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Problem 1

Consider the following system of equations:

$$\begin{cases} x_1 + 3x_2 + 5x_3 = 25 \\ 2x_2 + 4x_3 = 18 \\ 2x_1 + 4x_2 + 6x_3 = 32 \end{cases}$$

Assume now that $x \in \mathbb{R}^3$ is a vector as follows: $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

- a) Find a matrix A and a vector b of appropriate dimensions such that the system of equations can be written as follows:

$$Ax = b$$

- b) Find a solution $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ to the system of equations. Is the solution unique? Why or why not?

You can either solve the set of equations by hand or use any software to help you find a solution. In either way, please show your work and code, not just the solution.

- c) **(1 pt extra credit)** Characterize all of the possible solutions to the system of equations.

Problem 2

Show that the following n^{th} order linear time invariant differential equation of the form:

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1\dot{y} + a_0y = b_0u(t), \text{ where } n > 1$$

can be written as a state space system of the form:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$, and $y^{(n)} \triangleq \frac{d^n y}{dt^n}$.

Hint: Begin with $n = 2$, write the ODE, and place it in the form required. Repeat for $n = 3$ and see if you can find a pattern to write for a general n .

Problem 3

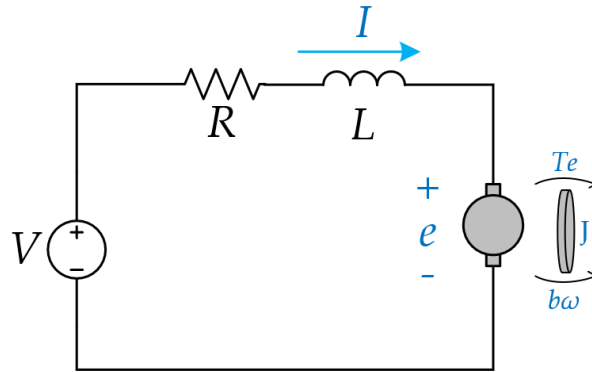


Figure 1: DC motor problem.

- a) Derive the state space model for the dc motor in Fig. 1. Assume the states are $x = \begin{pmatrix} I \\ \omega \end{pmatrix}$, the output is $y = \omega$, and the input is $u = V$. Place it into the form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

Assume the back-emf of the motor is given by $e = k_e \omega$ and the electrical torque is $T_e = k_t I$, where k_e and k_t are constants. Use variables only for this part.

Hint: To derive the equation for the motor speed, you can use Newton's second law $J\dot{\omega} = \text{sum of the torques}$. Assume clockwise is positive direction.

To derive the equation for the current, use KVL.

- b) Use the following values for the parameters: $R = 1 \, \Omega$, $L = 0.5 \, \text{H}$, $b = 0.1 \, \text{Nms}$, $J = 0.01 \, \text{kgm}^2$, and $k_e = k_t = 0.01$.

Use Matlab (m-file or Simulink) to plot the states when $x(0) = \begin{pmatrix} I(0) \\ \omega(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $V = 5 \, \text{V}$ are applied. Simulate for 4 s using a discrete time solver with timestep $T_s = 1 \, \text{ms}$.

Problem 4

Define state variables such that the n^{th} order linear time-varying differential equation:

$$y^{(n)} + a_{n-1}t^{-1}y^{(n-1)} + \cdots + a_1t^{-n+1}\dot{y} + a_0t^{-n}y = 0 \quad (2)$$

can be written as a time varying linear system of the form:

$$\dot{x}(t) = A(t)x(t) \quad (3)$$

where $x \in \mathbb{R}^n$ and $A(t) = t^{-1}A_0$ where $A_0 \in \mathbb{R}^{n \times n}$ is a constant matrix. The coefficients $a_0, \dots, a_{n-1} \in \mathbb{R}$ are constant, but notice that t is explicitly part of the differential equation.

Hint: Begin with $n = 2$, write the ODE, and place it in the form required as shown in (3). Repeat for $n = 3$ and see if you can find a pattern to write for a general n .