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I want a system that is able to construct and review polymorphic functions.
A polymorphic function can evaluate to or be applied to values of different types.

Let $f : A \rightarrow \mathcal{B}$ be a polymorphic function where

$A = \{A \in A \mid \text{there is some 'structure' to } A\}$ and

$B = \{B \in A \mid \text{there is some 'structure' to } B\}$

Let $A, A' \in A$ and $B, B' \in B$

So $f(A) = B$ and $f(A') = B'$

If this is all true then in our language A is isomorphic to A' .

We want the system to be able to construct polymorphic functions but that entails the prior construct of a system that is able to construct/recognize isomorphisms.

we have the example of the robot in the room.

The robot starts in the middle of the room.

The fourth time the robot goes forward it runs into the wall.

$t = 0 \ ?(i_0) \rightarrow m_1?(i_0)$

$t = 1 \ ?(m_1?(i_0)i_1) \rightarrow m_2?(m_1?(i_0)i_1)$

$t = 2 \ ?(m_2?(m_1?(i_0)i_1)i_2) \rightarrow m_3?(m_2?(m_1?(i_0)i_1)i_2)$

$t = 3 \ ?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3) \rightarrow m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)$

$t = 4 \ ?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4) \rightarrow m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)$

$t = 5 \ ?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5) \rightarrow m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)$

$t = 6 \ ?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7) \rightarrow m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)$

$t = 7 \ ?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7) \rightarrow m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)$

$t = 8 \ ?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8) \rightarrow$

$m_9?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8)$

$t = 9 \ ?(m_9?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8)i_9) \rightarrow$

$m_{10}?(m_9?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8)i_9)$

$t = 10 \ ?(m_{10}?(m_9?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8)i_9)i_{10}) \rightarrow$

$m_{11}?(m_{10}?(m_9?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8)i_9)i_{10})$

$t = 11 \ ?(m_{11}?(m_{10}?(m_9?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8)i_9)i_{10})i_{11})$

$m_{12}?(m_{11}?(m_{10}?(m_9?(m_8?(m_7?(m_6?(m_5?(m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)i_4)i_5)i_7)i_7)i_8)i_9)i_{10})i_{11})$

$t = 12 \ ?() \rightarrow m$

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so starting from the top we have

$$?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3) \rightarrow m_4?(m_3?(m_2?(m_1?(i_0)i_1)i_2)i_3)$$

third order logic.. ?

W

$$t = 0$$

$$?(i) \rightarrow m?(i)$$

$$\text{With } \mathcal{T} = \emptyset$$

$$t = 1$$

$$?(m?(i)w) \rightarrow m?(m?(i)w)$$

$$\text{With } \mathcal{T} = \{mi \rightarrow w\}$$

$$t = 2$$

$$?(m?(m?(i)w)w) \rightarrow m?(m?(m?(i)w)w)$$

$$\text{With } \mathcal{T} = \{\{mi \rightarrow w\}, \{mw \rightarrow w\}\}$$

$$t = 3$$

$$?(m?(m?(m?(i)w)w)w) \rightarrow m^{-1}?(m?(m?(m?(i)w)w)w)$$

$$\text{With } \mathcal{T} = \{\{mi \rightarrow w\}, \{mw \rightarrow w\}, \{mw \rightarrow w\}\}$$

$$\text{This implies With } \mathcal{T} = \{\{mi \rightarrow w\}, \{m\forall w \rightarrow w\}\}$$

$$\text{For brevity let } m?(m?(m?(i)w)w)w = \alpha$$

$$t = 4$$

$$?(m^{-1}?(\alpha)i) \rightarrow m^{-1}?(m^{-1}?(\alpha)i)$$

$$\text{With } \mathcal{T} = \{\{mi \rightarrow w\}, \{mw \rightarrow w\}, \{mw \rightarrow w\}\}$$

$$\text{This implies With } \mathcal{T} = \{\{mi \rightarrow w\}, \{m\forall w \rightarrow w\}\}$$