Neural Networks and Biological Modeling

Professor Wulfram Gerstner
Laboratory of Computational Neuroscience

Question set 8

Exercise 1: Continuous population model

We study a system with lateral connection $w(x-y)$ given by:

$$\tau \frac{\partial h(x,t)}{\partial t} = -h(x,t) + \int w(x-y) F[h(y,t)] dy + I_{ext}(x,t), \tag{1}$$

where $F[h(x,t)] = A(x,t)$ is the population's activity at the point x at time t.

1.1 Show that, for a constant current I_{ext}, the homogeneous stationary solution $h(x,t) = h_0$ leads to a constant activity A_0 given by:

$$A_0 = F(h_0) = \frac{h_0 - I_{ext}}{\bar{w}},$$

with $\bar{w} = \int w(x-y) dy$.

1.2 We set $h(x,t) = h_0 + \Delta h(x,t)$ where Δh is a small perturbation. Linearize the equation (1) around h_0, solve the Fourier transformed equation and obtain $\Delta h = \int g(k) dk$ where

$$g(k) = C(k) e^{ikx} e^{-\kappa(k) t/\tau}.$$

Identify the function κ. For which values of k do we get $\kappa < 0$?

1.3 Consider:

$$w(z) = \frac{\sigma_2 e^{-z^2/(2\sigma_1^2)} - \sigma_1 e^{-z^2/(2\sigma_2^2)}}{\sigma_2 - \sigma_1},$$

with $\sigma_1 = 1$ and $\sigma_2 = 10$. Sketch the qualitative behaviour of $w(z)$ and

$$\int w(z) \cos(kz) dz.$$

Determine graphically the stability condition.

Exercise 2: Stationary state in a network with lateral connections

Consider a neural network with lateral connections represented in figure 1: the interaction is locally excitatory and long range inhibitory:

$$w(x,x') = \begin{cases} 1 & |x-x'| \le \sigma \\ -b & |x-x'| > \sigma \end{cases} \tag{2}$$

Therefore σ corresponds to the range of the excitatory connections. The activity A of a neuron at position x is given by:

$$A(x) = F[h(x)], \tag{3}$$

where $h(x)$ is the total potential of the neuron at position x, defined as:

$$h(x) = \int w(x,x') A(x') dx' + I_{ext}(x). \tag{4}$$

The function $F(h)$ is a simple threshold function:

$$F(h) = \begin{cases} 1 & h > \theta \\ 0 & h \leq \theta \end{cases} \tag{5}$$

In this exercise we do not add any external input i.e. $I_{ext}(x) = 0$. The aim of the exercise is to find the neural activity $A(x)$. In order to do so, we assume that $A(x)$ may have a rectangular shape (of dimensions $2d \times 1$, as shown in figure 1) and we prove this assumption with the following passages.

2.1 Consider a point at location x_0 close to $x = 2d$ and calculate its input potential, assuming that $d > \sigma$.

(Hint: there is excitatory input from the right and there is excitatory and inhibitory input from the left).

2.2 Exploit that at $x_0 = 2d$ we must have $h(x_0) = \Theta$. Why? Calculate d.

2.3 Convince yourselves that the bump of size $2d$ is therefore a solution for the activity $A(x)$ and discuss its properties.

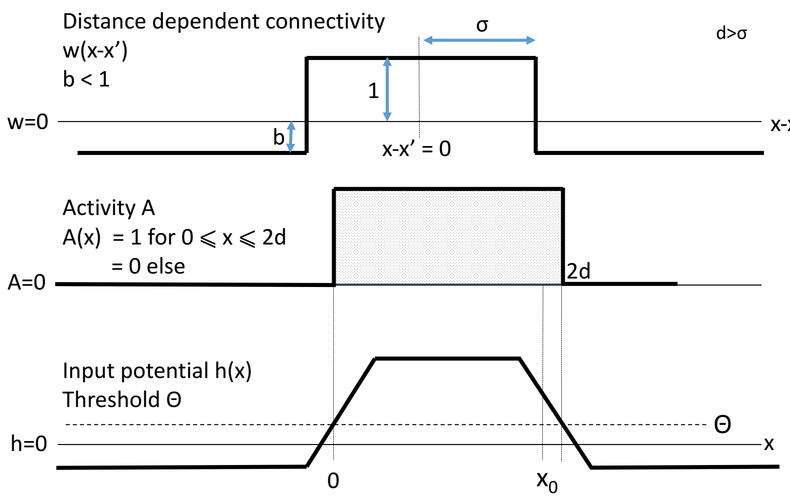

Figure 1: Spatial structure of the network.