Chapter 1

Introduction

1.1 Firing rate models

-phenomenological, not derived from spiking neuron [**?**]

1.2 Renewal spiking neuron model

Renewal processes keep memory of the last event, las firing time $\hat{t}$. For those processes the spikes are generated according to a stochastic intensity called the hazard rate

$$\rho(t|\hat{t}) = \rho(\tau) \tag{1.1}$$

which depends on the age of the neuron τ, i.e the time since the last spike $\tau = t - \hat{t}$. $\rho(\tau)$ define the probability to spike between $t + \Delta t$ knowing that there were no spike bewteen t and $\hat{t}$

The renewal theory allows to define the probability of the next event given the age of the system, to calculate the interspike-interval (ISI) distribution, i.e the probability to spike at age τ and no spike before.

$$P(\tau) = P(\hat{t} + \tau|\hat{t}) \tag{1.2}$$

The ISI distribution satisfy

$$\int_0^\infty P(\tau)\,\mathrm{d}\tau = 1 \tag{1.3}$$

and allows to compute the moment:

$$< \tau^n >= \int_0^\infty \tau^n P(\tau)\,\mathrm{d}\tau \tag{1.4}$$

Integration of $P(\tau)$ over time yields a probability as the interval distribution $P(\tau)$ is a probability density. The probability that neuron which has fired a spike at $\hat{t}$ and fires the next spike at between $\hat{t}$ and t is given by $\int_0^\tau P(s)\,\mathrm{d}s$.

The interspike-interval (ISI) distribution can be linked to the survivor function:

$$S(\tau) = 1 - \int_0^\tau P(s)\,\mathrm{d}s \tag{1.5}$$

The survivor function $S(\tau)$ define the probability that a neuron reach the age τ, so that a neuron "survive" without firing between $\hat{t}$ and t.

The hazard rate $\rho(\tau)$ corresponds to the rate of decay of the survivor function:

$$\rho(\tau) = -\frac{\frac{\mathrm{d}}{\mathrm{d}\tau}S(\tau)}{S(\tau)} \tag{1.6}$$

Integrating eq.1.6 yields to the survivor function:

$$S(\tau) = \exp\big[-\int_0^\tau \rho(s)\,\mathrm{d}s\big] \tag{1.7}$$

Taking the derivative of eq.(1.5), we can expressed the interspike-interval (ISI) distribution:

$$P(\tau) = -\frac{\mathrm{d}}{\mathrm{d}\tau}S(\tau) = \rho(\tau)S(\tau) \tag{1.8}$$

Eq.(1.9) describe that probability to spike at age τ and no spike before $P(\tau)$, is given by the product of the probability to survive until age τ times the momentary hazard $\rho(\tau)$. Inserting eq.(1.7) ineq.1.9 The interval distribution can be explicitly express in terms of the hazard, and is by itself normalized:

$$P(\tau) = -\frac{\mathrm{d}}{\mathrm{d}\tau}S(\tau) = \rho(\tau)\exp\big[-\int_0^\tau \rho(s)\,\mathrm{d}s\big] \tag{1.9}$$

1.2.1 Examples

Interval distribution and hazard functions have been measured in many experiments. Here are some examples widely used.

Simple model with recovery function

In the previous section we were implicitly considering stationary renewal system using the notation $\rho(t|\hat{t})$. In this section we will used the notation $\rho(\tau, h)$,with h a time dependent parameter, to show explicitly that$\rho(tau)$ can change in time.

$$\tau_m \dot{h} = -h + \mu(t) \tag{1.10}$$

Were $mu(t)$ is a time dependent external input.

The hazard rate, can be expressed using a recovery function $g(\tau)$

$$\rho(\tau, h) = \Phi(h)g(\tau) \tag{1.11}$$

With

(a)

(b)

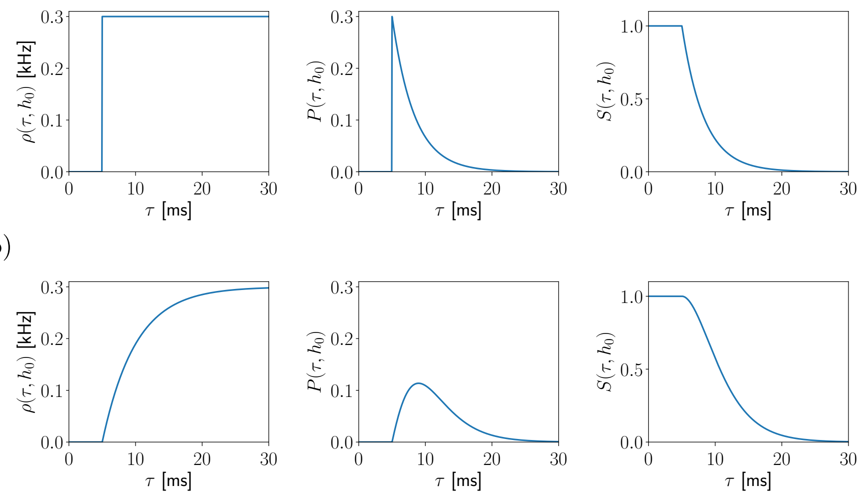

Figure 1.1: Hazard rate $\rho(\tau, h)$ (left), interval distribution $P(\tau, h)$ (middle) and survivor functio $S(\rho(\tau, h)$ (right) for different recovery function $g(\tau)$. (a) Recovery function corresponds to a Poisson neuron with absolute refractoriness Δ, with $Delta = 5$ ms, $h = h_0$, $\nu_{max} = 0.6$ kHz. (b) Recovery function defined by $g(\tau) = [1 - \exp(-\lambda(\tau - \Delta))]\,\theta(\tau - \Delta)$ Poisson neuron with absolute refractoriness Δ, with $\Delta = 5$ ms, $h = h_0$, $\nu_{max} = 0.6$ kHz.

$$\Phi(h) = \frac{\nu_{max}}{1 + \exp[-\beta(h - h_0)]} \tag{1.12}$$

The hazard rate, the survival probability, and the interval distribution are shown in Fig.1.2 for two Examples of recovery function g. Fig.1.2(a) corresponds to a poisson process with absolute refractoriness:

$$g(\tau) = \theta(\tau - \Delta) \tag{1.13}$$

The recovery function for Fig.1.2(b) is given by

$$g(\tau) = [1 - \exp(-\lambda(\tau - \Delta))]\,\theta(\tau - \Delta) \tag{1.14}$$

The main difference is that for the poisson neuron with absolute refractoriness the recovery function eq.(1.13) make a jump, whereas in eq.(1.14) the transition is smooth.

Gamma process

$\beta = 5$kHz $\gamma = 100$

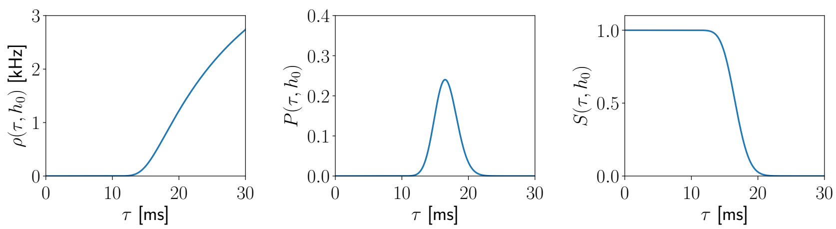

Figure 1.2: Hazard rate $\rho(\tau)$ (left), interval distribution $P(\tau)$ (middle) and survivor functio $S(\rho(\tau)$ (right) for different recovery function $g(\tau)$.

Perfect integrate-and-fire model driven by white noise

1.3 Populations of neurons and refractory density equations

1.3.1 Network model

- population activity definition
  - definition of network : all-to-all coupling
  -external input $\mu(t)$

$$\mu(t) = V_{rest} + RI_{ext}(t) + RI_{syn}(t) \tag{1.15}$$

  - and/or synaptic input

$$RI_{syn}(t) = \tau_m JA(t) \tag{1.16}$$

1.4 Spectral decomposition method

-main idea - everything known about the method that applies to a general operator L

-assume that neuron can be described by one state variable, e.g. membrane potential or age. p(v) or q(tau). For concreteness lets consider q(tau)

1.4.1 Refractory density equation

[**?**, **?**] -bounary conditions

1.4.2 Spectral decomposition for the refractory density equation

Chapter 2

Theory

-adjoint operator

2.0.1 Full Mattia 2002 system

2.1 Low-dimensional dynamics

2.1.1 Truncation Full Mattia 2002 system

2.1.2 Schaffer like

2.1.3 Mattias equation

Chapter 3

Spectral properties of specific models

3.1 Poisson neuron with absolute refractoriness

3.2 Gamma process

3.3 PIF neuron

3.4 General renewal neuron

Chapter 4

Population response to time-dependent input

–for uncoupled neurons –susceptibility

Chapter 5

Population dynamics of coupled neurons

- one population
  - two populations (E-I net)