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ORIGINAL RESEARCH

A Markov decision model for consumer term-loan collections

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Abstract
We examine how to efficiently schedule collection actions for consumer term-loan accounts over time using a Markov decision model. A consumer loan account at each age can be classified into different account states, including current, delinquent, early payoff, default, and bankrupt. We model state transitions of loan accounts using a Markov transition matrix, and develop an optimization method to determine the collection action at each state and age for each consumer type to maximize the lender's expected value. The optimization approach incorporates default risk and operational cost, and also addresses the time value of money, the tradeoff between interest revenue and borrowing cost, time consistency in optimization, competing risks between different account states, and penalty for late payment. Compared with a static collection policy, our method is demonstrably more valuable for accounts with high interest rates and medium to high loan amount, especially with stronger collection effects. We also demonstrate how the collection actions implemented under an optimal collection policy are affected by interest rate, loan amount, and collection effects.

Keywords Dynamic programming · Collection · Markov process · Optimization

JEL Classification G21 · G23

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1 Introduction

The past half century has witnessed a rapid worldwide growth in *consumer loans*. Consumer loans are also known as consumer credits or retail loans, and in this work specifically refer to money loaned to individuals for personal, family, or household use. Consumer loans can be classified into two categories: *nonrevolving* or *installment debt*, mainly including motor vehicle loans and other loans such as for mobile homes, education, boats, trailers, or vacations, and *revolving debt*, mainly including credit card loans. A nonrevolving debt has a fixed term, and is referred to as a term-loan in this work. Mortgage loans, typically for home purchases, are excluded from nonrevolving debt, since they are subject to different regulations. The U.S. consumer credit outstanding in 2012–2016 was 8.3, 7.9, 8.4, 7.7, and 6.7 trillion dollars, respectively, as reported by Federal Reserve statistical release (federalreserve.gov 2018). Widespread and large-scale approval of consumer loans creates concerns for credit companies over the risk of loss. Consequently, the credit collection and service business grows alongside the business of granting consumer loans. Credit companies consider the prevention of credit loss a key activity, and assign a significant amount of recourses to loan management (Rosenberg and Gleit 1994).

Focusing on nonrevolving consumer term-loans, this work originates from the loan collection business at a U.S. credit company. The company provides financing services to customers who purchase specific durable goods sold by the company's business partners. It is inevitable that certain consumers do not make payment as contracted, and thus it is important for the company to pursue an efficient collection policy to reduce credit loss. As technology advances, many new collection actions become implementable in the collection process with reduced operational cost. For example, text messaging, email, and automatic machine dialers can be used to contact consumers with little human intervention. However, the availability of multiple collection actions complicates the collection process. In this work, we focus on how to efficiently use the various collection methods to generate higher value for the lender, using a Markov decision model.

The intuition and appeal behind a Markov chain model for managing consumer loans is that an account naturally transits through different delinquency states in each time period. For example, an account in *current* state this month will be in *current* state again next month if a payment in the contracted amount is made by the due date, but will be in *1-month delinquent* state if no payment is received by the due date. The probability of transition from one state to another can be estimated using historical data, for example, via maximum likelihood estimate. When a collection action is applied to an account in a Markov chain, the probability of transition can differ from the case where no action or another action is applied. Here the decision problem faced by the lender is how to choose an action in each state and age for each consumer type, to maximize its expected net present value. This paper develops a systematic Markov decision modeling framework for optimal loan collection decisions.

Various modeling approaches have been applied to assist decision making regarding consumer and commercial loans. Rosenberg and Gleit (1994) review quantitative methods used to help managers make loan management decisions. The methods reviewed include discriminant analysis, decision tree, expert system, linear programming, dynamic programming, and Markov chain. Altman and Saunders (1998) review the literature on credit risk measurement of individual and portfolio loans over 1980–1998. Thomas et al. (2005) survey methodology development in consumer loan modeling over 1955–2005, and highlight certain critical issues in consumer loan management, such as designing consumer scoring systems, developing product feature selection models, devising policy inference techniques, and improving default forecasting accuracy.

Decision problems in loan management include: (1) whether to grant or extend a loan, (2) how much loan to grant or extend, (3) when a collection action should be initiated, and (4) what collection action should be used. The first two decision problems are categorized as *origination* decisions, and the final two are categorized as *collection* decisions. We emphasize that origination and collection decisions are not independent of each other; rather, they are closely related, because good origination decisions take future collection decisions into consideration, and efficient collections decisions incorporate loan specifications initiated in the origination stage.

Many researchers study decision making in loan origination, that is, determining whether and how much to lend or extend. Mehta (1968) applies sequential decision theory to a credit origination problem. In view of past experience, this author identifies the information required before a decision on credit extension can be made. Bierman and Hausman (1970) propose quantitative models for both single- and multi-period credit granting decisions. These authors develop a procedure that first incorporates an initial subjective prior estimate of default probability for a new customer and then revise this probability when new information is obtained, such as through collection. In their work, the probability of default is jointly considered with possible gains and losses, both at present and in the future, to establish an optimal credit granting policy. Betancourt (1999) applies Markov chains to estimate losses from mortgage portfolio. Trench et al. (2003) design a Markov decision process-based optimization system for the determination of credit line and annual percentage rate for Bank One credit cards.

In practice, origination decisions are often made on the basis of *credit score* (Capon 1982; Thomas 2000, 2009; Thomas et al. 2002). A credit score, computed for each applicant or existing account, is mainly designed to predict the likelihood of default within a predefined time period after loan origination, and is often used to support decisions on whether to grant a loan or to extend credit, to help in determining interest rates, and in choosing intervention strategies for delinquent accounts. Accordingly, a majority of the credit origination literature addresses credit scoring. For example, Desai et al. (1997) use neural networks and genetic algorithms to derive credit scores. Compared with traditional techniques, such as linear discriminant analysis and logistic regression, the neural networks and genetic algorithms have the advantage of capturing nonlinear relationships in data sets. West (2000) studies the credit scoring accuracy of five neural network models and five traditional methods using two real data sets. The results demonstrate that certain neural network models perform better than traditional methods, and logistic regression has the highest accuracy among the five traditional methods. Other researchers who investigate credit scoring/rating methods and applications include Curnow et al. (1997), Chen and Chiou (1999), Yobas et al. (2000), Lee et al. (2002), Malhotra and Malhotra (2003), Ong et al. (2005), Butera and Faff (2006), Min and Lee (2008), and Kukuk and Rönnberg (2013). Note that even though credit score is more frequently used in origination decisions, it can also help with collection decisions.

There are also many works studying credit collection decisions, that is, deciding when and what collection actions to adopt for delinquent accounts. Cyert et al. (1962) estimate *allowance* of doubtful accounts using a Markov chain model, where the allowance represents the estimated amount of receivable balance that will ultimately be uncollectable. This method helps with both origination and collection decisions. Liebman (1972) considers the selection of credit control policies that minimize total credit loss. This author formulates the credit control problem as an infinite horizon Markov decision model, which is transformed into an equivalent linear program. In that model, effects of collection actions on the probability of recovery are considered. However, this author does not differentiate customers according to their characteristics. Using the idea of Markov chain model, Makuch et al. (1992) develop a probabilistic account flow model for more efficient use of limited collection resource. Briat (2005) develops an approach to analyze and control credit card accounts, with the aim of finding a collection policy that maximizes the profit of the credit card issuer. This author's approach is based on a two-dimensional Markov decision process: one for the due status of the account, and the other for the unused credit limit of the account. De Almeida Filho et al. (2010) investigate the collection process for consumer loans at an operational level. The decision problems considered include selecting an appropriate collection action and selecting the time horizon over which the action should last. The transition process is deterministic over an infinite horizon, and the debtors are assumed homogeneous. These authors identify optimality properties that define an optimal collection policy under particular conditions, and conduct a case study using data from the collections department of a European bank. Our work is related to De Almeida Filho et al. (2010) in that we also use dynamic programming in a Markov chain model. The main difference is that de Almeida Filho et al. consider collection actions over defaulted debts, each in a fixed amount, while we consider collection actions over regular term-loans, each with the purchase of a durable good.

For both credit origination and collection decisions, the rates or probabilities of state transitions play an important role and have raised intensive research attention. Cyert et al. (1962) use a Markov chain model to project default rates and losses associated with revolving trade credit. In that work, the transition probabilities are assumed to be constant and do not change with time. Kallberg and Saunders (1983) use a Markov chain approach to analyze the behavior of retail loan consumers, and note that a current account with a small balance may be more likely to be paid off in the next period than one with a large balance. It becomes evident over time that transition probabilities among alternative states in credit accounts change as credit ages. Usually, the probability of default is very low at the initiation of the credit, then increases to a higher level at medium loan age and finally falls to a low level again at a later stage (Smith and Jin 2007). This phenomenon is also observed in the bond and mortgage fields. In their studies of bond portfolios, Altman (1989) and Asquith et al. (1989) find that the probability of default changes significantly as assets age. A similar phenomenon is observed by Kang and Zenios (1992) for home mortgages. In addition to loan age, many other factors contribute to the dynamic of state transition rates. Cunningham and Capone (1990) show that the determinants of delinquency, early payoff, and default for fixed rate home mortgages differ from those for adjustable rate mortgages. For mobile home credit, Lawrence et al. (1992) find that borrower delinquency patterns are dominant indicators of default risk. For home mortgages, Smith et al. (1996) propose a Markovian model with nonstationary transition probabilities to predict the incidence of prepayment, delinquency, default, and losses on defaulted loans. Smith et al. (2005) show that Markovian models with nonstationary transition probabilities outperform those with stationary transition probabilities in predicting financial performance of hybrid mortgage loans in Taiwan. Grimshaw and Alexander (2011) estimate a transition matrix for a Markov chain model for accounts receivable, where Bayes and empirical Bayes estimators for transition probabilities are derived, respectively. The effects of collection actions on state transition probabilities in a Markov transition matrix are studied by He et al. (2015), who use a specific parameterization process to identify the significance and magnitude of certain collection effects.

We find that, as also pointed out by De Almeida Filho et al. (2010), research on collection decisions in consumer loans at an operational level is limited relative to the important role those decisions play in reducing credit loss. Few works specifically deal with the collections of nonrevolving debt incurred in the purchases of durable goods and our work is intended to fill this gap. The contribution of this work is threefold. First, we provide a modeling framework for optimizing collection decisions by incorporating both the dynamic transitions of account states and the effects of collection actions. Second, we develop an efficient optimization method that can be used in managing loans granted for consumers' purchases of durable goods. Third, our model and solution approach are built based on a typical collection business in the U.S. and are tested using representative data, thus providing managerial insight on how the model and its desirable components can be used in practice and the advantages of using them. We show not only the relative performance of the optimization model relative to a static policy, but also how collection actions under an optimal collection policy are affected by certain important factors. In summary, our work builds a close connection between theoretical research and practical application in consumer loan collection.

The remainder of this paper proceeds as follows. The modeling framework and model setup are formally presented in Sect. 2. The Markov decision model is developed and solved in Sect. 3. In Sect. 4, we apply the model to a loan collection problem with data generated following the practices of credit companies in the U.S. Finally, in Sect. 5, we conclude the paper and provide directions for future research.

2 Model setup

We consider a seller who sells durable goods directly to consumers, and a lender who provides loans to consumers to purchase those goods. Let ρ denote the per-period *discount factor* for the lender, such that the net present value of 1 to be received one period in the future is ρ from the perspective of the lender. The discount factor represents the lender's borrowing or opportunity cost. The time period used for account management can be days, weeks, or months.

When a consumer is approved by the lender for a loan, the consumer signs a contract with the lender. The lender then establishes an account to track the details of the loan and the corresponding consumer information. In practice, an account may contain a rich set of information. We are interested in the following account parameters.

- M Market value of the durable good at the time of purchase.
- L Initial loan amount offered by the lender to the buyer.
- T Term of the loan, i.e., the number of periods to payoff the loan.
- r Per-period interest rate the consumer pays the lender.

Accordingly, a contracted per-period payment, z, can be computed from L, T and r, as follows:

$$z = \frac{L}{\sum_{t=1}^{T} \frac{1}{(1+r)^t}}. \tag{1}$$

Term T is equal to the number of payments in the amount of z to be paid by the consumer. Note that the per-period payment is paid at the end of each period.

For notational convenience, account characterization is labeled θ, which includes all the parameters defined above, and potentially other account information relevant to the collection decision, if available. When the meaning is clear from context, an account with characterization θ is referred to as account θ.

Let t denote the age of an account, which is the number of periods passed since the onset of the loan. At each age, an account can be at a specific payment status, which we label as *state*, denoted by s. At each age $t \geq 1$, an account state is determined by the state and payment behavior of the account at age $t-1$. To address typical business situations, we consider the following states of an account at age t when applicable.

- $s = 0$ *Current*, i.e., no past due payment at age $t-1$, for $t = 1, \ldots, T+1$. An account must be in state current when $t = 1$, because a down payment is required at the onset of the loan when $t = 0$. When a loan is paid off at age T, we have $s = 0$ at age $T+1$.
- $s = x$ for $x = 1, 2, 3, 4$ *x-period delinquent*, i.e., there are x contracted per-period payments not received at age $t-1$, for $t = x+1, \ldots, T+1$. Note that an account can never be in state $s = x$ at age t when $t \leq x$, because a maximum of $t-1$ per-period payments are due at age $t-1$.
- $s = -1$ *Early payoff*, i.e., the consumer pays off all the remaining balance at age $t-1$, for $t = 2, \ldots, T$. Note that an account cannot be in state $s = -1$ at $t = 1$, since otherwise no loan is granted. Also, when a consumer pays off the debt at age T, the end of term, the state at age $T+1$ is current.
- $s = -2$ *Default*, i.e., the consumer's purchased good is repossessed by the lender at age $t-1$. In practice, repossession occurs only when a customer misses several payments, and thus the default state can only occur when t is large enough.
- $s = -3$ *Bankrupt*, i.e., a third party certifies that the consumer is in a legal status rendering the consumer incapable of paying the remaining balance to the lender at age $t-1$. In this case, the loan contract loses its original effectiveness and the third party determines an amount of money that the consumer must pay the lender.

Note that for our problem there is no need to define the state of an account at time $t = 0$. In defining the states early payoff and bankrupt, *remaining balance* of an account is used. We denote γ as the per-period late payment penalty rate, charged over the per-period payment to the consumer by the lender. The remaining balance of an account θ in state s at age t is

$$\mathcal{R}(s, t, \theta) = \begin{cases} L, & \text{if} \quad t = 0, \\ \sum_{i=0}^{s} (1+\gamma)^i z + \sum_{i=t+1}^{T} z/(1+r)^{i-t}, & \text{if} \quad t = 1, \ldots, T, \end{cases}$$

where in the second case, the first term is the payment due by time period t plus past due with late payment penalty if any, and the second term is the remaining balance due by time period $t+1$ and later.

The above defined eight states model the most frequently occurring account statuses in practice, and simplify the collection problem in the following respects. First, essentially, a consumer can pay any amount no greater than the remaining balance. For ease of presentation, we assume a consumer pays either a multiple of the contracted per-period payment plus late payment penalty if any, or the entire remaining balance. Second, technically, an account could be delinquent for more than 4 periods. In this work, we assume that whenever an account is 4-period delinquent, the lender will repossess the purchased good from the consumer, and thus the state will not transit to 5-period delinquent. Third, it is possible that a consumer negotiates with the lender to change the loan contract, such as extending the term and/or reducing the interest rate. We do not consider contract changes in this work. However, the above-mentioned other payment amounts, delinquent by more periods, and changes to the loan contract can be modeled similarly in an enlarged state space and do not make a fundamental difference to our model.

Besides the payment status, risk status, specifically *risk level*, of an account at each age needs to be modeled. The lower the risk level, the more likely an account is to be in the state of current in the next age. Risk level is determined primarily by the past payment behavior of the account for the loan, and secondarily by the overall credit profile of the account. In practice, the lender tracks the account payments for the granted loan, and monitors the credit information of the account, such as the FICO score, to estimate the risk level of an account at each age.

The lender can implement a preventive or corrective collection action, denoted by a, over an account at a given age, if applicable. The following actions are considered in this work.

- $a = 0$ *Do-nothing*, leave the account as is.
- $a = 1$ *E-contact*, notify the consumer of account state via text message, email, or an automatic machine dialer.
- $a = 2$ *Account management*, mail or call the consumer to inform on account information, gather more information, and advise on future payment resolutions.
- $a = 3$ *Loss prevention*, communicate with the consumer intensively to discover why the account has been in delinquency states, and provide specific advice on how to retain the account in the state of current.
- $a = 4$ *Repossession*, repossess the consumer's good purchased through the loan. If the residual value of the good is lower than the sum of the remaining balance and the cost of repossession, then the lender incurs a loss in the amount of that difference; otherwise, the lender pays the difference back to the consumer. That is, the lender cannot earn money through repossession.

Note that there is an operational cost incurred by each action. Also, the repossession action relates to the *residual value* of an account. Let $R(t, \theta)$ denote the residual value of the purchased good at age t. That is, when the lender repossesses the good at age t, it can sell the good at price $R(t, \theta)$. In practice, the lender typically sells the repossessed good in a short time through auction at a price lower than the market value. Therefore, residual value is also referred to as auction value in the literature. We use a commonly used method for calculating residual value, as a function of the initial value of the purchased good and the age of the loan

$$R(t, \theta) = M \exp(-d_0 - dt), \tag{2}$$

where d_0 is a one-time depreciation factor at the time of purchase, and d is a per-period depreciation factor.

The *payback* policy of the extra residual value in repossession is required by laws to protect the interest of consumers. Theoretically, if the residual value of the repossessed good is insufficient to cover the remaining balance plus the repossession cost, then the consumer still owes the lender that difference. But practically, this gap is difficult to collect, and thus is ignored in our definition.

We assume that at most one action can be applied to an account in one time period, to be consistent with the assumption that the account state does not change within one time period. This assumption is not restricted, in that if the action requires time shorter than one time period, then a shorter time unit can be used in defining time period.

Now we consider how an account transits from one state to another as the account ages. A state transition happens at the time when an account becomes age $t+1$ from age t, for $t = 1, \ldots, T$. Recall that an account is always in the state of current at age $t = 1$ by our definition.

Let $P^a_{s,s'}(t, \theta)$ denote the probability of transition from state s to state s' for account θ that enters age $t+1$, under the application of collection action a. The transition probability defines a discrete Markov chain, where the account state at the next age depends only on the account state and the collection action applied on the account at the current age. The Markov transition matrix has the following structure:

$$\begin{pmatrix} P^a_{0,0}(t,\theta) & P^a_{0,1}(t,\theta) & 0 & 0 & 0 & P^a_{0,-1}(t,\theta) & 0 & P^a_{0,-3}(t,\theta) \\ P^a_{1,0}(t,\theta) & P^a_{1,1}(t,\theta) & P^a_{1,2}(t,\theta) & 0 & 0 & P^a_{1,-1}(t,\theta) & 0 & P^a_{1,-3}(t,\theta) \\ P^a_{2,0}(t,\theta) & P^a_{2,1}(t,\theta) & P^a_{2,2}(t,\theta) & P^a_{2,3}(t,\theta) & 0 & P^a_{2,-1}(t,\theta) & P^a_{2,-2}(t,\theta) & P^a_{2,-3}(t,\theta) \\ P^a_{3,0}(t,\theta) & P^a_{3,1}(t,\theta) & P^a_{3,2}(t,\theta) & P^a_{3,3}(t,\theta) & P^a_{3,4}(t,\theta) & P^a_{3,-1}(t,\theta) & P^a_{3,-2}(t,\theta) & P^a_{3,-3}(t,\theta) \\ P^a_{4,0}(t,\theta) & P^a_{4,1}(t,\theta) & P^a_{4,2}(t,\theta) & P^a_{4,3}(t,\theta) & P^a_{4,4}(t,\theta) & P^a_{4,-1}(t,\theta) & P^a_{4,-2}(t,\theta) & P^a_{4,-3}(t,\theta) \end{pmatrix}.$$

In the transition matrix, a 0 entry means that such a transition does not occur by definition. Specifically, an account cannot become x-period delinquent from the state of less than $(x-1)$-period delinquent, for $x = 2, 3, 4$, and a lender will not repossess a customer's purchased good when the account is less than 2-period delinquent. Note that states $s = -1, -2, -3$ do not transit to any other states and thus are omitted from the matrix as originating states. The econometric characterization of the transition matrix is tightly coupled with the envisioned dynamic credit risk and must be quantified specifically for the corresponding business problem. For quantification of the elements of such transition matrix, please refer to He et al. (2015).

There is a monetary value, either a gain or a loss, incurred by the lender during each state transition. Specifically, let $\pi^a_{s,s'}(t, \theta)$ denote the value received from account θ that transits from state s to state s' when entering age $t+1$ under action a. The structure of the per-period value matrix is as follows.

$$\begin{pmatrix} \pi^a_{0,0}(t,\theta) & \pi^a_{0,1}(t,\theta) & 0 & 0 & 0 & \pi^a_{0,-1}(t,\theta) & 0 & \pi^a_{0,-3}(t,\theta) \\ \pi^a_{1,0}(t,\theta) & \pi^a_{1,1}(t,\theta) & \pi^a_{1,2}(t,\theta) & 0 & 0 & \pi^a_{1,-1}(t,\theta) & 0 & \pi^a_{1,-3}(t,\theta) \\ \pi^a_{2,0}(t,\theta) & \pi^a_{2,1}(t,\theta) & \pi^a_{2,2}(t,\theta) & \pi^a_{2,3}(t,\theta) & 0 & \pi^a_{2,-1}(t,\theta) & \pi^a_{2,-2}(t,\theta) & \pi^a_{2,-3}(t,\theta) \\ \pi^a_{3,0}(t,\theta) & \pi^a_{3,1}(t,\theta) & \pi^a_{3,2}(t,\theta) & \pi^a_{3,3}(t,\theta) & \pi^a_{3,4}(t,\theta) & \pi^a_{3,-1}(t,\theta) & \pi^a_{3,-2}(t,\theta) & \pi^a_{3,-3}(t,\theta) \\ \pi^a_{4,0}(t,\theta) & \pi^a_{4,1}(t,\theta) & \pi^a_{4,2}(t,\theta) & \pi^a_{4,3}(t,\theta) & \pi^a_{4,4}(t,\theta) & \pi^a_{4,-1}(t,\theta) & \pi^a_{4,-2}(t,\theta) & \pi^a_{4,-3}(t,\theta) \end{pmatrix}.$$

The value matrix has the same 0 entries as the transition matrix. Each entry in the matrix is determined by account characteristics, the state transition, and the action applied. A more specific definition of the value matrix is provided in Sect. 3.

The lender's decision problem is to determine the collection action to be implemented at each state and age over each account or account type, with the objective of maximizing the expected net present value of each account.

3 Optimization

This section sets forth optimization methods to the Markov chain model defined in Sect. 2. In Sect. 3.1, we develop a dynamic programming algorithm to solve the general collection problem, and in Sect. 3.2, we provide a closed-form solution for a special case where the state and action spaces are simplified and the loan term is infinite.

3.1 General case

We begin by briefly discussing the estimation of the probabilities in the Markov transition matrix defined in Sect. 2. A detailed quantification method is provided by He et al. (2015). A transition probability can be parameterized. Specifically, let $S(s,\theta)$ be the set of states account θ can transit to from state s. Consider the parametrization

$$P^a_{s,s'}(t,\theta) = \frac{\exp(f^a_{s,s'}(t,\theta))}{\sum_{k\in S(s,\theta)}\exp(f^a_{s,k}(t,\theta))}, \tag{3}$$

such that the normalization $\sum_{k\in S(s,\theta)} P^a_{s,k}(t,\theta) = 1$ is satisfied. The value $f^a_{s,k}(t,\theta)$ can be a function of t and θ given start state s, end state k, and action a. Note that the transition probabilities are action dependent, or more generally collection policy dependent.

Next we specify entries in the per-period value matrix. Let $c^a_{s,s'}(\theta)$ denote the age-independent cost of applying action a to account θ that transits from state s to state s'. Also, denote as B the cost incurred by the lender when an account becomes bankrupt. Recall that $\mathcal{R}(s,t,\theta)$ denotes the remaining balance of account θ in state s at age t, and $R(t,\theta)$ denotes the residual value of the purchased good for account θ at age t. Now we are ready to quantify the per-period payoff.

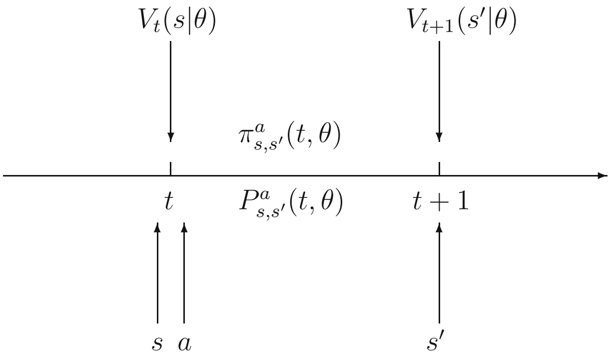
$$\begin{aligned}
&\pi^a_{s,s'}(t,\theta) = -c^a_{s,s'}(\theta) + z\sum_{i=s'}^{s}(1+\gamma)^i, \text{for } s\in\{0,1,2,3\} \text{ and } s'\le s+1;\\
&\pi^a_{s,-1}(t,\theta) = -c^a_{s,-1}(\theta) + \mathcal{R}(s,t,\theta),\ \text{ for } s\in\{0,\ldots,4\};\\
&\pi^a_{s,-2}(t,\theta) = \min\{-c^a_{s,-2}(\theta) + R(t,\theta), \mathcal{R}(s,t,\theta)\},\ \text{ for } s\in\{2,3,4\};\\
&\pi^a_{s,-3}(t,\theta) = -c^a_{s,-3}(\theta) + \max\{\mathcal{R}(s,t,\theta)\} - B, 0\},\ \text{ for } s\in\{0,\ldots,4\}.
\end{aligned}$$

Note that for an account that is in state s at age t, the consumer must pay $z\sum_{i=0}^{s}(1+\gamma)^i$, which includes late payment penalty, to have his account reinstated to the current state at age $t+1$. Also, when an account becomes bankrupt, the cost of bankrupt only applies when the remaining balance of the account is sufficient to cover the cost; otherwise, the lender will just write off the account with no value or cost.

The maximum expected net present value (ENPV) of account θ in state s at age t, taking the state transition and applicable action into consideration, is denoted by $V_t(s|\theta)$. To more clearly show how states transit in the Markov chain, we depict the timing convention in Fig. 1, where an action a is applied to account θ in state s at age t, and then with probability $P^a_{s,s'}(t,\theta)$ the account transits to state s', during which a value $\pi^a_{s,s'}(t,\theta)$, which can be either positive or negative, is received by the lender.

The value $V_t(s|\theta)$, and the optimal action that maximizes the ENPV of an account at each state and age can be found via the following Bellman equation through a standard backward induction:

Fig. 1 Timing convention of the Markov chain



$$V_t(s|\theta) = \max_{a\in A(s|\theta)} \left\{ \sum_{s'\in S(s|\theta)} P^a_{s,s'}(t,\theta)(\pi^a_{s,s'}(t,\theta) + \rho V_{t+1}(s'|\theta)) \right\} \quad \forall t \in \{1,2,\ldots,T\}, \tag{4}$$

where $A(s|\theta)$ is the action space for account θ in state s.

Note that the maximum ENPV at age 0, the onset of the loan, is $\rho V_1(0|\theta)$. The inclusion of discount factor ρ, together with the interest rate r, addresses the tradeoff between interest revenue and borrowing cost. Also, since $V_t(s|\theta)$ is optimized at each age t, the solution achieves time consistency in optimization. In addition, our definition of the transition and per-period value matrixes models competing risks between different account states, along with the penalty for late payment. Since the Markov chain model is widely applied for decisions on collection policy and action (Rosenberg and Gleit 1994; Thomas et al. 2005), we provide the following result with proof omitted.

Proposition 1 *Equation* (4) *finds an optimal action that maximizes the ENPV of account* θ *in state* s at age t.

More specifically, to maximize $V_t(s|\theta)$, *the recurrence relation of Eq.* (4) *defines an optimal action for the account at each combination of t and s*, i.e., an *optimal collection policy. To find the value of* $V_t(s|\theta)$, *the boundary condition of Eq.* (4), *i.e., the value of* $V_{T+1}(s|\theta)$, *must be specified based on the corresponding practical problem. We discuss how to specify the boundary condition in Sect.* 4.

The explicit appearance of account θ in Eq. (4) indicates that we can solve such an optimization problem for every account type, if the transition and per-period value matrixes are given according to account characteristics. In practice, account characteristics can be grouped to reduce the computing burden. Note that the action space $A(s|\theta)$ and the state space $S(s|\theta)$ at age t depend on both s and θ, which potentially excludes particular actions and states. Also, the applicable action may also depend on the actions applied at earlier ages; for example, a repossession action cannot be used unless a loss prevention action is applied at the preceding age. We can add a binary state variable to the value function V to model whether a loss prevention action is applied at the preceding age, and restrict the application of the repossession action at the current age accordingly. Given account characteristics, Eq. (4) has finite state and action spaces and finite horizon T, and thus can be easily solved numerically.

3.2 Special case

Consider a special case where the state and action spaces are simplified, the loan term is infinite, and consumers are homogeneous. Slightly abusing notations, let $\{s = 0, 1\}$ be the space of the states of an account, where $s = 0$ means that the account is current, and $s = 1$ means that the account is delinquent; and let $\{a = 0, 1\}$ be the space of the actions that can be implemented over an account, where $a = 0$ represents no action, and $a = 1$ represents a collection action. The action space of each state is defined as follows.

$$\begin{aligned} &A(s = 0) = \{a = 0\}, \\ &A(s = 1) = \{a = 0, a = 1\}. \end{aligned}$$

The transition matrixes from state s to s' when $a = 0$ or $a = 1$ are as follows.

$$\begin{aligned} P(s, s'|a = 0) &= \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}, \\ P(s, s'|a = 1) &= \begin{pmatrix} \# & \# \\ q & 1-q \end{pmatrix}, \end{aligned}$$

where $s, s' \in \{0, 1\}$, and p represent the probability that the account transits from state 0 to state 1 when no action is implemented, and q represents the probability that the account transits from state 1 to state 0 when the collection action is implemented. We assume that the collection action cannot be applied to a current account, and thus the corresponding probability has no practical meaning and is denoted by $\#$.

Next we define the per-period value matrix. For convenience of discussion, we normalize the payoff from an account that keeps at state 0 to be 0. The per-period value matrixes for an account with $a = 0$ or $a = 1$ are as follows.

$$\begin{aligned} \pi(s, s'|a = 0) &= \begin{pmatrix} 0 & -\alpha \\ \# & -\alpha \end{pmatrix}, \\ \pi(s, s'|a = 1) &= \begin{pmatrix} \# & \# \\ -\beta & -\alpha-\beta \end{pmatrix}, \end{aligned}$$

where α is the loss amount of a one-period payment from a delinquent account, and β represents the cost of implementing the action. Note that for entries with 0 value or no definition in the transition matrix, we do not need to define a value, so these entries are filled with $\#$.

Following the same logic as in Eq. (4), we may use a recurrence relation to find $V_t(s|\theta)$ for the special case. Since here we consider an infinite horizon and homogeneous consumers, we simplify $V_t(s|\theta)$ to $V(s)$.

$$V(s) = \max_{a \in A(s)} \left\{ \sum_{s' \in \{0,1\}} P^a_{s,s'} (\pi^a_{s,s'} + \rho V(s')) \right\}, \tag{5}$$

where $A(s)$ is the action space for an account in state s.

We next find a closed-form solution for Eq. (5), and specify the decision rule for implementing an optimal collection policy. Note that when $s = 0$, only action $a = 0$ is applicable, and we need to determine the optimal collection action when $s = 1$. We summarize our findings in the following proposition. Let $q_c = \beta[1-\rho(1-p)]/\alpha$ and $\beta_c = \alpha/[1-\rho(1-p)]$. Let a^* denote an optimal action.

Proposition 2

(1) *If* $q \le q_c$ *or* $\beta > \beta_c$, *then the optimal action is* $a^* = 0$ *when* $s = 1$, *and accordingly*

$$V(0) = \frac{-\alpha p}{(1-\rho)[1-(1-p)\rho]},$$
$$V(1) = \frac{-\alpha}{1-\rho};$$

(2) if $q > q_c$, *which implies* $\beta \le \beta_c$, *then the optimal action is* $a^* = 1$ *when* $s = 1$, *and accordingly*

$$V(0) = \frac{-p(\alpha+\rho\beta)}{(1-p)[1-\rho(1-p-q)]},$$
$$V(1) = \frac{-\alpha[1-q-\rho(1-p-q)] - \beta[1-\rho(1-p)]}{(1-\rho)[1-\rho(1-p-q)]}.$$

Proof First, we explain the meaning of q_c and how its value is obtained. According to Eq. (5), when $s = 0$, we have

$$\begin{aligned} V(0) &= \sum_{s' \in \{0,1\}} P^0_{0,s'}[\pi^0_{0,s'} + \rho V(s')] \\ &= (1-p)[0+\rho V(0)] + p[-\alpha+\rho V(1)], \end{aligned}$$

and then

$$V(0) = \frac{p[-\alpha+\rho V(1)]}{1-(1-p)\rho}. \tag{6}$$

□

When $s = 1$, we have

$$\begin{aligned} V(1) &= \max\left\{\sum_{s'} P^0_{1,s'}[\pi^0_{1,s'} + \rho V(s')], \sum_{s'} P^1_{1,s'}[\pi^1_{1,s'} + \rho V(s')]\right\} \\ &= \max\{-\alpha+\rho V(1), q[-\beta+\rho V(0)] + (1-q)[-\alpha-\beta+\rho V(1)]\}. \end{aligned} \tag{7}$$

Denote the first part in the braces of Eq. (7) as $V_{a=0}(1)$, the ENPV of the account when no action is implemented, and the second part as $V_{a=1}(1)$, the ENPV of the account when the collection action is implemented. The threshold value of q, denoted by q_c, which determines whether to implement the action, should solve $V_{a=0}(1) = V_{a=1}(1)$, and accordingly we have

$$q_c = \frac{\beta[1-\rho(1-p)]}{\alpha}.$$

Now we prove parts (1) and (2) of the proposition.

(1) If $q \le q_c$, then it can be easily deduced that $V_{a=0}(1) \ge V_{a=1}(1)$. Therefore, the optimal action is $a^* = 0$ in Eq. (7), and

$$V(1) = V_{a=0}(1) = -\alpha + \rho V(1),$$

from which we have

$$V(1) = \frac{-\alpha}{1-\rho}.$$

Plugging the value of $V(1)$ into Eq. (6), we have

$$V(0) = \frac{-\alpha p}{(1-\rho)[1-(1-p)\rho]}.$$

On the other hand, if $\beta > \alpha/[1-\rho(1-p)]$, then it can be easily verified that

$$q_c = \frac{\beta[1-\rho(1-p)]}{\alpha} > 1.$$

Note that q is a probability, and thus $q \leq q_c$. Hence, the optimal decision is $a^* = 0$.

(2) If $q > q_c$, then $V_{a=0}(1) < V_{a=1}(1)$, and thus the optimal action is $a^* = 1$ in Eq. (7). We have

$$V(1) = V_{a=1}(1) = q[-\beta + \rho V(0)] + (1-q)[-\alpha - \beta + \rho V(1)]. \tag{8}$$

Combining Eqs. (6) and (8), we obtain

$$V(0) = \frac{-p(\alpha + \rho\beta)}{(1-p)[1-\rho(1-p-q)]},$$
$$V(1) = \frac{-\alpha[1-q-\rho(1-p-q)] - \beta[1-\rho(1-p)]}{(1-\rho)[1-\rho(1-p-q)]}.$$

Thus, the proposition is proven. □

Proposition 2 recognizes the conditions under which the collection action should be implemented or not. The conditions first regard the action effect, as measured by q. When an account is in the state of delinquent, if the action effect is below a threshold value q_c, then the action should not be applied; and if the action effect is above q_c, then the action should be implemented. Also, the action cost β plays an important role in the collection decision. If the action cost is higher than a threshold value β_c, then the action should never be implemented.

The model in Proposition 2 does not explicitly consider interest rate. Observe that the one-period payment amount increases with interest rate, i.e., α increases with interest rate. By definition, the action effect threshold value q_c decreases with α, and thus we have that as interest rate increases, less action effect is needed for a collection action to be implemented. Also, the action cost threshold value β_c increases with α, which means that less cost efficiency is needed for a collection action to be implemented as the interest rate increases. Overall, we conclude that *the collection action should be more frequently implemented over delinquent accounts that have higher interest rates.*

It is straightforward to check that the value function $V(s)$ is continuously nondecreasing with q for $0 \leq q \leq 1$ and is continuously nonincreasing with β. Therefore, it is beneficial for the lender to design collection actions that are effective and cost-efficient. Contemporarily, it is widely acknowledged that new technology enables cost-efficient collections via electronic communication without much human interaction. However, the effects of these actions warrant thorough investigation at a quantitative level.

4 Computational study

In this section, we illustrate how our Markov chain optimization model applies to a designed loan maintenance problem. We show not only the relative performance of the optimization model relative to a static policy, but also how collection actions under an optimal collection policy are affected by certain important factors. Our experimental design is based on our understanding of the loan management practice faced by credit companies in the U.S. that offer term-loans. Problem parameters are estimated following the general collection practice, and thus the findings should be regarded as qualitative rather than quantitative. General business insights are provided sequentially. One benefit of using designed data is that our study is replicable for comparisons by other researchers.

The loan problem for high-value durable goods has an important feature that differentiates loans from other types of consumer loan. That is, the high and long lasting residual values of the purchased goods make the repossession action an important business practice in loan management. This feature significantly reduces the credit risk of the lender, but makes the loan collection problem more important, since companies may overly rely on repossession action and ignore other types of collection action.

We consider contracts with a 60-month term, and use month as the time unit. Using month as a planning unit is consistent with the typical business situation where payments are contracted to be paid per month. One disadvantage of using month as a period is that the outcomes of certain collection actions can be resolved much sooner than a month such that new actions can be implemented before the next month. This problem can be substantially alleviated if we use week or day as a planning unit.

Due to legal or business reasons, the applicable action space is smaller than the full action space. Specifically, we define the following action space:

$$A(0|\theta) = \{0\};$$
$$A(1|\theta) = \{0, 1, 2, 3\};$$
$$A(s|\theta) = \{0, 1, 2, 3, 4\}, \text{for}\, s \in \{2, 3\};$$
$$A(4|\theta) = \{4\}.$$

Let c^j denote the cost of action j, for $j = 1, \ldots, 4$. We assume $c^0 = 0$, $c^1 = 0.1$, $c^2 = 1$, $c^3 = 10$, $c^4 = 1000$, each in the unit of U.S. dollar. Note that, as discussed in Sect. 2, the cost of repossession can be covered by the residual value of the repossessed durable good if that value is higher than the remaining balance of the account. The residual value of the durable good is estimated using Eq. (2) with $d_0 = 0.3$ and $d = 0.02$. Also, we make the following assumptions on other relevant costs.

1. *Bankruptcy cost* The minimum of \$1,000 and the remaining balance of the bankrupt account;
2. *Delinquency penalty* 2% of the monthly payment, e.g., when a \$500 payment is 1 month past due, the consumer needs to pay a \$10 penalty.
3. *End-of-term value* All the late payments plus the late payment penalty at a 2% monthly penalty rate.

Note that for the end-of-term value, which will be used as a boundary condition in the Markov chain model, we assume that at the end of the term, the past due and the late payment penalty of a delinquent account are received by the lender. This assumption follows the practice that when the account is past due for a few months at the end of the term, the consumer is very likely to pay off all the debt in a short time, since the residual value of the consumer's durable good is much higher than the debt.

We use an internal discount factor $\rho = 1/(1 + 0.04/12)$, i.e., a yearly discount rate of 4%.

To determine the optimal collection policy, one must understand the effect of each action over an account. To model how a collection action affects the state transition of an account, we need to know how an account transits with no collection actions. We assume a parameterized function in terms of time period t, as follows:

$$P_{s,s'}(t) = \frac{\exp(f_{s,s'}(t)}{\sum_{k \in S(s)} \exp(f_{s,k}(t))}, \tag{9}$$

where we assume function $f_{s,s'}(t) \equiv u_{s,s'} + v_{s,s'}t$, i.e., a linear function of t, has the parameters defined in Table 1. Note that an account cannot go to state default $s = 4$ without repossession action, and must go to state default $s = 4$ when the repossession action applies to it. Also, action $a = 0$ has no effect. We assume that a repossession action applies whenever an account becomes 4-month delinquent and the account state becomes default $s = 4$ in the next period, and thus we have only $s = 0, 1, 2, 3$ in Table 1.

In general, collection effects are difficult to quantify due to account heterogeneity and the small sample size for certain types of data in practice. For simplicity, we make the following assumption on action effects over state transitions.

Table 1 Parameters for transition probabilities

s	s'	$u_{s,s'}$	$v_{s,s'}$
0	− 3	− 7.0	0.00
0	− 1	− 3.0	− 0.01
0	0	0.0	0.00
0	1	− 2.0	0.01
1	− 3	− 6.0	0.00
1	− 1	− 6.0	0.00
1	0	0.0	0.00
1	1	− 1.0	0.01
1	2	− 1.5	0.01
2	− 3	− 5.0	0.00
2	− 1	− 5.0	0.00
2	0	0.0	0.00
2	1	− 0.5	0.01
2	2	− 1.0	0.01
2	3	− 1.5	0.01
3	− 3	− 4.0	0.00
3	− 1	− 4.0	0.00
3	0	− 0.5	− 0.01
3	1	0.0	0.00
3	2	− 0.5	0.01
3	3	− 1.0	0.01
3	4	− 1.5	0.01

Assumption 1 Relative to action 0, actions 1, 2, and 3 (1) increase the probability of a delinquent account ($s = 1, 2, 3$) becoming current ($s = 0$), by $K\%$, $2K\%$ and $3K\%$, respectively; (2) do not affect the transition probability to early payoff ($s = -1$) and bankruptcy ($s = -3$); and (3) decrease the transition probabilities to other delinquent states ($s = 1, 2, 3, 4$), if applicable, by a common proportion such that with the application of an action, the total transition probability from any account state ($s = 1, 2, 3$) is equal to 1.

We consider $0 \le K \le 20$. Assumption 1 is consistent with the practice that the more intensively an action is applied to a delinquent account, the more likely the account becomes current. Also, early payoff and bankruptcy typically are caused by other factors and are not affected by collection actions.

From Eq. (9), Table 1, and Assumption 1, we can determine how transition probability is affected by collection actions. We illustrate in Table 2 the action effects, with $K = 15$.

Throughout our investigation, we assume the price of the purchased good is \$30,000.

To evaluate the benefit of implementing an optimal collection policy found by the Markov chain model, we compare that policy with a benchmark collection policy. The benchmark policy is *static* in that action $a = i$ is applied when an account reaches state $s = i$, for $i = 0, 1, 2, 3, 4$. The static policy is easy to understand and implement in practice. Note that our optimal collection policy is *dynamic* in the sense that, for the same state, actions can differ for different loan accounts or at different ages.

Under the assumed action effect, our Markov-chain-model-based optimal collection policy is applied to instances generated following the above-mentioned design. The solutions obtained through Eq. (4) and the static policy reveal that the relative performance of the optimal collection policy is affected by interest rate, loan amount, and action effect. Next we demonstrate how the performance of our optimal collection policy relative to the static collection policy is affected by each of the three factors.

First, we consider the factor of interest rate. Note that we assume an annual discount rate of 4%, and correspondingly we consider an annual interest rate from 4 to 20%. We fix the loan amount at L = \$18,000 and the action effect at $K = 15$. The extra monetary value generated by our optimal collection policy relative to the static collection policy is plotted against interest rate in Fig. 2. The extra value increases nearly linearly with interest rate. Intuitively, this is because higher interest rate potentially provides more interest revenue to the lender, and leaves more value for the optimal collection policy to improve over the static policy.

Second, we consider the factor of loan amount. Recall that we assume a price of the purchased good at \$30,000. Correspondingly, we consider loan amounts from \$3000 to \$30,000. We fix the annual interest rate at $r = 15\%$ and the action effect at $K = 15$. The extra monetary value generated by our optimal collection policy is plotted against loan amount in Fig. 3. It is interesting to note that the extra value offered by the optimal policy increases when the loan amount is not too high, and then first decreases and then increases slowly when the loan amount is above a particular amount. That is, the extra value

Table 2 Averaged action effects on transition probability

	$a = 0$	$a = 1$	$a = 2$	$a = 3$
$P^a_{1,0}$	0.552	0.635	0.717	0.800
$P^a_{2,0}$	0.377	0.434	0.491	0.547
$P^a_{3,0}$	0.144	0.165	0.187	0.208

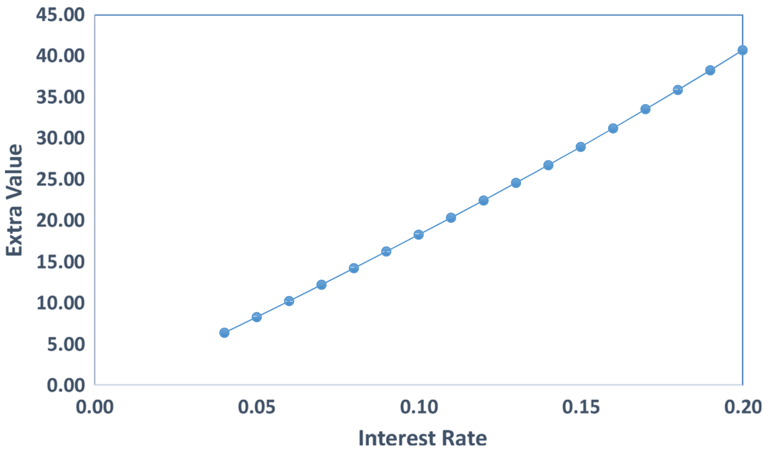


Fig. 2 Relative performance affected by interest rate

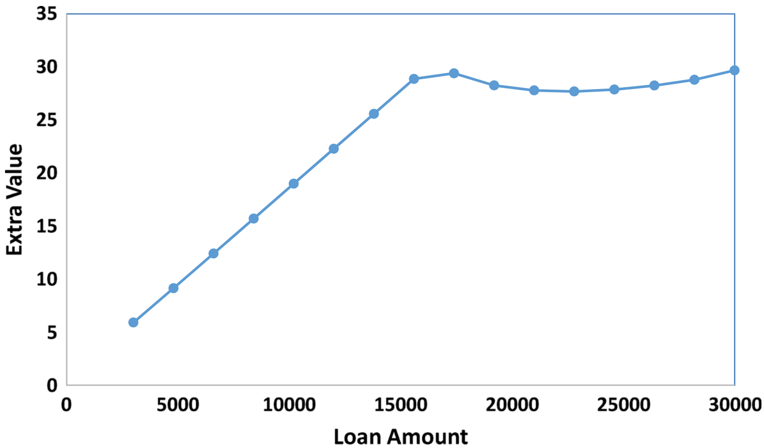


Fig. 3 Relative performance affected by loan amount

achieves its maximum at both a medium level and a very high level of the loan amount. This may be because when the loan amount is at a medium level, the tradeoff between interest revenue and loss of default is difficult to manage via a static collection policy, and when the loan amount is very high, high interest revenue provides more space for the optimal collection policy to improve over the static policy.

Third, we consider the factor of action effect. We consider action effect K from 0 to 20, and fix the annual interest rate at $r = 15\%$ and loan amount at L = \$18,000. The extra monetary value generated by our optimal collection policy is plotted against loan amount in Fig. 4. The relative performance of the optimal collection policy increases with action effects. Intuitively, the optimal collection policy becomes more valuable with larger collection effects relative to a static collection policy.

From the above results, we can see that compared with a static collection policy, the relative performance of the optimal collection policy is evidently affected by the three factors in consideration: interest rate, loan amount, and collection effect. Next for each of

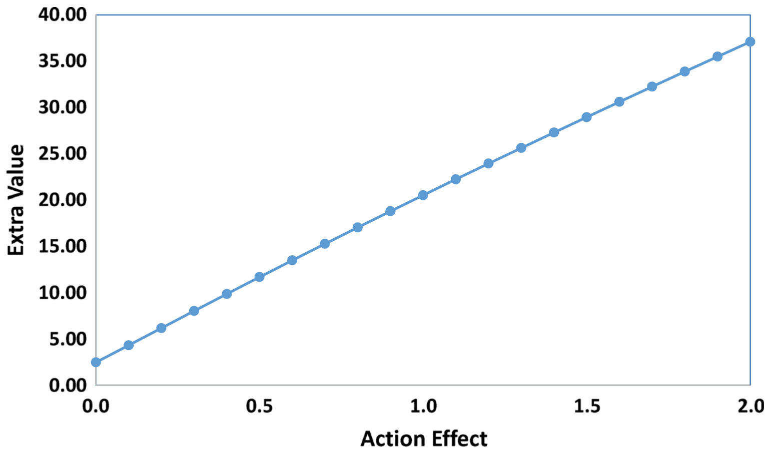


Fig. 4 Relative performance affected by action effect

these three factors, we provide a pair of instances to show how collection actions implemented under an optimal collection policy is affected.

First, consider the impacts of interest rate on collection actions under an optimal collection policy. We compare two accounts with the same default parameter setting except that one has a *low interest rate* of 5% and the other has a *high interest rate* of 15%. The optimal collection actions, i.e., the action to implement for each account at the combination of each state and age, are depicted in Fig. 5. We can see that the high interest account should be maintained under more intensive collection actions. Specifically, when at state 1 (1-month delinquent), no action is needed for both accounts. When at state 2 (2-month

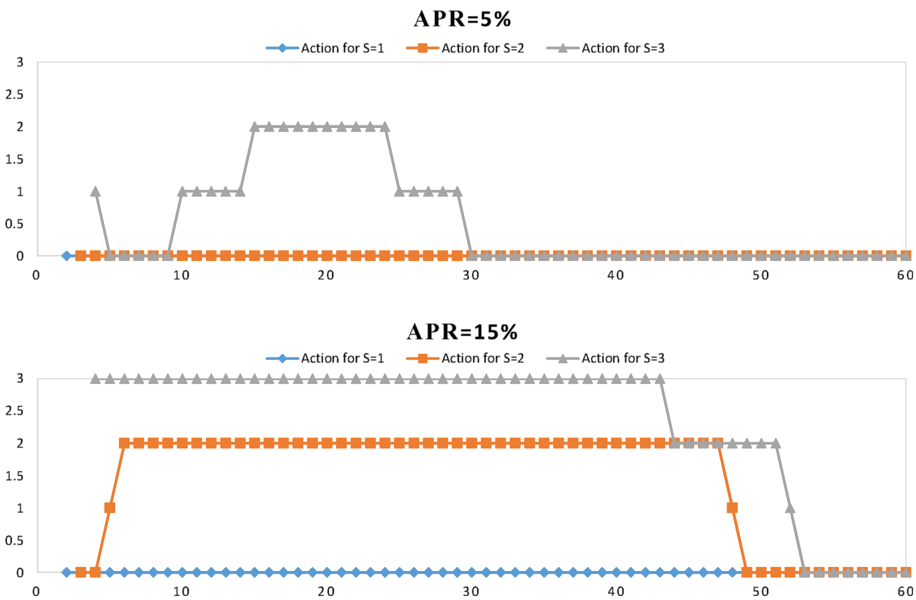


Fig. 5 Actions affected by interest rate

delinquent), still no action is needed for the low interest account, but action 2 (account management) should be applied to the high interest account unless the account is at very early or late ages. When at state 3 (3-month delinquent), even the low interest account needs action 1 (e-contact) or 2 at certain early ages, and the high interest account needs action 3 (loss prevention) except for at very late ages. Intuitively, the difference between the optimal collection policies for the two accounts is because that higher interest rate can generate greater interest revenue for an account at current state to cover collection costs.

Second, consider the impacts of loan amount on collection actions under an optimal collection policy. We compare two accounts with the same default parameter setting except that one has a *smaller loan amount* of $10,000 and the other has a *greater loan amount* of $25,000. Since we consider a fixed price $30,000 of the purchased good, considering different loan amounts is equivalent to considering different loan-to-value ratios. The optimal collection actions are depicted in Fig. 6. We can see that the account with greater loan amount should be more intensively maintained. Specifically, when at state 1, no action is needed for both accounts. When at state 2, no action is needed for the account with smaller loan amount, but action 2 should be applied to the account with greater loan amount unless the account is at very early or late ages. When at state 3, the account with smaller loan amount needs action 1 or 2 except for at very late ages, and the account with greater loan amount needs action 3 except for at late ages where action 2 may apply. Similar as for the impacts of interest rate, the difference between the optimal collection policies for the two accounts with different loan amounts is because that greater loan amount can bring higher interest revenue for an account at the current state to compensate collection costs.

Third, consider the impacts of collection effect on collection actions under an optimal collection policy. For an account with the default parameter setting, we consider two sets of actions with parameters $K = 5$ (namely, *weak action effect*) and 15 (namely, *strong*

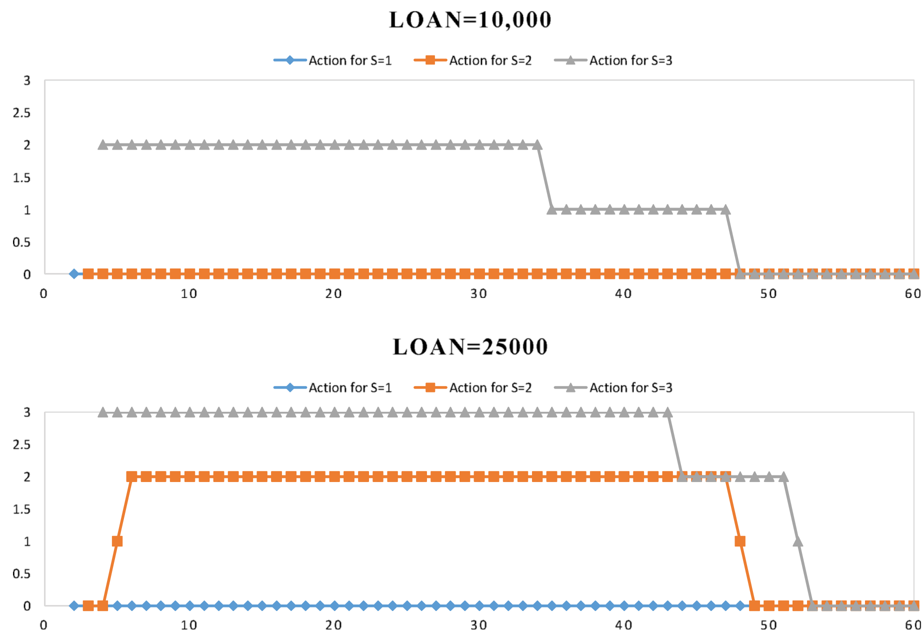


Fig. 6 Actions affected by loan amount

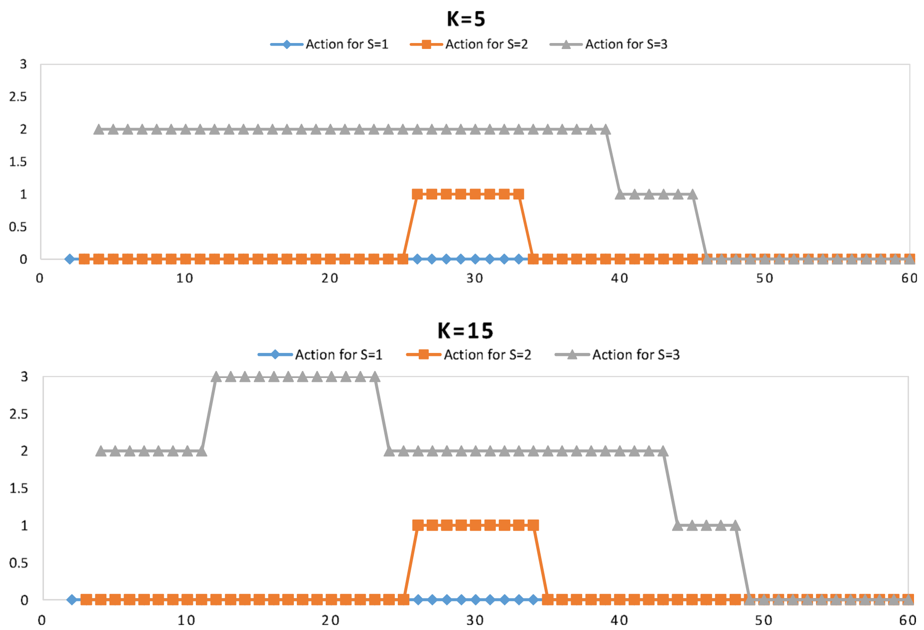


Fig. 7 Actions affected by action effect

action effect), respectively. The optimal collection actions are depicted in Fig. 7. We can see that collection actions with stronger effects should be more intensively applied. Specifically, when at state 1 or 2, the optimal collection actions are the same cross all the ages with the two action effects. However, when at state 3, collection actions with stronger actions effects should be applied more intensively. For example, action 3 never applies when the action effect is weak, but applies at certain earlier ages when the action effect is strong. Intuitively, the difference between the optimal collection policies with different action effects is because that action with stronger effects can more effectively keep an account in the current state and thus bring greater interest revenue, especially when the account is at an early age.

In summary, our computational results reveal that, compared with a static collection policy, it is evidently more valuable to adopt our optimal collection policy for term-loans that have high interest rate and medium to high loan amount, especially when collection actions have evident effects in helping delinquent accounts become current. We also demonstrate how the collection actions implemented under an optimal collection policy are affected by interest rate, loan amount, and collection effects. Note that in practice, the action to be applied over a loan account is contingent on consumer heterogeneity, because different consumers have different account characteristics, affecting probability and payoff in state transition, along with the effect of collection action.

5 Conclusions

The modeling framework proposed in this work is under the rubric of optimal Markov decision theory. Our model is based on a simplified business setting for consumer term-loan collection. The model can be viewed as a unifying framework for a number of simple collection decisions that are based on partial account information. This business setting is also consistent with how a dynamic credit risk model is envisioned based on a Markov chain modeling framework.

We find that the potential benefit of implementing an optimal collection policy based on account state and age, and consumer heterogeneity, can be substantial relative to a static collection policy based solely on account state. The three recognized factors that affect the relative performance of our dynamic collection policy are: interest rate, loan amount, and effects of collection actions. On average, compared with a static collection policy, the optimal collection policy is more valuable for accounts with high interest rate and medium to high loan amount, especially when collection actions are more intensive. We also demonstrate how the collection actions implemented under an optimal collection policy are affected by interest rate, loan amount, and collection effects.

There are multiple lines along which the present work can be extended, especially as motivated by practical collection problems. First, by enlarging the spaces of account states and collection actions, the Markov decision model can work for cases where: (1) information learned from the collection process can be incorporated into collection actions at later ages; and (2) correlation exists between residual value of the purchased good and account characteristics. Second, in collection problems with a large number of loan accounts, resource constraints on account maintenance need to be addressed in determining the appropriate collection actions. Third, it is important to experimentally quantify the effects of different collection actions, especially for those actions that are new to the business, such as electronic contact and automatic machine dialer. Finally, the criteria of contract rewriting during the collection process need thorough investigation.

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References

Altman EI (1989) Measuring corporate bond mortality and performance. J Finance 44:909–922
Altman EI, Saunders A (1998) Credit risk measurement: developments over the last 20 years. J Bank Finance 21:1721–1742
Asquith P, Mullins DW, Wolff ED (1989) Original issue high yield bonds: aging analysis of defaults, exchanges, and calls. J Finance 44:923–953
Betancourt L (1999) Using Markov chains to estimate losses from a portfolio of mortgages. Rev Quant Finance Account 12:303–318
Bierman H, Hausman WH (1970) The credit granting decision. Manag Sci 16:519–532
Briat P (2005) A Markovian approach to the analysis and optimization of a portfolio of credit card accounts. Master's degree thesis, Department of Industrial and Systems Engineering, National University of Singapore
Butera G, Faff R (2006) An integrated multi-model credit rating system for private firms. Rev Quant Finance Account 27:311–340
Capon N (1982) Credit scoring systems: a critical analysis. J Mark 46:82–91
Chen LH, Chiou TW (1999) A fuzzy credit-rating approach for commercial loans: a Taiwan case. Omega Int J Manag S 27:407–419
Cunningham DF, Capone CA (1990) The relative termination experience of adjustable to fixed rate mortgages. J Finance 45:1687–1703
Curnow G, Kochman G, Meester S, Sarkar D, Wilton K (1997) Automating credit and collections decisions at AT&T capital corporation. Interfaces 27:29–52
Cyert RM, Davidson HJ, Thompson GL (1962) Estimation of the allowance for doubtful accounts by Markov chains. Manag Sci 8:287–303

De Almeida Filho AT, Mues C, Thomas LC (2010) Optimizing the collections process in consumer credit. Prod Oper Manag 19:698–708
Desai VS, Convay DG, Crook JN, Overstreet GA (1997) Credit scoring models in the credit union environment using neural networks and genetic algorithms. IMA J Math Appl Bus Ind 8:323–346
federalreserve.gov (2018) Consumer Credit. http://www.federalreserve.gov/releases/g19/Current/. Accessed 2 Jan 2018
Grimshaw SD, Alexander WP (2011) Markov chain models for delinquency: Transition matrix estimation and forecasting. Appl Stoch Model Bus 27:267–279
He P, Hua Z, Liu Z (2015) A quantification method for collection effects on consumer term loans. J Bank Finance 57:17–26
Kallberg JG, Saunders A (1983) Markov chain approaches to the analysis of payment behavior of retail credit customers. Financ Manag 12:5–14
Kang P, Zenios S (1992) Complete prepayment models for mortgage-backed securities. Manag Sci 38:1665–1685
Kukuk M, Rönnberg M (2013) Corporate credit default models: a mixed logit approach. Rev Quant Finance Account 40:467–483
Lawrence EC, Smith LD, Rhoades M (1992) An analysis of default risk in mobile home credit. J Bank Finance 16:299–312
Lee TS, Chiu CC, Lu CJ, Chen IF (2002) Credit scoring using the hybrid neural discriminant technique. Expert Syst Appl 23:245–254
Liebman LH (1972) A Markov decision model for selecting optimal credit control policies. Manag Sci 18:519–525
Makuch WM, Dodge JL, Ecker JG, Granfors DC, Hahn GJ (1992) Managing consumer credit delinquency in the US economy: a multi-billion dollar management science application. Interfaces 22:90–109
Malhotra R, Malhotra DK (2003) Evaluating consumer loans using neural networks. Omega Int J Manag S 31:83–96
Mehta D (1968) The formulation of credit policy models. Manag Sci 15:30–50
Min JH, Lee YC (2008) A practical approach to credit scoring. Expert Syst Appl 35:1762–1770
Ong CS, Huang JJ, Tzeng GH (2005) Building credit scoring models using genetic programming. Expert Syst Appl 29:41–47
Rosenberg E, Gleit A (1994) Quantitative methods in credit management: a survey. Opre Res 42:589–613
Smith LD, Jin B (2007) Modeling exposure to losses on automobile leases. Rev Quant Finance Account 29:241–266
Smith LD, Sanchez SM, Lawrence EC (1996) A comprehensive model for managing risk on home mortgage portfolios. Decis Sci 27:291–317
Smith LD, Bilir C, Huang VM, Hung KY, Kaplan M (2005) Citibank models credit risk on hybrid mortgage loans in Taiwan. Interfaces 35:215–229
Thomas LC (2000) A survey of credit and behavioural scoring: forecasting financial risk of lending to consumers. Int J Forecast 16:149–172
Thomas LC (2009) Consumer credit models: pricing profit and portfolios. Oxford University Press, New York
Thomas LC, Edelman DB, Crook JN (2002) Credit scoring and its applications. Society for Industrial and Applied Mathematics, Philadelphia
Thomas LC, Oliver RM, Hand DJ (2005) A survey of the issues in consumer credit modelling research. J Oper Res Soc 56:1006–1015
Trench MS, Pederson SP, Lau ET, Ma L, Wang H, Nair SK (2003) Managing credit lines and prices for bank one credit cards. Interfaces 33:4–21
West D (2000) Neural network credit scoring models. Comput Oper Res 27:1131–1152
Yobas MB, Crook JN, Ross P (2000) Credit scoring using neural and evolutionary techniques. IMA J Math Appl Bus Ind 11:111–125