

## Infinite Discontinuities

In an *infinite* discontinuity, the left- and right-hand limits are infinite; they may be both positive, both negative, or one positive and one negative.

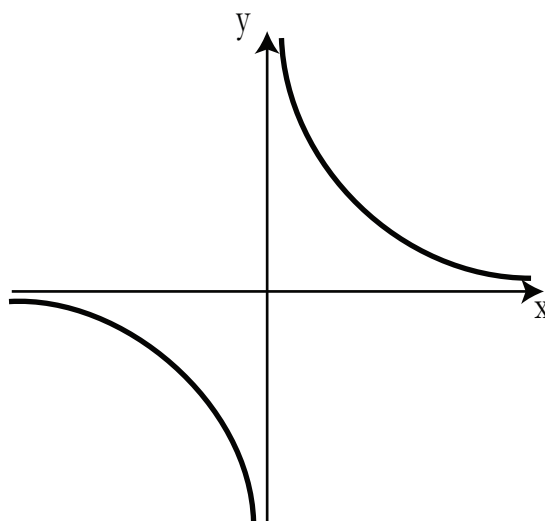


Figure 1: An example of an infinite discontinuity:  $\frac{1}{x}$

From Figure 1, we see that  $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$  and  $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$ . Saying that a limit is  $\infty$  is different from saying that the limit doesn't exist – the values of  $\frac{1}{x}$  are changing in a very definite way as  $x \rightarrow 0$  from either side. (Note that it's not true that  $\lim_{x \rightarrow 0} \frac{1}{x} = \infty$  because  $\infty$  and  $-\infty$  are different.)

There are two more things we can learn from this example. First, sketch the graph of  $\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$ ; it also has an infinite discontinuity at  $x = 0$ . Notice that the derivative of the function  $\frac{1}{x}$  is always negative. It may seem strange to you that the derivative is decreasing as  $x$  approaches 0 from the positive side while  $\frac{1}{x}$  is increasing, but very often the graph of the derivative will look nothing like the graph of the original function.

What the graph of the derivative  $-\frac{1}{x^2}$  is showing you is the slope of the graph of  $\frac{1}{x}$ . Where the graph of  $\frac{1}{x}$  is not very steep, the graph of  $-\frac{1}{x^2}$  lies close to the  $x$ -axis. Where the graph of  $\frac{1}{x}$  is steep, the graph of  $-\frac{1}{x^2}$  is far away from the  $x$ -axis. The value of  $-\frac{1}{x^2}$  is always negative, and the graph of  $\frac{1}{x}$  always slopes downward.

Finally,  $\frac{1}{x}$  is an odd function and  $-\frac{1}{x^2}$  is an even function. When you take the derivative of an odd function you always get an even function and vice-versa. If you can easily identify odd and even functions, this is a good way to check

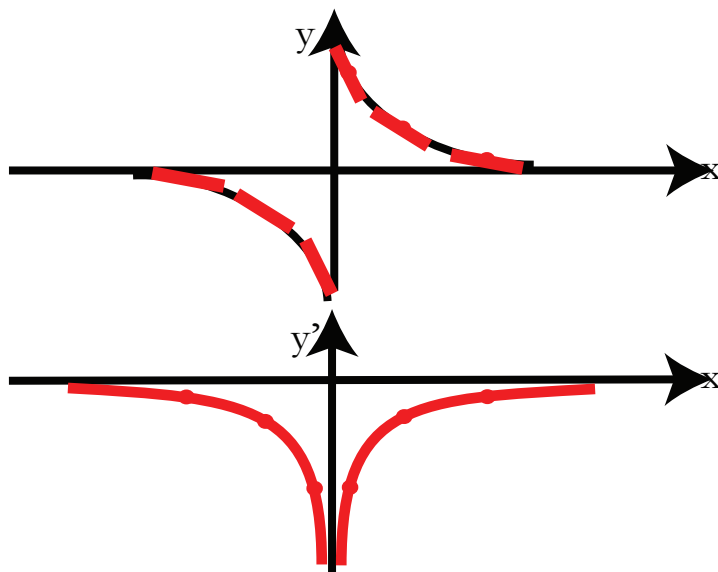


Figure 2: Top: graph of  $f(x) = \frac{1}{x}$  and Bottom: graph of  $f'(x) = -\frac{1}{x^2}$

your work.