

# Authority Propagation Models: PoP vs PoC and the Confused Deputy Problem

Nicola Gallo

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## 1 Model

Let  $P$  be a set of principals (e.g.,  $U$ ,  $PDF$ ,  $Storage$ ). Let  $O$  be a set of operations (e.g., convert, delete), and  $R$  a set of resources.

A privilege is a pair  $(o, r) \in O \times R$ . Each principal  $p$  has a privilege set:

$$Priv(p) \subseteq O \times R.$$

A request is a message  $req$  sent between principals and may contain a payload.

## 2 Confused Deputy

**Definition 1** (Confused Deputy). *A confused deputy occurs when there exist principals  $U$  (user) and  $D$  (deputy) such that:*

1.  $(o, r) \notin Priv(U)$ ,
2.  $(o, r) \in Priv(D)$ ,
3.  $U$  sends a request  $req$  to  $D$ ,
4. as a consequence of  $req$ ,  $D$  executes  $(o, r)$ .

This definition does not depend on implementation details, only on the mismatch of authority and causality.

## 3 Proof-of-Possession (PoP)

A token  $t$  grants a set of privileges:

$$Auth(t) \subseteq O \times R.$$

**PoP Semantics:** Possession implies usability: if a principal holds  $t$ , it may exercise all  $(o, r) \in \text{Auth}(t)$ .

PoP systems do not constrain authority by causality or provenance.

### 3.1 Vulnerability Condition

Assume:

$$(\text{delete}, r) \notin \text{Priv}(U) \quad (\text{H1})$$

$$(\text{delete}, r) \in \text{Priv}(\text{PDF}) \quad (\text{H2})$$

Further assume:

The payload of a request may influence control flow in  $\text{PDF}$ , including code paths that call internal privileges such as delete.

**Theorem 1** (PoP admits confused deputy). *Under assumptions (H1)–(H2), there exists a request  $\text{req}$  such that  $\text{PDF}$  executes  $(\text{delete}, r)$  as a result of processing  $\text{req}$ .*

*Proof.* Since  $(\text{delete}, r) \in \text{Priv}(\text{PDF})$  (H2), some code path in  $\text{PDF}$  invokes  $\text{Storage.delete}(r)$ . Because processing is influenced by payload, there exists an adversarial payload that triggers that path. Since  $U$  lacks  $(\text{delete}, r)$  (H1), and  $\text{PDF}$  acts on behalf of  $U$ , conditions for a confused deputy are satisfied.  $\square$

This applies to OAuth tokens and sealed capabilities: sealing protects transport, not authority semantics.

## 4 Proof-of-Continuity (PoC / PIC)

Execution is modeled as a causal chain of hops:

$$p_0 \rightarrow p_1 \rightarrow \dots \rightarrow p_n,$$

with  $p_0 = U$ .

Each hop transfers a privilege subset:

$$\text{ops}_i \subseteq O \times R$$

and must satisfy:

$$\text{ops}_{i+1} \subseteq \text{ops}_i. \quad (\text{C1})$$

Let  $\pi$  be a verifiable sequence:

$$\pi = \langle (p_0, \text{ops}_0), (p_1, \text{ops}_1), \dots, (p_n, \text{ops}_n) \rangle.$$

## 4.1 Authorization Rule

The final service authorizes  $(o, r)$  if and only if:

$$(o, r) \in \bigcap_{i=0}^n ops_i$$

and  $\pi$  is valid.

Since the chain is monotonic decreasing,

$$\bigcap_{i=0}^n ops_i = ops_n.$$

Thus no hop may acquire new authority not present at the origin.

## 5 Safety Property

**Definition 2** (Origin-bounded authority). *A model enforces origin-bounded authority if every executable privilege at hop  $n$  is a privilege originally granted by hop 0.*

**Theorem 2** (PoC prevents confused deputy). *If  $ops_0 = \{(convert, r)\}$  and (C1) holds for every hop, then  $(delete, r)$  can never be authorized at hop  $n$ .*

*Proof.* Authorization requires:

$$(delete, r) \in \bigcap_{i=0}^n ops_i.$$

But  $(delete, r) \notin ops_0$  and each  $ops_{i+1} \subseteq ops_i$ . Therefore  $(delete, r) \notin ops_i$  for all  $i$ , hence not in the intersection. The request is rejected.  $\square$

## 6 Discussion

PoP systems conflate authority and possession, enabling confused deputy attacks whenever privileged internal code paths exist. PIC/PoC systems propagate only non-expansive subsets of authority, ensuring that downstream services cannot perform operations not explicitly authorized at the origin.

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