Neural Networks and Biological Modeling

Professor Wulfram Gerstner
Laboratory of Computational Neuroscience

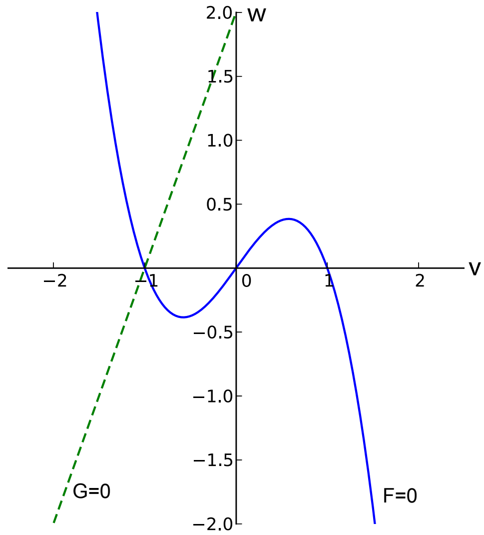
Questions set about the cable equation

Exercise 1: Inhibitory rebound

Consider the following two-dimensional Fitzhugh-Nagumo model:

$$\left[\begin{array}{lcl} \frac{du}{dt} & = & u\left(1-u^2\right)-w+I \equiv F(v,w) \\ \frac{dw}{dt} & = & \varepsilon\left(u-0.5w+1\right) \equiv \varepsilon G(v,w)\,, \end{array}\right. \quad (1)$$

where $\varepsilon \ll 1$.



1.1 Suppose that an inhibitory current step is applied,

$$I(t) \quad = \quad \begin{cases} -I_0 & t \leq 0 \\ 0 & t > 0 \end{cases}$$

How does the fixed point move?

1.2 What happens after the driving current is removed? Sketch the form of the trajectories for increasing values of I_0. What happens for large I_0?

Exercise 2: Phase Plane Analysis

In this exercise, we use the phase plane to study the dynamics of a two dimensional, nonlinear neuron model. The system is described by:

$$\begin{cases} \frac{d}{dt}u = F(u,w) \\ \frac{d}{dt}w = G(u,w) \end{cases} \quad (2)$$

where $F(u,w) = f(u) - w + I(t)$ and $G(u,w) = \epsilon(g(u) - w)$ with $\epsilon = 0.1$. $I(t)$ is an external current.

Figure 1 shows the u- and w-nullclines for the case $I(t) = 0$:

2.1 Given $F(u_4, 0) = 5, G(u_4, 0) = 1$, draw a few flow arrows along the two nullclines in figure 1.

2.2 Without doing any computation, can you determine the stability of the fixed point 2 (the one at (u_2, w_2))? Justify your answer.

2.3 Discuss the stability of the third fixed point (the one at (u_3, w_3)) analytically. That is, linearize the system at the fixed point 3 and discuss the evolution of a small perturbation around that point. For the numeric calculations, use $\epsilon = 0.1$ and approximate the values of $\frac{d}{du}f(u)_{|u_3}$ and $\frac{d}{du}g(u_3)_{|u_3}$ from figure 1.

2.4 Assume the neuron is at rest. Then, at t_0 we apply a pulse stimulus $I(t)$ to this system:

$$I(t) = (u_3 - u_1)\delta(t - t_0)$$

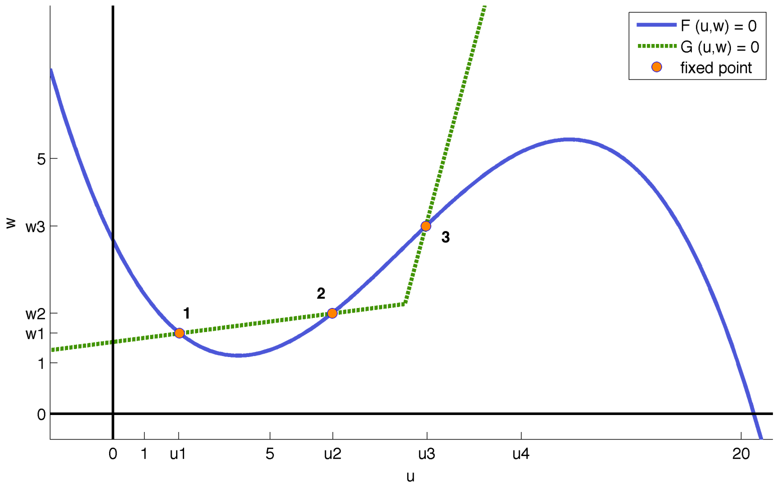


Figure 1: Phase Plane, $I(t) = 0$

(i) Sketch the trajectory $(u(t), w(t))$ in Figure 1.

(ii) Sketch the membrane potential $u(t)$ vs. time in a new figure.

Make sure you get the two plots qualitatively correct: Clearly indicate important states, for example at $t < t_0$, at t_0, and at $t > t_0$. Furthermore, in your $u(t)$ plot, fast and slow regions should be distinguishable.

2.5 Refering to figure 1, discuss the effect of injecting pulse currents $I(t) = q\delta(t - t_0)$ of different amplitudes q into the neuron. What happens if we gradually increase q ? Does this neuron model have a threshold?

2.6 Assume the neuron is at rest. We then apply a step current to the neuron:

$$I(t) = \begin{cases} 0 & if\ t \leq 0 \\ 3 & t > 0 \end{cases}$$

(i) Sketch the nullcline $\frac{d}{dt}u = 0$ for $t > 0$ in figure 2.

(ii) In figure 2, mark the state of the system at $t = 0$. Starting from that state, sketch the trajectory of the system for $t > 0$.

(iii) Qualitatively discuss the evolution of the system for $t \to \infty$.

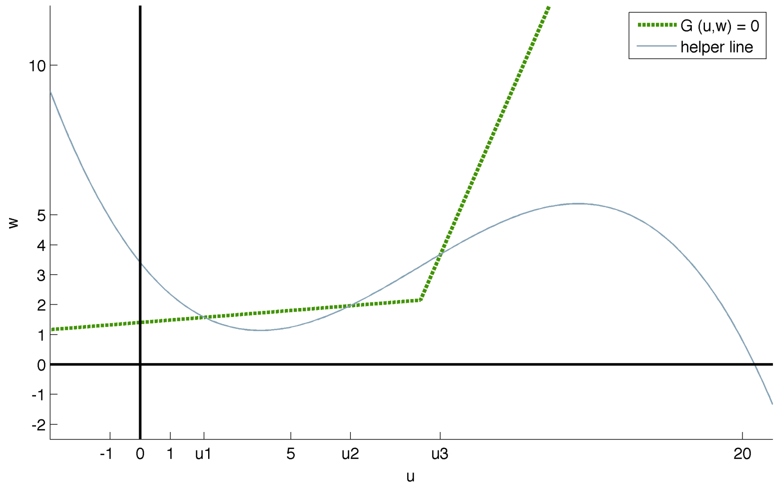


Figure 2: Phase Plane, $I(t) = 3$

Exercise 3: Impulse response

Consider the following system with separation of time scales:

$$\left[\begin{array}{ll} \dfrac{du}{dt} & = f(u) - w + I \\ \dfrac{dw}{dt} & = \varepsilon\,(bu - \gamma w) \end{array}\right.$$

where $\varepsilon \ll 1$ and

$$f(u) = \begin{cases} -u & \text{if } u < 1 \\ \dfrac{u-1}{a} - 1 & \text{if } 1 \leq u < 1 + 2a \\ 2(1+a) - u & \text{if } u > 1 + 2a \end{cases}$$

Assume that $b, \gamma, a > 0$ and $b/\gamma > 1/a$. Discuss the behaviour of the trajectories of $u(t)$ in response to a current pulse $I(t) = q\delta(t)$. Sketch these trajectories in the phase plane and in the temporal domain for a few values of q. Does the model exhibit a threshold-like behaviour?