x) = x^2 + 2 \geq 2$. Thus, the range is all real numbers ≥ 2.

22. Since we can divide by any number except for zero, we can have any x with $x \neq -1$ in $1/(x+1)$, so the domain is all real numbers $\neq -1$. As $x \to \infty$ or $x \to -\infty$, $1/(x+1) \to 0$, but it does not equal zero. Since $1/(x+1)$ takes all real values except zero, $f(x) = 1/(x+1) + 3$ takes all values except 3. So the range is all real numbers $\neq 3$.

23. The domain and range are both all real numbers.

24. Any number can be squared, so the domain is all real numbers. Since $(x-3)^2$ is always greater than or equal to zero, we see that $f(x) = (x-3)^2 + 2 \geq 2$. Thus, the range is all real numbers ≥ 2.

25. One way to do this is to combine two operations, one of which forces x to be non-negative, the other of which forces x not to equal 3. One possibility is

$$y = \frac{1}{x-3} + \sqrt{x}.$$

The fraction's denominator must not equal 0, so x must not equal 3. Further, the input of the square root function must not be negative, so x must be greater than or equal to zero. Other possibilities include

$$y = \frac{\sqrt{x}}{x-3}.$$

26.
- A function such as $y = \sqrt{x-4}$ is undefined for $x < 4$, because the input of the square root operation will negative for these x-values.
- A function such as $y = 1/(x-8)$ is undefined for $x = 8$.
- Combining two functions such as these, for example by adding or multiplying them, yields a function with the required domain. Thus, possible formulas include

$$y = \frac{1}{x-8} + \sqrt{x-4} \qquad \text{or} \qquad y = \frac{\sqrt{x-4}}{x-8}.$$

27. Since the restaurant opens at 2 pm, $t = 0$, and closes at 2 am, $t = 12$, a reasonable domain is $0 \le t \le 12$.
Since there cannot be fewer than 0 clients in the restaurant and 200 can fit inside, the range is $0 \le f(t) \le 200$.

28. The domain is all possible input values, namely $t = 1, 2, 3, \ldots, 12$.

29. We know that the theater can hold anywhere from 0 to 200 people. Therefore the domain of the function is the integers, n, such that $0 \le n \le 200$.
We know that each person who enters the theater must pay \$4.00. Therefore, the theater makes $(0) \cdot (\$4.00) = 0$ dollars if there is no one in the theater, and $(200) \cdot (\$4.00) = \$800.00$ if the theater is completely filled. Thus the range of the function would be the integer multiples of 4 from 0 to 800. (That is, $0, 4, 8, \ldots$.)
The graph of this function is shown in Figure 2.18.

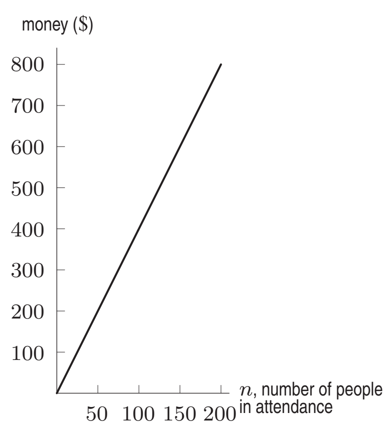


Figure 2.18

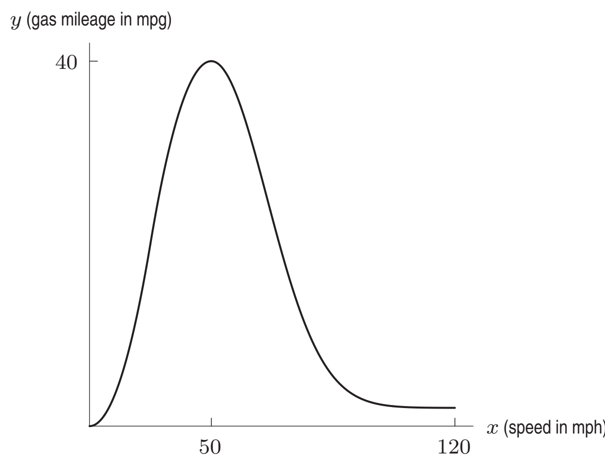


Figure 2.19

30. A possible graph of gas mileage (in miles per gallon, mpg) is shown in Figure 2.19. The function shown has a domain $0 \le x \le 120$ mph, as the car cannot have a negative speed and is not likely to go faster than 120 mph. The range of the function shown is $0 \le y \le 40$ mpg. A wide variety of other answers is possible.

31. (a) From the table we find that a 200 lb person uses 5.4 calories per minute while walking. So a half-hour, or a 30 minute, walk burns $30(5.4) = 162$ calories.

(b) The number of calories used per minute is approximately proportional to the person's weight. The relationship is an approximately linear increasing function, where weight is the independent variable and number of calories burned is the dependent variable.

(c) (i) Since the function is approximately linear, its equation is $c = b + mw$, where c is the number of calories and w is weight. Using the first two values in the table, the slope is

$$m = \frac{3.2 - 2.7}{120 - 100} = \frac{0.5}{20} = 0.025 \text{ cal/lb.}$$

Using the point $(100, 2.7)$ we have

$$\begin{aligned} 2.7 &= b + 0.025(100) \\ b &= 0.2. \end{aligned}$$

So the equation is $c = 0.2 + 0.025w$. See Figure 2.20. All the values given lie on this line with the exception of the last two which are slightly above it.

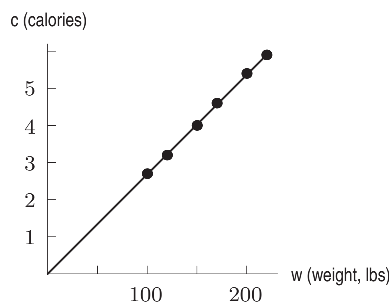


Figure 2.20

(ii) The intercept $(0, 0.2)$ is the number of calories burned by a weightless runner. Since 0.2 is a small number, most of the calories burned appear to be due to moving a person's weight. Other methods of finding the equation of the the line may give other values for the vertical intercept, but all values are close to 0.

(iii) Domain $0 < w$; range $0 < c$

(iv) Evaluating the function at $w = 135$,

$$\text{Calories} = 0.2 + 0.025(135) \approx 3.6.$$

32. We can put in any number for x except zero, which makes $1/x$ undefined. We note that as x approaches infinity or negative infinity, $1/x$ approaches zero, though it never arrives there, and that as x approaches zero, $1/x$ goes to negative or positive infinity. Thus, the range is all real numbers except a.

33. Since $(x-b)^{1/2} = \sqrt{x-b}$, we know that $x - b \geq 0$. Thus, $x \geq b$. If $x = b$, then $(x-b)^{1/2} = 0$, which is the minimum value of $\sqrt{x-b}$, since it can't be negative. Thus, the range is all real numbers greater than or equal to 6.

34. (a) We see that the 6^{th} listing has a last digit of 8. Thus, $f(6) = 8$.

(b) The domain of the telephone directory function is $n = 1, 2, 3, \ldots, N$, where N is the total number of listings in the directory. We could find the value of N by counting the number of listings in the phone book.

(c) The range of this function is $d = 0, 1, 2, \ldots, 9$, because the last digit of any listing must be one of these numbers.

35. (a) Substituting $t = 0$ into the formula for $p(t)$ shows that $p(0) = 50$, meaning that there were 50 rabbits initially. Using a calculator, we see that $p(10) \approx 131$, which tells us there were about 131 rabbits after 10 months. Similarly, $p(50) \approx 911$ means there were about 911 rabbits after 50 months.

(b) The graph in Figure 2.21 tells us that the rabbit population grew quickly at first but then leveled off at about 1000 rabbits after around 75 months or so. It appears that the rabbit population increased until it reached the island's capacity.

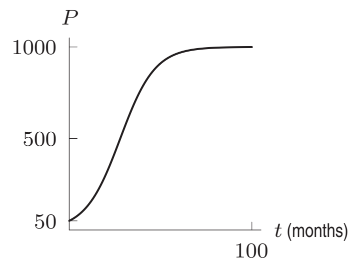


Figure 2.21

(c) From the graph in Figure 2.21, we see that the range is $50 \le p(t) \le 1000$. This tells us that (for $t \ge 0$) the number of rabbits is no less than 50 and no more than 1000.

(d) The smallest population occurred when $t = 0$. At that time, there were 50 rabbits. As t gets larger and larger, $(0.9)^t$ gets closer and closer to 0. Thus, as t increases, the denominator of

$$p(t) = \frac{1000}{1 + 19(0.9)^t}$$

decreases. As t increases, the denominator $1+19(0.9)^t$ gets close to 1 (try $t = 100$, for example). As the denominator gets closer to 1, the fraction gets closer to 1000. Thus, as t gets larger and larger, the population gets closer and closer to 1000. Thus, the range is $50 \le p(t) < 1000$.

36. (a) We can add as much copper to our alloy as we like, so, since positive x-values represent quantities of added copper, x can be as big as we please. But, since the alloy starts off with only 3 kg of copper, we can remove no more than this. Therefore, the domain of f is $x \ge -3$.

For the range of f, note that the output of f is a percentage of copper. Since the alloy can contain no less than 0% copper (as would be the case if all 3 kg were removed), we see that $f(x)$ must be greater than (or equal to) 0%. On the other hand, no matter how much copper we add, the alloy will always contain 6 kg of tin. Thus, we can never obtain a pure, 100%-copper alloy. This means that if $y = f(x)$,

$$0\% \le y < 100\%,$$

or

$$0 \le y < 1.$$

(b) By definition, $f(x)$ is the percentage of copper in the bronze alloy after x kg of copper are added (or removed). We have

$$\begin{array}{c}\text{Percentage of copper}\\ \text{in the bronze alloy}\end{array} = \frac{\text{quantity of copper in the alloy}}{\text{total quantity of alloy}}.$$

Since x is the quantity of copper added or removed, this gives

$$f(x) = \frac{\text{initial quantity of copper} + x}{\text{initial quantity of alloy} + x},$$

and since the original 9 kg of alloy contained 3 kg of copper, we have

$$f(x) = \frac{3 + x}{9 + x}.$$

(c) If we think of the formula $f(x) = (3 + x)/(9 + x)$ as defining a function, but not as a model of an alloy of bronze, then the way we think about its domain and range changes. For example, we no longer need to ask, "Does this x-value make sense in the context of the model?" We need only ask "Is $f(x)$ algebraically defined for this value of x?" or "If we use this x-value for input, will there be a corresponding y-value as output?"

For the domain of f, we see that $y = (3 + x)/(9 + x)$ is defined for any x-value other than $x = -9$. Thus, the domain of f is any value of x such that $x \neq -9$.

To find the range of this function, we solve $y = f(x)$ for x in terms of y:

$$\begin{aligned}
y &= \frac{3+x}{9+x} \\
y(9+x) &= 3+x && \text{(multiply both sides by denominator)} \\
9y + xy &= 3+x && \text{(expand parentheses)} \\
xy - x &= 3 - 9y && \text{(collect all terms with } x \text{ at left)} \\
x(y-1) &= 3 - 9y && \text{(factor out } x\text{)} \\
x &= \frac{3-9y}{y-1} && \text{(divide by } y-1\text{).}
\end{aligned}$$

In solving for x, at the last step we had to divide by $y - 1$. This is valid if and only if $y \neq 1$, for otherwise we would be dividing by zero. There is no x-value resulting in a y-value of 1. Consequently, the range of f is any value of y such that $y \neq 1$.

Notice the difference between this situation and the situation where f is being used as a model for bronze.

Solutions for Section 2.3

Exercises

1. $f(x) = \begin{cases} -1, & -1 \leq x < 0 \\ 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$ is shown in Figure 2.22.

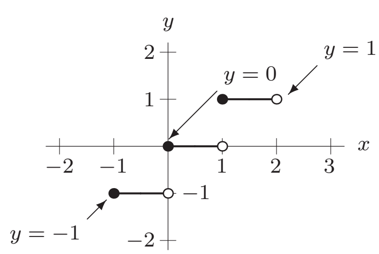


Figure 2.22

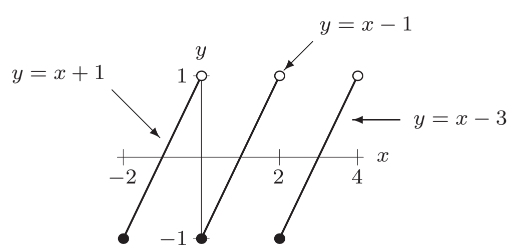


Figure 2.23

2. $f(x) = \begin{cases} x+1, & -2 \leq x < 0 \\ x-1, & 0 \leq x < 2 \\ x-3, & 2 \leq x < 4 \end{cases}$ is shown in Figure 2.23.

3. The graph of $f(x) = \begin{cases} x+4, & x \leq -2 \\ 2, & -2 < x < 2 \\ 4-x, & x \geq 2 \end{cases}$ is shown in Figure 2.24.

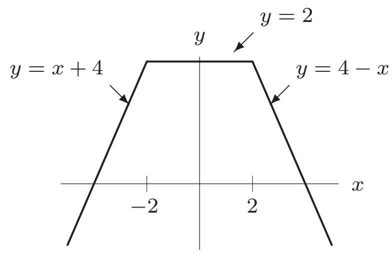


Figure 2.24

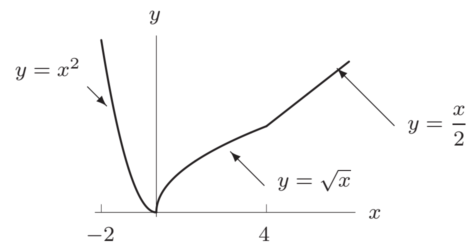


Figure 2.25

4. The graph of $f(x)=\begin{cases} x^2, & x \le 0 \\ \sqrt{x}, & 0 < x < 4 \\ x/2, & x \ge 4 \end{cases}$ is shown in Figure 2.25.

5. Since $G(x)$ is defined for all x, the domain is all real numbers. For $x < -1$ the values of the function are all negative numbers. For $-1 \ge x \ge 0$ the functions values are $4 \ge G(x) \ge 3$, while for $x > 0$ we see that $G(x) \ge 3$ and the values increase to infinity. The range is $G(x) < 0$ and $G(x) \ge 3$.

6. Since $F(x)$ is defined for all x, the domain is all real numbers. For $x \le 1$ the values of the function are $-\infty \le F(x) \le 1$, while for $x > 1$ we see that $0 < F(x) < 1$, so the range is $F(x) \le 1$.

7. We find the formulas for each of the lines. For the first, we use the two points we have, $(1,4)$ and $(3,2)$. We find the slope: $(2-4)/(3-1) = -1$. Using the slope of -1, we solve for the y-intercept:

$$\begin{aligned} 4 &= b - 1 \cdot 1 \\ 5 &= b. \end{aligned}$$

Thus, the first line is $y = 5 - x$, and it is for the part of the function where $x < 3$. Notice that we do not use this formula for the value $x = 3$.

We follow the same method for the second line, using the points $(3, \frac{1}{2})$ and $(5, \frac{3}{2})$. We find the slope: $(\frac{3}{2} - \frac{1}{2})/(5 - 3) = \frac{1}{2}$. Using the slope of $\frac{1}{2}$, we solve for the y-intercept:

$$\begin{aligned} \frac{1}{2} &= b + \frac{1}{2} \cdot 3 \\ -1 &= b. \end{aligned}$$

Thus, the second line is $y = -1 + \frac{1}{2}x$, and it is for the part of the function where $x \ge 3$.

Therefore, the function is:

$$y = \begin{cases} 5 - x & \text{for } x < 3 \\ -1 + \frac{1}{2}x & \text{for } x \ge 3. \end{cases}$$

8. We find the formulas for each of the lines. For the first, we use the two points we have, $(1, 6.5)$ and $(3, 5.5)$. We find the slope: $(5.5 - 6.5)/(3-1) = -\frac{1}{2}$. Using the slope of $-\frac{1}{2}$, we solve for the y-intercept:

$$\begin{aligned} 6.5 &= b - \frac{1}{2} \cdot 1 \\ 7 &= b. \end{aligned}$$

Thus, the first line is $y = 7 - \frac{1}{2}x$, and it is for the part of the function where $x \le 3$.

We follow the same method for the second line, using the points $(3,2)$ and $(5,2)$. Noting that the y values are the same, we know the slope is zero and that the line is $y = 2$ for the part of the function where $3 < x \le 5$.

We follow the same method for the third line, using the points $(5,7)$ and $(7,3)$. We find the slope: $(3-7)/(7-5) = -2$. Using the slope of -2, we solve for the y-intercept:

$$\begin{aligned} 3 &= b - 2 \cdot 7 \\ 17 &= b. \end{aligned}$$

Thus, the second line is $y = 17 - 2x$, and it is for the part of the function where $x > 5$.

Therefore, the function is:

$$y = \begin{cases} 7 - \frac{1}{2}x & \text{for } x \leq 3 \\ 2 & \text{for } 3 < x \leq 5 \\ 17 - 2x & \text{for } x > 5. \end{cases}$$

9. We find the formulas for each of the lines. For the first, we use the two points we have, $(1, 3.5)$ and $(3, 2.5)$. We find the slope: $(2.5 - 3.5)/(3 - 1) = -\frac{1}{2}$. Using the slope of $-\frac{1}{2}$, we solve for the y-intercept:

$$\begin{aligned} 3.5 &= b - \frac{1}{2} \cdot 1 \\ 4 &= b. \end{aligned}$$

Thus, the first line is $y = 4 - \frac{1}{2}x$, and it is for the part of the function where $1 \leq x \leq 3$.

We follow the same method for the second line, using the points $(5, 1)$ and $(8, 7)$. We find the slope: $(7-1)/(8-5) = 2$. Using the slope of 2, we solve for the y-intercept:

$$\begin{aligned} 1 &= b + 2 \cdot 5 \\ -9 &= b. \end{aligned}$$

Thus, the second line is $y = -9 + 2x$, and it is for the part of the function where $5 \leq x \leq 8$.

Therefore, the function is:

$$y = \begin{cases} 4 - \frac{1}{2}x & \text{for } 1 \leq x \leq 3 \\ -9 + 2x & \text{for } 5 \leq x \leq 8. \end{cases}$$

10. We find the formulas for each of the lines. For the first, we use the two points we have, $(1, 4)$ and $(3, 2)$. We find the slope: $(2 - 4)/(3 - 1) = -1$. Using the slope of -1, we solve for the y-intercept:

$$\begin{aligned} 4 &= b - 1 \cdot 1 \\ 5 &= b. \end{aligned}$$

Thus, the first line is $y = 5 - x$, and it is for the part of the function where $x \leq 3$.

We follow the same method for the second line, using the points $(3, 2)$ and $(5, 4)$. We find the slope: $(4-2)/(5-3) = 1$. Using the slope of 1, we solve for the y-intercept:

$$\begin{aligned} 4 &= b + 1 \cdot 5 \\ -1 &= b. \end{aligned}$$

Thus, the second line is $y = -1 + x$, and it is for the part of the function where $x \geq 3$.

Therefore, the function is:

$$y = \begin{cases} 5 - x & \text{for } x \leq 3 \\ -1 + x & \text{for } x \geq 3. \end{cases}$$

Notice that the value of $y = 2$ at $x = 3$ can be obtained from either formula.

Alternatively, this is the graph of the absolute value function,

$$y = |x - 3| + 2.$$

Problems

11. **(a)** Yes, because every value of x is associated with exactly one value of y.

(b) No, because some values of y are associated with more than one value of x.

(c) $y = 1, 2, 3, 4.$

12. (a) Figures 2.27 and 2.26 show the two functions x and $\sqrt{x^2}$. Because the two functions do not coincide for $x < 0$, they cannot be equal. The graph of $\sqrt{x^2}$ looks like the graph of $|x|$.

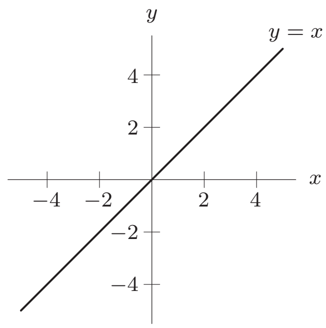


Figure 2.26

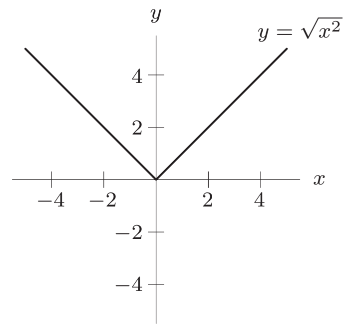


Figure 2.27

(b) Table 2.3 is the complete table. Because the two functions do not coincide for $x < 0$ they cannot be equal. The table for $\sqrt{x^2}$ looks like a table for of $|x|$.

Table 2.3

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$\sqrt{x^2}$	5	4	3	2	1	0	1	2	3	4	5

(c) If $x > 0$, then $\sqrt{x^2} = x$, whereas if $x < 0$ then $\sqrt{x^2} = -x$. This is the definition of $|x|$. Thus we have shown $\sqrt{x^2} = |x|$.

(d) We see nothing because $\sqrt{x^2} - |x| = 0$, and the graphing calculator or computer has drawn a horizontal line on top of the x-axis.

13. (a) Figure 2.28 shows the function $u(x)$. Some graphing calculators or computers may show a near vertical line close to the origin. The function seems to be -1 when $x < 0$ and 1 when $x > 0$.

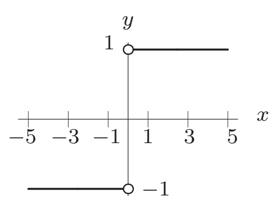


Figure 2.28

(b) Table 2.4 is the completed table. It agrees with what we found in part (a). The function is undefined at $x = 0$.

Table 2.4

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$\lvert x\rvert/x$	-1	-1	-1	-1	-1		1	1	1	1	1

(c) The domain is all x except $x = 0$. The range is -1 and 1.

(d) $u(0)$ is undefined, not 0. The claim is false.

14. **(a)** Up to $1/4$ mile, the cost is \$2.50. The next $1/4$ mile, (up to $2/4$ mile) adds \$0.40, giving a fare of \$2.90. For a journey of $3/4$ mile, another \$0.40 is added for a fare of \$3.30. Each additional $1/4$ mile gives an another increment of \$0.40. See Table 2.5.

Table 2.5

Miles	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Cost	0	2.50	2.90	3.30	3.70	4.10	4.50	4.90	5.30

(b) The table shows that the cost for a 1.25 mile trip is \$4.10.
(c) From the table, the maximum distance one can travel for \$5.30 is 2.00 mile.
(d) See Figure 2.29

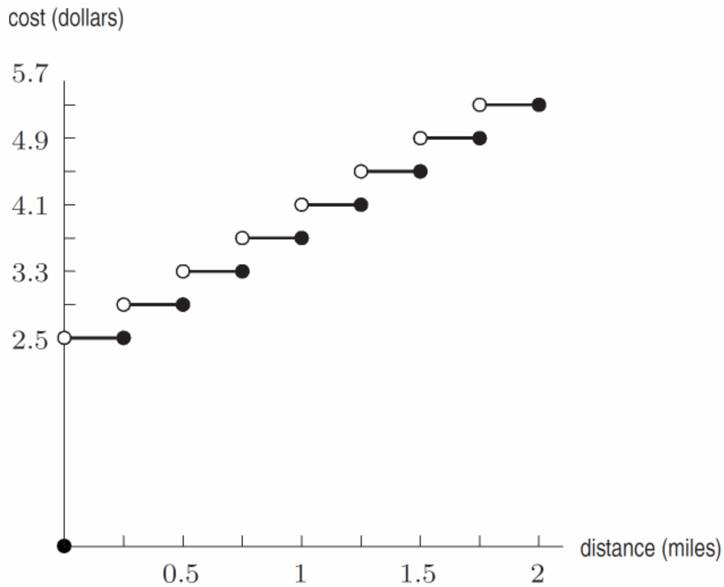


Figure 2.29

15. **(a)** The dots in Figure 2.30 represent the graph of the function.

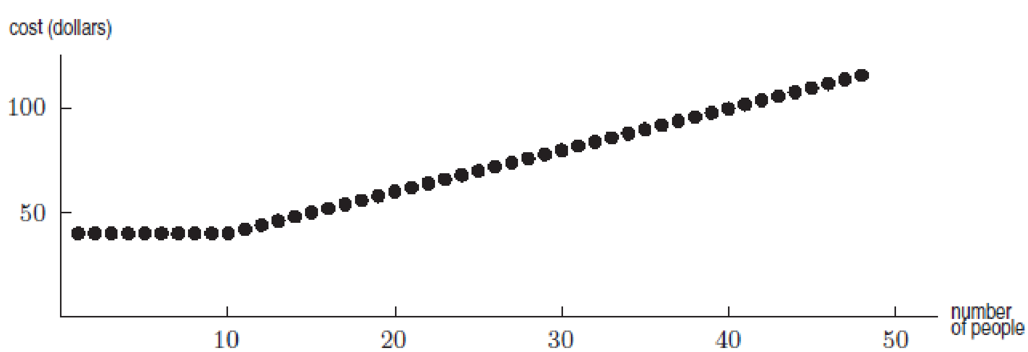


Figure 2.30

(b) Since admission is charged for whole numbers of people between 1 and 50, the domain is the integers from 1 to 50. The minimum cost is \$40. The maximum occurs for 50 people and is $\$40 + 40(\$2) = \$120$. Since the lowest cost is \$40, and each additional person costs \$2, the range only includes numbers which are multiples of 2. Thus, the range is all the even integers from 40 to 120.

16. (a) Let $y = f(x)$ be the cost of a stripping and refinishing job for a floor which is x square feet in area. When the area is less than or equal to 150 square feet, the price is \$1.83 times the number of square feet. Thus, for x-values up through 150, we have $f(x) = 1.83x$. However, if the area is more than 150 square feet, the extra cost of toxic waste disposal is added, giving $f(x) = 1.83x + 350$. The maximum total area for a job is 1000 square feet, so the formula is

$$f(x) = \begin{cases} 1.83x, & 0 \leq x \leq 150 \\ 1.83x + 350, & 150 < x \leq 1000 \end{cases}$$

(b) The graph is in Figure 2.31. Note that when $x = 150$ sq ft, $y = 1.83(150) = \$274.5$. When x goes above 150 sq ft, the cost jumps by \$350 to \$624.5.

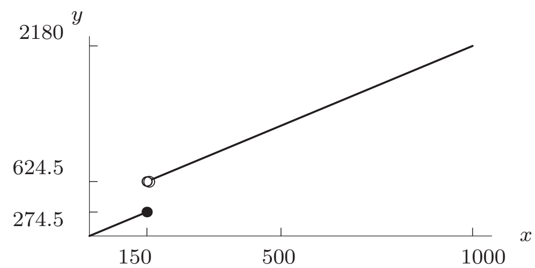


Figure 2.31

No floor has a negative area and the company will refinish any floor whose area is less than or equal to 1000 square feet, so

$$\text{Domain is} \quad 0 \leq x \leq 1000.$$

As the size of the floor gets bigger, the cost increases. The smallest value of the range occurs when $x = 0$ and the largest value occurs when $x = 1000$. So the smallest value is $f(0) = 0$ and the largest is $f(1000) = 2180$. There is a gap, though, in the values of the range. The value of $f(x)$ jumps from 274.5, when $x = 150$, to more than 624.5 when x is just slightly more than 150. Putting all these pieces together, we have

$$\text{Range is} \quad 0 \leq y \leq 274.5 \text{ or } 624.5 < y \leq 2180.$$

17. (a) The depth of the driveway is 1 foot or 1/3 of a yard. The volume of the driveway is the product of the three dimensions, length, width and depth. So,

$$\begin{array}{c}\text{Volume of} \\ \text{gravel needed}\end{array} = \text{Length} \cdot \text{Width} \cdot \text{Depth} = (L)(6)(1/3) = 2L.$$

Since he buys 10 cubic yards more than needed,

$$n(L) = 2L + 10.$$

(b) The length of a driveway is not less than 5 yards, so the domain of n is all real numbers greater than or equal to 5. The contractor can buy only 1 cubic yd at a time so the range only contains integers. The smallest value of the range occurs for the shortest driveway, when $L = 5$. If $L = 5$, then $n(5) = 2(5) + 10 = 20$. Although very long driveways are highly unlikely, there is no upper limit on L, so no upper limit on $n(L)$. Therefore the range is allintegers greater than or equal to 20. See Figure 2.32.

(c) If $n(L) = 2L + 10$ was not intended to represent a quantity of gravel, then the domain and range of n would be all real numbers.

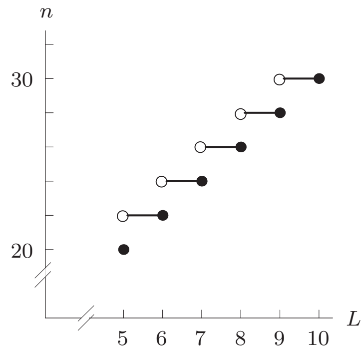


Figure 2.32

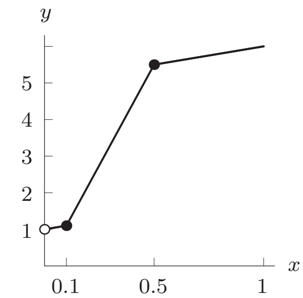


Figure 2.33

18. (a) The smaller the difference, the smaller the refund. The smallest possible difference is \$0.01. This translates into a refund of $\$1.00 + \$0.01 = \$1.01$.

(b) Looking at the refund rules, we see that there are three separate cases to consider. The first case is when 10 times the difference is less than \$1. If the difference is more than 0 but less than 10¢, and you will receive \$1 plus the difference. The formula for this is:

$$y = 1 + x \quad \text{for} \quad 0 < x < 0.10.$$

In the second case, 10 times the difference is between \$1 and \$5. This will be true if the difference is between 10¢and 50¢. The formula for this is:

$$y = 10x + x \quad \text{for} \quad 0.10 \leq x \leq 0.50.$$

In the third case, 10 times the difference is more than \$5. If the difference is more than 50¢, then you receive \$5 plus the difference or:

$$y = 5 + x \quad \text{for} \quad x > 0.50.$$

Putting these cases together, we get:

$$y = \begin{cases} 1 + x & \text{for } 0 < x < 0.1 \\ 10x + x & \text{for } 0.1 \leq x \leq 0.5 \\ 5 + x & \text{for } x > 0.5. \end{cases}$$

(c) We want x such that $y = 9$. Since the highest possible value of y for the first case occurs when $x = 0.09$, and $y = 1 + 0.09 = \$1.09$, the range for this case does not go high enough. The highest possible value for the second case occurs when $x = 0.5$, and $y = 10(0.5) + 0.5 = \$5.50$. This range is also not high enough. So we look to the third case where $x > 0.5$ and $y = 5 + x$. Solving $5 + x = 9$ we find $x = 4$. So the price difference would have to be \$4.

(d) See Figure 2.33.

19. (a) Each signature printed costs \$0.14, and in a book of p pages, there are at least $p/16$ signatures. In a book of 128 pages, there are

$$\frac{128}{16} = 8 \text{ signatures,}$$
$$\text{Cost for 128 pages } = 0.14(8) = \$1.12.$$

A book of 129 pages requires 9 signatures, although the ninth signature is used to print only 1 page. Therefore,

$$\text{Cost for 129 pages } = \$0.14(9) = \$1.26.$$

To find the cost of p pages, we first find the number of signatures. If p is divisible by 16, then the number of signatures is $p/16$ and the cost is

$$C(p) = 0.14\left(\frac{p}{16}\right).$$

If p is not divisible by 16, the number of signatures is $p/16$ rounded up to the next highest integer and the cost is 0.14 times that number. In this case, it is hard to write a formula for $C(p)$ without a symbol for "rounding up."

(b) The number of pages, p, is greater than zero. Although it is possible to have a page which is only half filled, we do not say that a book has 124 1/2 pages, so p must be an integer. Therefore, the domain of $C(p)$ is $p > 0$, p an integer. Because the cost of a book increases by multiples of \$0.14 (the cost of one signature), the range of $C(p)$ is $C > 0$, C an integer multiple of \$0.14,

(c) For 1 to 16 pages, the cost is \$0.14, because only 1 signature is required. For 17 to 32 pages, the cost is \$0.28, because 2 signatures are required. These data are continued in Table 2.6 for $0 \le p \le 128$, and they are plotted in Figure 2.34. A closed circle represents a point included on the graph, and an open circle indicates a point excluded from the graph. The unbroken lines in Figure 2.34 suggest, erroneously, that *fractions* of pages can be printed. It would be more accurate to draw each step as 16 separate dots instead of as an unbroken line.

Table 2.6 *The cost C for printing a book of p pages*

p, pages	C, dollars
1-16	0.14
17-32	0.28
33-48	0.42
49-64	0.56
65-80	0.70
81-96	0.84
97-112	0.98
113-128	1.12

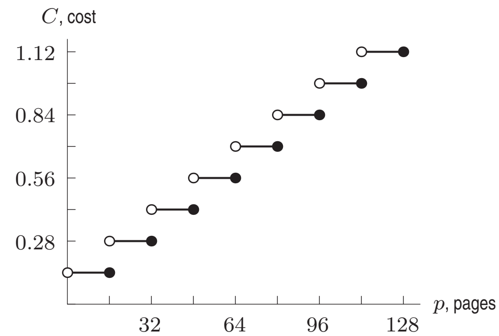


Figure 2.34: Graph of the cost C for printing a book of p pages

20. (a) Figure 2.35 shows the rates for the first and last periods of the year. Figure 2.37 shows the rates for holiday periods (Dec 25-Jan 3, Jan16-18, Feb 3-21) and Figure 2.36 shows the rates for the other times.

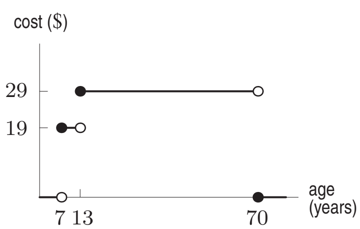


Figure 2.35: Opening/Closing

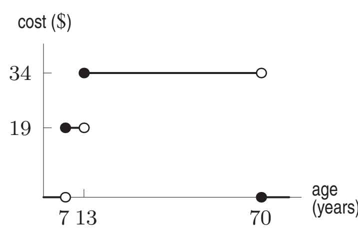


Figure 2.36: Regular rates

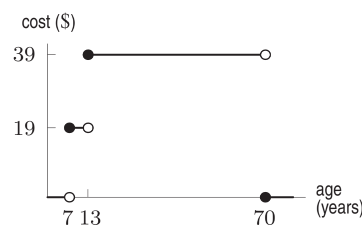


Figure 2.37: Holiday rates

(b) Ages 13-69.

(c) See Figure 2.38.

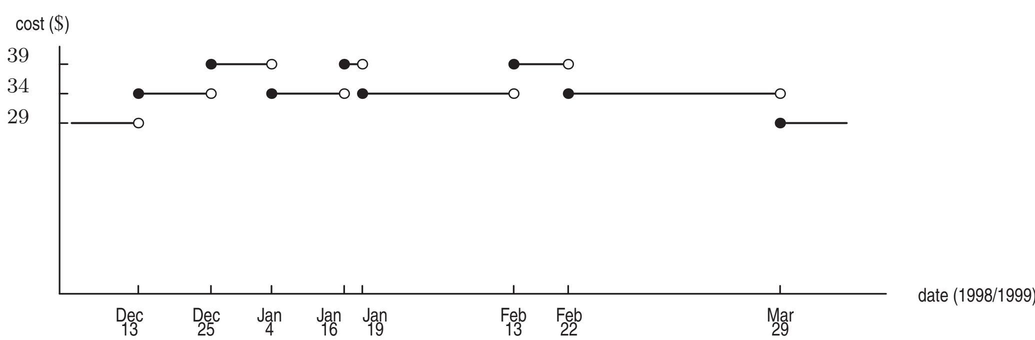


Figure 2.38: Cost as a function of date for 13-69 years old

(d) Rates through 12 December and after 19 March represent early-season and late-season rates, respectively; these are off-peak rates; it makes economic sense to cut rates when there are fewer skiers. Holiday rates took effect from 25 December through 3 January because of the Christmas/New Year's holiday; they took effect from 16 January through 18 January for Martin Luther King's holiday; they took effect from 13 February through 21 February for the Presidents' Week holiday; it makes economic sense to charge over peak rates during the holidays, as more skiers are available to use the facility. Other times represent rates during the heart of the winter skiing season; these are the regular rates.

Solutions for Section 2.4

Exercises

1. $A(f(t))$ is the area, in square centimeters, of the circle at time t minutes.

2. $R(f(p))$ is the revenue, in millions of dollars, when the price of oil is p dollars/barrel.

3. $a(g(w))$ is the acceleration in meters/sec^2 when the wind speed is w meters/second.

4. $P(f(t))$ is the period, in seconds, of the pendulum at time t minutes.

5. $f(g(0)) = f(2 \cdot 0 + 3) = f(3) = 3^2 + 1 = 10.$

6. $f(g(1)) = f(2 \cdot 1 + 3) = f(5) = 5^2 + 1 = 26.$

7. $g(f(0)) = g(0^2 + 1) = g(1) = 2 \cdot 1 + 3 = 5.$

8. $g(f(1)) = g(1^2 + 1) = g(2) = 2 \cdot 2 + 3 = 7.$

9. $f(g(x)) = f(2x + 3) = (2x + 3)^2 + 1 = 4x^2 + 12x + 10.$

10. $g(f(x)) = g(x^2 + 1) + 3 = 2(x^2 + 1) + 3 = 2x^2 + 5.$

11. $f(f(x)) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 2.$

12. $g(g(x)) = g(2x + 3) = 2(2x + 3) + 3 = 4x + 9.$

13. The inverse function, $f^{-1}(P)$, gives the year in which population is P million. Units of $f^{-1}(P)$ are years.

14. The inverse function, $f^{-1}(T)$, gives the temperature in $^\circ$F needed if the cake is to bake in T minutes. Units of $f^{-1}(T)$ are $^\circ$F.

15. The inverse function, $f^{-1}(N)$, is the number of days for N inches of snow to fall. Units of $f^{-1}(N)$ are days.

16. The inverse function, $f^{-1}(V)$, gives the time at which the speed is V. Units of $f^{-1}(V)$ are seconds.

17. The inverse function, $f^{-1}(I)$, gives the interest rate that gives \$$I$ in interest. Units of $f^{-1}(I)$ is percent per year.

18. Since $y = 2t + 3$, solving for t gives

$$\begin{aligned} 2t + 3 &= y \\ t &= \frac{y-3}{2} \\ f^{-1}(y) &= \frac{y-3}{2}. \end{aligned}$$

19. Since $Q = x^3 + 3$, solving for x gives

$$\begin{aligned} x^3 + 3 &= Q \\ x^3 &= Q - 3 \\ x &= (Q-3)^{1/3} \\ f^{-1}(Q) &= (Q-3)^{1/3} \end{aligned}$$

20. Since $y = \sqrt{t} + 1$, solving for t gives

$$\begin{aligned} \sqrt{t} + 1 &= y \\ \sqrt{t} &= y - 1 \\ t &= (y-1)^2 \\ g^{-1}(y) &= (y-1)^2. \end{aligned}$$

21. Since $P = 14q - 2$, solving for q gives

$$\begin{aligned} 14q - 2 &= P \\ q &= \frac{P+2}{14} \\ f^{-1}(P) &= \frac{P+2}{14}. \end{aligned}$$

22. **(a)** Since the vertical intercept of the graph of f is $(0, 2)$, we have $f(0) = 2$.
(b) Since the horizontal intercept of the graph of f is $(-3, 0)$, we have $f(-3) = 0$.
(c) The function f^{-1} goes from y-values to x-values, so to evaluate $f^{-1}(0)$, we want the x-value corresponding to $y = 0$. This is $x = -3$, so $f^{-1}(0) = -3$.
(d) Solving $f^{-1}(?) = 0$ means finding the y-value corresponding to $x = 0$. This is $y = 2$, so $f^{-1}(2) = 0$.

23. **(a)** Since the vertical intercept of the graph of f is $(0, b)$, we have $f(0) = b$.
(b) Since the horizontal intercept of the graph of f is $(a, 0)$, we have $f(a) = 0$.
(c) The function f^{-1} goes from y-values to x-values, so to evaluate $f^{-1}(0)$, we want the x-value corresponding to $y = 0$. This is $x = a$, so $f^{-1}(0) = a$.
(d) Solving $f^{-1}(?) = 0$ means finding the y-value corresponding to $x = 0$. This is $y = b$, so $f^{-1}(b) = 0$.

Problems

24. Since

$$n = \frac{A}{250},$$

solving for A gives

$$A = 250n.$$

Thus, $A = g(n) = 250n$.

25. Since $f(A) = A/250$ and $f^{-1}(n) = 250n$, we have

$$\begin{aligned} f^{-1}(f(A)) &= f^{-1}\left(\frac{A}{250}\right) = 250\frac{A}{250} = A. \\ f(f^{-1}(n)) &= f(250n) = \frac{250n}{250} = n. \end{aligned}$$

To interpret these results, we use the fact that $f(A)$ gives the number of gallons of paint needed to cover an area A, and $f^{-1}(n)$ gives the area covered by n gallons. Thus $f^{-1}(f(A))$ gives the area which can be covered by $f(A)$ gallons; that is, A square feet. Similarly, $f(f^{-1}(n))$ gives the number of gallons needed for an area of $(f^{-1}(n)$; that is, n gallons.

26. **(a)** To find values of f, read the table from top to bottom, so

(i) $f(0) = 2$ (ii) $f(1) = 0$.

To find values of f^{-1}, read the table in the opposite direction (from bottom to top), so

(iii) $f^{-1}(0) = 1$ (iv) $f^{-1}(2) = 0$.

(b) Since the values $(0, 2)$ are paired in the table, we know $f(0) = 2$ and $f^{-1}(2) = 0$. Thus, knowing the answer to (i) (namely, $f(0) = 2$) tells us the answer to (iv). Similarly, the answer to (ii), namely $f(1) = 0$, tells us that the values $(1, 0)$ are paired in the table, so $f^{-1}(0) = 1$ too.

27. The input values in the table of values of g^{-1} are the output values for g. See Table 2.7.

Table 2.7

y	7	12	13	19	22
$g^{-1}(y)$	1	2	3	4	5

28. We solve the equation $C = g(x) = 600 + 45x$ for x. Subtract 600 from both sides and divide both sides by 45 to get

$$x = \frac{1}{45}(C - 600).$$

So

$$g^{-1}(C) = \frac{1}{45}(C - 600).$$

29. We solve the equation $V = f(r) = \frac{4}{3}\pi r^3$ for r. Divide both sides by $\frac{4}{3}\pi$ and then take the cube root to get

$$r = \sqrt[3]{\frac{3V}{4\pi}}.$$

So

$$f^{-1}(V) = \sqrt[3]{\frac{3V}{4\pi}}.$$

30. **(a)** $f(10) = 100 + 0.2 \cdot 10 = 102$ thousand dollars, the cost of producing 10 kg of the chemical.

(b) $f^{-1}(200)$ is the quantity of the chemical which can be produced for 200 thousand dollars. Since

$$\begin{aligned} 200 &= 100 + 0.2q \\ 0.2q &= 100 \\ q &= \frac{100}{0.2} = 500 \text{ kg}, \end{aligned}$$

we have $f^{-1}(200) = 500$.

(c) To find $f^{-1}(C)$, solve for q:

$$\begin{aligned} C &= 100 + 0.2q \\ 0.2q &= C - 100 \\ q &= \frac{C}{0.2} - \frac{100}{0.2} = 5C - 500 \\ f^{-1}(C) &= 5C - 500. \end{aligned}$$

31. **(a)** $f(3) = 4 \cdot 3 = 12$ is the perimeter of a square of side 3.

(b) $f^{-1}(20)$ is the side of a square of perimeter 20. If $20 = 4s$, then $s = 5$, so $f^{-1}(20) = 5$.

(c) To find $f^{-1}(P)$, solve for s:

$$\begin{aligned} P &= 4s \\ s &= \frac{P}{4} \\ f^{-1}(P) &= \frac{P}{4}. \end{aligned}$$

32. **(a)** $G(11)$ is the output corresponding to the input of $t = 11$. So $G(11)$ represents the GDP for the year 2001.

(b) The input to the G^{-1} function is billions of dollars, so its output is a time in years after 1990. Thus, $G^{-1}(9873)$ represents the number of years after 1990 at which the GDP is 9873 billion dollars.

33. (a) To write s as a function of A, we solve $A = 6s^2$ for s

$$s^2 = \frac{A}{6} \qquad \text{so} \qquad s = f(A) = +\sqrt{\frac{A}{6}} \qquad \text{Because the length of a side of a cube is positive.}$$

The function f gives the side of a cube in terms of its area A.

(b) Substituting $s = f(A) = \sqrt{A/6}$ in the formula $V = g(s) = s^3$ gives the volume, V, as a function of surface area, A,

$$V = g(f(A)) = s^3 = \left(\sqrt{\frac{A}{6}}\right)^3.$$

34. Since $n = f(A)$, in $f(100)$ we have $A = 100$ ft^2. Evaluating $f(100)$ tells us how much paint is needed for 100 ft^2. Since

$$n = f(100) = \frac{100}{250} = 0.4,$$

we know that it takes 0.4 gallon of paint to cover 100 ft^2.

In $f^{-1}(100)$, the 100 is the number of gallons, so $f^{-1}(100)$ represents the area which can be painted by 100 gallons:

$$A = f^{-1}(100) = 250 \cdot 100 = 25{,}000 \text{ ft}^2.$$

35. (a) The cost of producing 5000 loaves is \$653.

(b) $C^{-1}(80)$ is the number of loaves of bread that can be made for \$80, namely 0.62 thousand or 620.

(c) The solution is $q = 6.3$ thousand. It costs \$790 to make 6300 loaves.

(d) The solution is $x = 150$ dollars, so 1.2 thousand, or 1200, loaves can be made for \$150.

36. (a) Since the deck is square, $f(s) = s^2$.

(b) Since a can costs \$29.50 and covers 200 ft^2, we know

$$\text{Cost of stain for 1 ft}^2 = \frac{29.50}{200} = 0.1475 \text{ dollars.}$$

Thus

$$\text{Cost of stain for } A \text{ ft}^2 = 0.1475A \text{ dollars,}$$

so

$$C = g(A) = 0.1475A.$$

(c) Substituting for $A = f(s) = s^2$ into g gives

$$C = g(f(s)) = 0.1475s^2.$$

The function $g(f(s))$ gives the cost in dollars of staining a square deck of side s feet.

(d) (i) $f(8) = 8^2 = 64$ square feet; the area of a deck of side 8 feet.

(ii) $g(80) = 0.1475 \cdot 80 = 11.80$ dollars; the cost of staining a deck of area 80 ft^2.

(iii) $g(f(10)) = 0.1475 \cdot 10^2 = 14.75$ dollars; the cost of staining a deck of side 10 feet.

37. Since $V = \frac{4}{3}\pi r^3$ and $r = 50 - 2.5t$, substituting r into V gives

$$V = f(t) = \frac{4}{3}\pi(50 - 2.5t)^3.$$

38. Since the oil slick is circular, $A = \pi r^2$, so substituting $r = 2t - 0.1t^2$ into the formula for A gives

$$A = f(t) = \pi(2t - 0.1t^2)^2.$$

39. (a) Since $t = 0$ represents 1998, we see that $0 \le t \le 5$ is the domain. The corresponding outputs are the range, $365 \le C(t) \le 375$.

(b) $C(4)$ is the concentration of carbon dioxide in the earth's atmosphere when $t = 4$ or 2002.

(c) $C^{-1}(370)$ is the number of years after 1998 when the concentration was 370 ppm. From the data given in the problem, the actual number of years cannot be determined.

40. **(a)** This is the fare for a ride of 3.5 miles. $C(3.5) \approx \$6.25$.
(b) This is the number of miles you can travel for \$3.50. Between 1 and 2 miles the increase in cost is \$1.50. Setting up a proportion we have:

$$\frac{1 \text{ additional mile}}{\$1.50 \text{ additional fare}} = \frac{x \text{ additional miles}}{\$3.50 - \$2.50 \text{ additional fare}}$$

and $x = 0.67$ miles. Therefore

$$C^{-1}(\$3.5) \approx 1.67.$$

41. **(a)** $P = f(s) = 4s$.
(b) $f(s+4) = 4(s+4) = 4s + 16$. This the perimeter of a square whose side is four meters larger than s.
(c) $f(s) + 4 = 4s + 4$. This is the perimeter of a square whose side is s, plus four meters.
(d) Meters.

Solutions for Section 2.5

Exercises

1. To determine concavity, we calculate the rate of change:

$$\frac{\Delta f(x)}{\Delta x} = \frac{1.3 - 1.0}{1 - 0} = 0.3$$
$$\frac{\Delta f(x)}{\Delta x} = \frac{1.7 - 1.3}{3 - 1} = 0.2$$
$$\frac{\Delta f(x)}{\Delta x} = \frac{2.2 - 1.7}{6 - 3} \approx 0.167.$$

The rates of change are decreasing, so we expect the graph of $f(x)$ to be concave down.

2. To determine concavity we calculate the rate of change:

$$\frac{\Delta f(t)}{\Delta t} = \frac{10 - 20}{1 - 0} = -10.$$
$$\frac{\Delta f(t)}{\Delta t} = \frac{6 - 10}{2 - 1} = -4.$$
$$\frac{\Delta f(t)}{\Delta t} = \frac{3 - 6}{3 - 2} = -3.$$
$$\frac{\Delta f(t)}{\Delta t} = \frac{1 - 3}{4 - 3} = -2.$$

It appears that a graph of this function would be concave up, because the average rate of change becomes less negative as t increases.

3. The graph appears to be concave up, as its slope becomes less negative as x increases.

4. The graph appears to be concave down, as its slope becomes less positive as x increases.

5. The slope of $y = x^2$ is always increasing, so its graph is concave up. See Figure 2.39.

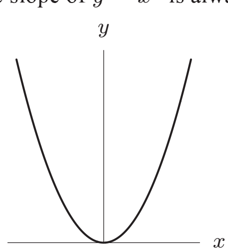


Figure 2.39

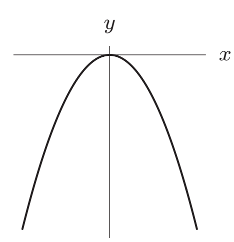


Figure 2.40

6. The slope of $y = -x^2$ is always decreasing, so its graph is concave down. See Figure 2.40.

7. The slope of $y = x^3$ is always increasing on the interval $x > 0$, so its graph is concave up. See Figure 2.41.

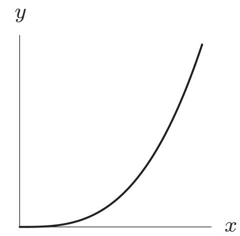


Figure 2.41

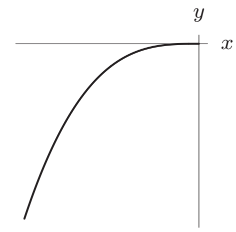


Figure 2.42

8. The slope of $y = x^3$ is always decreasing on the interval $x < 0$, so its graph is concave down. See Figure 2.42.

9. The rate of change between $t = 0.2$ and $t = 0.4$ is

$$\frac{\Delta p(t)}{\Delta t} = \frac{-2.32 - (-3.19)}{0.4 - 0.2} = 4.35.$$

Similarly, we have

$$\frac{\Delta p(t)}{\Delta t} = \frac{-1.50 - (-2.32)}{0.6 - 0.4} = 4.10$$
$$\frac{\Delta p(t)}{\Delta t} = \frac{-0.74 - (-1.50)}{0.8 - 0.6} = 3.80.$$

Thus, the rate of change is decreasing, so we expect the graph to be concave down.

10. The rate of change between $x = 12$ and $x = 15$ is

$$\frac{\Delta H(x)}{\Delta x} = \frac{21.53 - 21.40}{15 - 12} \approx 0.043.$$

Similarly, we have

$$\frac{\Delta H(x)}{\Delta x} = \frac{21.75 - 21.53}{18 - 15} \approx 0.073$$
$$\frac{\Delta H(x)}{\Delta x} = \frac{22.02 - 21.75}{21 - 18} \approx 0.090.$$

The rate of change is increasing, so we expect the graph of $H(x)$ to be concave up.

11. A possible graph is in Figure 2.43.

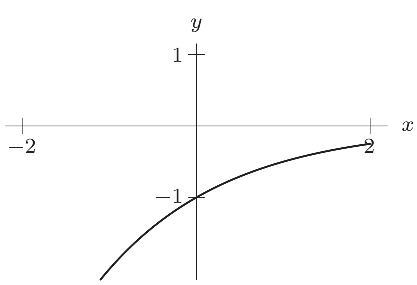


Figure 2.43

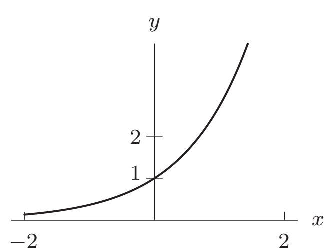


Figure 2.44

12. A possible graph is in Figure 2.44.

Problems

13. This function is increasing throughout and the rate of increase is increasing, so the graph is concave up.

14. This function is decreasing. As the coffee cools off, the temperature decreases at a slower rate. Since the rate of change is less negative, the graph is concave up.

15. The function is increasing throughout. At first, the graph is concave up. As more and more people hear the rumor, the rumor spreads more slowly, which means that the graph is then concave down.

16. Since more and more of the drug is being injected into the body, this is an increasing function. However, since the rate of increase of the drug is slowing down, the graph is concave down.

17. Since new people are always trying the product, it is an increasing function. At first, the graph is concave up. After many people start to use the product, the rate of increase slows down and the graph becomes concave down.

18. **(a)** This describes a situation in which y is increasing rapidly at first, then very slowly at the end. In Table (E), y increases dramatically at first (from 20 to 275) but is hardly growing at all by the end. In Graph (I), y is increasing at a constant rate, while in Graph (II), it is increasing faster at the end. Graph (III) increases rapidly at first, then slowly at the end. Thus, scenario (a) matches with Table (E) and Graph (III).

(b) Here, y is growing at a constant rate. In Table (G), y increases by 75 units for every 5-unit increase in x. A constant increase in y relative to x means a straight line, that is, a line with a constant slope. This is found in Graph (I).

(c) In this scenario, y is growing at a faster and faster rate as x gets larger. In Table (F), y starts out by growing by 16 units, then 30, then 54, and so on, so Table (F) refers to this case. In Graph (II), y is increasing faster and faster as x gets larger.

19. **(a)** This is a case in which the rate of decrease is constant, i.e., the change in y divided by the change in x is always the same. We see this in Table (F), where y decreases by 80 units for every decrease of 1 unit in x, and graphically in Graph (IV).

(b) Here, the change in y gets smaller and smaller relative to corresponding changes in x. In Table (G), y decreases by 216 units for a change of 1 unit in x initially, but only decreases by 6 units when x changes by 1 unit from 4 to 5. This is seen in Graph (I), where y is falling rapidly at first, but much more slowly for longer values of x.

(c) If y is the distance from the ground, we see in Table (E) that initially it is changing very slowly; by the end, however, the distance from the ground is changing rapidly. This is shown in Graph (II), where the decrease in y is larger and larger as x gets bigger.

(d) Here, y is decreasing quickly at first, then decreases only slightly for a while, then decreases rapidly again. This occurs in Table (H), where y decreases from 147 units, then 39, and finally by another 147 units. This corresponds to Graph (III).

20. The graphical representation of the data is misleading because in the graph the number of violent crimes is put on the horizontal axis which give the graph the appearance of leveling out. This can fool us into believing that crime is leveling out. Note that it took from 1998 to 2000, about 2 years, for the number of violent crimes to go from 500 to 1,000, but it took less than 1/2 a year for that number to go from 1,500 to 2,000, and even less time for it to go from 2,000 to 2,500. In actuality, this graph shows that crime is growing at an increasing rate. If we were to graph the number of crimes as a function of the year, the graph would be concave up.

21. **(a)** From O to A, the rate is zero, so no water is flowing into the reservoir, and the volume remains constant. From A to B, the rate is increasing, so the volume is going up more and more quickly. From B to C, the rate is holding steady, but water is still going into the reservoir—it's just going in at a constant rate. So volume is increasing on the interval from B to C. Similarly, it is increasing on the intervals from C to D and from D to E. Even on the interval from E to F, water is flowing into the reservoir; it is just going in more and more slowly (the *rate* of flow is decreasing, but the total amount of water is still increasing). So we can say that the volume of water increases throughout the interval from A to F.

(b) The volume of water is constant when the rate is zero, that is from O to A.

(c) According to the graph, the rate at which the water is entering the reservoir reaches its highest value at $t = D$ and stays at that high value until $t = E$. So the volume of water is increasing most rapidly from D to E. (Be careful. The rate itself is increasing most rapidly from C to D, but the volume of water is increasing fastest when the rate is at its highest points.)

(d) When the rate is negative, water is leaving the reservoir, so its volume is decreasing. Since the rate is negative from F to I, we know that the volume of water *decreases* on that interval.

22. (a) If l is the length of one salmon and its speed is u, then

$$u = 19.5\sqrt{l}.$$

Suppose the speed of the longer salmon is U and its length is $4l$. Then

$$U = 19.5\sqrt{4l} = 2 \cdot 19.5\sqrt{l} = 2u.$$

Thus, the larger one swims twice as fast as the smaller one.

(b) A typical graph is in Figure 2.45. Notice that the graphs are all of the shape of $y = \sqrt{x} = x^{1/2}$. All the graphs are increasing and concave down.

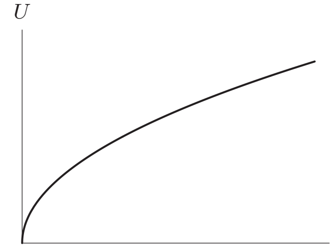


Figure 2.45

(c) The function $U = \sqrt{l}$ is an increasing function. Because $\sqrt{l}$ is an increasing function the equation predicts that larger salmon swim faster than smaller ones.

(d) The graph of $U = \sqrt{l}$ is concave down. Because the graph is concave down equal changes in l give smaller changes in U for the larger l. Thus, the difference in speed between the two smaller fish is greater than the difference in speed between the two larger fish.

Solutions for Section 2.6

Exercises

1. Yes. We rewrite the function giving

$$\begin{aligned} f(x) &= 2(7-x)^2 + 1 \\ &= 2(49 - 14x + x^2) + 1 \\ &= 98 - 28x + 2x^2 + 1 \\ &= 2x^2 - 28x + 99. \end{aligned}$$

So $f(x)$ is quadratic with $a = 2$, $b = -28$ and $c = 99$.

2. Yes.We rewrite the function giving

$$L(P) = (P+1)(1-P) = 1 - P^2 = (-1)P^2 + 0 \cdot P + 1.$$

So $L(P)$ is quadratic with $a = -1$, $b = 0$ and $c = 1$.

3. Yes.We rewrite the function giving

$$\begin{aligned} g(m) &= m(m^2 - 2m) + 3\left(14 - \frac{m^3}{3}\right) + \sqrt{3}m \\ &= m^3 - 2m^2 + 42 - m^3 + \sqrt{3}m \\ &= -2m^2 + \sqrt{3}m + 42. \end{aligned}$$

So $g(m)$ is quadratic with $a = -2$, $b = \sqrt{3}$ and $c = 42$.

4. Yes.We rewrite the function giving

$$h(t) = -16(t-3)(t+1) = -16(t^2-2t-3) = -16t^2+32t+48$$

So $h(t)$ is quadratic with $a=-16$, $b=32$ and $c=48$.

5. No. We rewrite the function giving

$$\begin{aligned} R(q) &= \frac{1}{q^2}(q^2+1)^2 \\ &= \frac{1}{q^2}(q^4+2q^2+1) \\ &= q^2+2+\frac{1}{q^2} \\ &= q^2+2+q^{-2}. \end{aligned}$$

So $R(q)$ is not quadratic since it contains a term with q to a negative power.

6. No. The function $K(x)$ is not quadratic since the term 13^x has the variable in the exponent.

7. Yes.We rewrite the function giving

$$T(n) = \sqrt{5}+\sqrt{3n^4}-\sqrt{\frac{n^4}{4}} = \sqrt{5}+\sqrt{3}n^2-\frac{n^2}{2} = \left(\sqrt{3}-\frac{1}{2}\right)n^2+\sqrt{5}$$

So $T(n)$ is quadratic with $a=\sqrt{3}-1/2$, $b=0$ and $c=\sqrt{5}$.

8. We solve for r in the equation by factoring

$$\begin{aligned} 2r^2-6r-36 &= 0 \\ 2(r^2-3r-18) &= 0 \\ 2(r-6)(r+3) &= 0. \end{aligned}$$

The solutions are $r=6$ and $r=-3$.

9. We solve for x in the equation $5x-x^2+3=0$ using the quadratic formula with $a=-1$, $b=5$ and $c=3$.

$$\begin{aligned} x &= \frac{-5\pm\sqrt{(-5)^2-4(-1)3}}{2(-1)} \\ x &= \frac{-5\pm\sqrt{37}}{-2}. \end{aligned}$$

The solutions are $x\approx-0.541$ and $x\approx5.541$.

10. The function $f(x)=(x-1)(x-2)$ has zeros $x=1$ and $x=2$. To get another function with the same zeros, we can multiply $f(x)$ by any constant: for example, let $g(x)=-7(x-1)(x-2)$. In general, any function of the form $y=a(x-1)9x-2)$ will do.

11. **(a)** Rewriting $6x-\frac{1}{3}=3x^2$ in the form $ax^2+bx+c=0$, we get $3x^2-6x+\frac{1}{3}=0$, as shown in Figure 2.46. Applying the quadratic formula, we obtain

$$\begin{aligned} x &= \frac{6\pm\sqrt{6^2-4\cdot3\cdot\frac{1}{3}}}{2\cdot3} \\ x &= \frac{6\pm\sqrt{36-4}}{6} = \frac{6\pm\sqrt{32}}{6} \\ x &= 1\pm\frac{4\sqrt{2}}{6} \\ x &\approx 0.057 \quad \text{or} \quad x\approx1.943. \end{aligned}$$

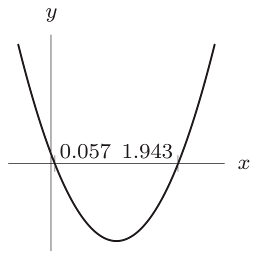


Figure 2.46: The graph of $y = 3x^2 - 6x + \frac{1}{3}$ crosses x-axis at $x \approx 0.057$, and $x \approx 1.943$

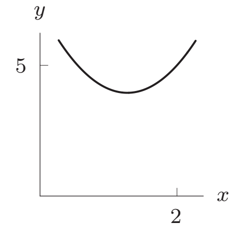


Figure 2.47: The graph of $y = 2x^2 - 5.1x + 7.2$ does not cross the x-axis

(b) Rewriting $2x^2 + 7.2 = 5.1x$ in the form $ax^2 + bx + c = 0$, we get $2x^2 - 5.1x + 7.2 = 0$. Applying the quadratic formula, we obtain

$$x = \frac{5.1 \pm \sqrt{5.1^2 - 4 \cdot 2 \cdot 7.2}}{2 \cdot 2}.$$

Notice that $5.1^2 - 4 \cdot 2 \cdot 7.2 = -31.59$, so the number under the square root sign is negative. Thus, there are no real solutions. (See Figure 2.47.)

12. Factoring gives $y = 3x^2 - 16x - 12 = (3x + 2)(x - 6)$. So the zeros are $x = -\frac{2}{3}$ and $x = 6$, the axis of symmetry is halfway between the zeros at $x = \dfrac{-\frac{2}{3} + 6}{2} = \dfrac{8}{3}$. The y-coordinates of the vertex is

$$y = 3\left(\frac{8}{3}\right)^2 - 16\left(\frac{8}{3}\right) - 12 = -\frac{100}{3}$$

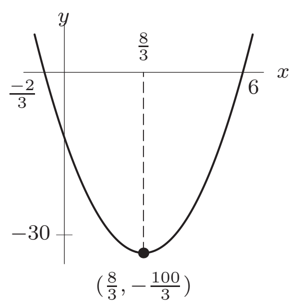


Figure 2.48: $y = 3x^2 - 16x - 12$

13. Setting the factors equal to zero, we have

$$\begin{aligned} 2 - x &= 0 \\ x &= 2 \\ \text{and} \quad 3 - 2x &= 0 \\ x &= 3/2, \end{aligned}$$

so the zeros are $x = 2, 3/2$.

14. To find the zeros, we solve the equation

$$0 = 2x^2 + 5x + 2.$$

We see that this is factorable, as follows:

$$y = (2x + 1)(x + 2).$$

Therefore, the zeros occur where $x = -2$ and $x = -\frac{1}{2}$.

15. To find the zeros, we solve the equation

$$0 = 4x^2 - 4x - 8.$$

We see that this is factorable, as follows:

$$\begin{aligned} 0 &= 4(x^2 - x - 2) \\ 0 &= 4(x-2)(x+1). \end{aligned}$$

Therefore, the zeros occur where $x = 2$ and $x = -1$.

16. To find the zeros, we solve the equation

$$0 = 7x^2 + 16x + 4.$$

We see that this is factorable, as follows:

$$0 = (7x+2)(x+2).$$

Therefore, the zeros occur where $x = -\frac{2}{7}$ and $x = -2$.

17. To find the zeros, we solve the equation

$$0 = 9x^2 + 6x + 1.$$

We see that this is factorable, as follows:

$$\begin{aligned} y &= (3x+1)(3x+1) \\ y &= (3x+1)^2. \end{aligned}$$

Therefore, there is only one zero at $x = -\frac{1}{3}$.

18. This can be factored as follows:

$$\begin{aligned} 6x^2 - 17x + 12 &= 6x^2 \underbrace{-8x - 9x}_{-17x} + 12 \\ &= 2x(3x-4) - 3(3x-4) \\ &= (2x-3)(3x-4). \end{aligned}$$

Setting these factors equal to zero, we find that the zeros of this function are $x = 3/2, 4/3$.

19. Using the quadratic formula, we have

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - 4(5)(-1)}}{2(5)} \\ &= \frac{-2 \pm \sqrt{24}}{10} \\ &= \frac{-1 \pm \sqrt{6}}{5}. \end{aligned}$$

20. Using the quadratic formula, we have

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(6)}}{2(3)} \\ &= \frac{2 \pm \sqrt{-68}}{6}, \end{aligned}$$

so there are no real-valued zeros.

21. To find the zeros, we solve the equation $0 = -17x^2 + 23x + 19$. This does not appear to be factorable. Thus, we use the quadratic formula with $a = -17$, $b = 23$, and $c = 19$:

$$x = \frac{-23 \pm \sqrt{23^2 - 4(-17)(19)}}{2(-17)}$$
$$x = \frac{-23 \pm \sqrt{1821}}{-34}.$$

Therefore, the zeros occur where $x = (-23 \pm \sqrt{1821})/(-34) \approx 1.932$ or -0.579.

22. To find the zeros, we solve the equation $0 = 89x^2 + 55x + 34$. This does not appear to be factorable. Thus, we use the quadratic formula with $a = 89$, $b = 55$, and $c = 34$:

$$x = \frac{-55 \pm \sqrt{55^2 - 4(89)(34)}}{2(89)}$$
$$x = \frac{-55 \pm \sqrt{-9079}}{178}$$

Since $\sqrt{-9079}$ is undefined, there are no zeros.

23. Letting $z = x^2$, we have $y = z^2 + 5z + 6$. This can be factored, giving

$$\begin{aligned} y &= (z+2)(z+3) \\ &= (x^2+2)(x^2+3). \end{aligned}$$

Setting the factors equal to zero, we have $x^2 + 2 = 0$, which has no solution, and $x^2 + 3 = 0$, which also has no solution, so this function has no real-valued zeros. Another way to see this is to notice that both x^4 and $5x^2$ are either positive or 0, so y can not be less than 6.

24. Letting $z = \sqrt{x}$, we have $z^2 = x$, which gives $y = z^2 - z - 12$. Factoring, we have

$$\begin{aligned} y &= (z-4)(z+3) \\ &= (\sqrt{x}-4)(\sqrt{x}+3). \end{aligned}$$

Setting the factors equal to zero, we have $\sqrt{x} = 4$, so $x = 16$, and $\sqrt{x} = -3$, so $x = 9$. Checking our answers, we have $16 - \sqrt{16} - 12 = 16 - 4 - 12 = 0$, so $x = 16$ is a zero of the original function. However, we also see that $9 - \sqrt{9} - 12 = 9 - 3 - 12 = -6$, so $x = 9$ is not a zero of the original function. Thus, $x = 16$ is the only zero.

Problems

25. We solve the equation $f(t) = -16t^2 + 64t + 3 = 0$ using the quadratic formula

$$\begin{aligned} -16t^2 + 64t + 3 &= 0 \\ t &= \frac{-64 \pm \sqrt{64^2 - 4(-16)3}}{2(-16)}. \end{aligned}$$

Evaluating gives $t = -0.046$ sec and $t = 4.046$ sec; the value $t = 4.046$ sec is the time we want. The baseball hits the ground 4.046 sec after it was hit.

26. No, there is not. The shape of a non-trivial quadratic function is a parabola, and a parabola cannot intersect the x-axis more than twice, whereas a function with zeros $x = 1$, $x = 2$, and $x = 3$ would intersect the x-axis three times.

27. Between $x = -1$ and $x = 1$

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{(4 - 1^2) - (4 - (-1)^2)}{2} = 0.$$

Between $x = 1$ and $x = 3$

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(3) - f(1)}{3 - 1} = \frac{(4 - 3^2) - (4 - 1^2)}{2} = -4.$$

Between $x = 3$ and $x = 5$

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(5) - f(3)}{5 - 3} = \frac{(4 - 5^2) - (4 - 3^2)}{2} = -8.$$

Since rates of change are decreasing, the graph of $f(x)$ is concave down.

28. For example, we can use $y = (x+2)(x-3)$. See Figure 2.49.

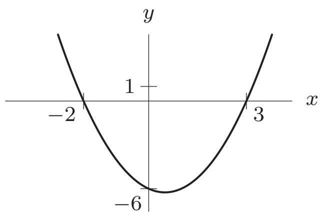


Figure 2.49

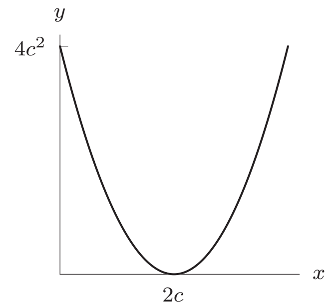


Figure 2.50: $y = -4cx + x^2 + 4c^2$ for $c > 0$

29. Factoring gives $y = -4cx + x^2 + 4c^2 = x^2 - 4ck + 4c^2 = (x-2c)^2$. Since $c > 0$, this is the graph of $y = x^2$ shifted to the right $2c$ units. See Figure 2.50.

30. **(a)** $h(2) = 80(2) - 16(2)^2 = 160 - 64 = 96$. This means that after 2 seconds, the ball's height is 96 feet.

(b) $h(t) = 80$ has 2 solutions, as you can see from Figure 2.51. One way to find these solutions is by using a graphing calculator. Another way is to solve

$$80t - 16t^2 = 80$$
$$16t^2 - 80t + 80 = 0.$$

Divide both sides of the equation by 16:

$$t^2 - 5t + 5 = 0.$$

Use the quadratic formula

$$t = \frac{5 \pm \sqrt{25 - 4 \cdot 5}}{2} = \frac{5 \pm \sqrt{5}}{2}.$$

The solutions are $t \approx 1.382$ and $t \approx 3.618$. This means that the ball reaches the height of 80 ft once on the way up, after approximately 1.382 seconds, and once on the way down, after 3.618 seconds.

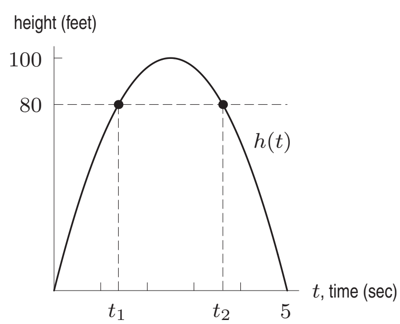


Figure 2.51

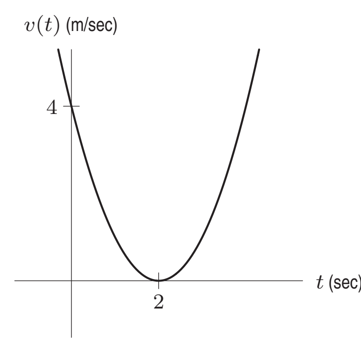


Figure 2.52

31. **(a)** The initial velocity is the velocity when $t = 0$. So $v(0) = 0^2 - 4 \cdot 0 + 4 = 4$ meters per second.

(b) The object is not moving when its velocity is zero. This time is found by factoring $t^2 - 4t + 4 = (t-2)^2 = 0$. The solution, $t = 2$, tells us that the object is not moving at 2 seconds.

(c) From the graph of the velocity function in Figure 2.52, we can see that it is concave up.

32. To show that the data in the table is approximated by the formula $p(x) = -0.8x^2 + 8.8x + 7.2$, we substitute $x = 0, 1, 2, 3, 4$ (for years 1992-1996) into the formula:

$$p(0) = 7.2,\ p(1) = 15.2,\ p(2) = 21.6,\ p(3) = 26.4,\ p(4) = 29.6.$$

Our results approximate the table. In the year 2004, $x = 12$, and $p(12) = -2.4$, so the model predicts -2.4% of schools will have videodisc players in 2004. This is a reasonable model for the period 1992 to 1996, but not for the year 2004 since -2.4% does not make sense. Since the x^2 term in $p(x)$ has a negative coefficient, as x increases beyond 12, the values of $p(x)$ become more negative, and so are not reasonable predictions for the percentage of schools with a videodisc player. Thus, $p(x)$ is not a good model for predicting the future.

33. **(a)** In this window we see the expected parabolic shapes of $f(x)$ and $g(x)$. Both graphs open upward, so their shapes are similar and the end behaviors are the same. The differences in $f(x)$ and $g(x)$ are apparent at their vertices and intercepts. The graph of $f(x)$ has one intercept at $(0, 0)$. The graph of $g(x)$ has x-intercepts at $x = -4$ and $x = 2$, and a y-intercept at $y = -8$. See Figure 2.53.

(b) As we extend the range to $y = 100$, the difference between the y-intercepts for $f(x)$ and $g(x)$ becomes less significant. See Figure 2.54.

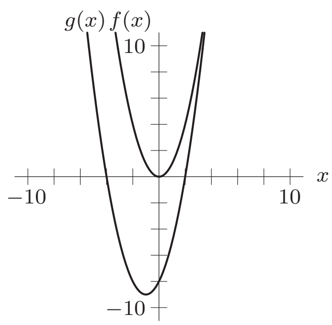


Figure 2.53

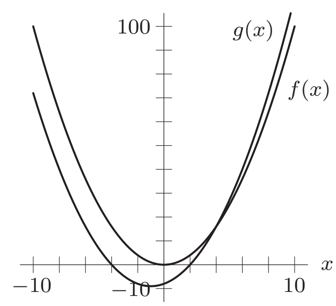


Figure 2.54

(c) In the window $-20 \le x \le 20, -10 \le y \le 400$, the graphs are still distinguishable from one another, but all intercepts appear much closer. In the next window, the intercepts appear the same for $f(x)$ and $g(x)$. Only a thickening along the sides of the parabola gives the hint of two functions. In the last window, the graphs appear identical. See Figure 2.55.

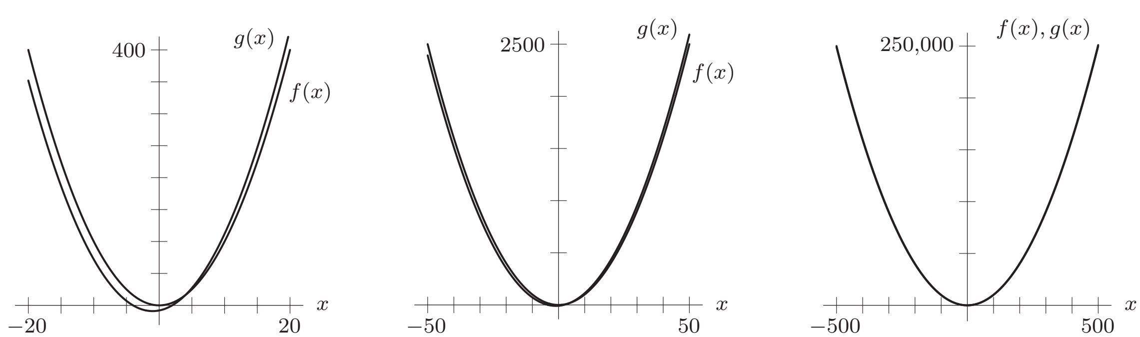


Figure 2.55

34. (a) According to the figure in the text, the package was dropped from a height of 5 km.

(b) When the package hits the ground, $h = 0$ and $d = 4430$. So, the package has moved 4430 meters forward when it lands.

(c) Since the maximum is at $d = 0$, the parabola is of the form $h = ad^2 + b$. Since $h = 5$ at $d = 0$, $5 = a(0)^2 + b = b$, so $b = 5$. We now know that $h = ad^2 + 5$. Since $h = 0$ when $d = 4430$, we have $0 = a(4430)^2 + 5$, giving $a = \dfrac{-5}{(4430)^2} \approx -0.000000255$. So $h \approx -0.000000255d^2 + 5$.

35. (a) Since the function is quadratic, we take $y = ax^2+bx+c$. We know that $y = 1$ when $x = 0$, so $y = a(0)^2+b(0)+c = 1$. Thus, $c = 1$ and the formula is $y = ax^2 + bx + 1$. We know that $y = 3.01$ when $x = 1$, which gives us $y = a(1)^2 + b(1) + 1 = 3.01$, so $a + b + 1 = 3.01$, or $b = 2.01 - a$. Similarly, $y = 5.04$ when $x = 2$ suggests that $y = a(2)^2 + b(2) + 1 = 5.04$, which simplifies to $4a + 2b = 4.04$, or $2a + b = 2.02$. In this case, $b = 2.02 - 2a$. Since $b = 2.01 - a$ and $b = 2.02 - 2a$, we can say that

$$\begin{aligned} 2.01 - a &= 2.02 - 2a \\ a &= 0.01 \end{aligned}$$

and

$$b = 2.01 - a = 2.01 - 0.01 = 2.$$

Since $a = 0.01$, $b = 2$, and $c = 1$, we know that

$$y = 0.01x^2 + 2x + 1$$

is the quadratic function passing through the first three data points. If $x = 50$, then

$$y = 0.01(50)^2 + 2(50) + 1 = 126,$$

so the fifth data point also satisfies the quadratic model.

(b) A linear function through (1, 3.01) and (2, 5.04) has slope $m = \frac{5.04-3.01}{2-1} = \frac{2.03}{1} = 2.03$, so $y = 2.03x + b$. We combine this with the knowledge that $(1, 3.01)$ lies on the line to get

$$\begin{aligned} 3.01 &= 2.03(1) + b \\ 3.01 &= 2.03 + b \end{aligned}$$

so

$$b = 0.98.$$

Thus, a linear model using just the second two data points is

$$y = 2.03x + 0.98.$$

(c) Using this linear function, when $x = 3$,

$$y = 2.03(3) + 0.98 = 7.07.$$

Since the value of the quadratic function at $x = 3$ is 7.09, the difference between the quadratic and linear models at $x = 3$ is

$$7.09 - 7.07 = 0.02.$$

(d) At $x = 50$, the linear function has a value of $y = 2.03(50) + 0.98 = 102.48$. The quadratic function gives 126 when $x = 50$. The difference in output is $126 - 102.48 = 23.52$.

(e) If we want the difference to be less than 0.05, we want

$$|(0.01x^2 + 2x + 1) - (2.03x + 0.98)| = |0.01x^2 - 0.03x + 0.02| \leq 0.05.$$

Using a graphing calculator or computer, we graph $y = 0.01x^2 - 0.03x + 0.02$ and look for the values of x for which $|y| \leq 0.05$. These occur when x is between -0.791 and 3.791.

Solutions for Chapter 2 Review

Exercises

1. To evaluate $p(7)$, we substitute 7 for each r in the formula:

$$p(7) = 7^2 + 5 = 54.$$

2. To evaluate $p(x) + p(8)$, we substitute x and 8 for each r in the formula and add the two expressions:

$$p(x) + p(8) = (x^2 + 5) + (8^2 + 5) = x^2 + 74.$$

3. **(a)** $h(1) = (1)^2 + b(1)^2 + c = b + c + 1$
(b) Substituting $b + 1$ for x in the formula for $h(x)$:

$$\begin{aligned} h(b+1) &= (b+1)^2 + b(b+1) + c \\ &= (b^2 + 2b + 1) + b^2 + b + c \\ &= 2b^2 + 3b + c + 1 \end{aligned}$$

4. **(a)** $g(100) = 100\sqrt{100} + 100 \cdot 100 = 100 \cdot 10 + 100 \cdot 100 = 11,000$
(b) $g(4/25) = 4/25 \cdot \sqrt{4/25} + 100 \cdot 4/25 = 4/25 \cdot 2/5 + 16 = 8/125 + 16 = 16.064$
(c) $g(1.21 \cdot 10^4) = g(12100) = (12100)\sqrt{12100} + 100 \cdot (12100) = 2,541,000$

5. We solve for l in the equation $7l - l^2 = 0$ by factoring to obtain $l(7 - l) = 0$, so $l = 0$ and $l = 7$ are the zeros.

6. **(a)** If $x = a$ is not in the domain of f there is no point on the graph with x-coordinate a. For example, there are no points on the graph of the function in Figure 2.56 with x-coordinates greater than 2. Therefore, $x = a$ is not in the domain of f for any $a > 2$.

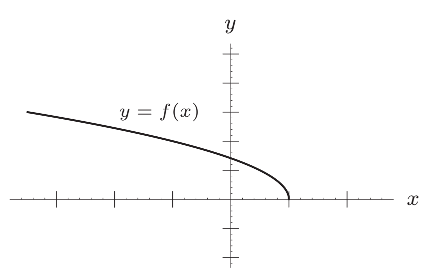


Figure 2.56

(b) If $x = a$ is not in the domain of f the formula is undefined for $x = a$. For example, if $f(x) = 1/(x - 3)$, $f(3)$ is undefined, so 3 is not in the domain of f.

7. Since $h(x)$ is defined for any value that we might choose for x, the domain of $h(x)$ consists of all real numbers. Using a graphing calculator or computer, we get the graph in Figure 2.57. We see that $h(x)$ is an upward-opening curve whose lowest point is $(-4, -16)$. Thus, the domain of $y = h(x)$ is all the real numbers and the range is $h(x) \geq -16$.

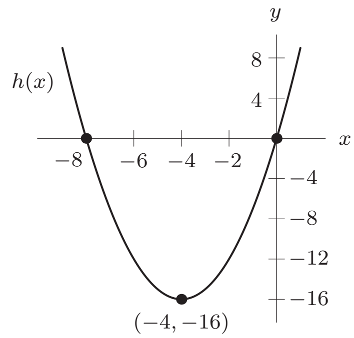


Figure 2.57

8. The square root of a negative number is undefined, and so x must not be less than 4, but it can have any value greater than or equal to 4. Since $f(4) = 0$, and $f(x)$ increases as x increases, $f(x)$ is greater than or equal to zero. Thus, the domain of $f(x)$ is $x \geq 4$, and the range is $f(x) \geq 0$.

9. We know that $x \geq 4$, for otherwise $\sqrt{x-4}$ would be undefined. We also know that $4 - \sqrt{x-4}$ must not be negative. Thus we have

$$\begin{aligned} 4 - \sqrt{x-4} &\geq 0 \\ 4 &\geq \sqrt{x-4} \\ 4^2 &\geq \left(\sqrt{x-4}\right)^2 \\ 16 &\geq x - 4 \\ 20 &\geq x. \end{aligned}$$

Thus, the domain of $r(x)$ is $4 \leq x \leq 20$.

We use a computer or graphing calculator to find the range of $r(x)$. Graphing over the domain of $r(x)$ gives Figure 2.58.

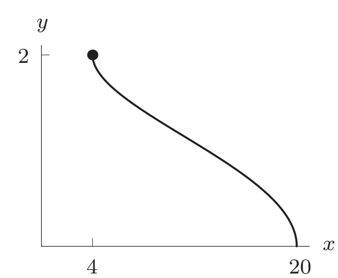


Figure 2.58

Because $r(x)$ is a decreasing function, we know that the maximum value of $r(x)$ occurs at the left end point of the domain, $r(4) = \sqrt{4 - \sqrt{4-4}} = \sqrt{4-0} = 2$, and the minimum value of $r(x)$ occurs at the right end point, $r(20) = \sqrt{4 - \sqrt{20-4}} = \sqrt{4 - \sqrt{16}} = \sqrt{4-4} = 0$. The range of $r(x)$ is thus $0 \leq r(x) \leq 2$.

10. Since x^2 is always greater than or equal to zero, $4/(4+x^2)$ is defined for all real numbers. So the domain of $g(x)$ is the real numbers.

As the denominator gets larger, the whole fraction gets smaller, so the maximum value of the function occurs when the denominator is smallest. Since $4 + x^2$ is smallest when $x = 0$, the maximum value of the function is $4/(4+0^2) = 1$. As x grows larger or becomes more and more negative, the denominator gets very large and the whole fraction gets closer and closer to zero.

(Note that because both its numerator and denominator are positive, $4/(4+x^2)$ is always greater than zero.) So the range of this function is $0 < g(x) \leq 1$.

11. **(a)** $-3g(x) = -3(x^2 + x)$.
(b) $g(1) - x = (1^2 + 1) - x = 2 - x$.
(c) $g(x) + \pi = (x^2 + x) + \pi = x^2 + x + \pi$.
(d) $\sqrt{g(x)} = \sqrt{x^2 + x}$.
(e) $g(1)/(x + 1) = (1^2 + 1)/(x + 1) = 2/(x + 1)$.
(f) $(g(x))^2 = (x^2 + x)^2$.

12. **(a)** $2f(x) = 2(1 - x)$.
(b) $f(x) + 1 = (1 - x) + 1 = 2 - x$.
(c) $f(1 - x) = 1 - (1 - x) = x$.
(d) $(f(x))^2 = (1 - x)^2$.
(e) $f(1)/x = (1 - 1)/x = 0$.
(f) $\sqrt{f(x)} = \sqrt{1 - x}$.

13. $f(g(x)) = f(x^3 + 1) = 3(x^3 + 1) - 7 = 3x^3 - 4$.

14. $g(f(x)) = g(3x - 7) = (3x - 7)^3 + 1$.

15. Since $y = 3x - 7$, solving for x gives

$$\begin{aligned} 3x - 7 &= y \\ x &= \frac{y + 7}{3} \\ f^{-1}(y) &= \frac{y + 7}{3}. \end{aligned}$$

16. Since $y = x^3 + 1$, solving for x gives

$$\begin{aligned} x^3 + 1 &= y \\ x^3 &= y - 1 \\ x &= (y - 1)^{1/3} \\ f^{-1}(y) &= (y - 1)^{1/3}. \end{aligned}$$

17. **(a)** Since the vertical intercept of the graph of f is $(0, 1.5)$, we have $f(0) = 1.5$.
(b) Since the horizontal intercept of the graph of f is $(2.2, 0)$, we have $f(2.2) = 0$.
(c) The function f^{-1} goes from y-values to x-values, so to evaluate $f^{-1}(0)$, we want the x-value corresponding to $y = 0$. This is $x = 2.2$, so $f^{-1}(0) = 2.2$.
(d) Solving $f^{-1}(?) = 0$ means finding the y-value corresponding to $x = 0$. This is $y = 1.5$, so $f^{-1}(1.5) = 0$.

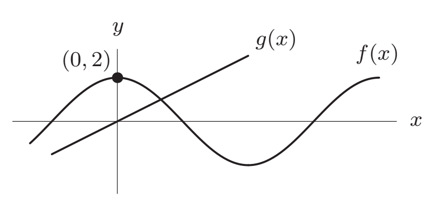
18. The composition, $P = g(f(t))$ gives the daily electricity consumption in megawatts at time t.

19. If $P = f(t)$, then $t = f^{-1}(P)$, so $f^{-1}(P)$ gives the time in years at which the population is P million.

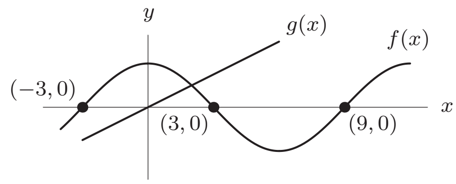
20. If $E = g(P)$, then $P = g^{-1}(E)$ so $g^{-1}(E)$ gives the population leading to a daily electricity consumption of E megawatts.

Problems

21.



22.



23. Curves cross at $x = 2$. See Figure 2.59. We do not know the y-coordinate of this point.

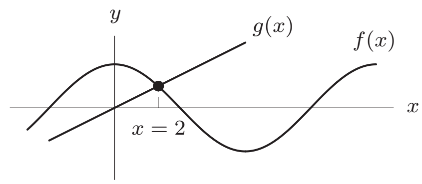


Figure 2.59

24. Graph of $g(x)$ is above graph of $f(x)$ for x to the right of 2. See Figure 2.60.

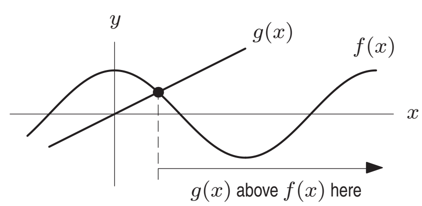


Figure 2.60

25. $f(a) = \dfrac{a \cdot a}{a+a} = \dfrac{a^2}{2a} = \dfrac{a}{2}$.

26. $f(1-a) = \dfrac{a(1-a)}{a+(1-a)} = a(1-a) = a - a^2$

27. $f\left(\dfrac{1}{1-a}\right) = \dfrac{a\dfrac{1}{1-a}}{a+\dfrac{1}{1-a}} = \dfrac{\dfrac{a}{1-a}}{\dfrac{a(1-a)+1}{1-a}} = \dfrac{a}{1-a} \cdot \dfrac{1-a}{a-a^2+1} = \dfrac{a}{a-a^2+1}$.

28. **(a)** Using Pythagoras' Theorem, we see that the diagonal d is given in terms of s by

$$\begin{aligned} d^2 &= 2s^2 \\ s &= \sqrt{\frac{d^2}{2}} = \frac{d}{\sqrt{2}} \\ s &= f(d) = \frac{d}{\sqrt{2}}. \end{aligned}$$

(b) $A = g(s) = s^2$.

(c) Substituting $s = d/\sqrt{2}$ in g gives

$$A = g(s) = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}.$$

(d) The function h is the composition of f and g, with f as the inside function, that is $h(d) = g(f(d))$.

29. **(a)** The cost of producing 5000 loaves is \$653.
(b) $C^{-1}(80)$ is the number of loaves of bread that can be made for \$80, namely 0.62 thousand or 620.
(c) The solution is $q = 6.3$ thousand. It costs \$790 to make 6300 loaves.
(d) The solution is $x = 150$ dollars, so 1.2 thousand, or 1200, loaves can be made for \$150.

30. We can find the inverse function by solving for t in our equation:

$$\begin{aligned} H &= \frac{5}{9}(t-32) \\ \frac{9}{5}H &= t-32 \\ \frac{9}{5}H + 32 &= t. \end{aligned}$$

This function gives us the temperature in degrees Fahrenheit if we know the temperature in degrees Celsius.

31. **(a)** We substitute zero into the function, giving:

$$H = f(0) = \frac{5}{9}(0-32) = -\frac{160}{9} = -17.778.$$

This means that zero degrees Fahrenheit is about -18 degrees Celsius.

(b) In Exercise 30, we found the inverse function. Using it with $H = 0$, we have:

$$t = f^{-1}(0) = \frac{9}{5}0 + 32 = 32.$$

This means that zero degrees Celsius is equivalent to 32 degrees Fahrenheit (the temperature at which water freezes).

(c) We substitute 100 into the function, giving:

$$H = f(100) = \frac{5}{9}(100-32) = \frac{340}{9} = 37.778.$$

This means that 100 degrees Fahrenheit is about 38 degrees Celsius.

(d) In Exercise 30, we found the inverse function. Using it with $H = 100$, we have:

$$t = f^{-1}(100) = \frac{9}{5}100 + 32 = 212.$$

This means that 100 degrees Celsius is equivalent to 212 degrees Fahrenheit (the temperature at which water boils).

32. We have

$$H = f(g(n)) = f(68 + 10 \cdot 2^{-n}) = \frac{5}{9}\left(68 + 10 \cdot 2^{-n} - 32\right) = 20 + \frac{50}{9}2^{-n},$$

and $f(g(n))$ gives the temperature, H, in degrees Celsius after n hours.

33. Since

$$T = 2\pi\sqrt{\frac{l}{g}},$$

solving for l gives

$$\begin{aligned} T^2 &= 4\pi^2\frac{l}{g} \\ l &= \frac{gT^2}{4\pi^2}. \end{aligned}$$

Thus,

$$f^{-1}(T) = \frac{gT^2}{4\pi^2}.$$

The function $f^{-1}(T)$ gives the length of a pendulum of period T.

34. **(a)** $A = f(r) = \pi r^2$
(b) $f(0) = 0$
(c) $f(r+1) = \pi(r+1)^2$. This is the area of a circle whose radius is 1 cm more than r.
(d) $f(r)+1 = \pi r^2 + 1$. This is the area of a circle of radius r, plus 1 square centimeter more.
(e) Centimeters.

35. **(a)** To evaluate $f(2)$, we determine which value of I corresponds to $w = 2$. Looking at the graph, we see that $I \approx 7$ when $w = 2$. This means that ≈ 7000 people were infected two weeks after the epidemic began.
(b) The height of the epidemic occurred when the largest number of people were infected. To find this, we look on the graph to find the largest value of I, which seems to be approximately 8.5, or 8500 people. This seems to have occurred when $w = 4$, or four weeks after the epidemic began. We can say that at the height of the epidemic, at $w = 4$, $f(4) = 8.5$.
(c) To solve $f(x) = 4.5$, we must find the value of w for which $I = 4.5$, or 4500 people were infected. We see from the graph that there are actually two values of w at which $I = 4.5$, namely $w \approx 1$ and $w \approx 10$. This means that 4500 people were infected after the first week when the epidemic was on the rise, and that after the tenth week, when the epidemic was slowing, 4500 people remained infected.
(d) We are looking for all the values of w for which $f(w) \geq 6$. Looking at the graph, this seems to happen for all values of $w \geq 1.5$ and $w \leq 8$. This means that more than 6000 people were infected starting in the middle of the second week and lasting until the end of the eighth week, after which time the number of infected people fell below 6000.

36. **(a)** $r(0) = 800 - 40(0) = 800$ means water is entering the reservoir at 800 gallons per second at time $t = 0$. Since we don't know how much water was in the reservoir originally, this is not the amount of water in the reservoir.
$r(15) = 800 - 40(15) = 800 - 600 = 200$ means water is entering the reservoir at 200 gallons per second at time $t = 15$.
$r(25) = 800 - 40(25) = 800 - 1000 = -200$ means water is leaving the reservoir at 200 gallons per second at time $t = 25$.
(b) The intercepts occur at $(0, 800)$ and $(20, 0)$. The first tells us that the water is initially flowing in at the rate of 800 gallons per second. The other tells us that at 20 seconds, the flow has stopped.
The slope is $(800 - 0)/(0 - 20) = 800/-20 = -40$. This means that the rate at which water enters the reservoir decreases by 40 gallons per second each second. The water is flowing in at a decreasing rate.

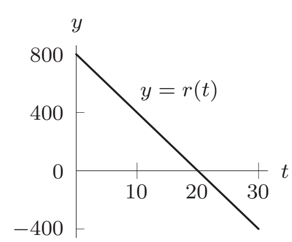


Figure 2.61

(c) The reservoir has more and more water when the rate is positive, because then water is being added. Water is being added until $t = 20$ when it starts flowing out. This means at $t = 20$, the most water is in the reservoir. The reservoir has water draining out between $t = 20$ and $t = 30$, but this amount is not as much as the water that entered between $t = 0$ and $t = 20$. Thus, the reservoir had the least amount of water at the beginning when $t = 0$. Remember the graph shows the rate of flow, not the amount of water in the reservoir.
(d) The domain is the number of seconds specified; $0 \leq t \leq 30$. The rate varies from 800 gallons per second at $t = 0$ to -400 gallons per second at $t = 30$, so the range is $-400 \leq r(t) \leq 800$.

37. **(a)** Since $f(2) = 3$, $f^{-1}(3) = 2$.
(b) Unknown
(c) Since $f^{-1}(5) = 4$, $f(4) = 5$.

38. **(a)** $j(h(4)) = h^{-1}(h(4)) = 4$
(b) We don't know $j(4)$

(c) $h(j(4)) = h(h^{-1}(4)) = 4$
(d) $j(2) = 4$
(e) We don't know $h^{-1}(-3)$
(f) $j^{-1}(-3) = 5$, since $j(5) = -3$
(g) We don't know $h(5)$
(h) $(h(-3))^{-1} = \left(j^{-1}(-3)\right)^{-1} = 5^{-1} = 1/5$
(i) We don't know $(h(2))^{-1}$

39. **(a)** To evaluate $f(1)$, we need to find the value of f which corresponds to $x = 1$. Looking in the table, we see that that value is 2. So we can say $f(1) = 2$. Similarly, to find $g(3)$, we see in the table that the value of g which corresponds to $x = 3$ is 4. Thus, we know that $g(3) = 4$.

(b) The values of $f(x)$ increase by 3 as x increases by 1. For $x > 1$, the values of $g(x)$ are consecutive perfect squares. The entries for $g(x)$ are symmetric about $x = 1$. In other words, when $x < 1$ the values of $g(x)$ are the same as the values when $x > 1$, but the order is reversed.

(c) Since the values of $f(x)$ increase by 3 as x increases by 1 and $f(4) = 11$, we know that $f(5) = 11 + 3 = 14$. Similarly, $f(x)$ decreases by three as x goes down by one. Since $f(-1) = -4$, we conclude that $f(-2) = -4-3 = -7$.

The values of $g(x)$ are consecutive perfect squares. Since $g(4) = 9$, then $g(5)$ must be the next perfect square which is 16, so $g(5) = 16$. Since the values of $g(x)$ are symmetric about $x = 1$, the value of $g(-2)$ will equal $g(5)$ (since -2 and 4 are both a distance of 3 units from 1). Thus, $g(-2) = g(4) = 9$.

(d) To find a formula for $f(x)$, we begin by observing that $f(0) = -1$, so the value of $f(x)$ that corresponds to $x = 0$ is -1. We know that the value of $f(x)$ increases by 3 as x increases by 1, so

$$\begin{aligned}
f(1) &= f(0) + 3 = -1 + 3 \\
f(2) &= f(1) + 3 = (-1 + 3) + 3 = -1 + 2 \cdot 3 \\
f(3) &= f(2) + 3 = (-1 + 2 \cdot 3) + 3 = -1 + 3 \cdot 3 \\
f(4) &= f(3) + 3 = (-1 + 3 \cdot 3) + 3 = -1 + 4 \cdot 3.
\end{aligned}$$

The pattern is

$$f(x) = -1 + x \cdot 3 = -1 + 3x.$$

We can check this formula by choosing a value for x, such as $x = 4$, and use the formula to evaluate $f(4)$. We find that $f(4) = -1 + 3(4) = 11$, the same value we see in the table.

Since the values of $g(x)$ are all perfect squares, we expect the formula for $g(x)$ to have a square in it. We see that x^2 is not quite right since the table for such a function would look like Table 2.8.

Table 2.8

x	-1	0	1	2	3	4
x^2	1	0	1	4	9	16

But this table is very similar to the one that defines g. In order to make Table 2.8 look identical to the one given in the problem, we need to subtract 1 from each value of x so that $g(x) = (x-1)^2$. We can check our formula by choosing a value for x, such as $x = 2$. Using our formula to evaluate $g(2)$, we have $g(2) = (2-1)^2 = 1^2 = 1$. This result agrees with the value given in the problem.

40. **(a)** To find a point on the graph $k(x)$ with an x-coordinate of -2, we substitute -2 for x in the formula for $k(x)$. We obtain $k(-2) = 6 - (-2)^2 = 6 - 4 = 2$. Thus, we have the point $(-2, k(-2))$, or $(-2, 2)$.

(b) To find these points, we want to find all the values of x for which $k(x) = -2$. We have

$$\begin{aligned}
6 - x^2 &= -2 \\
-x^2 &= -8 \\
x^2 &= 8 \\
x &= \pm 2\sqrt{2}.
\end{aligned}$$

Thus, the points $(2\sqrt{2}, -2)$ and $(-2\sqrt{2}, -2)$ both have a y-coordinate of -2.

(c) Figure 2.62 shows the desired graph. The point in part (a) is $(-2, 2)$. We have called this point A on the graph in Figure 2.62. There are two points in part (b): $(-2\sqrt{2}, -2)$ and $(2\sqrt{2}, -2)$. We have called these points B and C, respectively, on the graph in Figure 2.62.

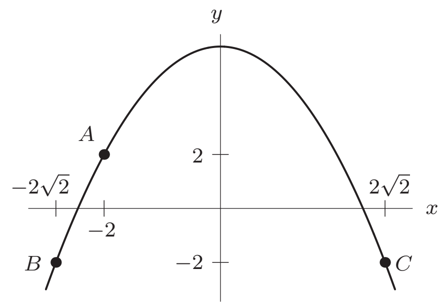


Figure 2.62

(d) For $p = 2$, $k(p) - k(p-1) = k(2) - k(1)$. Now $k(2) = 6 - 2^2 = 6 - 4 = 2$, while $k(1) = 6 - (1)^2 = 6 - 1 = 5$, thus, $k(2) - k(1) = 2 - 5 = -3$.

41. **(a)** $f(2) = 4.98$ means that 2 pounds of apples cost $\$4.98$.
(b) $f(0.5) = 1.25$ means that $1/2$ pound of apples cost $\$1.25$.
(c) $f^{-1}(0.62) = 0.25$ means that $\$0.62$ buys $1/4$ pound of apples.
(d) $f^{-1}(12.45) = 5$ means that $\$12.45$ buys 5 pounds of apples.

42. **(a)** $t(400) = 272$.
(b) (i) It takes 136 seconds to melt 1 gram of the compound at a temperature of $800°$C.
(ii) It takes 68 seconds to melt 1 gram of the compound at a temperature of $1600°$C.
(c) This means that $t(2x) = t(x)/2$, because if x is a temperature and $t(x)$ is a melting time, then $2x$ would be double this temperature and $t(x)/2$ would be half this melting time.

43. **(a)** (i) From the table, $N(150) = 6$. When 150 students enroll, there are 6 sections.
(ii) Since $N(75) = 4$ and $N(100) = 5$, and 80 is between 75 and 100 students, we choose the higher value for $N(s)$. So $N(80) = 5$. When 80 students enroll, there are 5 sections.
(iii) The quantity $N(55.5)$ is not defined, since 55.5 is not a possible number of students.
(b) (i) The table gives $N(s) = 4$ sections for $s = 75$ and $s = 50$. For any integer between those in the table, the section number is the higher value. Therefore, for $50 \leq s \leq 75$, we have $N(s) = 4$ sections. We do not know what happens if $s < 50$.
(ii) First evaluate $N(125) = 5$. So we solve the equation $N(s) = 5$ for s. There are 5 sections when enrollment is between 76 and 125 students.

44. This represents the change in average hurricane intensity at average Caribbean Sea surface temperature after CO_2 levels rise to future projected levels.

45. This represents the change in average hurricane intensity at current CO_2 levels if sea surface temperature rises by $1°$C.

46. **(a)** Increasing until year 60 and then decreasing.
(b) The average rate of change of the population is given in Table 2.9.

Table 2.9 *The population of Ireland from 1780 to 1910*

Year (years)	Population (millions)	$\Delta P/\Delta t$ (millions/year)
0 to 20	4.0 to 5.2	0.060
20 to 40	5.2 to 6.7	0.075
40 to 60	6.7 to 8.3	0.080
60 to 70	8.3 to 6.9	-0.140
70 to 90	6.9 to 5.4	-0.075
90 to 110	5.4 to 4.7	-0.035
110 to 130	4.7 to 4.4	-0.015

(c) The average rate of change is increasing until between years 40 and 60. At year 60, the sign abruptly changes, but after 60, the rate of change is still increasing. Thus, the graph is concave up, although something strange is happening near year 60.

(d) The rate of change of the population was greatest between 40 and 60, that is 1820-1840. The rate of change of the population was least (most negative) between 60 and 70, that is 1840-1850. At this time the population was shrinking fastest.
Since the greatest rate of increase was directly followed by the greatest rate of decrease, something catastrophic must have happened to cause the population not only to stop growing, but to start shrinking.

(e) Figures 2.63 and 2.64 show the population of Ireland from 1780 to 1910 as a function of the time with two different curves dashed in—either of which could be correct. From the graphs we can see that the curve is increasing until about year 60, and then decreases. Also, it is concave up most of the time except, possibly, for a short time interval near year 60.

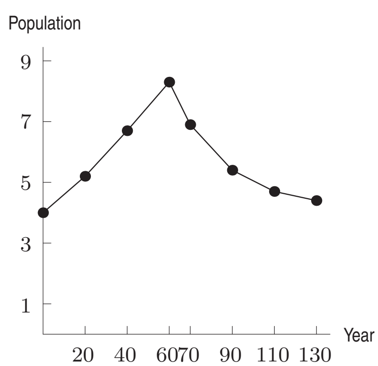


Figure 2.63: The population of Ireland from 1780 to 1910

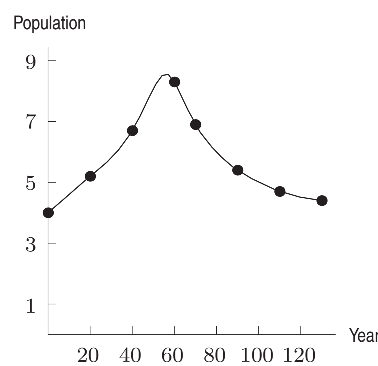


Figure 2.64: The population of Ireland from 1780 to 1910

(f) Something catastrophic happened in Ireland about year 60—that is, 1840. This is when the Irish potato famine took place.

47. (a) If $V = \pi r^2 h$ and $V = 355$, then $\pi r^2 h = 355$. So $h = (355)/(\pi r^2)$. Thus, since

$$A = 2\pi r^2 + 2\pi r h,$$

we have

$$A = 2\pi r^2 + 2\pi r\left(\frac{355}{\pi r^2}\right),$$

and

$$A = 2\pi r^2 + \frac{710}{r}.$$

(b)

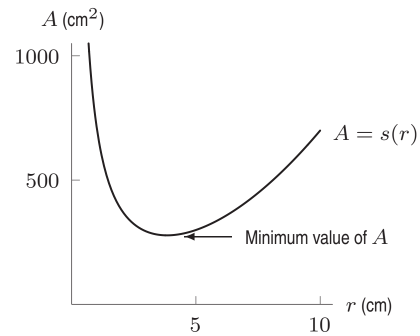


Figure 2.65: Graph of $A(r)$ for $0 < r \leq 10$

(c) The domain is any positive value or $r > 0$, because (in practice) a cola can could have as large a radius as you wanted (it would just have to be very short to maintain its 12 oz size). From the graph in (b), the value of A is never less than about 277.5 cm^2. Thus, the range is $A > 277.5$ cm^2 (approximately).

(d) They need a little more than 277.5 cm^2 per can. The minimum A-value occurs (from graph) at $r \approx 3.83$ cm, and since $h = 355/\pi r^2$, $h \approx 7.7$ cm.

(e) Since the radius of a real cola can is less than the value required for the minimum value of A, it must use more aluminum than necessary. This is because the minimum value of A has $r \approx 3.83$ cm and $h \approx 7.7$ cm. Such a can has a diameter of $2r$ or 7.66 cm. This is roughly equal to its height—holding such a can would be difficult. Thus, real cans are made with slightly different dimensions.

CHECK YOUR UNDERSTANDING

1. False. $f(2) = 3 \cdot 2^2 - 4 = 8$.
2. True. Functions are evaluated by substituting a known value or variable, here b, for the independent variable, here x.
3. False. $f(x + h) = (x + h)^2 = x^2 + 2xh + h^2$.
4. True. If $q = 1/\sqrt{z^2 + 5} = 1/3$, then

$$\begin{aligned} \frac{1}{z^2+5} &= \frac{1}{9} \\ z^2 + 5 &= 9 \\ z^2 &= 4 \\ z &= \pm 2. \end{aligned}$$

5. False. $W = (8 + 4)/(8 - 4) = 3$.
6. True. $f(0) = 0^2 + 64 = 64$.
7. False. For example, if $f(x) = x - 3$, then $f(x) = 0$ for $x = 3$ but not for $x = 0$.
8. False. For example, $f(1) = 10$ but $f(-1) = 6$.
9. True. A fraction can only be zero if the numerator is zero.
10. False. $h(3) + h(4) = (-6 \cdot 3 + 9) + (-6 \cdot 4 + 9) = -9 + (-15) = -24$ but $h(7) = -6 \cdot 7 + 9 = -33$.
11. True. This is the definition of the domain.
12. True. This is the common practice.
13. False. The domain consists of all real numbers x, $x \neq 3$
14. False. The domain consists of all real numbers $x \leq 2$.

15. False. The range does not include zero, since $1/x$ does not equal zero for any x.

16. False. If $x < 0$, then $y > 4$.

17. True. Since f is an increasing function, the domain endpoints determine the range endpoints. We have $f(15) = 12$ and $f(20) = 14$.

18. True. The $x^2 + 1$ inside the square root is positive for all x so $f(x)$ is defined for all x.

19. True. It has a slope of -1 to the left of the origin, goes through the origin, and continues as an increasing function with slope 1 to the right of the origin.

20. True. It is defined for all x.

21. True. $|x| = |-x|$ for all x.

22. False. For example, if $x = 1$ then $f(1) = 1$ and $g(1) = -1$.

23. False. For example, if $x = -1$ then $y = -1$.

24. False. Since $0 \leq 3 < 4$, the middle formula must be used. So $f(3) = 3^2 = 9$.

25. True. If $x < 0$, then $f(x) = x < 0$, so $f(x) \neq 4$. If $x > 4$, then $f(x) = -x < 0$, so $f(x) \neq 4$. If $0 \leq x \leq 4$, then $f(x) = x^2 = 4$ only for $x = 2$. The only solution for the equation $f(x) = 4$ is $x = 2$.

26. False. If f is invertible, we know $f^{-1}(5) = 3$, but nothing else.

27. True. This is the definition of the inverse function.

28. False. Check to see if $f(0) = 8$, which it does not.

29. True. To find $f^{-1}(R)$, we solve $R = \frac{2}{3}S + 8$ for S by subtracting 8 from both sides and then multiplying both sides by $(3/2)$.

30. False. For example, if $f(x) = x + 1$ then $f^{-1}(x) = x - 1$ but $(f(x))^{-1} = \dfrac{1}{x+1}$.

31. True. Since $t^{-1} = 1/t$ this is a direct substitution for the independent variable x.

32. False. The output units of a function are the same as the input units of its inverse.

33. False. Since

$$f(g(x)) = 2\left(\frac{1}{2}x - 1\right) + 1 = x - 1 \neq x,$$

the functions do not undo each other.

34. True. The quantity of rice required is $f(x)$ tons. The cost of this quantity is $g(f(x))$ dollars. Thus, the cost to feed x million people for a year is $g(f(x))$ dollars.

35. True.

36. False. The composite function $g(f(t))$ gives the volume of the ball in meter^3 after t seconds. Thus, the units of $g(f(t))$ are meter^3.

37. True. Since the function is concave up, the average rate of change increases as we move right.

38. True. The rates of change are increasing:

$$\frac{f(0) - f(-2)}{0 - (-2)} = \frac{6 - 5}{2} = \frac{1}{2}.$$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{8 - 6}{2} = 1.$$

$$\frac{f(4) - f(2)}{4 - 2} = \frac{12 - 8}{2} = 2.$$

39. True. The rates of change are decreasing:

$$\frac{g(1) - g(-1)}{1 - (-1)} = \frac{8 - 9}{2} = -\frac{1}{2}.$$

$$\frac{g(3) - g(1)}{3 - 1} = \frac{6 - 8}{2} = -1.$$

$$\frac{g(5)-g(3)}{5-3} = \frac{3-6}{2} = -\frac{3}{2}.$$

40. False. A straight line is neither concave up nor concave down.

41. True. For $x > 0$, the function $f(x) = -x^2$ is both decreasing and concave down.

42. False. For $x < 0$, the function $f(x) = x^2$ is both concave up and decreasing.

43. True. It is of the form $f(x) = a(x-r)(x-s)$ where $a = 1, r = 0$ and $s = -2$.

44. False. The zeros are -1 and -2.

45. False. It has a minimum but no maximum.

46. False. All quadratic equations have the form $y = ax^2 + bx + c$.

47. False. The time when the object hits the ground is when the height is zero, $s(t) = 0$. The value $s(0)$ gives the height when the object is launched at $t = 0$.

48. True. Solving $f(x) = 0$ gives the zeros of $f(x)$.

49. False. For example, $x^2 + 1 = 0$ has no solutions, and $x^2 = 0$ has one solution.

50. False. The functions $f(x) = x^2 - 4$ and $g(x) = -x^2 + 4$ both have zeros at $x = -2, x = 2$.

51. False. A quadratic function may have two, one, or no zeros.

Solutions to Tools for Chapter 2

1. $3(x+2) = 3x + 6$

2. $5(x-3) = 5x - 15$

3. $2(3x-7) = 6x - 14$

4. $-4(y+6) = -4y - 24$

5. $12(x+y) = 12x + 12y$

6. $-7(5x-8y) = -35x + 56y$

7. $x(2x+5) = 2x^2 + 5x$

8. $3z(2x-9z) = 6xz - 27z^2$

9. $-10r(5r+6rs) = -50r^2 - 60r^2s$

10. $x(3x-8) + 2(3x-8) = 3x^2 - 8x + 6x - 16 = 3x^2 - 2x - 16$

11. $5z(x-2) - 3(x-2) = 5xz - 10z - 3x + 6$

12. $(x+1)(x+3) = x^2 + 3x + x + 3 = x^2 + 4x + 3$

13. $(x-2)(x+6) = x^2 + 6x - 2x - 12 = x^2 + 4x - 12$

14. $(5x-1)(2x-3) = 10x^2 - 15x - 2x + 3 = 10x^2 - 17x + 3$

15. $(x+2)(3x-8) = 3x^2 - 8x + 6x - 16 = 3x^2 - 2x - 16$

16. $(y+1)(z+3) = yz + 3y + z + 3$

17. $(12y-5)(8w+7) = 96wy + 84y - 40w - 35$

18. $(5z-3)(x-2) = 5xz - 10z - 3x + 6$

19. $-(x-3) - 2(5-x) = -x + 3 - 10 + 2x = x - 7.$

20. $(x-5)6 - 5(1-(2-x)) = 6x - 30 - 5(1-2+x) = 6x - 30 + 5 - 5x = x - 25.$

21. First we multiply 4 by the terms $3x$ and $-2x^2$, and expand $(5+4x)(3x-4)$. Therefore,

$$\begin{aligned}\left(3x-2x^2\right)(4) + (5+4x)(3x-4) &= 12x - 8x^2 + 15x - 20 + 12x^2 - 16x \\ &= 4x^2 + 11x - 20.\end{aligned}$$

22. In this example, we distribute the factors $50t$ and $2t$ across the two binomials t^2+1 and $25t^2+125$, respectively. Thus,

$$\begin{aligned}(t^2+1)(50t)-(25t^2+125)(2t) &= 50t^3+50t-(50t^3+250t)\\ &= 50t^3+50t-50t^3-250t=-200t.\end{aligned}$$

23. The order of operations tell us to expand $(p-3q)^2$ first and then multiply the result by p. Therefore,

$$\begin{aligned}P(p-3q)^2 &= P(p-3q)(p-3q)\\ &= P(p^2-3pq-3pq+9q^2)=P(p^2-6pq+9q^2)\\ &= Pp^2-6Ppq+9Pq^2.\end{aligned}$$

24. Expanding $(A^2-B^2)^2=(A^2-B^2)(A^2-B^2)$, we get

$$A^4-2A^2B^2+B^4.$$

25. The order of operations tells us to expand $(x-3)^2$ first and then multiply the result by 4. Therefore,

$$\begin{aligned}4(x-3)^2+7 &= 4(x-3)(x-3)+7\\ &= 4(x^2-3x-3x+9)+7=4(x^2-6x+9)+7\\ &= 4x^2-24x+36+7=4x^2-24x+43.\end{aligned}$$

26. First we square $\sqrt{2x}+1$ and then take the negative of this result. Therefore,

$$\begin{aligned}-\left(\sqrt{2x}+1\right)^2 &= -\left(\sqrt{2x}+1\right)\left(\sqrt{2x}+1\right)=-\left(2x+\sqrt{2x}+\sqrt{2x}+1\right)\\ &= -(2x+2\sqrt{2x}+1)=-2x-2\sqrt{2x}-1.\end{aligned}$$

27. Multiplying from left to right we obtain:

$$\begin{aligned}u\left(u^{-1}+2^u\right)2^u &= \left(u^0+u\cdot 2^u\right)2^u=(1+u\cdot 2^u)\,2^4\\ &= 2^u+u\cdot 2^u\cdot 2^u=2^u+u\cdot 2^{2u}.\end{aligned}$$

28. $2x+6=2(x+3)$
29. $3y+15=3(y+5)$
30. $5z-30=5(z-6)$
31. $4t-6=2(2t-3)$
32. $10w-25=5(2w-5)$
33. $u^2-2u=u(u-2)$
34. $3u^4-4u^3=u^3(3u-4)$
35. $3u^7+12u^2=3u^2(u^5+4)$
36. $12x^3y^2-18x=6x(2x^2y^2-3)$
37. $14r^4s^2-21rst=7rs(2r^3s-3t)$
38. $x^2+3x+2=(x+2)(x+1)$
39. Can be factored no further.
40. $x^2-3x+2=(x-2)(x-1)$
41. Can be factored no further.

42. Can be factored no further.

43. $x^2 - 2x - 3 = (x-3)(x+1)$

44. Can be factored no further.

45. $x^2 + 2x - 3 = (x+3)(x-1)$

46. $2x^2 + 5x + 2 = (2x+1)(x+2)$

47. $3x^2 - x - 4 = (3x-4)(x+1)$

48. Since each term has a common factor of 2, we write:

$$\begin{aligned} 2x^2 - 10x + 12 &= 2\left(x^2 - 5x + 6\right) \\ &= 2(x-3)(x-2). \end{aligned}$$

49. $x^2 + 3x - 28 = (x+7)(x-4)$

50. $x^3 - 2x^2 - 3x = x(x^2 - 2x - 3) = x(x-3)(x+1)$

51. $x^3 + 2x^2 - 3x = x(x^2 + 2x - 3) = x(x+3)(x-1)$

52. $ac + ad + bc + bd = a(c+d) + b(c+d) = (c+d)(a+b)$.

53. $x^2 + 2xy + 3xz + 6yz = x(x+2y) + 3z(x+2y) = (x+2y)(x+3z)$.

54. $x^2 - 1.4x - 3.92 = (x+1.4)(x-2.8)$

55. $a^2x^2 - b^2 = (ax-b)(ax+b)$

56. The common factor is πr. Therefore,

$$\pi r^2 + 2\pi rh = \pi r(r + 2h).$$

57. We notice that the only factors of 24 whose sum is -10 are -6 and -4. Therefore,

$$B^2 - 10B + 24 = (B-6)(B-4).$$

58. $c^2 + x^2 - 2cx = (x-c)^2$

59. The expression $x^2 + y^2$ cannot be factored.

60. We factor and observe that $a^2 - 4$ is the difference of perfect squares. Thus,

$$\begin{aligned} a^4 - a^2 - 12 &= \left(a^2 - 4\right)\left(a^2 + 3\right) \\ &= (a-2)(a+2)\left(a^2 + 3\right). \end{aligned}$$

61. This example is factored as the difference of perfect squares. Thus,

$$\begin{aligned} (t+3)^2 - 16 &= ((t+3) - 4)\,((t+3) + 4) \\ &= (t-1)(t+7). \end{aligned}$$

Alternatively, we could arrive at the same answer by multiplying the expression out and then factoring it.

62. $x^2 + 4x + 4 - y^2 = (x+2)^2 - (y)^2 = (x+2+y)(x+2-y)$.

63. $a^3 - 2a^2 + 3a - 6 = a^2(a-2) + 3(a-2) = (a-2)(a^2+3)$.

64.

$$\begin{aligned} b^3 - 3b^2 - 9b + 27 &= b^2(b-3) - 9(b-3) \\ &= (b-3)(b^2 - 9) \\ &= (b-3)(b-3)(b+3) \\ &= (b-3)^2(b+3). \end{aligned}$$

65.

$$\begin{aligned} c^2d^2 - 25c^2 - 9d^2 + 225 &= c^2(d^2 - 25) - 9(d^2 - 25) \\ &= (d^2 - 25)(c^2 - 9) \\ &= (d + 5)(d - 5)(c + 3)(c - 3). \end{aligned}$$

66. By grouping the terms hx^2 and $-4hx$, we find a common factor of hx and for the terms 12 and $-3x$, we find a common factor of -3. Therefore,

$$\begin{aligned} hx^2 + 12 - 4hx - 3x = hx^2 - 4hx + 12 - 3x &= hx(x - 4) - 3(-4 + x) \\ &= hx(x - 4) - 3(x - 4) = (hx - 3)(x - 4). \end{aligned}$$

67. The idea here is to rewrite the second expression $-2(s - r)$ as $+2(r - s)$. This latter expression shares a common factor of $r - s$ with the first expression $r(r - s)$. Thus,

$$r(r - s) - 2(s - r) = r(r - s) + 2(r - s) = (r + 2)(r - s).$$

68. Factor as:

$$y^2 - 3xy + 2x^2 = (y - 2x)(y - x).$$

69. The common factor is xe^{-3x}. Therefore,

$$x^2e^{-3x} + 2xe^{-3x} = xe^{-3x}(x + 2).$$

70. $t^2e^{5t} + 3te^{5t} + 2e^{5t} = e^{5t}(t^2 + 3t + 2) = e^{5t}(t + 1)(t + 2)$.

71. Difference of squares: $(s + 2t)^2 - 4p^2 = (s + 2t + 2p)(s + 2t - 2p)$.

72. The two expressions $P(1 + r)^2$ and $P(1 + r)^2r$ share a common factor of $P(1 + r)^2$. So,

$$P(1 + r)^2 + P(1 + r)^2r = P(1 + r)^2(1 + r) = P(1 + r)^3.$$

73. $x^2 - 6x + 9 - 4z^2 = (x - 3)^2 - (2z)^2 = (x - 3 + 2z)(x - 3 - 2z)$.

74. $dk + 2dm - 3ek - 6em = d(k + 2m) - 3e(k + 2m) = (k + 2m)(d - 3e)$.

75. $\pi r^2 - 2\pi r + 3r - 6 = \pi r(r - 2) + 3(r - 2) = (r - 2)(\pi r + 3)$.

76. $8gs - 12hs + 10gm - 15hm = 4s(2g - 3h) + 5m(2g - 3h) = (2g - 3h)(4s + 5m)$.

77.

$$\begin{aligned} x^2 + 7x + 6 &= 0 \\ (x + 6)(x + 1) &= 0 \\ x + 6 &= 0 \quad \text{or} \quad x + 1 = 0 \\ x &= -6 \quad \text{or} \quad x = -1 \end{aligned}$$

78.

$$\begin{aligned} y^2 - 5y - 6 &= 0 \\ (y + 1)(y - 6) &= 0 \\ y + 1 &= 0 \quad \text{or} \quad y - 6 = 0 \\ y &= -1 \quad \text{or} \quad y = 6 \end{aligned}$$

79.

$$\begin{aligned}2w^2+w-10&=0\\(2w+5)(w-2)&=0\\2w+5=0\quad&\text{or}\quad w-2=0\\w=\frac{-5}{2}\quad&\text{or}\quad w=2\end{aligned}$$

80.

$$\begin{aligned}x&=\frac{-3\pm\sqrt{3^2-4(4)(-15)}}{2(4)}\\x&=\frac{-3\pm\sqrt{249}}{8}\end{aligned}$$

81.

$$\begin{aligned}\frac{2}{x}+\frac{3}{2x}&=8\\\frac{4+3}{2x}&=8\\16x&=7\\x&=\frac{7}{16}\end{aligned}$$

82.

$$\begin{aligned}\frac{3}{x-1}+1&=5\\\frac{3}{x-1}&=4\\4(x-1)&=3\\4x-4&=3\\4x&=7\\x&=\frac{7}{4}\end{aligned}$$

83.

$$\begin{aligned}\sqrt{y-1}&=13\\y-1&=169\\y&=170\end{aligned}$$

84.

$$\begin{aligned}\sqrt{5y+3}&=7\\5y+3&=49\\5y&=46\\y&=\frac{46}{5}\end{aligned}$$

85.

$$\begin{aligned}\sqrt{2x-1}+3&=9\\\sqrt{2x-1}&=6\\2x-1&=36\\2x&=37\\x&=\frac{37}{2}\end{aligned}$$

86.

$$\begin{aligned}
\frac{21}{z-5} - \frac{13}{z^2-5z} &= 3 \\
\frac{21}{z-5} - \frac{13}{z(z-5)} &= 3 \\
\frac{21z-13}{z(z-5)} &= 3 \\
21z - 13 &= 3z(z-5) \\
21z - 13 &= 3z^2 - 15z \\
3z^2 - 36z + 13 &= 0
\end{aligned}$$

$$\begin{aligned}
z &= \frac{-(-36) \pm \sqrt{(-36)^2 - 4(3)(13)}}{2(3)} \\
&= \frac{36 \pm \sqrt{1140}}{6} \\
&= \frac{36 \pm \sqrt{4 \cdot 285}}{6} \\
&= \frac{36 \pm 2\sqrt{285}}{6} \\
&= \frac{18 \pm \sqrt{285}}{3}
\end{aligned}$$

87.

$$\begin{aligned}
-16t^2 + 96t + 12 &= 60 \\
-16t^2 + 96t - 48 &= 0 \\
t^2 - 6t + 3 &= 0
\end{aligned}$$

$$\begin{aligned}
t &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)} \\
t &= \frac{6 \pm \sqrt{48}}{2} = \frac{6 \pm 4\sqrt{3}}{2} \\
t &= 3 \pm 2\sqrt{3}
\end{aligned}$$

88. Rewrite the equation $r^3 - 6r^2 = 5r - 30$ with a zero on the right side and factor.

$$\begin{aligned}
r^3 - 6r^2 - 5r + 30 &= 0 \\
r^2(r-6) - 5(r-6) &= 0 \\
(r-6)(r^2-5) &= 0.
\end{aligned}$$

So, $r - 6 = 0$ or $r^2 - 5 = 0$. Thus, $r = 6$ or $r = \pm\sqrt{5}$.

89. Rewrite the equation $g^3 - 4g = 3g^2 - 12$ with a zero on the right side and factor completely.

$$\begin{aligned}
g^3 - 3g^2 - 4g + 12 &= 0 \\
g^2(g-3) - 4(g-3) &= 0 \\
(g-3)(g^2-4) &= 0 \\
(g-3)(g+2)(g-2) &= 0.
\end{aligned}$$

So, $g - 3 = 0$, $g + 2 = 0$, or $g - 2 = 0$. Thus, $g = 3, -2$, or 2.

90. First multiply both sides by (-1):
$$-1(8+2x-3x^2)=(-1)(0).$$

$$\begin{aligned}3x^2-2x-8&=0\\(3x+4)(x-2)&=0\\3x+4=0 \quad &\text{or} \quad x-2=0\\x=-\frac{4}{3} \quad &\text{or} \quad x=2.\end{aligned}$$

91. By grouping the first two and the last two terms, we obtain:

$$\begin{aligned}\left(2p^3+p^2\right)-18p-9&=0\\\left(2p^3+p^2\right)-(18p+9)&=0\\p^2(2p+1)-9(2p+1)&=0\\\left(p^2-9\right)(2p+1)&=0\\(p-3)(p+3)(2p+1)&=0\\p=3, \text{ or } p=-3, &\text{ or } p=-\frac{1}{2}.\end{aligned}$$

92.

$$\begin{aligned}N^2-2N-3&=2N(N-3)\\N^2-2N-3&=2N^2-6N\\N^2-4N+3&=0\\(N-3)(N-1)&=0\\N=3 \text{ or } N&=1\end{aligned}$$

93. Do not divide both sides by t, because you would lose the solution $t=0$ in that case. Instead, set one side $=0$ and factor.

$$\begin{aligned}\frac{1}{64}t^3&=t\\\frac{1}{64}t^3-t&=0\\t(\frac{1}{64}t^2-1)&=0\\t=0 \text{ or } \frac{1}{64}t^2-1&=0\end{aligned}$$

The second equation still needs to be solved for t:

$$\begin{aligned}\frac{1}{64}t^2-1&=0\\\frac{1}{64}t^2&=1\\t^2&=64\\t&=\pm 8.\end{aligned}$$

So the final answer is $t=0$ or $t=8$ or $t=-8$.

94. We write $x^2 - 1 = 2x$ or $x^2 - 2x - 1 = 0$ which does not factor. Employing the quadratic formula, we have $a = 1$, $b = -2$, $c = -1$. Therefore

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} \\ &= \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}. \end{aligned}$$

95.

$$\begin{aligned} 4x^2 - 13x - 12 &= 0 \\ (x-4)(4x+3) &= 0 \\ x = 4 \ \text{ or } \ x &= -\frac{3}{4} \end{aligned}$$

96. We rewrite the quadratic equation in standard form and use the quadratic formula. So

$$\begin{aligned} 60 &= -16t^2 + 96t + 12 \\ 16t^2 - 96t + 48 &= 0 \\ t^2 - 6t + 3 &= 0 \\ t &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2} = \frac{6 \pm \sqrt{36 - 12}}{2} \\ &= \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}. \end{aligned}$$

97. Rewrite the equation $n^5 + 80 = 5n^4 + 16n$ with a zero on the right side and factor completely.

$$\begin{aligned} n^5 - 5n^4 - 16n + 80 &= 0 \\ n^4(n-5) - 16(n-5) &= 0 \\ (n-5)(n^4 - 16) &= 0 \\ (n-5)(n^2-4)(n^2+4) &= 0 \\ (n-5)(n+2)(n-2)(n^2+4) &= 0. \end{aligned}$$

So, $n - 5 = 0$, $n + 2 = 0$, $n - 2 = 0$, or $n^2 + 4 = 0$. Note that $n^2 + 4 = 0$ has no real solutions, so, $n = 5, -2$, or 2.

98. Rewrite the equation $5a^3 + 50a^2 = 4a + 40$ with a zero on the right side and factor.

$$\begin{aligned} 5a^3 + 50a^2 - 4a - 40 &= 0 \\ 5a^2(a+10) - 4(a+10) &= 0 \\ (a+10)(5a^2 - 4) &= 0. \end{aligned}$$

So, $a + 10 = 0$ or $5a^2 - 4 = 0$. Thus, $a = -10$ or $a = \dfrac{\pm 2\sqrt{5}}{5}$.

99. Using the quadratic formula for

$$y^2 + 4y - 2 = 0, \quad a = 1,\ b = 4,\ c = -2,$$

we obtain,

$$\begin{aligned} y &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-2)}}{2} = \frac{-4 \pm \sqrt{16+8}}{2} \\ &= \frac{-4 \pm \sqrt{24}}{2} = \frac{-4 \pm 2\sqrt{6}}{2} = -2 \pm \sqrt{6}. \end{aligned}$$

100. To find the common denominator, we factor the second denominator

$$\begin{aligned}\frac{2}{z-3}+\frac{7}{z^2-3z}&=0\\ \frac{2}{z-3}+\frac{7}{z(z-3)}&=0\end{aligned}$$

which produces a common denominator of $z(z-3)$. Therefore:

$$\begin{aligned}\frac{2z}{z(z-3)}+\frac{7}{z(z-3)}&=0\\ \frac{2z+7}{z(z-3)}&=0\\ 2z+7&=0\\ z&=-\frac{7}{2}.\end{aligned}$$

101. First we combine like terms in the numerator.

$$\begin{aligned}\frac{x^2+1-2x^2}{(x^2+1)^2}&=0\\ \frac{-x^2+1}{(x^2+1)^2}&=0\\ -x^2+1&=0\\ -x^2&=-1\\ x^2&=1\\ x&=\pm 1\end{aligned}$$

102.

$$\begin{aligned}4-\frac{1}{L^2}&=0\\ 4&=\frac{1}{L^2}\\ 4L^2&=1\\ L^2&=\frac{1}{4}\\ L&=\pm\frac{1}{2}\end{aligned}$$

103. The common denominator for this fractional equation is $(q+1)(q-1)$. If we multiply both sides of this equation by $(q+1)(q-1)$, we obtain:

$$\begin{aligned}2+\frac{1}{q+1}-\frac{1}{q-1}&=0\\ 2(q+1)(q-1)+1(q-1)-1(q+1)&=0\\ 2\left(q^2-1\right)+q-1-q-1&=0\\ 2q^2-2+q-1-q-1&=0\\ 2q^2-4&=0\\ 2q^2&=4\\ q^2&=2\\ q&=\pm\sqrt{2}.\end{aligned}$$

104. We can solve this equation by squaring both sides.

$$\begin{aligned}\sqrt{r^2+24} &= 7\\ r^2+24 &= 49\\ r^2 &= 25\\ r &= \pm 5\end{aligned}$$

105. We can solve this equation by cubing both sides of this equation.

$$\begin{aligned}\frac{1}{\sqrt[3]{x}} &= -2\\ \left(\frac{1}{\sqrt[3]{x}}\right)^3 &= (-2)^3\\ \frac{1}{x} &= -8\\ x &= -\frac{1}{8}\end{aligned}$$

106. We can solve this equation by squaring both sides.

$$\begin{aligned}3\sqrt{x} &= \frac{1}{2}x\\ 9x &= \frac{1}{4}x^2\\ \frac{1}{4}x^2-9x &= 0\\ x(\frac{1}{4}x-9) &= 0\\ x=0 \text{ or } &\frac{1}{4}x=9\\ x=0 \text{ or } &x=36\end{aligned}$$

107. We can solve this equation by squaring both sides.

$$\begin{aligned}10 &= \sqrt{\frac{v}{7\pi}}\\ 100 &= \frac{v}{7\pi}\\ 700\pi &= v\end{aligned}$$

108. Multiply by $(x-5)(x-1)$ on both sides of the equation, giving

$$(3x+4)(x-2)=0.$$

So, $3x+4=0$, or $x-2=0$, that is,

$$x=-\frac{4}{3}, \quad x=2.$$

109. We begin by squaring both sides of the equation in order to eliminate the radical.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{l}{g}} \\ T^2 &= 4\pi^2\left(\frac{l}{g}\right) \\ \frac{gT^2}{4\pi^2} &= l \end{aligned}$$

110. First solve for b^5, then take fifth root:

$$\begin{aligned} Ab^5 &= C \\ b^5 &= \frac{C}{A} \\ b &= \sqrt[5]{\frac{C}{A}}. \end{aligned}$$

111. We have $2x+1=7$ or $2x+1=-7$, that is, $2x=6$ or $2x=-8$. So,

$$x = 3, \quad x = -4.$$

112. For a fraction to equal zero, the numerator must equal zero. So, we solve

$$x^2 - 5mx + 4m^2 = 0.$$

Since $x^2-5mx+4m^2=(x-m)(x-4m)$, we know that the numerator equals zero when $x=4m$ and when $x=m$. But for $x=m$, the denominator will equal zero as well. So, the fraction is undefined at $x=m$, and the only solution is $x=4m$.

113. We substitute -3 for y in the first equation.

$$\begin{aligned} y &= 2x - x^2 \\ -3 &= 2x - x^2 \\ x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x &= 3 \quad \text{and} \quad y = 2(3) - 3^2 = -3 \quad \text{or} \\ x &= -1 \quad \text{and} \quad y = 2(-1) - (-1)^2 = -3 \end{aligned}$$

114. We set the equations $y=1/x$ and $y=4x$ equal to one another.

$$\begin{aligned} \frac{1}{x} &= 4x \\ 4x^2 &= 1 \\ x^2 &= \frac{1}{4} \\ x &= \frac{1}{2} \quad \text{and} \quad y = \frac{1}{\frac{1}{2}} = 2 \quad \text{or} \\ x &= -\frac{1}{2} \quad \text{and} \quad y = \frac{1}{-\frac{1}{2}} = -2 \end{aligned}$$

115. Substituting the value of y from the second equation into the first equation, we obtain

$$\begin{aligned} x^2 + (x-3)^2 &= 36 \\ x^2 + x^2 - 6x + 9 &= 36 \\ 2x^2 - 6x - 27 &= 0, \end{aligned}$$

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-27)}}{(2)(2)} \\ &= \frac{6 \pm \sqrt{252}}{4} \\ &= \frac{6 \pm \sqrt{4 \cdot 63}}{4} \\ &= \frac{6 \pm 2\sqrt{63}}{4} \\ &= \frac{3 \pm \sqrt{63}}{2}. \end{aligned}$$

Now we substitute the values of x into the second equation:

$$\begin{aligned} y &= \frac{3 \pm \sqrt{63}}{2} - 3 \\ &= \frac{-3 \pm \sqrt{63}}{2}. \end{aligned}$$

116. We substitute the expression $4 - x^2$ for y in the second equation.

$$\begin{aligned} y - 2x &= 1 \\ 4 - x^2 - 2x &= 1 \\ -x^2 - 2x + 3 &= 0 \\ x^2 + 2x - 3 &= 0 \\ (x+3)(x-1) &= 0 \\ x &= -3 \quad \text{and} \quad y = 4 - (-3)^2 = -5 \quad \text{or} \\ x &= 1 \quad \text{and} \quad y = 4 - 1^2 = 3 \end{aligned}$$

117. These equations cannot be solved exactly. A calculator gives the solutions as

$$x = 2.081, \quad y = 8.013 \quad \text{and} \quad x = 4.504, \quad y = 90.348.$$

118. Using the point-slope formula for the equation of a line, we have

$$y - 0 = 3(x - 0)$$

or

$$y = 3x.$$

We need to find the points where this line intersects $y = x^2$. This means we want points such that

$$x^2 = 3x \quad \text{or} \quad x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0 \quad \text{or} \quad x = 3.$$

So the points are $(0, 0)$ and $(3, 9)$.

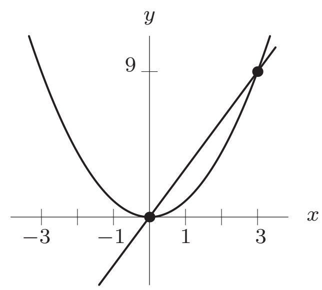


Figure 2.66

119. We substitute $y = x - 1$ in the equation $x^2 + y^2 = 25$.

$$\begin{aligned}
x^2 + (x-1)^2 &= 25 \\
x^2 + x^2 - 2x + 1 &= 25 \\
2x^2 - 2x - 24 &= 0 \\
x^2 - x - 12 &= 0 \\
(x-4)(x+3) &= 0 \\
x = 4 \quad \text{and} \quad y = 4 - 1 = 3 \quad &\text{or} \\
x = -3 \quad \text{and} \quad y = -3 - 1 = -4
\end{aligned}$$

So the points of intersection are $(4, 3)$, $(-3, -4)$.

120. Solving $y = x^2$ and $y = 15 - 2x$ simultaneously, we have

$$\begin{aligned}
x^2 &= 15 - 2x \\
x^2 + 2x - 15 &= 0 \\
(x+5)(x-3) &= 0 \\
x &= -5, 3.
\end{aligned}$$

Thus, the points of intersection are $(-5, 25)$, $(3, 9)$.