

# Neural Networks and Biological Modeling

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## QUESTION SET 10

### Exercise 1: Poisson neuron

We consider a neuron that fires stochastically. Its firing rate is described by a Poisson process of rate  $\rho$ . In other words, in every small time interval  $\Delta t$ , the probability that the neuron fires is given by  $\rho\Delta t$ .

**1.1** What is the probability that the neuron does *not* fire during a time of arbitrarily large length  $t$ ?  
Hint: Consider first the probability of not firing during a short interval  $\Delta t$ .

**1.2** Suppose that the neuron has fired at time  $t_0$ . Calculate the distribution of intervals  $P(s)$ , i.e., the probability density that the neuron fires its next spike after a time  $s$ .

**1.3** Suppose that the neuron is driven by some input. For  $t < t_0$ , the input is weak, so that its firing rate is  $\rho_0 = 2\text{Hz}$ . For  $t_0 < t < t_1 = t_0 + 100\text{ms}$ , the input is strong and the neuron fires at  $\rho_1 = 20\text{Hz}$ .

(i) Calculate the interval distributions for weak and strong stimuli.

(ii) What is the probability of having a “burst” consisting of two intervals of less than 20 ms each if the input is weak/strong?

(iii) Suppose that the onset time  $t_0$  of the strong input is unknown; can an observer, who is looking at the neuron’s output, decide whether the input is weak or strong?

**1.4** Suppose that a Poisson neuron with a constant rate of 20 Hz emits in a trial of 5 second duration 100 spikes at times  $t^{(1)}, t^{(2)}, \dots, t^{(100)}$ . The experiment is repeated such that a second spike train with a duration of 5 seconds is observed.

What is the percentage of spikes that coincide between the first and second trial with a precision of  $\pm 2\text{ms}$ ? More generally, what percentage of spikes coincide between two trials of a Poisson neuron with arbitrary rate  $\rho_0$  under the assumption that trials are sufficiently long?

### Exercise 2: Stochastic spike arrival

Consider a neuron with a passive membrane,

$$\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + RI(t). \quad (1)$$

**2.1** The neuron receives synaptic input at a rate  $\nu$  such that

$$I(t) = q \sum_f \delta(t - t^f). \quad (2)$$

Calculate the average value of membrane potential as a function of the presynaptic rate  $\nu$ , assuming

stochastic (Poisson) spike arrival.

**Hint:** Integrate Eq. 1 keeping explicitly the  $\delta$ -function. Under the assumption of stochastic spike arrival we have  $\left\langle \sum_f \delta(t - t^f) \right\rangle = \nu$ .

**2.2** Calculate the average value of membrane potential as a function of the presynaptic rate  $\nu$  if the current coming from the presynaptic activity is:

$$I(t) = \sum_f \alpha(t - t^f). \quad (3)$$

**Hint:** As before, integrate Eq. 1 keeping the  $\delta$ -function explicit.

### Exercise 3: Homework

**3.1** The poisson neuron has a probability to fire in a very small interval  $\Delta t$  equal to  $\nu \Delta t$ . What will be the probability to observe exactly  $k$  spikes in the time interval  $T = N \Delta t$  ( $P_k(T)$ )? Start with the probability to observe  $k$  events in  $N$  slots (the binomial distribution):

$$P(k, N) = \frac{N!}{k!(N-k)!} p_1^{N-k} p_2^k$$

where  $p_1$  and  $p_2$  are the probability to spike or to remain silent in one  $\Delta t$  slot. Take the continuous time limit with Stirling's approximation ( $N! \approx (N/e)^N$  for large  $N$ ) to obtain the Poisson distribution:

$$P_N = \frac{(\nu T)^N}{N!} \exp(-\nu T)$$

Verify that this distribution predicts an average number of spikes  $\langle N \rangle = \nu T$ .