

## Main Formula

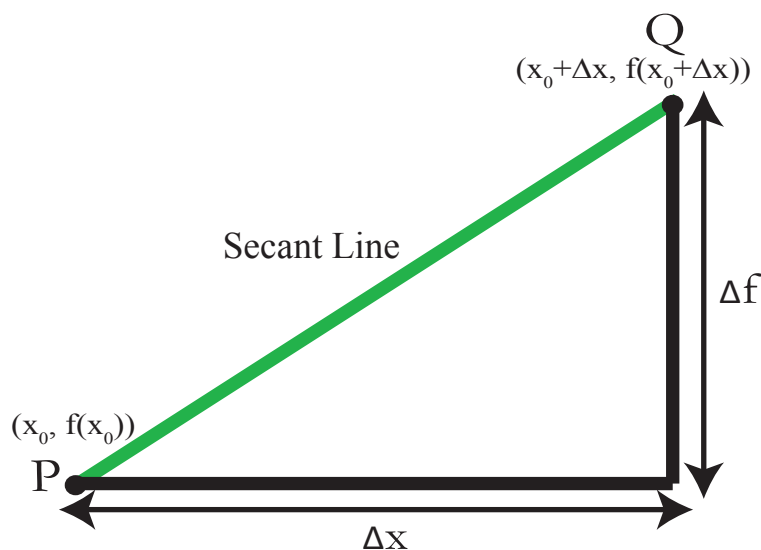


Figure 1: Geometric definition of the derivative

We started with a point  $P$  on the graph of  $y = f(x)$  which had coordinates  $(x_0, f(x_0))$ . We then found a point  $Q$  on the the graph which was  $\Delta x$  units to the right of  $P$ . The coordinates of  $Q$  must be  $(x_0 + \Delta x, f(x_0 + \Delta x))$ . We can now write the following formula for the derivative:

$$m = \underbrace{f'(x_0)}_{\text{derivative of } f \text{ at } x_0} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \rightarrow 0} \underbrace{\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}}_{\text{difference quotient}}$$

This is by far the most important formula in Lecture 1; it is the formula that we use to compute the derivative  $f'(x_0)$ , which equals the slope of the tangent line to the graph at  $P$ . A machine could use this formula together with the coordinates  $(x_0, f(x_0))$  of the point  $P$  to draw the tangent line to the graph of  $y = f(x)$  at the point  $P$ .