表現論の組合せ論的側面とその周辺

日程: 2023 年 10 月 23 日（月）〜27 日（金）

会場: 早稲田大学西早稲田キャンパス 62W 号館 1 階大会議室 A

アブストラクト

10/23 (月)

- 14:30 – 15:30 淺井 聡太 (東京大学)

  Faces of interval neighborhoods of silting cones

  Abstract: This talk is based on joint work with Osamu Iyama. In the representation theory of a finite dimensional algebra A over a field, it is important to study certain subcategories called torsion classes of the category $\mathsf{mod}A$ of finitely generated A-modules. Baumann-Kamnitzer-Tingley associated two semistable torsion classes to each element θ in the real Grothendieck group $K_0(\mathsf{proj}A)_{\mathbb{R}}$ of the category $\mathsf{proj}A$ of finitely generated projective A-modules. This real Grothendieck group is the Euclidean space whose canonical basis is given by the isoclasses of indecomposable projective A-modules. By using semistable torsion classes, I introduced an equivalence relation called TF equivalence on $K_0(\mathsf{proj}A)_{\mathbb{R}}$. A typical example of TF equivalence classes is the silting cone $C^\circ(U)$ of each 2-term presilting complex U, which is the cone generated by the g-vectors of the indecomposable direct summands of U. In general, there can be other TF equivalence classes, and we are studying them. For this purpose, we use the interval neighborhood N_U of each silting cone $C^\circ(U)$. The closure $\overline{N_U}$ is a rational polyhedral cone in the Euclidean space $K_0(\mathsf{proj}A)_{\mathbb{R}}$, so it is natural to consider its faces. I will talk on some nice representation-theoretical and combinatorial properties of faces of $\overline{N_U}$.

- 16:00 – 17:00 山根 宏之 (富山大学)

  一般化された有限ルート系のワイル亜群のケイリーグラフおよびハミルトン閉路

  Title (English): Cayley graphs of Weyl groupoids of finite generalized root systems and their Hamilton circuits.

  Abstract: Roughly speaking, the Weyl groupoid of a Lie superalgebra of type A-G is generated by simple reflections of its Weyl group and odd reflections. Similarly we also have the Weyl groupoid of a generalized quantum group, or the quantum double of a Nichols algebra of diagonal-type. In 2008, Heckenberger and the speaker showed that the Weyl groupoid can be presented by simple-reflection-type generators and Coxeter-type relations. In order to properly define the Weyl groupoid, we need a generalized root system. In 2015, Cuntz and Heckenberger classified the finite generalized root systems. The speaker explain the existence of a Hamilton circuit of the Cayley graph of the Weyl groupoid of a finite generalized root system. This talk is based on a joint work with Takato Inoue.

10/24 (火)

- 9:30 – 10:30 西山 雄太 (熊本大学)

  歪 mitosis 作用素の組合せ論

  Abstract: A 型のシューベルト計算における組合せ論的なモデルとして，pipe dream と呼ばれる図形が用いられる．それぞれの pipe dream には対称群の元が対応しており，それぞれの元に対応する pipe dream たちは mitosis 作用素を用いて構成できる．この pipe dream の C 型への拡張として歪 pipe dream が知られており，それぞれの歪 pipe dream には C 型 Weyl 群の元が対応する．本講演では，C 型 Weyl 群の各元に対応する歪 pipe dream たちが歪 mitosis 作用素を用いて構成できることを紹介する．なお本講演の内容は藤田直樹氏（熊本大学）との共同研究に基づく．

  Title (English): Combinatorics of skew mitosis operators

Abstract (English): Diagrams called pipe dreams are used as a combinatorial model in Schubert calculus in type A. Each pipe dream corresponds to an element of the symmetric group, and the set of pipe dreams corresponding to each element of the symmetric group can be constructed with mitosis operators. Skew pipe dreams are known as an extension of pipe dreams to type C, and each skew pipe dream corresponds to an element of Weyl group of type C. In this talk, we show that the set of skew pipe dreams corresponding to each element of Weyl group can be constructed with skew mitosis operators. This talk is based on a joint work with Naoki Fujita (Kumamoto University).

- 11:00 – 12:00 岡田 聡一 (名古屋大学)

  Proof of Mizukawa–Nakajima–Yamada's Conjecture on Q-functions

  Abstract: Sato and Mori introduced a family of weighted homogeneous polynomials in their study of the Korteweg–de Vries (KdV) and modified KdV (mKdV) equations. Later Mizukawa, Nakajima and Yamada conjectured that these polynomials are Schur's Q-functions in the duplicated Sato variables. In this talk, we prove their conjecture in full generality. Our proof uses the minor-summation formula of Ishikawa and Wakayama.

- 14:00 – 15:00 山口　航平 (名古屋大学)

  Equivariant K-homology of the symplectic affine Grassmannian

  Abstract: G を斜交群 $\mathrm{Sp}_{2n}(\mathbb{C})$ とし, そのアフィン・グラスマニアン Gr_G を考える. 本研究の目的はトーラス同変 K ホモロジー環 $K_*^T(\mathrm{Gr}_G)$ に対して, 対称関数を用いた実現を与えることである. 中川・成瀬は無限次元のラグランジアン・グラスマニアンのシューベルト類を表す多項式として K 理論的 (同変) 双対 P 関数を導入した. これらの関数によって張られる環を考えるとき, その中で "ピエリ元" によって生成される部分環が $K_*^T(\mathrm{Gr}_G)$ と同型であることを示す. この同型を介して, Gr_G のシューベルト多様体の構造層に対応する関数はデマジュール作用素を用いて具体的に計算できる. 本講演の内容は池田岳氏と Mark Shimozono 氏, 岩尾慎介氏との共同研究に基づく.

  Abstract (English): Let G be the symplectic group $\mathrm{Sp}_{2n}(\mathbb{C})$, and Gr_G the affine Grassmannian of G. The purpose of the study is to provide a realization of the torus equivariant K-homology ring $K_*^T(\mathrm{Gr}_G)$ in terms of symmetric functions. Nakagawa and Naruse introduced the K-theoretic (equivariant) dual P-function as a polynomial representing the Schubert class of the infinite dimensional Lagrangian Grassmannian. When considering the ring generated by these functions, we prove that the subring generated by the "Pieri elements" is isomorphic to $K_*^T(\mathrm{Gr}_G)$. Via this isomorphism, the function corresponding to the structure sheaf of a Schubert variety of Gr_G can be computed explicitly using the Demazure operators. This talk is based on joint work of Takeshi Ikeda, Mark Shimozono, and Shinsuke Iwao.

- 15:30 – 16:30 河野 隆史 (早稲田大学)

  Parabolic K-Peterson isomorphism for the Lagrangian Grassmannian

  Abstract: Let G be the symplectic group $\mathrm{Sp}_{2n}(\mathbb{C})$ and Gr_G the affine Grassmannian associated to G. We denote by $\mathrm{LG}(n)$ the Lagrangian Grassmannian. By the parabolic K-theoretic Peterson isomorphism, we know that a subring of the equivarivant quantum K-ring $QK_T(\mathrm{LG}(n))$ of $\mathrm{LG}(n)$ is isomorphic to a quotient of the equivariant K-homology ring $K_*^T(\mathrm{Gr}_G)$ of Gr_G under suitable localization. We give an explicit generators of the ideal J of $K_*^T(\mathrm{Gr}_G)$ such that $K_*^T(\mathrm{Gr}_G)/J$ is isomorphic to a subring of $QK_T(\mathrm{LG}(n))$ under localization. This talk is based on a joint work with Takeshi Ikeda.

10/25 (水)

- 9:30 – 10:30 降旗 駿 (京都大学)

  On the Beem–Nair Conjecture

  Abstract: Given a simple linear algebraic group G, we have an open immersion (constructed by Beem and Nair) from the cover of the Kostant–Toda lattice associated to G into the universal centralizer of G. They expected that a free field realization of the chiral universal centralizer of G at the critical level will be obtained by the

chiralization of this immersion. In this talk, we will verify this conjecture is true by constructing an embedding from the chiral universal centralizer of G into an appropriate vertex operator algebra at any level.

- 11:00 – 12:00 元良 直輝 (東京大学)

  W 代数の段階的還元

  Abstract: TBA

- 14:00 – 15:00 桑原 敏郎 (筑波大学)

  Vertex superalgebras associated with Hilbert schemes of points in the plane

  Abstract: Hilbert scheme of points on the complex plane is a fundamental example of symplectic resolution of the symplectic singularity $\mathbb{C}^{2N}/S_N$, where S_N denotes the symmetric group. To each complex reflection group G one can associate a canonical symplectic singularity denoted by M_G. Motivated by the 4D/2D duality, Bonetti, Meneghelli and Rastelli conjectured the existence of a supersymmetric vertex operator superalgebra W_G whose associated variety is isomorphic to M_G. In this talk, we construct such a vertex operator superalgebra W_G for $G = S_N$ using BRST reduction. This is associated with the Hamiltonian reduction which realize the Hilbert scheme as a Nakajima quiver variety. This is joint work with Tomoyuki Arakawa and Sven Möller.

- 15:30 – 16:30 楫 元 (早稲田大学)

  Push-Forward Formula for Grassmann Bundles and Its Application

  Abstract: I explain a push-forward formula for Grassmann bundles, which is a joint work with T. Terasoma (Hosei University), and compute the degree of 2-step flag varieties, as an application.

10/26 (木)

- 9:30 – 10:30 Duc-Khanh Nguyen (沖縄科学技術大学院大学)

  A generalization of the Murnaghan–Nakayama rule for K-k-Schur and k-Schur functions

  Abstract: The K-k-Schur functions and k-Schur functions appeared in the study of K-theoretic and affine Schubert Calculus as polynomial representatives of Schubert classes. In this paper, we introduce a new family of symmetric functions $\mathcal{F}_\lambda^{(k)}$, that generalizes the constructions via the Pieri rule of K-k-Schur functions and k-Schur functions. Then we obtain the Murnaghan-Nakayama rule for the generalized functions. The rule is described explicitly in the cases of K-k-Schur functions and k-Schur functions, with concrete descriptions and algorithms for coefficients. Our work recovers the result of Bandlow, Schilling, and Zabrocki for k-Schur functions, and explains it as a degeneration of the rule for K-k-Schur functions. In particular, many other special cases and connections promise to be detailed in the future.

- 11:00 – 12:00 Martín Forsberg Conde (沖縄科学技術大学院大学)

  Specht Module の Carter–Payne 準同型の一般化

  Abstract: 有限群の表現論において、対照群の正標数の Specht Module の間の準同型の叙述は大事な未解決問題。Carter と Payne の定理の下、二つの Specht Module の分割の差が一つの”小さい”行のみの場合、その Specht Module の間に準同型が存在すると知られている。KLR 代数の方法によって Carter–Payne 定理の一般化について述べる。

  Abstract (English): Describing the homomorphism spaces between Specht modules of the symmetric groups in positive characteristic is an important open problem in representation theory. Carter and Payne gave a remarkable result describing homomorphisms between Specht modules whose associated partitions differ by a single row. We describe the use of techniques from KLR algebras to achieve a generalization of the Carter-Payne theorem.

- 14:00 – 15:00 足利 涼介 (北海道大学)

  Construction of dynamical reflection maps

  Abstract: In this talk, I will introduce the way to construct dynamical reflection maps, solutions to the reflection equation in the tensor category SetH, by use of left quasigroups. This talk is based on a joint work with Youichi Shibukawa.

- 15:30 – 16:30 大本 豊数 (岡山大学), 高山 義輝 (岡山大学)

  交代符号行列に関連した半順序集合上のコイン裏返しゲーム

  Abstract: 有限型不偏ゲームは, 各局面に対して非負整数を値に持つグランディ数 (Grandy value) と呼ばれる関数が定義され, このグランディ数によって局面が先手必勝局面または後手必勝局面のいずれかであることが決定することができる組合せゲームである. 有限型不偏ゲームのひとつとして「半順序集合上のコイン裏返しゲーム」というものが“石取りゲームの数学” (佐藤文広著) の中で紹介されている. 今回は交代符号行列に関連した半順序集合上のあるコイン裏返しゲームについて考察する. この半順序集合は $X_n = \{(x, y, z) \in \mathbb{Z}^3 \mid x, y, z \geq 0 \text{ and } x + y + z \leq n - 2\}$ (順序は後に定める) によって定義され, 交代符号行列に関連して J. Striker によって導入されたものである. このゲームに対してグランディ数を決定することができたので紹介する.

  Title (English): A poset game associated with alternating sign matrices

  Abstract (English): A finite impartial game is a two-player game in which players take turns to make moves and finishes in a finite number of moves. In this kind of combinatorial games one can define the Grundy value which determine which player will win. In the book “Mathematics of NIM” (in Japanese) written by F. Sato, the author introduced so-called “poset game” which is an example of two-player impartial games. In this talk we consider a poset game in which the poset is defined by J. Striker related to the alternating sign matrices, that is to say the base set is defined by $X_n = \{(x, y, z) \in \mathbb{Z}^3 \mid x, y, z \geq 0 \text{ and } x + y + z \leq n - 2\}$ whose cover relations are defined later. We completely determine the Grundy value for this game which is introduced by us.

10/27 (金)

- 9:30 – 10:30 渡邉 英也 (大阪公立大学)

  A new branching rule from $GL_{2n}(\mathbb{C})$ to $Sp_{2n}(\mathbb{C})$

  Abstract: In this talk, I will introduce a new branching rule from the general linear group $GL_{2n}(\mathbb{C})$ to the symplectic group $Sp_{2n}(\mathbb{C})$ by establishing a simple algorithm which gives rise to a bijection from the set of semistandard tableaux of a fixed shape to a disjoint union of several copies of the sets of symplectic tableaux of various shapes. The algorithm arises from representation theory of a quantum symmetric pair of type $A\mathrm{II}_{2n-1}$, which is a q-analogue of the classical symmetric pair $(\mathfrak{gl}_{2n}(\mathbb{C}), \mathfrak{sp}_{2n}(\mathbb{C}))$.

- 11:00 – 12:00 宮地 兵衛 (大阪公立大学)

  Hecke 環における二つの相互律について

  Abstract: Hecke 代数の 2 つの相互律について話す。(1) 一つは、岩堀 Hecke 代数の Kazhdan-Lusztig cells についてである。ここでは cell は、Weyl 群の元としての意味での左 cell を意味している。この cell に関して Mackey 公式が成立することを報告する。(2) 群環において既約加群と直既約射影加群に関して Robinson の相互律という定理が知られている。 これの次数付き版の拡張定理を構成してので報告する。主だった用途は、巡回的箙 Hecke 代数やその他の型の箙 Hecke 代数である。この 2 つ目の相互律は、岩堀 Hecke 代数や巡回的 Hecke 代数についてでも新しいものである。

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