Sliding Mode Control: An Approach to Control a Quadrotor

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Sliding Mode Control: An approach to Control a Quadrotor

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***Abstract*— This paper describes the synthesis and application of controller based on sliding mode theory to a quadrotor. A PD sliding surface is considered for vertical take-off and landing aircraft, also changes in angles are done, and some disturbances are included. Therefore, the controller can be implemented using a PD controller as the sliding surface, and adding some algebra the complete controller algorithm is presented. The controller is tested by simulations.**

***Keywords*— Sliding Mode Control, Control theory, UAV's, Nonlinear control.**

I. INTRODUCTION

Quadrotor, also known as Unmanned Aerial Vehicles (UAV), is widely used in several applications, in both military and civilian markets. UAV is nonlinear, unstable and vulnerable to external disturbances. It presents an interesting dynamics where the uncertainties on physical parameters and variable dynamic driving as the drone make it an attractive and challenging control problem. Different methodologies and strategies have been recommended to control a quadrotor. The following approaches have been used: linear quadratic regulator (LQR) control [1], proportional-integral derivative (PID) control [1-4], fuzzy logic (FL) control [5].

Sliding Mode Control (SMC) is a simple robust technique that permits designing controllers for linear and nonlinear processes [6-8]. SMC has become one of the selected solutions for many practical control designs such as electronics devices, robotics, chemical processes and so on [8, 9]. SMC consists in two parts: firstly, a discontinuous control law to enforce the error vector toward a decision rule, called sliding surface, during the reaching phase. This part the control is switching on the different sides of the sliding surface equation and secondly, once the error vector is confined to the sliding surface, an equivalent part of the controller acts to follow the dynamics imposed by the equations describing the sliding surface [6].

There are some works of SMC applied to tracking and regulation tasks for quadrotors, let us make a briefly mention of them, for instances: [10], in this paper is proposed a control scheme based on backstepping and its combination with fuzzy system for chattering reduction. [11], the authors present a controller approach where it is obtained by dividing the design in two steps: firstly, a high gain observer is considered and secondly, a dynamic sliding mode controller (DSMC) is proposed. [12] in this article a direct adaptive sliding mode control is developed. The proposed controller is applied without considering disturbances and parameter uncertainties. The design of the adaptation laws is based on Lyapunov design principle. [13], in this article, backstepping and sliding mode as control schemes are presented and compared. [14] In this paper a controller based on high order sliding mode control (HOSMC) is designed for trajectory tracking. [15] Describes a feedback linearization-based controller with a high-order sliding mode observer. The observer works as an observer and estimator of the external disturbances effect. [16], in this paper; the principal attention is based on the dynamic modelling of quadrotor taking into account the high-order nonholonomic constraints. Using the Backstepping approach for the synthesis of tracking errors and Lyapunov functions, a sliding mode controller is obtained. [17], in this paper an adaptive backstepping approach is utilized to design the sliding mode control.

The objective of this paper consists in designing a sliding mode control and applying it by simulation in a quadrotor. A PD sliding surface is considered for vertical take-off and landing aircraft, also changes in angles are done, and some disturbances are included. Therefore, the controller can be implemented using a PD controller as the sliding surface, and adding some algebra the complete controller algorithm is presented.

This article is structured as follows. In Section II the quadrotor model is presented. Section III, describes concepts about SMC method. Section IV shows the controller synthesis. In Section V the proposed controller is applied by simulations in the quadrotor, tracking and regulation tasks are proved. Section 6 concludes the paper

II. QUADROTOR DYNAMIC MODEL

The scheme of the quadrotor studied here is shown in Fig. 1. The equations of motion are given in [1, 18,19].

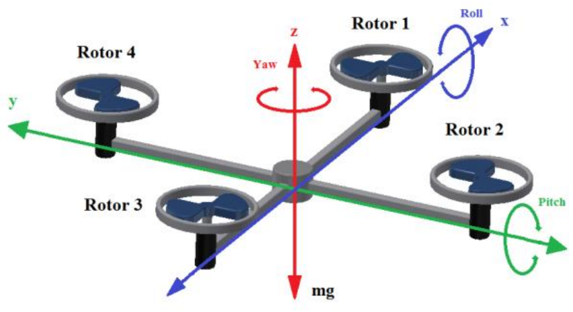

Fig.1.Scheme of the quadrotor

The quadrotor has twelve states, which are the following:

$$X = [x\ \dot{x}\ y\ \dot{y}\ z\ \dot{z}\ \phi\ \dot{\phi}\ \theta\ \dot{\theta}\ \psi\ \dot{\psi}\ ] \qquad (1)$$

Where, x, y and z are the position in the x, y, and z axes. $\dot{x}$, $\dot{y}$ and $\dot{z}$ are the speed in the axes. ϕ, θ, ψ Are the roll, pitch, and yaw angles respectively, and the parameters are $\dot{\phi}, \dot{\theta}, \dot{\psi}$ the speed for roll, pitch and yaw.

$$\ddot{x} = \frac{cos(\phi)sin(\theta)cos(\psi) + sin(\phi)sin(\psi)}{m} U_1 \qquad (2)$$

$$\ddot{y} = \frac{cos(\phi)sin(\theta)sin(\psi) - sin(\phi)cos(\psi)}{m} U_1 \qquad (3)$$

$$\ddot{z} = -g + \frac{cos(\phi)cos(\theta)}{m} U_1 \qquad (4)$$

$$\ddot{\phi} = \dot{\phi}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{Jr}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}U_2 \qquad (5)$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) + \frac{Jr}{I_y}\dot{\theta}\Omega + \frac{l}{I_y}U_3 \qquad (6)$$

$$\ddot{\psi} = \dot{\theta}\dot{\phi}\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}U_4 \qquad (7)$$

The input signal U_1 is the total drag of the rotors. $U_2, U_3\ and\ U_4$ Are the moments for pitch, roll and yaw respectively. m represents the mass of the quad rotor, J_r is the inertia of the rotor and I_x, I_y and I_z are the inertia of the quad rotor in 'x', 'y' and 'z' respectively.

The input signals are described from the equation (8) to (11).

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \qquad (8)$$

$$U_2 = b(-\Omega_2^2 + \Omega_4^2) \qquad (9)$$

$$U_3 = b(-\Omega_1^2 + \Omega_3^2) \qquad (10)$$

$$U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \qquad (11)$$

The angular speed for each rotor is noted as $\Omega_1, \Omega_2, \Omega_3, \Omega_4$. l is the distance between the rotor and the center of the quadrotor, and b is the drag factor. All of these parameters involved in equations (2) to (11) are stated in [18].

From the equations (8), (9),(10) and (11) the Ω factor can be obtained as follows:

$$\begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -b & 0 & b \\ -b & 0 & b & 0 \\ -d & d & -d & d \end{bmatrix}^{-1} * \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

And finally the value of Ω is:

$$\Omega = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \qquad (12)$$

III. SLIDING MODE CONTROLLLER DESIGN

Sliding Mode Control is a procedure that arises from Variable Structure Control (VSC)[3]. SMC objective involves two parts: in the first one, a control law to enforce the error vector toward a decision rule, called sliding surface, during the reaching phase is designed, this part the control is switching on the different sides of this sliding surface and secondly, once the error vector is restricted in the sliding surface, it tracks the dynamics imposed by the equations describing the sliding surface, this second part of the controller is called equivalent control [6].

Four control equations are used to keep the quadrotor on the reference value in spite of external disturbances. The signal U_1 is used to guarantee that the altitude follows the reference value, although the signals U_2, U_3 and U_4 are used to control the roll, pitch and yaw of the system. Looking at the equations (2-7), we found four control input signals. The action of these input signals makes the quadrotor moves forwards, backward, to the left, to the right, upwards or down.

As was mentioned before, the sliding surface or decision rule must be selected. This selection is made based on performance criteria, since the sliding surface equation determines the dynamics of the system. Therefore, it can be written as:

$$s = \dot{e} + \lambda e \qquad (13)$$

Where λ is a tuning parameter design greater than zero.

The control law, *U(t)*, is composed by two parts: a continuous part, u_{eq}(t), and a discontinuous part, $u_D(t)$. That is

$$U(t) = u_{eq}(t) + u_D(t) \quad (14)$$

The first part is a function of the controlled variable, and the reference value. The second part includes a nonlinear element that contains a switching element.

$$u_D(t) = k_D \Theta(s(t)) \quad (15)$$

k_D is the tuning parameter responsible for the reaching phase , and $\Theta(s(t))$ is a nonlinear function of *s(t)*. Then, Eq.(15) can be rewritten as follows:

$$u_D(t) = k_D \, sign(s(t)) \quad (16)$$

Chattering problem [6,7] can be reduced if Eq. (16) is represented as:

$$u_D(t) = k_D \frac{s(t)}{|s(t)| + \delta} \quad (17)$$

δ is a tuning parameter used to reduce the chattering effect.

In order to find $u_{eq}(t)$ the equivalent control procedure is followed:

The sliding surface is defined as in equation (13), for the altitude of the quadrotor the error would be.

$$e = z_d - z \quad (18)$$

z is measured state and z_d is the desired state.

By replacing (18) in (13) the following result is obtained.

$$s = (\dot{z}_d - \dot{z}) + \lambda(z_d - z) \quad (19)$$

By applying the sliding condition, $\dot{s} = 0$, it is obtained:

$$\dot{s} = \ddot{e} + \lambda\dot{e} = 0 \quad (20)$$

And, it is obtained:

$$\dot{s} = (\ddot{z}_d - \ddot{z}) + \lambda(\dot{z}_d - \dot{z}) \quad (21)$$

Substituting (4), into (21), it can be found:

$$\dot{s} = (\ddot{z}_d + g - \frac{cos(\phi)cos(\psi}{m} U_1) + \lambda(\dot{z}_d - \dot{z}) \quad (22)$$

Considering $U_1 = u_{eq}$, since the system is in sliding condition, the above equation becomes.

$$\dot{s} = (\ddot{z}_d + g - \frac{cos(\phi)cos(\psi}{m} U_{eq}) + \lambda(\dot{z}_d - \dot{z}) \quad (23)$$

Since, $\dot{s} = 0$; u_{eq} becomes:

$$u_{eq} = [g + \lambda(\dot{z}_d - \dot{z}) + \ddot{z}_d] \frac{m}{cos(\phi)\, cos(\psi)} \quad (24)$$

And, the complete controller can be written as follows:

$$U_1 = [g + \lambda\dot{e} + \ddot{z}_d] \frac{m}{cos(\phi)\, cos(\psi)} + k_D sign(s) \quad (25)$$

To design u_D, a Lyapunov function *V* is defined. This function must be positive-definite.

$$V = \frac{1}{2}s^2 > 0 \quad (28)$$

The derivative of the function *V* must be negative-definite.

$$\dot{V} = s\dot{s} < 0 \quad (29)$$

In order to satisfy Eq.(29), u_D should be:

$$u_D = k_D sign(s) \quad (30)$$

Where *s* is defined by (13).

The reaching condition is given by the following inequality $\dot{V} > 0$. Therefore, to satisfy it:

$$k_D > 0 \text{ for all } t > 0 \quad (31)$$

To avoid the chattering problem, u_D is re-defined as follows:

$$u_D(t) = k_D \frac{s}{|s| + \delta} \quad (32)$$

Thus, the complete controller equation is

$$U_1 = [g + \lambda\dot{e} + \ddot{z}_d] \frac{m}{cos(\phi)\, cos(\psi)} + k_D \frac{s}{|s| + \delta} \quad (33)$$

The same procedure is followed to get the controllers for pitch, roll and yaw.

$$u_{eq\phi} = [\lambda(\dot{\phi}_d - \dot{\phi}) + \ddot{\phi}_d - \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) + \frac{J_r}{I_x}\dot{\theta}\Omega]\frac{I_x}{l} \quad (34)$$

$$u_{eq\theta} = [\lambda(\dot{\theta}_d - \dot{\theta}) + \ddot{\theta}_d - \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\theta}\Omega]\frac{I_y}{l} \quad (35)$$

$$u_{eq\psi} = [\lambda(\dot{\psi}_d - \dot{\psi}) + \ddot{\psi}_d - \dot{\theta}\dot{\phi}\left(\frac{I_x - I_y}{I_z}\right)]I_z \quad (36)$$

IV. SIMULATION RESULTS

This Section shows the proposed controller performance. The next figures present how the system responds to different set points, such as, altitude, angles ϕ, Θ and ψ. Also the performance of the system is tested to disturbances.

The implementation of the controller for the altitude gives the result showed in the Fig. 3. The controller tuning parameters are presented in Table 1.

TABLE 1 TUNING PARAMETERS

tunings	Altitude and Angle Parameters			
	z	ϕ	θ	ψ
λ	68	5	5	5
k_D	25	100	100	100
δ	0.3	0.3	0.3	0.3

Fig.2 shows how the quadrotor takes off and reaches two different set points without roll pitch or yaw angles. The controller does not have chattering in the control signal (as it is shown in Fig.3).

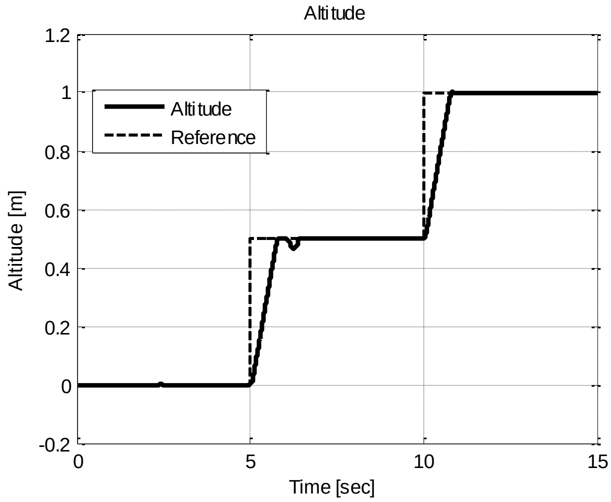

Fig 2. Quadrotor take off and tracking task

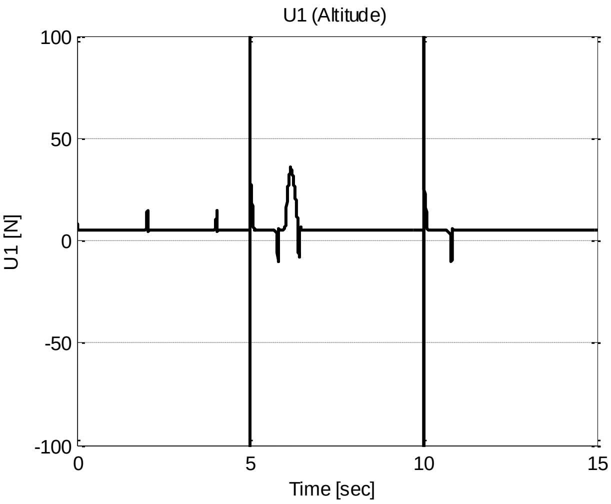

Fig.3. Quadrotor taking off. Controller output

The performance of the system during the landing is shown in Fig.4 and the control signal related to it is Fig.5

The next figures show regulation task to disturbances. The values considered for the simulations are z = 1m, Roll=5° (pi/36 rad), Pitch=5°(pi/36 rad), Yaw=5° (pi /36). The disturbances appear at time 2s, 4s and 6s respectively.

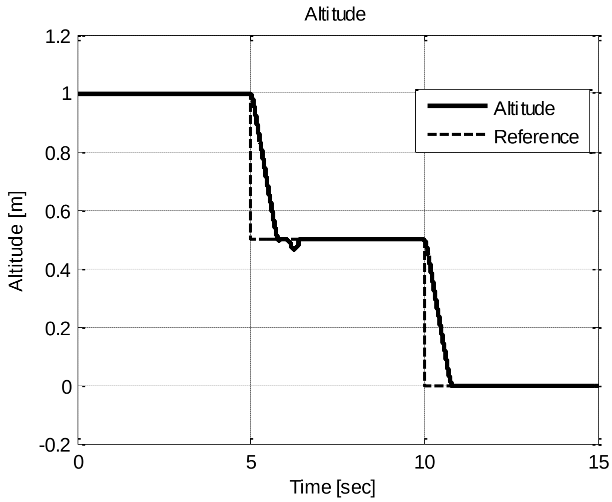

Fig.4 Quadrotor Landing

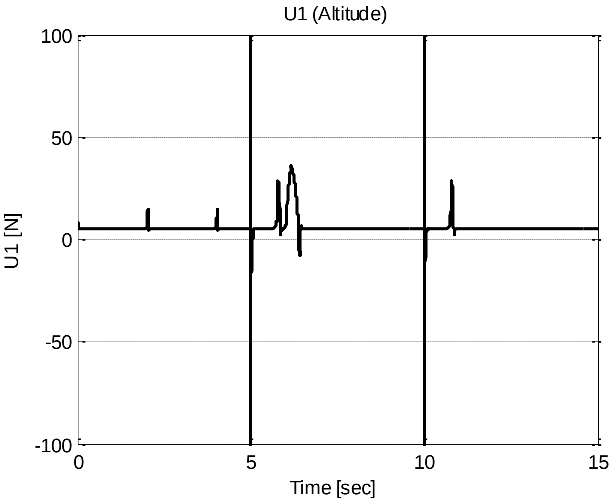

Fig.5. Quadrotor landing. Controller output

In Fig. 6 shows how the quadrotor takes off and stays on its reference value 1[m], in spite of the external disturbances at 4s and 8s.

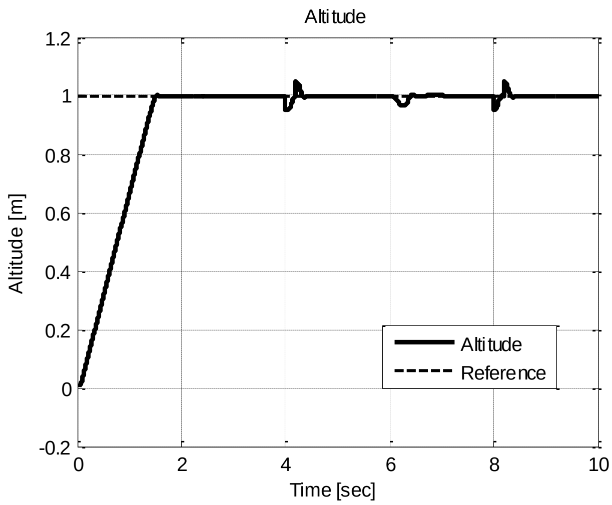

Fig 6. Quadrotor taking off with disturbances

The roll (ϕ), pitch (Θ) and yaw (ψ) appear in order every two seconds as can be seen in Fig.7, Fig.8 and Fig.9.

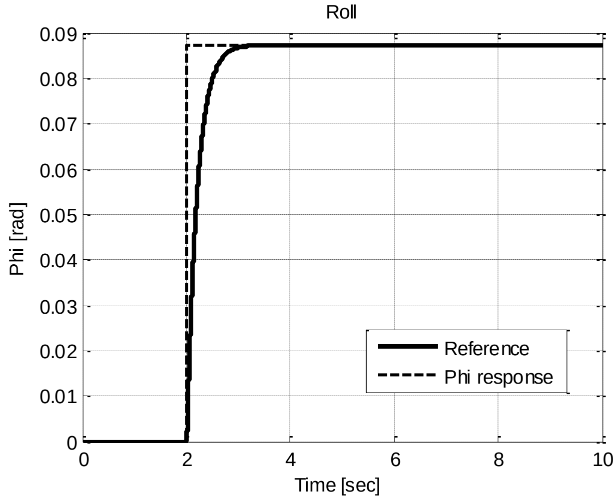

Fig.7. Roll response

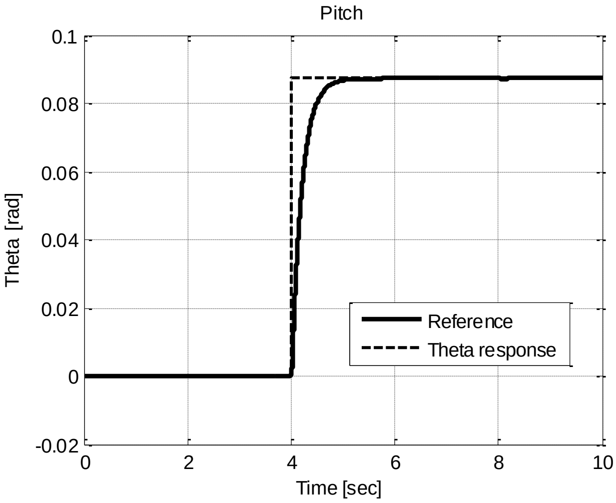

Fig.8 Pitch response

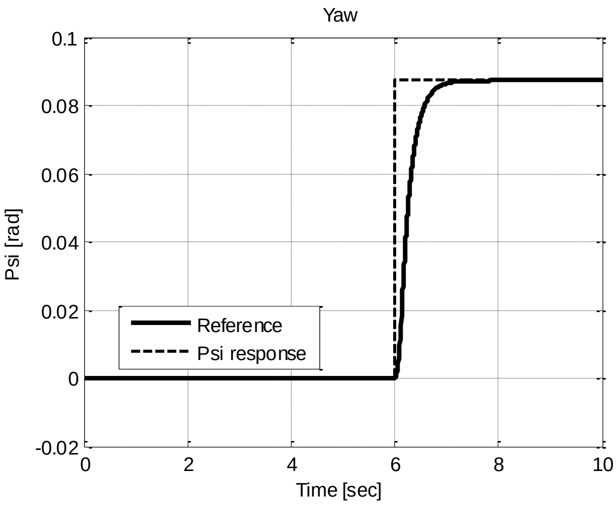

Fig. 9 Yaw response

Control signals for altitude, roll, pitch and yaw are shown in Fig.10

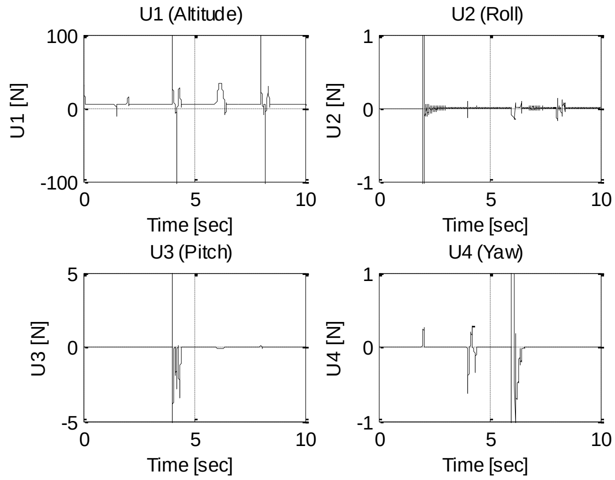

Fig.10. Quarodrotor performance to disturbances and angles variations

The path followed by the quadrotor given by the angles is in Fig.11.

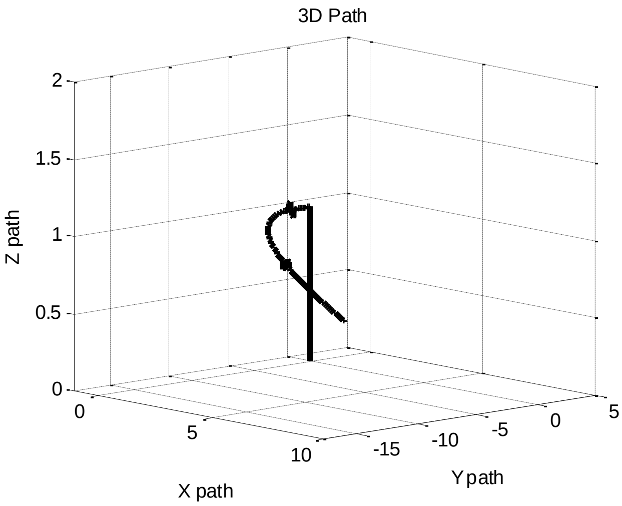

Fig.11. 3D Path given by roll, pitch and yaw angles

V. CONCLUSIONS

The presented work shows a full control for a quadrotor using a SMC from a PD sliding surface. From the PD sliding surface a SMC is developed and tested by simulations. The test results show a good tracking and regulation performance. The controller presented is easy to implement based on a PD controller used as a sliding surface.

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References

[1] Samir Bouabdallah. "Design and control of quad rotors with application to autonomous flying". Ph.D. Thesis. École Polytechnique Federale de Lausanne. 2007.

[2] C Balas. Modelling and Linear Control of a Quadrotor. MSc Thesis. Cranfield University. School Of Engineering. Academic year 2006-2007.

[3] Lucas M. Argentim, Willian C. Rezende, Paulo E. Santos, Renato A. Aguiar. PID, LQR And LQR-PID On A Quadcopter Platform.

[4] G. Szafranski, R. Czyba . Different Approaches of PID Control UAV Type Quadrotor. Proceedings of the International Micro Air Vehicles conference 2011 summer edition

[5] Deepak Gautam and Cheolkeun Ha. Control of a Quadrotor Smart Self-Tuning Fuzzy Controller Control of a Quadrotor Using a Smart Self-Tuning Fuzzy PID Controller. International Journal of Advanced Robotic Systems. Int. j. adv. robot. syst., 2013, Vol. 10, 380:2013

[6] Utkin, V. I., (1977), "Variable Structure Systems With Sliding Modes", Transactions of IEEE on Automatic Control, AC – 22, pp. 212 – 222.

[7] Slotine, J.J., and W. Li, (1991), Applied Nonlinear Control, Prentice-Hall, New Jersey.

[8] Zinober, A. S. I., (1994). Variable Structure And Liapunov Control, Spring – Verlag, London

[9] Camacho, O., Rojas, R., 2000, "A General Sliding Mode Controller for Nonlinear Chemical Processes", ASME Transactions, Journal of Dynamic Systems, Measurement and Control, pp 650-655

[10] Mohd Ariffanan Mohd Basri, Abdul Rashid Husain, Kumeresan A. Danapalasingam . Robust Chattering Free Backstepping Sliding Mode Control Strategy for Autonomous Quadrotor Helicopter. International Journal of Mechanical & Mechatronics Engineering.Vol:14 No. 03, 2014

[11] M'hammed GUISSER and Hicham MEDROMI. A High Gain Observer and Sliding Mode controller for an Autonomous Quadrotor Helicopter. International Journal Of Intelligent Control And Systems VOL.14, No.3, September 2009, 204-212.

[12] Hakim Bouadi, Sebasti~ao Simoes Cunha, Antoine Drouin, Felix Mora-Camino. Adaptive Sliding Mode Control For Quadrotor Attitude Stabilization And Altitude Tracking. CINTI 2011, IEEE 12th International Symposium on Computational Intelligence and Informatics, Nov 2011, Budapest,Hungary. pp 449 - 455

13] Swarup and Sudhir. Comparison of Quadrotor Performance Using Backstepping and Sliding Mode Control. Proceedings of the 2014 International Conference on Circuits, Systems and Control.

[14] Nader Jamali Soufi Amlashi, Mohammad Rezaei, Hossein Bolandi and Ali Khaki Sedigh. Robust Second Order Sliding Mode Control For A Quadrotor Considering Motor Dynamics. International Journal of Control Theory and Computer Modeling. Vol.4, No.1/2, April 2014

[15] Benallegue,y A. Mokhtari and L. Fridman. High-Order Sliding-Mode Observer For A Quadrotor UAV. International Journal Of Robust And Nonlinear Control (in press).

[16] H. Bouadi, M. Bouchoucha, and M. Tadjine. Sliding Mode Control based on Backstepping Approach for an UAV Type-Quadrotor. World Academy of Science, Engineering and Technology International Journal of Mechanical, Aerospace, Industrial and Mechatronics Engineering Vol:1 No:2, 2007

[17] Abderrahmane KACIMI, Abdellah MOKHTARI, Benatman KOUADRI. Sliding Mode Control Based on Adaptive Backstepping Approch for a Quadrotor Unmanned Aerial Vehicle. Przegl ˛ad Elektrotechniczny (Electrical Review), ISSN 0033-2097, R. 88 NR 6/2012

[18] Diaz Cano J. M. "Modelo Y Control LQR De Una Aeronave De Cuatro Motores" Bachelor Thesis.Universidad de Sevilla. 2007.

[19] Gomez, A. "Real-Time Robustness analysis of Quadrotor UAV's". Master of Science Thesis. University of Glasgow. 2012