Geometric Interpretation of Differentiation

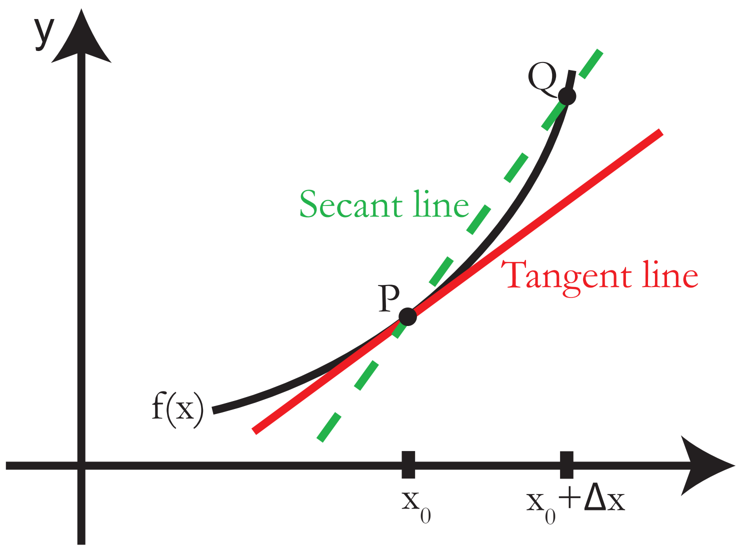

Figure 1: A graph with secant and tangent lines.

The derivative of $f(x)$ at $x = x_0$ is the slope of the tangent line to the graph of $f(x)$ at the point $(x_0, f(x_0))$. But what is a tangent line?

- It is NOT just a line that meets the graph at one point.
- It is the *limit* of the secant lines joining points $P = (x_0, f(x_0))$ and Q on the graph of $f(x)$ as Q approaches P.

The tangent line touches the graph at $(x_0, f(x_0))$; the slope of the tangent line matches the direction of the graph at that point. The tangent line is the straight line that best approximates the graph at that point.

Given a graph of our function, it's not hard for us to draw the tangent line to the graph. However, we'll want to do computations involving the tangent line and so will need a computational method of finding the tangent line.

How do we compute the equation of the line tangent to the graph of the function $f(x)$ at a point $P = (x_0, y_0)$? We know that the equation of the straight line with slope m through the point (x_0, y_0) is $y - y_0 = m(x - x_0)$, so in the abstract we know the equation of the tangent line.

To get a specific equation for the line, we'll need to know the coordinates x_0 and y_0 of the point P. If we know x_0 we can find $y_0 = f(x_0)$ by substituting the value x_0 in to the expression for $f(x)$. The second thing we need to know is the slope, $m = f'(x_0)$, which we call *the derivative of* f.

Definition: The derivative $f'(x_0)$ of f at x_0 is the slope of the tangent line to $y = f(x)$ at the point $P = (x_0, f(x_0)$.