Biological Modeling of Neural Networks

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Week 1: **A first simple neuron model/ neurons and mathematics**
Week 2: **Hodgkin-Huxley models and biophysical modeling**
Week 3: **Two-dimensional models and phase plane analysis**
Week 4: **Two-dimensional models, type I and type II models**
Week 5,6: **Associative Memory, Hebb rule, Hopfield**
Week 7,8: **Noise models, noisy neurons and coding**
Week 9: **Estimating neuron models for coding and decoding: GLM**
Week 10-13: **Networks, cognition, learning**
Week x: **Online video: Dendrites/Biophysics**

LEARNING OUTCOMES

- Solve linear one-dimensional differential equations
- Analyze two-dimensional models in the phase plane
- Develop a simplified model by separation of time scales
- Analyze connected networks in the mean-field limit
- Formulate stochastic models of biological phenomena
- Formalize biological facts into mathematical models
- Prove stability and convergence
- Apply model concepts in simulations
- Predict outcome of dynamics
- Describe neuronal phenomena

Look at samples of past exams

Use a textbook, (Use video lectures) don't use slides (only)

Transversal skills

- Plan and carry out activities in a way which makes optimal use of available time and other resources.
- Collect data.
- Write a scientific or technical report.

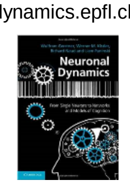
miniproject

Biological Modeling of Neural Networks

Written Exam (2/3) + miniproject (1/3)

http://neuronaldynamics.epfl.ch/

Textbook:



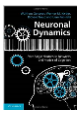
Videos (for half the material):
http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html

Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Reading for week 1:
NEURONAL DYNAMICS
- Ch. 1 (without 1.3.6 and 1.4)
- Ch. 5 (without 5.3.1)



Cambridge Univ. Press

1.1 **Neurons and Synapses:**
Overview
1.2 **The Passive Membrane**
- Linear circuit
- Dirac delta-function
1.3 **Leaky Integrate-and-Fire Model**
1.4 **Generalized Integrate-and-Fire Model**
1.5. **Quality of Integrate-and-Fire Models**

Biological Modeling of Neural Networks

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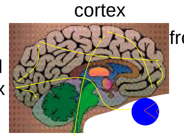
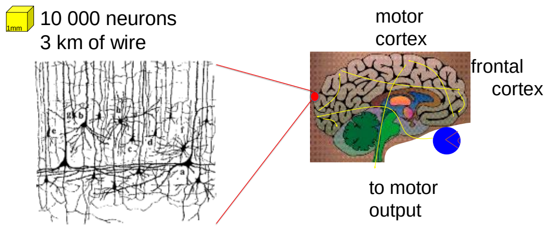
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

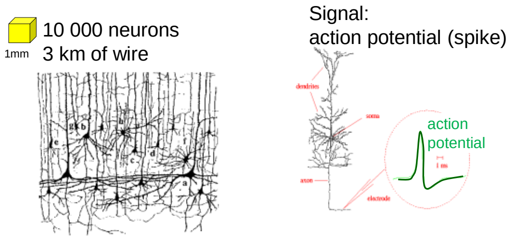
How do we recognize things?
Models of cognition
Weeks 10-14



Neuronal Dynamics – 1.1. Neurons and Synapses/Overview



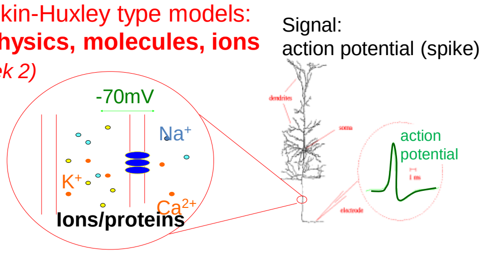
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview



Ramon y Cajal

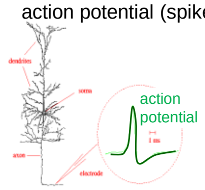
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

Hodgkin-Huxley type models:
Biophysics, molecules, ions
(week 2)



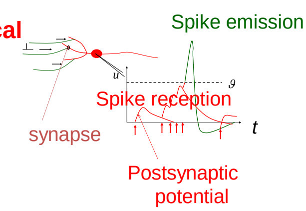
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

Signal:
action potential (spike)



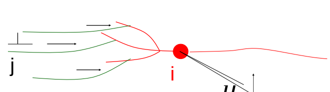
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

Integrate-and-fire models:
Formal/phenomenological
(week 1 and week 7-9)



-spikes are events
-triggered at threshold
-spike/reset/refractoriness

Noise and variability in integrate-and-fire models



Output
-spikes are **rare** events
-triggered at threshold

Subthreshold regime:
-trajectory of potential shows fluctuations

Neuronal Dynamics – membrane potential fluctuations

Spontaneous activity *in vivo*

What is noise?

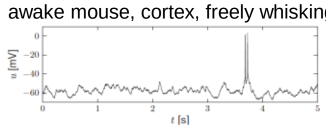
What is the neural code?

(week 7-9)

electrode

Brain

awake mouse, cortex, freely whisking,



Lab of Prof. C. Petersen, EPFL *Crochet et al., 2011*

Biological Modeling of Neural Networks – Quiz 1.1

A cortical neuron sends out signals which are called:
- [ ] action potentials
- [ ] spikes
- [ ] postsynaptic potential

The dendrite is a part of the neuron
- [ ] where synapses are located
- [ ] which collects signals from other neurons
- [ ] along which spikes are sent to other neurons

In an integrate-and-fire model, when the voltage hits the threshold:
- [ ] the neuron fires a spike
- [ ] the neuron can enter a state of refractoriness
- [ ] the voltage is reset
- [ ] the neuron explodes

In vivo, a typical cortical neuron exhibits
- [ ] rare output spikes
- [ ] regular firing activity
- [ ] a fluctuating membrane potential

Multiple answers possible!

Biological Modeling of Neural Networks

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EPFL, Lausanne, Switzerland

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Biological modeling of Neural Networks

Course: Monday : 9:15-13:00

A typical Monday:
1st lecture 9:15-9:50
1st exercise 9:50-10:00 — *paper and pencil*
2nd lecture 10:15-10:35
2nd exercise 10:35-11:00 — *paper and pencil*
3rd lecture 11:15 – 11:40
3rd exercise 11:45-12:40 — *paper and pencil OR interactive toy examples on computer*

have your laptop with you

Course of 4 credits = *6 hours of work per week*
4 'contact' + 2 homework

http://lcn.epfl.ch/~gerstner/ *moodle.eplf.ch*

Week 1 – part 2: The Passive Membrane

Biological Modeling of Neural Networks

Week 1 – neurons and mathematics:
a first simple neuron model

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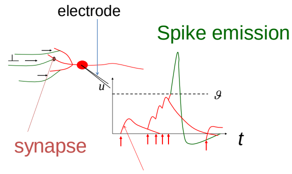
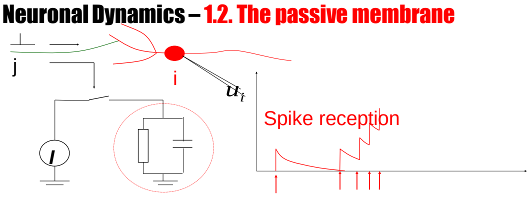
Neuronal Dynamics – 1.2. The passive membrane



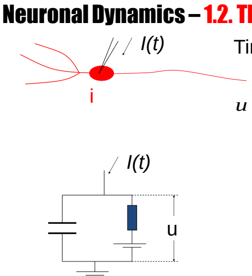
Integrate-and-fire model

Neuronal Dynamics – 1.2. The passive membrane

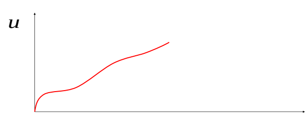


Subthreshold regime
- linear
- passive membrane
- RC circuit

Neuronal Dynamics – 1.2. The passive membrane

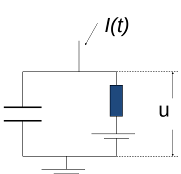


Time-dependent input

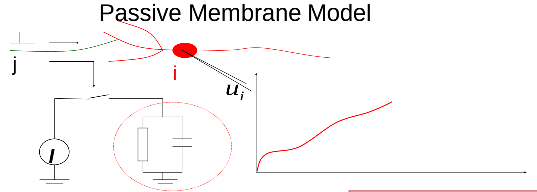


Math development:
Derive equation

Passive Membrane Model



Passive Membrane Model



$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

$$\tau \cdot \frac{d}{dt}V = -V + RI(t); \quad V = (u - u_{rest})$$

Math Development: Voltage rescaling

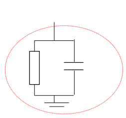
Passive Membrane Model

$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

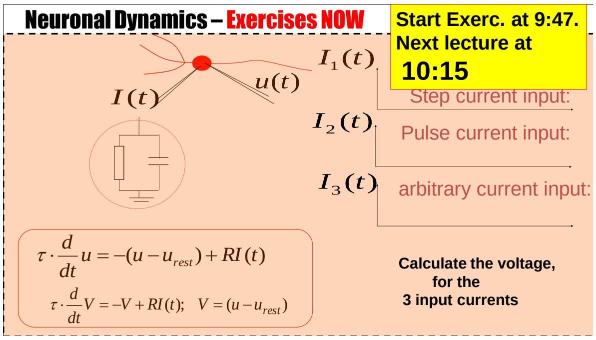
$$\tau \cdot \frac{d}{dt}V = -V + RI(t); \qquad V = (u - u_{rest})$$

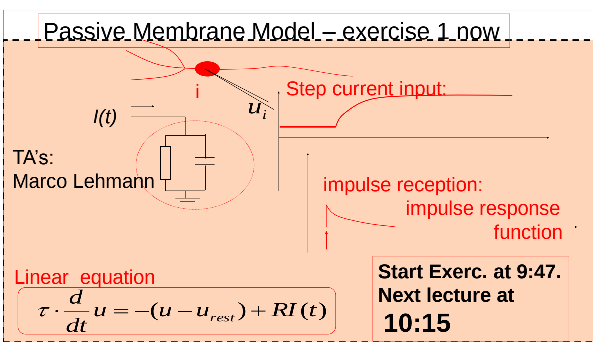
Passive Membrane Model/Linear differential equation

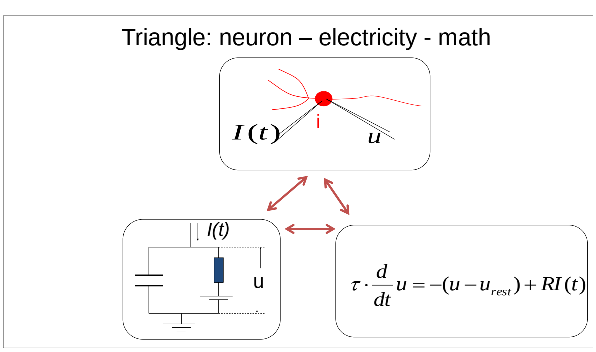
$$\tau \cdot \frac{d}{dt}V = -V + R\cancel{I}(t);$$



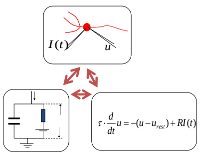
Free solution:
exponential decay


Neuronal Dynamics – Exercises NOW
Start Exerc. at 9:47.
Next lecture at
10:15
$I(t)$
$u(t)$
$I_1(t)$
Step current input:
$I_2(t)$
Pulse current input:
$I_3(t)$
arbitrary current input:
$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$
$\tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{rest})$
Calculate the voltage,
for the
3 input currents


Passive Membrane Model – exercise 1 now
i
u_i
I(t)
Step current input:
TA's:
Marco Lehmann
impulse reception:
impulse response
function
Linear equation
$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$
Start Exerc. at 9:47.
Next lecture at
10:15


Triangle: neuron – electricity - math
$I(t)$
i
u
I(t)
u
$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$

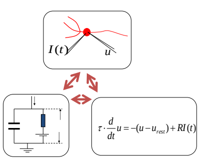
Pulse input – charge – delta-function



$$I(t) = q \cdot \delta(t - t_0)$$

Pulse current input

Dirac delta-function



$$I(t) = q \cdot \delta(t - t_0)$$

$$1 = \int_{t_0 - a}^{t_0 + a} \delta(t - t_0)\, dt$$

$$f(t_0) = \int_{t_0 - a}^{t_0 + a} f(t)\delta(t - t_0)\, dt$$

Neuronal Dynamics – Solution of Ex. 1 – arbitrary input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Arbitrary input

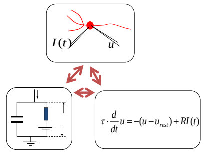
$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Single pulse

$$\Delta u(t) = \frac{q}{C} e^{-(t-t_0)/\tau}$$

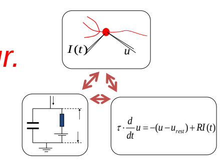
you need to know the solutions of linear differential equations!

Passive membrane, linear differential equation



Passive membrane, linear differential equation

If you have difficulties, watch lecture 1.2detour.



Three prerequisites:
-Analysis 1-3
-Probability/Statistics
-Differential Equations or Physics 1-3 or Electrical Circuits

LEARNING OUTCOMES
•Solve linear one-dimensional differential equations
•Analyze two-dimensional models in the phase plane
•Develop a simplified model by separation of time scales
•Analyze connected networks in the mean-field limit
•Formulate stochastic models of biological phenomena
•Formalize biological facts into mathematical models
•Prove stability and convergence
•Apply model concepts in simulations
•Predict outcome of dynamics
•Describe neuronal phenomena

Look at samples of past exams

Use a textbook, (Use video lectures) don't use slides (only)

Transversal skills
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miniproject

Biological Modeling of Neural Networks

Written Exam (2/3)
+ miniproject (1/3)

http://neuronaldynamics.epfl.ch/

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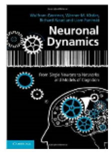

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Questions?

Videos (for half the material):
http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html

Week 1 – part 3: Leaky Integrate-and-Fire Model

Biological Modeling of Neural Networks

Week 1 – neurons and mathematics:
a first simple neuron model

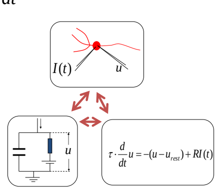
Wulfram Gerstner
EPFL, Lausanne, Switzerland

√ 1.1 **Neurons and Synapses:**
Overview
√ 1.2 **The Passive Membrane**
- Linear circuit
- Dirac delta-function
- Detour: solution of 1-dim linear differential equation

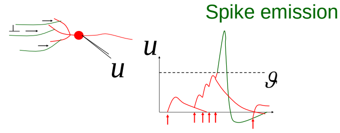
1.3 **Leaky Integrate-and-Fire Model**
1.4 **Generalized Integrate-and-Fire Model**
1.5. **Quality of Integrate-and-Fire Models**

Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



Neuronal Dynamics – Integrate-and-Fire type Models



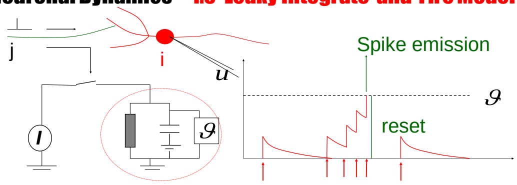
Input spike causes an EPSP
= excitatory postsynaptic potential

Simple
Integate-and-Fire Model:
passive membrane
+ threshold

Leaky Integrate-and-Fire Model

-output spikes are events
-generated at threshold
-after spike: reset/refractoriness

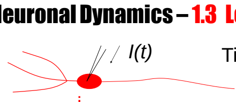
Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



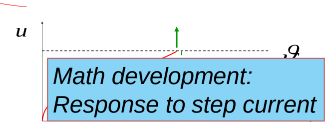
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$ linear

$u(t) = \vartheta \Rightarrow$ Fire+reset $u \to u_r$ threshold

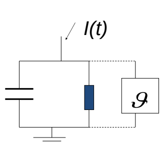
Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



Time-dependent input



Math development:
Response to step current



-spikes are events
-triggered at threshold
-spike/reset/refractoriness

Week1 – Quiz 2.

Take 90 seconds:

Consider the linear differential equation $\tau \cdot \frac{d}{dt} x = -x + x_c$

with initial condition $at\ t = 0 : x = 0$

The solution for t>0 is

(i) $x(t) = x_c \exp(t/\tau)$

(ii) $x(t) = x_c \exp(-t/\tau)$

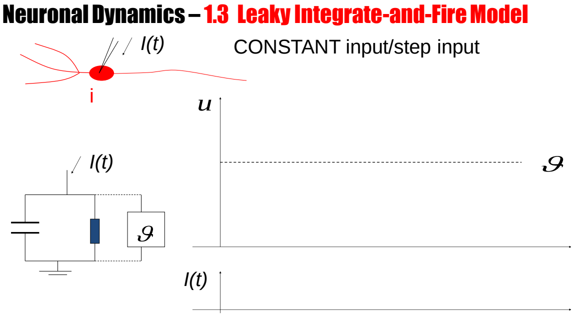
(iii) $x(t) = x_c[1 - \exp(-t/\tau)]$

(iv) $x(t) = 0.5 x_c[1 + \exp(-t/\tau)]$

You will have to use the results/response to constant input/step input again and again

Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

CONSTANT input/step input



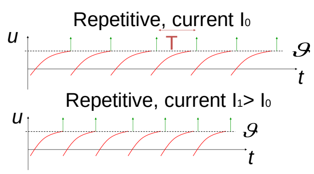
Leaky Integrate-and-Fire Model (LIF)

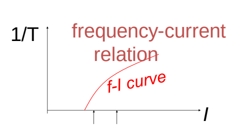
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI_0$$

LIF

If $u(t) = \vartheta \Rightarrow \ u \to u_r$

'Firing'

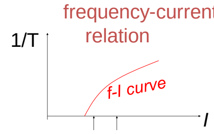




Neuronal Dynamics – First week, Exercise 2

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

frequency-current relation



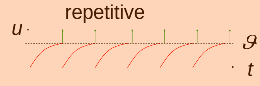
EXERCISE 2 NOW: Leaky Integrate-and-fire Model (LIF)

LIF $\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI_0$ If firing: $u \rightarrow u_r$

$assume: \; u_r = u_{rest}$

Exercise!
Calculate the interspike interval *T* for constant input *I*.
Firing rate is *f=1/T.*
Write *f* as a function of *I*.
What is the frequency-current curve *f=g(I)* of the LIF?

repetitive



Start Exerc. at 10:53.
Next lecture at 11:15

Week 1 – part 4: Generalized Integrate-and-Fire Model

Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

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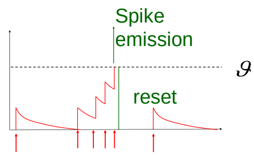
EPFL, Lausanne, Switzerland

Neuronal Dynamics – 1.4. Generalized Integrate-and Fire



Integrate-and-fire model

LIF: linear + threshold

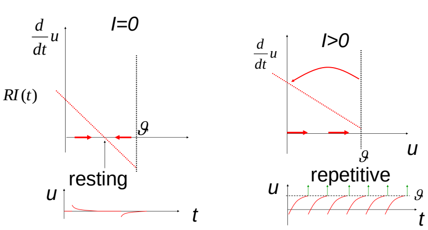
Neuronal Dynamics – 1.4. Leaky Integrate-and Fire revisited

LIF

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

If firing:

$u \rightarrow u_r$



Neuronal Dynamics – 1.4. Nonlinear Integrate-and Fire

LIF

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

NLIF

$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

If firing:

$u \rightarrow u_{reset}$

Neuronal Dynamics – 1.4. Nonlinear Integrate-and Fire

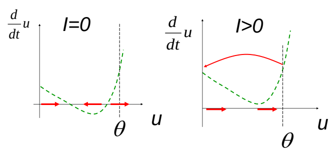
Nonlinear Integrate-and-Fire

NLIF

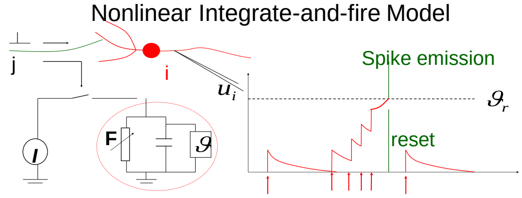
$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

firing: $u(t) = \theta \Rightarrow$

$u \rightarrow u_r$



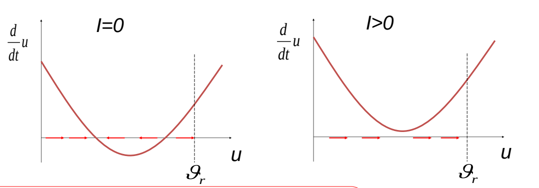
Nonlinear Integrate-and-fire Model



$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$ NONlinear

$u(t) = \vartheta_r \Rightarrow$ Fire+reset threshold

Nonlinear Integrate-and-fire Model



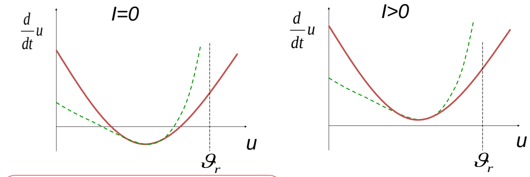
$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$ NONlinear

$u(t) = \vartheta_r \Rightarrow$ Fire+reset threshold

Quadratic I&F:

$F(u) = c_2 (u - c_1)^2 + c_0$

Nonlinear Integrate-and-fire Model



$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

$u(t) = \vartheta_r \Rightarrow$ Fire+reset

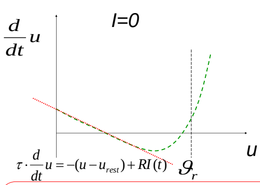
Quadratic I&F:

$F(u) = c_2 (u - c_1)^2 + c_0$

exponential I&F:

$F(u) = -(u - u_{rest}) + c_0 \exp(u - \vartheta)$

Nonlinear Integrate-and-fire Model



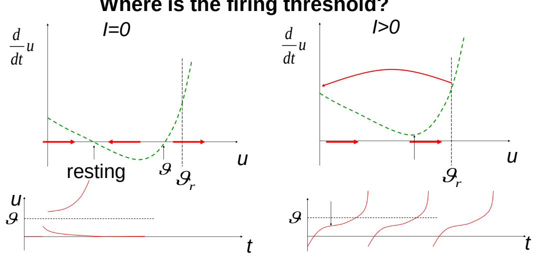
$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$ NONlinear

$u(t) = \vartheta_r \Rightarrow$ Fire+reset threshold

exponential I&F:

$F(u) = -(u - u_{rest})$
$+ c_0 \exp(u - \vartheta)$

Nonlinear Integrate-and-fire Model
Where is the firing threshold?



$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

Week 1 – part 5: How good are Integrate-and-Fire Model?

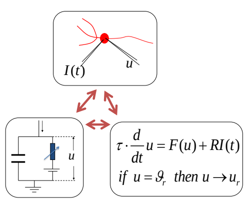
Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

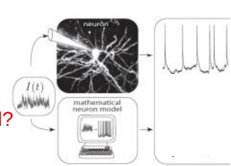
Neuronal Dynamics – 1.5.How good are integrate-and-fire models?

Can we compare neuron models with experimental data?



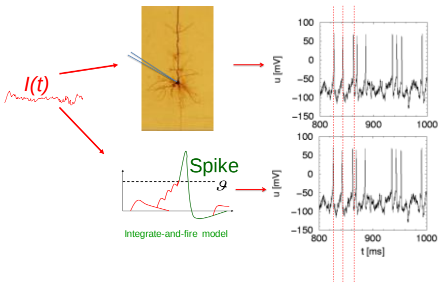
Neuronal Dynamics – 1.5.How good are integrate-and-fire models?

What is a good neuron model?

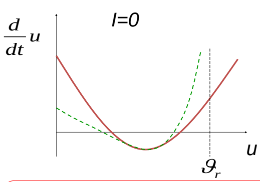


Can we compare neuron models with experimental data?

Neuronal Dynamics – 1.5.How good are integrate-and-fire models?



Nonlinear Integrate-and-fire Model



Can we measure the function *F(u)*?

$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

$$u(t) = \vartheta_r \Rightarrow \text{ Fire+reset}$$

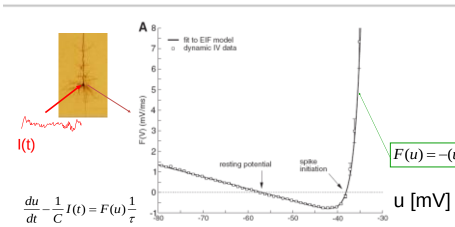
Quadratic I&F:

$$F(u) = c_2 (u - c_1)^2 + c_0$$

exponential I&F:

$$F(u) = -(u - u_{rest}) + c_0 \exp(u - \vartheta)$$

Neuronal Dynamics – 1.5.How good are integrate-and-fire models?



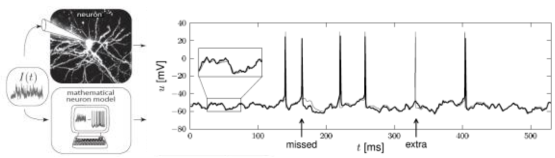
$$\frac{du}{dt} - \frac{1}{C} I(t) = F(u) \frac{1}{\tau}$$

$$F(u) = -(u - u_{rest}) + \Delta \exp(\tfrac{u-\vartheta}{\Delta})$$

u [mV]

Badel et al., J. Neurophysiology 2008

Neuronal Dynamics – 1.5.How good are integrate-and-fire models?



Neuronal Dynamics – 1.5.How good are integrate-and-fire models?

Computer ecercises: Python

Nonlinear integrate-and-fire models are good

Mathematical description → prediction

Need to add
- adaptation
- noise
- dendrites/synapses

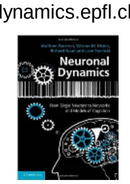
Biological Modeling of Neural Networks

TA's:
Marco Lehmann
Vasiliki Liakoni
Florian Colombo

http://neuronaldynamics.epfl.ch/

Textbook:

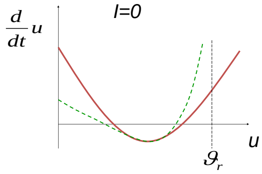
Lecture today:
-Chapter 1
-Chapter 5



Exercises today:
-Install PYTHON for Computer Exercises
-Exercise 3, on sheet

Videos (for today: 'week 1'):
http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html

Biological Modeling of Neural Networks – week1/Exercise 3



$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

Homework!

First week – References and Suggested Reading

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski, *Neuronal Dynamics: from single neurons to networks and models of cognition.* Chapter 1*: Introduction*. Cambridge Univ. Press, 2014

Selected references to linear and nonlinear integrate-and-fire models

- Lapicque, L. (1907). *Recherches quantitatives sur l'excitation electrique des nerfs traitee comme une polarization.* J. Physiol. Pathol. Gen., 9:620-635.

-Stein, R. B. (1965). A theoretical analysis of neuronal variability. Biophys. J., 5:173-194.

-Ermentrout, G. B. (1996). *Type I membranes, phase resetting curves, and synchrony*. Neural Computation, 8(5):979-1001.

-Fourcaud-Trocme, N., Hansel, D., van Vreeswijk, C., and Brunel, N. (2003). *How spike generation mechanisms determine the neuronal response to fluctuating input.* J. Neuroscience, 23:11628-11640.

-Badel, L., Lefort, S., Berger, T., Petersen, C., Gerstner, W., and Richardson, M. (2008). Biological Cybernetics, 99(4-5):361-370.

- Latham, P. E., Richmond, B., Nelson, P., and Nirenberg, S. (2000). *Intrinsic dynamics in neuronal networks. I. Theory.* J. Neurophysiology, 83:808-827.

First week

THE END (of main lecture)

MATH DETOUR SLIDES
(for online VIDEO)

Week 1 – part 2: Detour/Linear differential equation

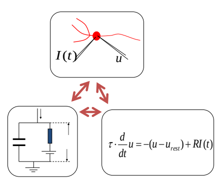
Neuronal Dynamics: Computational Neuroscience of Single Neurons

Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Neuronal Dynamics – 1.2Detour – Linear Differential Eq.

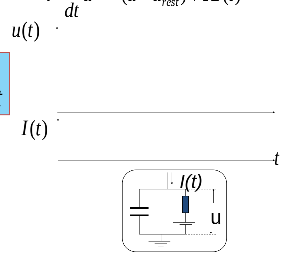
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



Neuronal Dynamics – 1.2Detour – Linear Differential Eq.

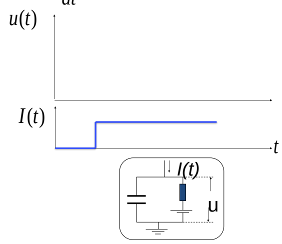
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Math development:
Response to step current



Neuronal Dynamics – 1.2Detour – Step current input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



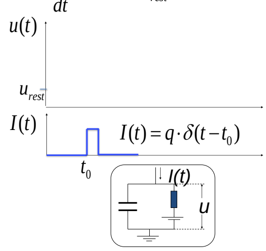
Neuronal Dynamics – 1.2Detour – Short pulse input

$$u(t) = u_{rest} + RI_0 \left[1 - e^{-(t-t_0)/\tau}\right]$$

short pulse: $(t - t_0) << \tau$

Math development: Response to short current pulse

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

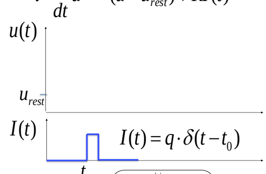


Neuronal Dynamics – 1.2Detour – Short pulse input

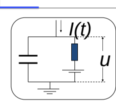
$$u(t) = u_{rest} + RI_0 \left[1 - e^{-(t-t_0)/\tau}\right]$$

short pulse: $(t - t_0) << \tau$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



$$u(t) = u_{rest} + \frac{q}{C} e^{-(t-t_0)/\tau}$$



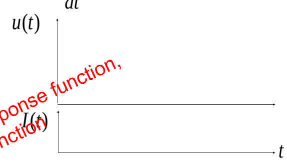
Neuronal Dynamics – 1.2Detour – arbitrary input

Single pulse

$$u(t) = u_{rest} + \frac{q}{C} e^{-(t-t_0)/\tau}$$

Multiple pulses:

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



Impulse response function,
Green's function

$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Neuronal Dynamics – 1.2Detour – Greens function

Single pulse

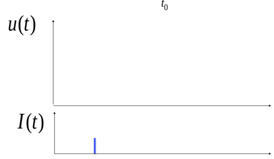
$$\Delta u(t) = q \frac{1}{C} e^{-(t-t_0)/\tau}$$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Multiple pulses:

$$u(t) = u_{rest} + [u(t_0) - u_{rest}] + \int_{t_0}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Impulse response function,
Green's function



$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Neuronal Dynamics – 1.2Detour – arbitrary input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

*If you don't feel at ease yet, spend **10 minutes** on these mathematical exercise*
And quiz 2 in week 1.

Arbitrary input

$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{c} e^{-(t-t')/\tau} I(t')dt'$$

Single pulse

$$\Delta u(t) = \frac{q}{c} e^{-(t-t_0)/\tau}$$

you need to know the solutions of linear differential equations!