Biological Modeling of Neural Networks

Week 3 – Reducing detail:
Two-dimensional neuron models

Wulfram Gerstner
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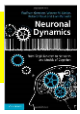
Reading for week 3:
NEURONAL DYNAMICS
- Ch. 4.1- 4.3



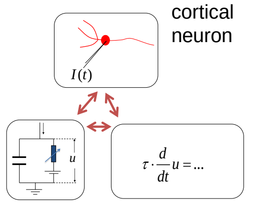
Cambridge Univ. Press

3.1 **From Hodgkin-Huxley to 2D**
- Overview: From 4 to 2 dimensions
- MathDetour 1: Exploiting similarities
- MathDetour 2: Separation of time scales

3.2 **Phase Plane Analysis**
- Role of nullclines

3.3 **Analysis of a 2D Neuron Model**
- constant input vs pulse input
- MathDetour 3: Stability of fixed points

3.1. Review of week 2 :Hodgkin-Huxley Model



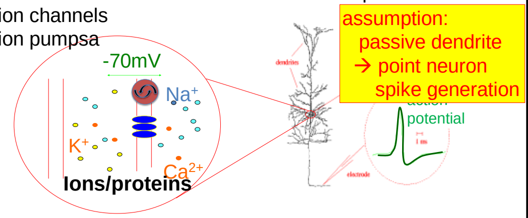
→Hodgkin-Huxley model
→Compartmental models

3.1 Review of week 2 : Hodgkin-Huxley Model

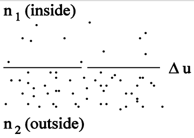
Week 2:
Cell membrane contains
- ion channels
- ion pumpsa

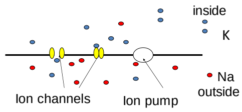
Dendrites (week x:video):
Active processes?

assumption:
passive dendrite
→ point neuron
spike generation



3.1. Review of week 2 :Hodgkin-Huxley Model





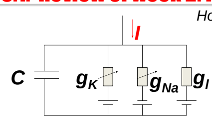
$$\Delta u = u_1 - u_2 = \frac{-kT}{q} \ln \frac{n(u_1)}{n(u_2)}$$

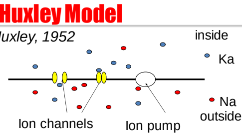
Reversal potential

ion pumps →concentration difference ⇔ voltage difference

3.1. Review of week 2: Hodgkin-Huxley Model

Hodgkin and Huxley, 1952





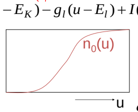
stimulus

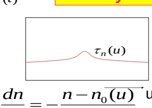
$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + I(t)$$

4 equations
= 4D system

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$





$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$

Week 3 – 3.1. Overview and aims

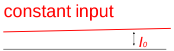
Can we understand the dynamics of the HH model?

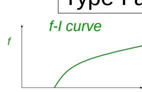
- mathematical principle of Action Potential generation?
- constant input current vs pulse input?
- Types of neuron model (type I and II)? (next week)
- threshold behavior? (next week)

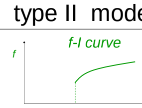
→ Reduce from 4 to 2 equations

Type I and type II models

ramp input/
constant input







Week 3 – 3.1. Overview and aims

Can we understand the dynamics of the HH model?

→ Reduce from 4 to 2 equations

Week 3 – Quiz 3.1.

A - A biophysical point neuron model
with 3 ion channels,
each with activation and inactivation,
has a total number of equations equal to
[ ] 3 or
[ ] 4 or
[ ] 6 or
[ ] 7 or
[ ] 8 or more

Next week:
No class!

Watch video:
Video Week 2.5
Video Week 3.1-3.5
(82 minutes total)

http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html

Week 3 – 3.1. Overview and aims

Toward a
two-dimensional neuron model

-**Reduction of Hodgkin-Huxley to 2 dimension**
-step 1: separation of time scales

-step 2: exploit similarities/correlations

3.1. Reduction of Hodgkin-Huxley model

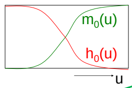
stimulus

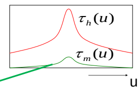
$$C\frac{du}{dt} = -\overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} - \overbrace{g_K n^4(u-E_K)}^{I_K} - \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$





MathDetour

1) dynamics of *m* are fast $\longrightarrow$ $m(t) = m_0(u(t))$

Reduction of dimensionality: Separation of time scales

$$\tau_1\frac{dx}{dt} = -x + c(t)$$

Exercise 1 (week 3)
(later today !)

Two coupled differential equations

$$\tau_1\frac{dx}{dt} = -x + h(y)$$

$$\tau_2\frac{dy}{dt} = f(y) + g(x)$$

Separation of time scales

$\tau_1 \ll \tau_2 \rightarrow$ *x=h(y)*

Reduced 1-dimensional system

$$\tau_2\frac{dy}{dt} = f(y) + g(h(y))$$

3.1. Reduction of Hodgkin-Huxley model

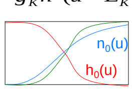
stimulus

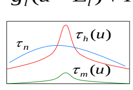
$$C\frac{du}{dt} = -\overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} - \overbrace{g_K n^4(u-E_K)}^{I_K} - \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$





1) dynamics of *m* are fast $\longrightarrow$ $m(t) = m_0(u(t))$

2) dynamics of *h* and *n* are similar

3.1. Reduction of Hodgkin-Huxley model

Reduction of Hodgkin-Huxley Model to 2 Dimension

-step 1:
separation of time scales

-step 2:
exploit similarities/correlations

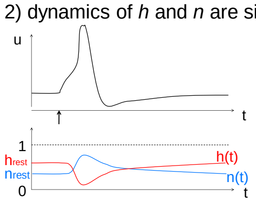
Now !

3.1. Reduction of Hodgkin-Huxley model

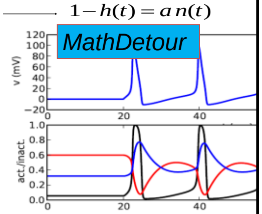
stimulus

$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na}) - g_K n^4(u-E_K) - g_l(u-E_l) + I(t)$$

2) dynamics of *h* and *n* are similar $\quad 1-h(t) = a\,n(t)$



MathDetour

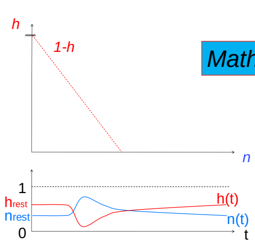


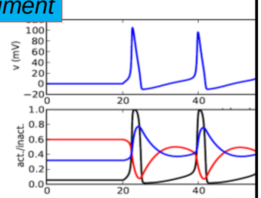
3.1 Detour 1. Exploit similarities/correlations

dynamics of *h* and *n* are similar

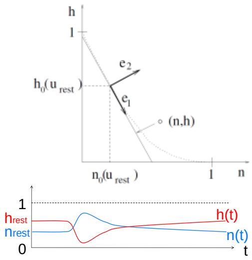
$$1-h(t) = a\,n(t)$$

Math. argument





3.1 Detour 1. Exploit similarities/correlations



dynamics of *h* and *n* are similar

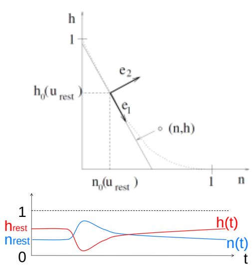
$$1-h(t)=a\,n(t)$$

at rest

$$\frac{dh}{dt}=-\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt}=-\frac{n-n_0(u)}{\tau_n(u)}$$

3.1 Detour 1. Exploit similarities/correlations



dynamics of *h* and *n* are similar

(i) Rotate coordinate system
(ii) Suppress one coordinate
(iii) Express dynamics in new variable

$$1-h(t)=a\,n(t)=w(t)$$

$$\frac{dh}{dt}=-\frac{h-h_0(u)}{\tau_h(u)} \qquad \frac{dw}{dt}=-\frac{w-w_0(u)}{\tau_{eff}(u)}$$

$$\frac{dn}{dt}=-\frac{n-n_0(u)}{\tau_n(u)}$$

3.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt}=\overbrace{-g_{Na}[m(t)]^3h(t)(u(t)-E_{Na})}^{I_{Na}}\;\overbrace{-g_K[n(t)]^4(u(t)-E_K)}^{I_K}\;\overbrace{-g_l(u(t)-E_l)}^{I_{leak}}+I(t)$$

$$C\frac{du}{dt}=-g_{Na}\,m_0(u)^3(1-w)(u-E_{Na})-g_K[\frac{w}{a}]^4(u-E_K)-g_l(u-E_l)+I(t)$$

1) dynamics of *m* are fast —— $m(t)=m_0(u(t))$

2) dynamics of *h* and *n* are similar —— $\underbrace{1-h(t)}_{w(t)}=\underbrace{a\,n(t)}_{w(t)}$

$$\frac{dh}{dt}=-\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt}=-\frac{n-n_0(u)}{\tau_n(u)} \quad\text{——}\quad \frac{dw}{dt}=-\frac{w-w_0(u)}{\tau_{eff}(u)}$$

3.1. Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = \overbrace{-g_{Na}\, m_0(u)^3 (1-w)(u-E_{Na})}^{I_{Na}} \overbrace{-g_K (\frac{w}{a})^4 (u-E_K)}^{I_K} \overbrace{-g_l (u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_{eff}(u)}$$

$$\tau \frac{du}{dt} = F(u(t), w(t)) + R\, I(t)$$

$$\tau_w \frac{dw}{dt} = G(u(t), w(t))$$

3.1. Reduction to 2 dimensions

2-dimensional equation

$$C\frac{du}{dt} = f(u(t), w(t)) + I(t)$$

$$\frac{dw}{dt} = g(u(t), w(t))$$

Enables graphical analysis! → Phase plane analysis

-Discussion of threshold
- Constant input current *vs* pulse input
-Type I and II
- Repetitive firing

Week 3 – Quiz 3.2-similar dynamics

Exploiting similarities:

A sufficient condition to replace two gating variables *r,s* by a single gating variable *w* is

[ ] Both *r* and *s* have the same time constant (as a function of u)
[ ] Both *r* and *s* have the same activation function
[ ] Both *r* and *s* have the same time constant (as a function of u) AND the same activation function
[ ] Both *r* and *s* have the same time constant (as a function of u) AND activation functions that are identical after some additive rescaling
[ ] Both *r* and *s* have the same time constant (as a function of u) AND activation functions that are identical after some multiplicative rescaling

Biological Modeling of Neural Networks

Week 3 – Reducing detail:
Two-dimensional neuron models

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3.2. Reduced Hodgkin-Huxley model

$$C\frac{du}{dt} = \underbrace{-g_{Na}\, m_0(u)^3(1-w)(u-E_{Na})}_{I_{Na}} \underbrace{- g_K(\frac{w}{a})^4(u-E_K)}_{I_K} \underbrace{- g_l(u-E_l)}_{I_{leak}} + I(t)$$

$$\frac{dw}{dt} = -\frac{w-w_0(u)}{\tau_w(u)}$$

stimulus

$$\tau\frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w\frac{dw}{dt} = G(u,w)$$

3.2. Phase Plane Analysis/nullclines

2-dimensional equation

stimulus

$$\tau\frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w\frac{dw}{dt} = G(u,w)$$

Enables graphical analysis!
-Discussion of threshold
-Type I and II

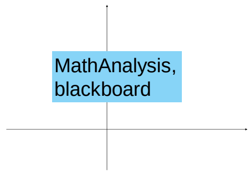
First step:

u-nullcline:
all points with *du/dt=0*

w-nullcline:
all points with *dw/dt=0*

3.2. FitzHugh-Nagumo Model

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$
$$= u - \frac{1}{3}u^3 - w + RI(t)$$
$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$



u-nullcline

w-nullcline

3.2. flow arrows

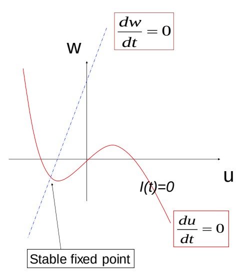
$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$ Stimulus I=0

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Consider change in small time step

Flow on nullcline

Flow in regions between nullclines

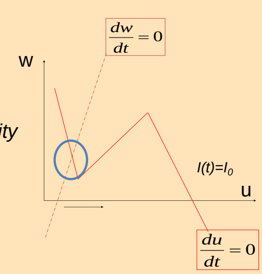


NOW Exercise 2.1: Stability of Fixed Point in 2D

$$\frac{du}{dt} = \alpha u - w$$
$$\frac{dw}{dt} = \beta u - w$$

- calculate *stability*
- compare $\frac{dx}{dt} = -\frac{x}{\tau}$

Exercises:
1.1-1.5 now!
2.1 start now!
2.2 homework
(you may start if you have time)



NOW Exercises 1.1-1.5: separation of time scales

$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na}) - g_K n^4(u-E_K)$$

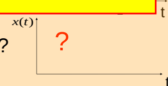
Exerc. 10:15-11:00
***Next lecture*: 11h15**
Please stop Ex. 1 at 10:50 and start Ex. 2

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

Exercises:
1.1-1.5 *now!*
2.1 start now!
2.2 *homework*
(you may start if you have time)

A: $\frac{dx}{dt} = -$

- calculate *x(t)!*
- what if τ is small?



$$\frac{dm}{dt} = -\frac{m-c(u)}{\tau_m}$$
$$\frac{du}{dt} = f(u) - m$$

B: -calculate *m(t)* if τ is small!
- reduce to 1 eq.

Biological Modeling of Neural Networks

Week 3 – Reducing detail: Two-dimensional neuron models

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- MathDetour 1: Exploiting similarities
- MathDetour 2: Separation of time scales

3.2 **Phase Plane Analysis**
- Role of nullclines

3.3 **Analysis of a 2D Neuron Model**
- constant input vs pulse input
- MathDetour 3: Stability of fixed points

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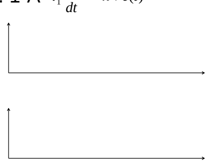
Discussion Exercise 1 – MathDetour 3.1: Separation of time scales

Exercise 1 (week 3)

Ex. 1-A $\tau_1\frac{dx}{dt} = -x + c(t)$



Draw graph, blackboard

Two coupled differential equations

$$\tau_1\frac{dx}{dt} = -x + c(t)$$
$$\tau_2\frac{dy}{dt} = f(y) + g(x)$$

Separation of time scales

$$\tau_1 \ll \tau_2$$

Reduced 1-dimensional system

$$\tau_2\frac{dy}{dt} = f(y) + g(c(t))$$

Discussion Exercise 1 – MathDetour 3.1 Separation of time scales

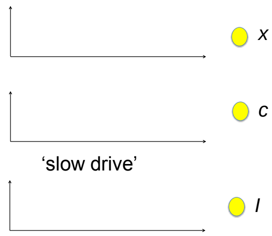
Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + c(t)$$

$$\tau_2 \frac{dc}{dt} = -c + f(x)\, a + I(t)$$

a=0
a=1

$$\tau_1 \ll \tau_2$$



Draw graph, blackboard

Discuss Exercise 1 – MathDetour 3.1: Separation of time scales

Two coupled differential equations

$$\tau_1 \frac{dx}{dt} = -x + h(y)$$

$$\tau_2 \frac{dy}{dt} = f(y) + g(x)$$

Exercise 1 (week 3)

even more general

Separation of time scales

$$\tau_1 \ll \tau_2 \rightarrow x = h(y)$$

Reduced 1-dimensional system

$$\tau_2 \frac{dy}{dt} = f(y) + g(h(y))$$

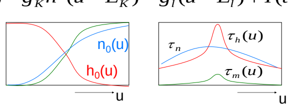
Discuss exercise 1 – Reduction of Hodgkin-Huxley model

$$C\frac{du}{dt} = -\underbrace{g_{Na} m^3 h (u - E_{Na})}_{I_{Na}} - \underbrace{g_K n^4 (u - E_K)}_{I_K} - \underbrace{g_l (u - E_l)}_{I_{leak}} + \underbrace{I(t)}_{\text{stimulus}}$$

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n - n_0(u)}{\tau_n(u)}$$



dynamics of *m* is fast $\longrightarrow$ $m(t) = m_0(u(t))$

Fast compared to what?

Neuronal Dynamics – Quiz 3.3.

A- Separation of time scales:
We start with two equations

$$\tau_1 \frac{dx}{dt} = -x + y + I(t)$$

$$\tau_2 \frac{dy}{dt} = -y + x^2 + A$$

[ ] If $\tau_1 << \tau_2$ then the system can be reduded to

$$\tau_2 \frac{dy}{dt} = -y + [y + I(t)]^2 + A$$

[ ] If $\tau_2 << \tau_1$ then the system can be reduded to

$$\tau_1 \frac{dx}{dt} = -x + x^2 + A + I(t)$$

[ ] None of the above is correct.

Pay attention to *I(t)*

Week 3 – Quiz 3.2-similar dynamics

Exploiting similarities:

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Neuronal Dynamics – 3.2. flow arrows

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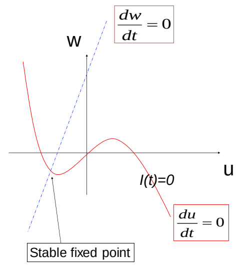
$$\tau_w \frac{dw}{dt} = G(u,w)$$

Consider change in small time step

Flow on nullcline

Flow in regions between nullclines



Week 3 – Quiz 3.4

Take 1 minute

A. u-Nullclines

[ ] On the u-nullcline, arrows are always vertical
[ ] On the u-nullcline, arrows point always vertically upward
[ ] On the u-nullcline, arrows are always horizontal
[ ] On the u-nullcline, arrows point always to the left
[ ] On the u-nullcline, arrows point always to the right

B. w-Nullclines

[ ] On the w-nullcline, arrows are always vertical
[ ] On the w-nullcline, arrows point always vertically upward
[ ] On the w-nullcline, arrows are always horizontal
[ ] On the w-nullcline, arrows point always to the left
[ ] On the w-nullcline, arrows point always to the right
[ ] On the w-nullcline, arrows can point in an arbitrary direction

3.2. FitzHugh-Nagumo Model

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$= u - \frac{1}{3}u^3 + RI(t) \text{ -}w$$

$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$

change b_0 , b_1

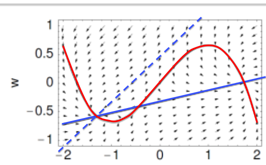


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

3.2. Nullclines of reduced HH model

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

u-nullcline

w-nullcline

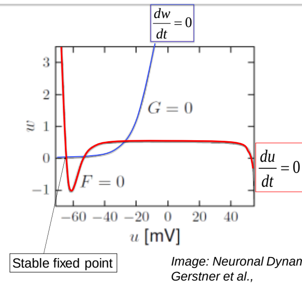


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

3.2. Phase Plane Analysis

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Enables graphical analysis! → Application to neuron models

Important role of
- nullclines
- flow arrows

Week 3 – part 3: Analysis of a 2D neuron model

3.1 **From Hodgkin-Huxley to 2D**

3.2 **Phase Plane Analysis**
- Role of nullcline

$\boxed{\text{3.3 Analysis of a 2D Neuron Model}}$
- pulse input
- constant input
- MathDetour 3: Stability of fixed points

3.3. Analysis of a 2D neuron model

2-dimensional equation

stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

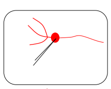
$$\tau_w \frac{dw}{dt} = G(u,w)$$

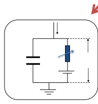
Enables graphical analysis!

2 important input scenarios

- Pulse input
- Constant input

3.3. 2D neuron model : Pulse input





$$\tau \cdot \frac{d}{dt} u = F(u,w) + RI$$

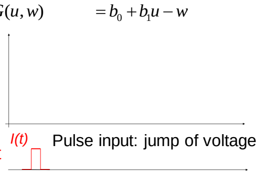
$$\tau_w \frac{d}{dt} w = G(u,w)$$

pulse input

3.3. FitzHugh-Nagumo Model : Pulse input

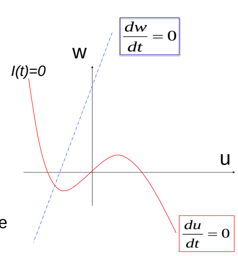
$$\tau \frac{du}{dt} = F(u,w) + RI(t) = u - \frac{1}{3}u^3 - w + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w) \qquad = b_0 + b_1 u - w$$

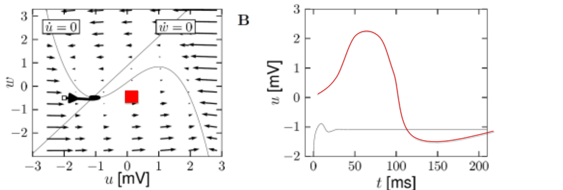


pulse input

Pulse input: jump of voltage



3.3. FitzHugh-Nagumo Model : Pulse input



FN model with $b_0 = 0.9; b_1 = 1.0$

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014)

Pulse input: jump of voltage/initial condition

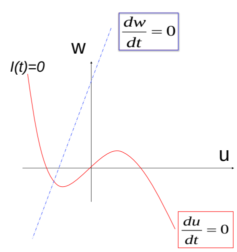
3.3. FitzHugh-Nagumo Model – 2 different inputs

Pulse input: DONE!
- jump of voltage
- 'new initial condition'
- spike generation for large input pulses

2 important input scenarios

constant input:
- graphics?
- spikes?
- repetitive firing?

Now



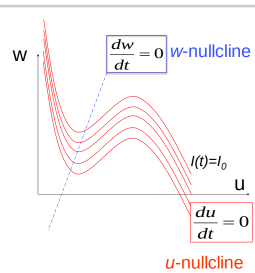
3.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$= u - \frac{1}{3}u^3 - w + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$

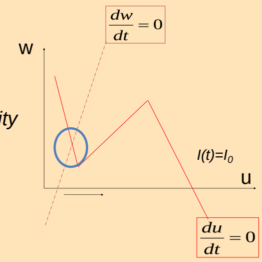
Intersection point (fixed point)
-moves
-changes Stability



NOW Exercise 2.1: Stability of Fixed Point in 2D

$$\frac{du}{dt} = \alpha u - w$$
$$\frac{dw}{dt} = \beta u - w$$

- calculate *stability*
- compare $\frac{dx}{dt} = -\frac{x}{\tau}$



Exercises:
2.1***now!***
2.2 *homework*

Next lecture:
11:42

Week 3 – part 3: Analysis of a 2D neuron model

3.1 **From Hodgkin-Huxley to 2D**

3.2 **Phase Plane Analysis**
- Role of nullcline

3.3 **Analysis of a 2D Neuron Model**
- pulse input
- constant input
-MathDetour 3: Stability of fixed points

Discussion of exercise 2 Detour. Stability of fixed points

2-dimensional equation

stimulus

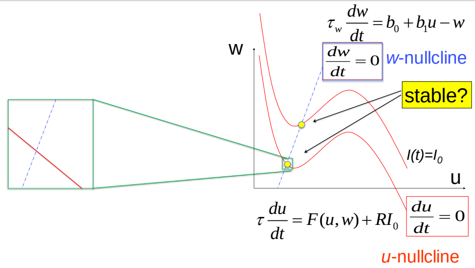
$$\tau \frac{du}{dt} = F(u, w) + RI_0$$
$$\tau_w \frac{dw}{dt} = G(u, w)$$

How to determine stability of fixed point?

Discussion of exercise 2 - Detour: Stability of fixed points.

$$\tau_w \frac{dw}{dt} = b_0 + b_1 u - w$$

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

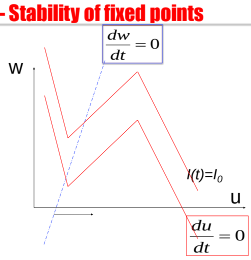


Discussion of Exercise 2: Detour - Stability of fixed points

stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$

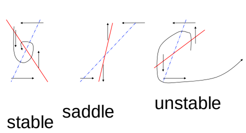


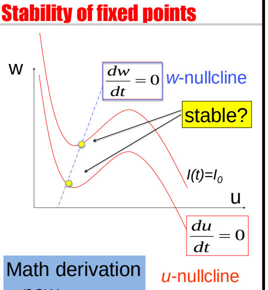
Discussion of Exercise 2: Detour. Stability of fixed points

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

zoom in:





Math derivation now

Discussion of Exercise 2 Detour. Stability of fixed points

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$
$$\tau_w \frac{dw}{dt} = G(u,w)$$

Fixed point at (u_0, w_0)

At fixed point

$$0 = F(u_0, w_0) + RI_0$$
$$0 = G(u_0, w_0)$$

zoom in:

$$x = u - u_0$$
$$y = w - w_0$$

Discussion of Exercise 2 - Detour. Stability of fixed points

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$
$$\tau_w \frac{dw}{dt} = G(u,w)$$

Fixed point at (u_0, w_0)

At fixed point

$$0 = F(u_0, w_0) + RI_0$$
$$0 = G(u_0, w_0)$$

zoom in:

$$x = u - u_0$$
$$y = w - w_0$$

$$\tau \frac{dx}{dt} = F_u x + F_w y$$
$$\tau_w \frac{dy}{dt} = G_u x + G_w y$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x}.$$

Discussion of Exercise 2 Detour. Stability of fixed points

Linear matrix equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x}.$$

Search for solution

$$\boldsymbol{x}(t) = \boldsymbol{e}\,\exp(\lambda t)$$

Two solution with Eigenvalues λ_+, λ_-

$$\lambda_+ + \lambda_- = F_u + G_w$$
$$\lambda_+ \lambda_- = F_u G_w - F_w G_u$$

Discussion of Exercise 2: Detour. Stability of fixed points

Linear matrix equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x}$$

Search for solution

$$\boldsymbol{x}(t) = \boldsymbol{e}\,\exp(\lambda t)$$

Two solution with Eigenvalues λ_+, λ_-

$$\lambda_+ + \lambda_- = F_u + G_w$$

$$\lambda_+ \lambda_- = F_u G_w - F_w G_u$$

Stability requires:

$\lambda_+ < 0$ *and* $\lambda_- < 0$

$F_u + G_w < 0$

and

$F_u G_w - F_w G_u > 0$

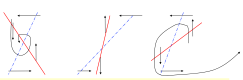
Discussion of exercise 2: Detour. Stability of fixed points

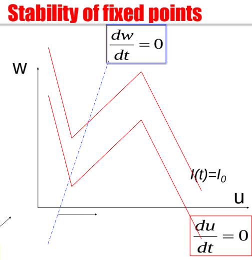
stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$

$$\lambda_{+/-} =$$





3.3. Neuron models and Stability of fixed points

Now Back:

Application to our neuron model

2-dimensional equation

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

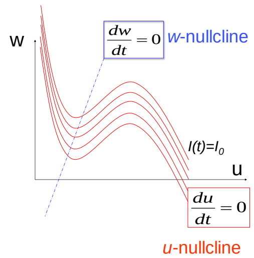
Stability characterized by Eigenvalues of linearized equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} \boldsymbol{x}$$

3.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$
$$= u - \frac{1}{3}u^3 - w + RI_0$$
$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$

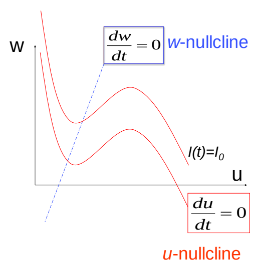
Intersection point (fixed point)
-moves
-changes Stability



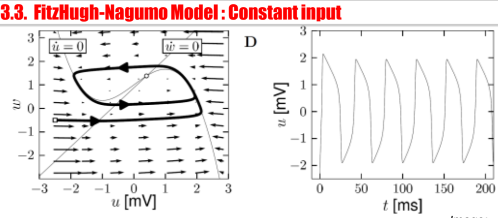
3.3. FitzHugh-Nagumo Model: Constant input

$$\tau \frac{du}{dt} = F(u,w) + RI_0$$
$$= u - \frac{1}{3}u^3 - w + RI_0$$
$$\tau_w \frac{dw}{dt} = G(u,w) = b_0 + b_1 u - w$$

Intersection point (fixed point)
-moves
-changes Stability



3.3. FitzHugh-Nagumo Model : Constant input



FN model with $b_0 = 0.9; b_1 = 1.0; RI_0 = 2$

constant input: u-nullcline moves
limit cycle

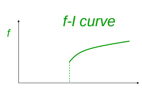


Image: Neuronal Dynamics, Gerstner et al., Cambridge (2014)

Neuronal Dynamics – Quiz 3.5.

A. Short current pulses. In a 2-dimensional neuron model, the effect of a delta current pulse can be analyzed

[ ] By moving the u-nullcline vertically upward
[ ] By moving the w-nullcline vertically upward
[ ] As a potential change in the stability or number of the fixed point(s)
[ ] As a new initial condition
[ ] By following the flow of arrows in the appropriate phase plane diagram

B. Constant current. In a 2-dimensional neuron model, the effect of a constant current can be analyzed

[ ] By moving the u-nullcline vertically upward
[ ] By moving the w-nullcline vertically upward
[ ] As a potential change in the stability or number of the fixed point(s)
[ ] By following the flow of arrows in the appropriate phase plane diagram

Computer exercise now

Can we understand the dynamics of the 2D model?

The END for today

Now: computer exercises

Type I and type II models