

Neural Networks and Biological Modeling

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QUESTION SET 12

Exercise 1: Synaptic Plasticity: the BCM rule

A neuron receives 20 inputs that are organized in two groups of 10 inputs. The two groups fire in alternation: when group 1 is active, group 2 is silent; when group 2 is active, group 1 is silent. The input switches between the two groups every second (see figure 1(a)). All initial weights are $w_{ij} = 1$, but weights can change according to the BCM rule (eq. 1 with $\vartheta = 20Hz$). The firing rate of the postsynaptic neuron ν_i^{post} is given by eq. 2. The shape of Φ is shown in figure 1(b).

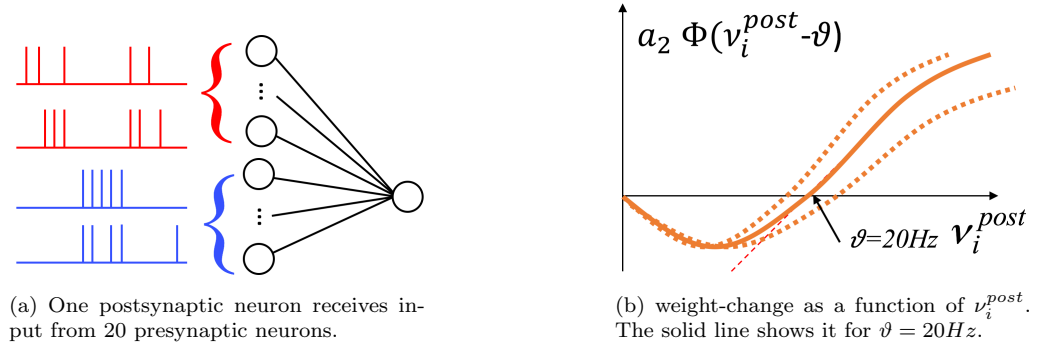


Figure 1: Network and weight-change

$$\frac{d}{dt}w_{ij} = a_2^{corr}\Phi(\nu_i^{post} - \vartheta)\nu_j^{pre} \quad (1)$$

$$\nu_i^{post} = g(I_i) = \sum_j^N w_{ij}\nu_j^{pre} \quad (2)$$

- Assume that group 1 fires at 3Hz, then group 2 at 1 Hz, then again group 1 etc. How do the weights of both groups evolve?
- Assume that group 1 fires at 3Hz, then group 2 at 2.5 Hz, then again group 1 etc. How do the weights of both groups evolve?
- The inputs are as in part b, but now you are free to choose theta. Suppose that the synapse can measure the time-average postsynaptic rate $\bar{\nu}$. What would you propose as model of ϑ so that the weight-pattern becomes non-trivial?

Exercise 2: Hopfield networks and Hebbian learning

Here we explore how we may obtain a Hopfield network with M stored prototypes through Hebbian plasticity instead of fixing the weights explicitly.

This is achieved by presenting the patterns to a fully connected network and apply a plasticity rule:

$$\frac{d}{dt}w_{ij} = a_2^{\text{corr}}(\nu_i^{\text{post}}(t) - \vartheta)(\nu_j^{\text{pre}}(t) - \vartheta), \quad (3)$$

where a_2 and ϑ are parameters of the plasticity model; $\nu_i^{\text{post}}(t)$ and $\nu_j^{\text{pre}}(t)$ are the activities of neurons i and j at time t .

We present a pattern μ to the network in the following way: Each pixel j of pattern μ , $p_j^\mu \in \{-1, +1\}$, stimulates exactly one neuron j in the network. That neuron's firing rate ν_j depends on the pattern: $\nu_j = 0 \text{ Hz}$ if $p_j^\mu = -1$; $\nu_j = 20 \text{ Hz}$ if $p_j^\mu = +1$.

During that presentation, the network learns the pattern by adjusting its weights according to the plasticity rule given in equation 3. We assume initial weights $w_{ij} = 0$. For this exercise, we use a constant threshold $\vartheta = 10 \text{ Hz}$.

2.1 We now have the network learn M patterns. Each one is presented once for 0.5 seconds. Show that, for an appropriate choice of a_2 , the final weights are given by

$$w_{ij} = \sum_{\mu} p_i^\mu p_j^\mu. \quad (4)$$

Hint: Begin by calculating the weight change induced by presenting a single pattern for 0.5s.

2.2 How does this learning rule map to the general formulation

$$\frac{d}{dt}w_{ij} = a_0 + a_1^{\text{pre}}\nu_j^{\text{pre}} + a_1^{\text{post}}\nu_i^{\text{post}} + a_2^{\text{corr}}\nu_j^{\text{pre}}\nu_i^{\text{post}} + \dots? \quad (5)$$

2.3 Would you describe this learning procedure as reinforcement or unsupervised learning?

Exercise 3: Spike-time dependent plasticity by local variables

The goal of this exercise is to show that it is possible to account for the asymmetry in the STDP window using a simple microscopic model of synaptic plasticity.

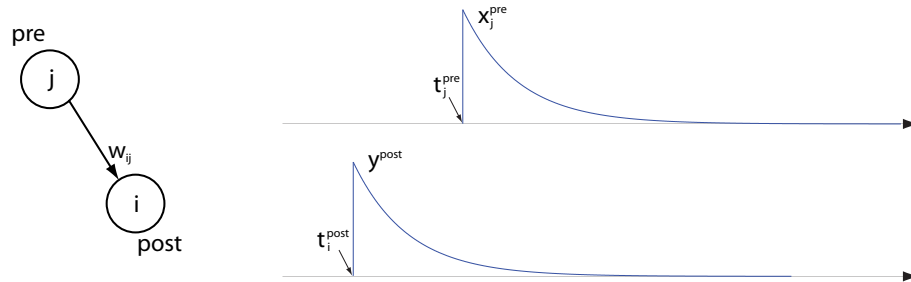


Figure 2: Memory traces of pre- and post-synaptic spike trains.

Suppose that the change in synaptic weight is controlled by the local concentration of two molecules x^{pre} and y^{post} . The substance x^{pre} acts as a memory trace of presynaptic spikes in the sense that each presynaptic spike triggers an increase in the concentration of x^{pre} :

$$\tau_+ \frac{d}{dt} x_j^{\text{pre}} = -x_j^{\text{pre}} + \delta(t - t_j^{\text{pre}}). \quad (6)$$

Similarly, y^{post} is the trace left by the postsynaptic spike train,

$$\tau_- \frac{d}{dt} y^{\text{post}} = -y^{\text{post}} + \delta(t - t_i^{\text{post}}). \quad (7)$$

Calculate the form of the learning window $\Delta w = f(\Delta t)$ – where $\Delta t = t_j^{\text{pre}} - t_i^{\text{post}}$ assuming that the synaptic weights are updated according to the rule

$$\frac{d}{dt} w_{ij} = a_+ x_j^{\text{pre}} \delta(t - t_i^{\text{post}}) - a_- y^{\text{post}} \delta(t - t_j^{\text{pre}}). \quad (8)$$

The constants a_+ and a_- are both positive.

Hint: Calculate the weight change for a pair of pre/post spikes. Consider the two cases $\Delta t > 0$ and $\Delta t < 0$.

Exercise 4: From spike-time dependent plasticity to rate models

Suppose that we have pair-based plasticity with an STDP window $W(t_i^f - t_j^{f'})$. The window decays exponentially and the slowest time scale of the decay is τ_- . Every presynaptic spike interacts with every postsynaptic spike as long as the timing is close enough to fall within the above time window.

4.1 Assume presynaptic spike trains generated by a homogeneous Poisson process with rate ν_j . Assume postsynaptic spike trains generated by another, independent, Poisson process with constant rate ν_i . How much is the expected weight change Δw_{ij} in a time T , if $T \gg \tau_-$?

Hint: Write the weight change as an integral over spike trains. Link the expectation over spike trains to the firing rate.

4.2 Assume presynaptic spike trains generated by a homogeneous Poisson process with rate ν_j . Assume postsynaptic spike trains generated by another Poisson process with rate:

$$\nu_i(t) = \sum_k w_{ik} \sum_f \epsilon(t - t_k^f) = \sum_k w_{ik} \int_0^\infty \epsilon(s) S_k(t - s) ds.$$

How much is the expected weight change $\Delta w_{ij}/T$ in a time T , if $T \gg \tau_-$?

Hints:

- (i) Exploit the autocorrelation of the Poisson process.
- (ii) The output spikes are generated with rate ν_i , but this rate depends on the input.
- (ii) Treat the input from synapse j explicitly. Note that the output spike train depends on the input spikes: If a spike has arrived at time t_j^f the postsynaptic rate is higher than 'on average'.