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Optimal vaccination choice, vaccination games, and rational exemption: An appraisal

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ABSTRACT

A threat for vaccination policies might be the onset of "rational" exemption, i.e. the family's decision not to vaccinate children after a seemingly rational comparison between the perceived risk of infection and the perceived risk of vaccine side effects. We study the implications of rational exemption by models of vaccination choice. By a simple model of individual choice we first prove the "elimination impossible" result in presence of informed families, i.e. aware of herd immunity, and suggest that limited information might explain patterns of universal vaccination. Next, we investigate vaccination choice in a game-theoretic framework for communities stratified into two groups, "pro" and "anti" vaccinators, having widely different perceived costs of infection and of vaccine side effects. We show that under informed families neither a Nash nor a Stackelberg behaviour (characterized, respectively, by players acting simultaneously and by an asymmetric situation with a "leader" and a "follower") allow elimination, unless "pro-vaccinators" assign no costs to vaccine side effects. Elimination turns out to be possible when cooperation is encouraged by a social planner, provided, however, he incorporates in the "social loss function" the preferences of anti-vaccinators only. This allows an interpretation of the current Italian vaccination policy.

1. Introduction

Despite the fundamental role played in history by vaccines, second only to potable water in the reduction of mortality and morbidity, various forms of exemption to vaccination (conscientious, religious, philosophical) have always been documented [32]. In more recent times episodes of decline in vaccine uptake have been associated to vaccine scares, e.g. the cases of the whole-cell pertussis vaccine [19], of thimerosal and HBV vaccine [24], up to the MMR scare [17,35,39]. In such cases a significant role of anti-vaccinators' movements in raising and spreading concerns about vaccine safety was also documented [19,28].

Today, developed countries are increasingly facing the challenge of rational exemption (RE). By RE we mean, in regimes of voluntary vaccination, the parents' decision not to immunize children after a seemingly rational comparison between the perceived utility of vaccination, i.e. protection from the risk of infection – perceived as very low as a consequence of the high herd immunity due to decades of successful vaccination policies – with its disutility, i.e. the risk of vaccine-associated side effects. RE is often considered as a form of "free riding" [36]. Such a behaviour, resulting from the optimization performed by rational agents, might well turn out to be "myopically" rational, since it considers only the current perceived risk of disease, and not the risk of its future resurgence due to declining coverage. Some evidence of rational exemption behaviour is documented by surveys of vaccination lifestyles [2,24,39,17].

Theoretical papers based on traditional epidemiological models have investigated the implications of RE for the dynamics and control of vaccine preventable diseases [20,9,11]. This literature, which has pointed out the critical interplay between information and vaccinating as well as other disease-related behaviour [12], has shown that RE might make elimination a "mission impossible" unless the fraction of those who practice RE is small, i.e. below the susceptibility threshold ensuring endemic persistence. Moreover, it has shown that if the information set used by individuals includes past information, then the disease dynamics might yield epidemic waves with very long period. The negative implications for rubella

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control of a free vaccines market have also been investigated by more realistic epidemiological models [38].

These results call for explanations of vaccination choices [21–22,26,34,40] by behavioural variables, e.g. economic and psychological, typically neglected in epidemiological models [14]. Besides some old pioneering work [7,16], a series of recent intriguing works by Bauch and co-workers have attempted to explain RE in its most appropriate framework, i.e. game theory [4,5,8,29]. These papers have provided the first game-theoretic proof of the "elimination impossible" result, and various dynamic implications of RE. These implications suggest potential difficulties, both at the national and international level, for global eradication plans [3,13]. In [27] it was suggested that elimination might become possible when more realistic contact network structures including local information are considered. Various applications to influenza control by vaccination, e.g. the role of adaptive behaviour [37], and the interplay between perceived and real costs in vaccination choices [18], have also been considered. A two-population epidemic game related to that studied here was analyzed in [30].

In this paper we discuss RE in developed countries by simple static models of voluntary vaccination behaviour. In this case the actual coverage p is the sum of the vaccination *choices* of the individual families. Families make their choices "rationally", by minimizing a loss function taking into account both the perceived risk of infection and the perceived risk of vaccine-associated side effects (VSE).

We first consider the simple optimization problem where homogenous "representative" families make their choice without taking into account other families' choices. We distinguish two cases, i.e. *informed families* who know the principle of herd immunity, and *not-fully informed ones*, who believe that 100% coverage is necessary to avoid any risk of infection. We prove that the "elimination impossible" result arises as soon as informed families perceive any, however small, cost of VSE. The case of not-fully informed families allows us to conclude that patterns of universal vaccination are more likely to arise when lack of knowledge on herd immunity concurs with low perceived costs of VSE.

Next, we model vaccination choices as a game where families interact "strategically" by incorporating the other family choices in their loss function [25,33]. We assume, as in the theoretical paper [8], the coexistence of two groups of social actors (labeled as *pro-* and *anti-vaccinators*) having widely different perceived costs of infection and of VSE. This is broadly consistent with the current situation of developed countries.

Unlike [4] where only the Nash equilibrium in a simultaneous game was considered, we discuss the implications of all the three classical "types" of agents' interaction, i.e. also the cases of non-simultaneous (Stackelberg) and "social planner" games [25]. In the Nash case agents play simultaneously, and take each other's action as given. In the Stackelberg (or sequential) case, a few active agents "lead" the game. This seems to be the case of anti-vaccination groups which appear to be very active in the acquisition of information, and optimize their action by taking into account pro-vaccinators actions [2]. In the "social planner" case a "Deus ex machina" ("social planner") seeks an agreed solution between the two groups by minimizing a "social" loss function given by an "average" of the two groups' loss functions. We prove that under informed families, the "elimination impossible" result continues to hold in both Nash and Stackelberg cases. The only case where elimination is feasible occurs when pro-vaccinators do not perceive any cost from vaccine side effects, and moreover their group is large enough to allow elimination even if the other group does not vaccinate at all. In particular, the Stackelberg case with anti-vaccinators leadership always leads to a lower coverage compared to Nash behaviour. Finally, we show that even in the "social planner" case elimination is possible only when, provided the "pro-vaccination" group is large enough, the social planner assigns to anti-vaccinators preferences the 100% of the weight in the social loss function. This allows a nice interpretation of the current Italian situation.

The paper is organised as follows. Section 2 deals with the basic model of vaccination choice. Section 3 discusses models of strategic interaction. Concluding remarks follow.

2. A simple model of optimal family behaviour without strategic interaction

We consider a common Susceptible-Infective-Removed (SIR) vaccine preventable infection, such as measles or mumps, in a stationary homogeneously mixing population [1]. Infection can give rise to serious but non-fatal sequelae. The vaccine is *perfect*, i.e. providing 100% effective lifelong immunity. Vaccination is voluntary, and it is administered at birth. No "recuperation" strategy, such as vaccination at a later date during epochs of higher perceived risk, is allowed [9]. The infection is characterised by a basic reproduction number $R_0 > 1$. This ensures endemic circulation as far as the vaccine uptake is below the *critical threshold* $p_c = 1 - 1/R_0$ allowing disease elimination [1].

In this section we assume that families behave non-strategically (i.e. we rule out any game-theoretic consideration). Families are fully homogeneous in their preferences toward vaccination, i.e. we consider a single *representative* family [25]. This implies identical individual decisions, which in turn straightforwardly imply the equality between the individual propensity to vaccinate p and the collective coverage. Real world families have only two options: to vaccinate or not their children at birth. In our formulation each family determines its optimal propensity to vaccinate p, i.e. its *vaccine demand*, as the quantity that minimises its loss function L. The propensity to vaccinate represents the agent's probability (i.e. $0 \le p \le 1$) to take the decision to vaccinate. The loss function summarises the cost families will suffer as a consequence of their decisions. As loss function we take a quadratic additive function, which is the commonest type of loss function in Economics [25], increasing in the perceived risk of infection $\rho_I(\rho)$, and in the perceived risk of vaccine-associated side effect $\rho_V(p)$:

$$L = \rho_I^2 + \beta\rho_V^2(p). \tag{1}$$

In (1) $\beta > 0$ is the *relative cost of vaccine side effects*, which is given by the ratio between the cost of VSE and the cost of infection. These parameters summarise the whole set of costs (economic, psychological, etc.) following from serious episodes of disease or of VSE. The ideas underlying the simple quadratic additive loss (1) are that: (a) agents have two *objective* variables, i.e. the two perceived risks (from infection and from VSE); (b) there are *target* values ($\rho_I = \rho_V = 0$) such that any deviation from such targets results in a loss for the agent. The case $\rho_I = \rho_V = 0$ corresponds to the situation where the disease has been eliminated, so that the risk of infection has not only been forced to zero, but also the need to vaccinate has been eventually removed, as it has been the case of smallpox, so that also the risk of VSE is driven to zero; (c) deviations of the objective variable from the target are penalised in a more than proportional way; (d) simple additivity straightforwardly reflects the trade-off between the two objectives, e.g. the fact that if agent A increases his/her propensity to vaccinate (for example as a consequence of an external rumour which increases his perceived risk of infection), then A expects to observe a reduction in the risk of disease, but at the cost of increasing the risk of VSE.

A major problem is to what extent families are informed about the disease and the vaccine, and therefore how they evaluate risks. Families hardly know "technical" quantities as R_0. Nonetheless, it seems reasonable to assume that they are aware that (a) more transmissible diseases need a stronger vaccination effort to be controlled; (b) a higher *collective* coverage implies a lower *individual* risk of infection; (c) a coverage of 100% by a perfect vaccine is certainly sufficient to eliminate the disease. What is unclear is whether families know the existence of the critical coverage p_c, i.e. the herd immunity principle, based on the intuitive concept that "if everyone vaccinates I do not need to" ([1] Chapter 4). Though this does not seem to be true in general, it might occur in some cases [2]. Knowledge of herd immunity was postulated in [4]. For generality, we therefore consider two cases: in the first case families are "informed", i.e. they do know herd immunity; in the second one they do not, and therefore believe universal vaccination to be the only strategy to surely avoid infection.

2.1. The case of "informed" families

Informed families are assumed to exactly know the critical threshold p_c, and to estimate the risk of infection as a decreasing function of collective coverage p, for p below the critical threshold p_c, and zero above it, since in this case the disease is eliminated. The simpler form is:

$$\rho_I(p) = \begin{cases} H(p_c - p), & p < p_c \\ 0, & p_c \le p \le 1 \end{cases} \tag{2}$$

where $H > 0$ is a constant. In particular $\rho_I(0) = Hp_c$ represents the family's perceived risk of infection in absence of any immunization at the population level. In [4] perfectly informed agents were postulated to know in detail the underlying epidemiological model, in particular the functional form $\pi(p) = (p_c - p)/(1 - p)$ of the lifetime risk of infection at equilibrium, and to estimate $\rho_I(p)$ by $\pi(p)$. This choice and ours are qualitatively equivalent (both predict a vanishing risk of infection for $p = p_c$), but ours has the advantage of being simpler, and also less demanding in terms of abstraction. Though (2) is formally equivalent to the equilibrium force of infection in the SIR model [1], we prefer to interpret (2) as a coarse qualitative estimate of the lifetime risk of infection, consistently with the above made assumption that agents do not know "technical" quantities.

The perceived risk ρ_V of a VSE is modelled as the product αp of the family propensity p to vaccinate children, times the conditional probability $\alpha > 0$ (exogenously given) of suffering a side effect given vaccination relative to the corresponding probability of suffering serious disease following infection.

The informed representative family aims therefore at choosing the value of p which minimizes the loss function[1]:

$$L(p) = \begin{cases} H^2(p_c - p)^2 + \beta\alpha^2 p^2, & p < p_c \\ \beta\alpha^2 p^2, & p \ge p_c \end{cases} \tag{3}$$

Note that $L(p)$ has a continuous first derivative. The idea underlying the quadratic loss, as (3), is that families perceive two types of costs, one arising from the risk of infection, the other one from the risk of VSE, and want to make both them as low as possible. In [4] the expected loss $L(p) = \rho_I(p)(1 - p) + \beta\rho_V(p)$ was used.[2]

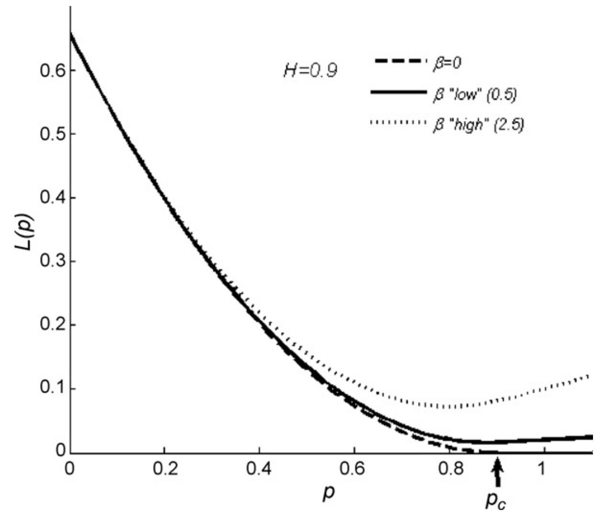

Fig. 1. Shapes of the individual family loss as a function of the level p of vaccine "demanded" by the family for different costs of vaccine side effects β ($\beta = 0$, $\beta = 0.5$, $\beta = 2.5$). The point where the graph achieves its minimum is the optimal individual solution p_{opt}. For $\beta = 0$ the loss is minimal (i.e. zero) for any choice equal or in excess of the critical coverage p_c. Other parameters values are: $H = 0.9$, and $\alpha = 0.2$.

Note first that if $\beta = 0$, i.e. if no cost is perceived from VSE, then (3) becomes:

$$L(p) = \begin{cases} H^2(p_c - p)^2, & p < p_c \\ 0, & p \ge p_c \end{cases} \tag{4}$$

One immediately notes that for $p < p_c$, the graph of $L(p)$ is an arc of parabola decreasing to zero as p tends to p_c, implying that any choice p equal to, or above the critical coverage p_c is optimal for the individual. In other words, if no costs arise from VSE it is optimal for the individual to achieve the elimination threshold.

For $\beta > 0$ the loss function is decreasing for small p and increasing for $p \ge p_c$, implying that the optimal solution can never exceed p_c. This means that policies in excess of the critical level are more costly also at the individual level, as they expose people to unnecessarily high risks of VSE. By focusing on the case $p < p_c$, differentiating the loss with respect to p, and equating to zero, we get a unique global solution of "minimum loss"

$$p_{opt} = \frac{H^2}{\beta\alpha^2 + H^2} p_c = \varepsilon p_c \tag{5}$$

where $0 < \varepsilon < 1$. Expression (5) shows that the *optimal propensity to vaccinate* p_{opt} is strictly lower than the critical coverage for any, however small, risk of VSE α. In particular p_{opt} is decreasing in both the risk, and the cost of VSE. This is the simplest proof that the family's optimal solution always implies a collective coverage below the elimination threshold, and therefore that elimination is not possible in a free vaccination market. We illustrate this (Fig. 1) for a measles-like disease with $R_0 = 10$, implying a critical coverage $p_c = 0.90$. We consider distinct values ($\beta = 0$, $\beta = 0.5$, $\beta = 2.5$) of the relative cost of VSE; $\beta = 2.5$ means that the cost attributed to a damage from VSE is 2.5 times higher than that attributed to disease. In correspondence of the "low" relative cost of VSE ($\beta = 0.5$) the optimal propensity to vaccinate p_{opt} is about 88%, i.e. very close to p_c, whereas for the "high" value of β ($\beta = 2.5$) p_{opt} declines to about 80%, i.e. substantially less than p_c. In other words if the cost families attribute to VSE largely exceeds the cost of infection, the vaccine demand might become quite low. In the real world families will then have to implement these "theoretical" propensities into a binary decision ("vaccinate" or "not vaccinate"). If families vaccinate independently then, unless in very small communities, the actual "realized" coverage, i.e. the average of families' realised

[1] More general results can be obtained by more general formulations allowing interpolation between the linear and quadratic loss formulations, as suggested by an anonymous referee.

[2] It is possible to see that eventually the two formulations yield the same conclusions. The only difference is that the expected loss allows a dichotomy result, i.e. no one will vaccinate if the cost of VSE is always higher than the cost of infection, while in the opposite case some will vaccinate. Though the former case is of theoretical interest as it yields a "no vaccination" equilibrium, it does not seem to have practical relevance.

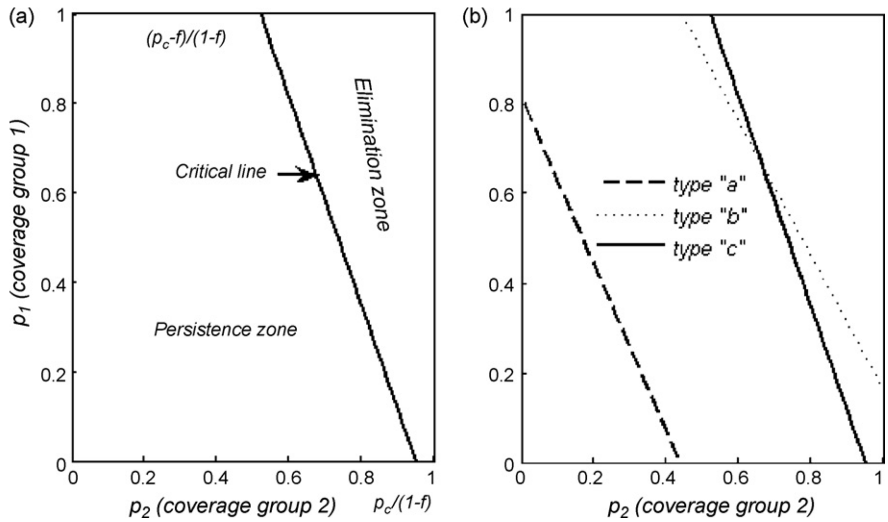

Fig. 2. (a) Vaccination square, critical line, and "elimination zone" for a disease with $R_0 = 5$ ($p_c = 0.80$) in a community where group 2 is majority ($f_2 = 0.90$); (b) Different shapes of the CL corresponding to the three different socio-epidemiological "types": type "a": $R_0 = 1.4, f_1 = 0.35$; type "b": $R_0 = 3, f_1 = 0.4$; type "c": $R_0 = 3, f_1 = 0.3$.

decisions, will differ little from p_{opt}. It is to be noted that p_{opt} could vary significantly among different diseases because individuals naturally tend to rank both the costs of diseases and VSE [6].

2.2. Not-fully-informed families

By analogy with the case of informed families we use the following simple form for the perceived risk of infection:

$$\rho_I = K(1-p) \tag{6}$$

where $K > 0$. This amounts to assuming that families do not know the critical threshold and believe instead that universal vaccination is the only strategy surely avoiding infection. In this case the representative family minimizes:

$$L = (K(1-p))^2 + \beta\alpha^2 p^2, \quad 0 \le p \le 1 \tag{7}$$

yielding the optimal solution (the graph of the loss function is again an arc of parabola):

$$p_{opt} = \frac{K^2}{K^2 + \beta\alpha^2}. \tag{8}$$

If, for comparison purposes, we keep $K = H$, as in the case of informed families, then $p_{opt} = \varepsilon$, i.e. this case allows to achieve coverages that are systematically higher compared to the case of informed families, and in particular close to 100% if the cost of VSE is negligible. Under the same parameters assignments of the example above we find that for $\beta = 0.5$, the optimal coverage is near 98%, and even for $\beta = 2.5$ it is about 89%, i.e. still quite close to p_c. It follows that lack of information on the existence of the infection threshold seems therefore to provide a simple explanation of how patterns of universal vaccination might emerge in actual circumstances.

2.3. Only a fraction of the population is eligible

An interesting case is when only a fraction f_1 of the population is eligible for vaccination, for example because of the presence of a group practicing consciencious exemption. In this case the overall vaccination coverage is defined as $p_1 = pf_1$, and the critical coverage that needs to be achieved in the eligible population is $p_c^* = (p_c/f_1) > p_c$, which might be greater than one if the eligible fraction is small. In the case of informed families we get

$$p_{opt} = \begin{cases} \dfrac{H^2 f_1^2}{\beta\alpha^2 + H^2 f_1^2}\dfrac{p_c}{f_1} = \varepsilon_1 p_c^*, & \varepsilon_1 p_c^* < 1 \\ 1, & \text{elsewhere} \end{cases} \tag{9}$$

This result will be useful in the next section.

3. Implications of strategic behaviour: the game-theoretic approach

3.1. A preliminary: the critical elimination line for multigroup populations

Let us now consider a SIR-type disease in a population subdivided into two groups with frequencies f_1, f_2, and heterogeneous coverages p_1, p_2. If the population is homogeneously mixing the collective coverage is $p = p_1 f_1 + p_2 f_2$ and the elimination rule ($p \ge p_c$) for vaccination at birth becomes:

$$p_1 f_1 + p_2 f_2 \ge p_c \tag{10}$$

showing that the possibility of elimination depends not only on groups' actual coverages but also on their frequencies. Taking the frequencies as given, (10) states that elimination requires coverages p_1, p_2 in the two groups lying above a *critical line* (CL). The CL is the portion lying in the unit vaccination square $[0,1] \times [0,1]$ (i.e. the space of admissible vaccination policies), of the straight line $p_1 f_1 + p_2 f_2 = p_c$, which can be written as:

$$p_1 = p_{1,c}^* - \frac{f_2}{f_1} p_2 \tag{11}$$

where $p_{1,c}^* = p_c/f_1$. Any pair (p_1, p_2) lying above the CL would eliminate the infection. The CL has negative slope ("an increase in critical coverage in group 2 allows a reduction in group 1 without compromising elimination") $m = -f_2/f_1$, reflecting the relative size of the two groups. The intercept $p_{1,c}^*$ and the foot $p_{2,c}^* = p_c/f_2$ represent the vaccination effort that would be required for either group to eliminate infection when the other group does not vaccinate. If either $p_{1,c}^*, p_{2,c}^*$ exceeds 1 elimination is unfeasible without some "cooperation".

By means of the CL we may answer useful questions related to the containment of the impact of anti-vaccinators on control programmes, e.g. if group 2 vaccinates above threshold, to what extent can group 1 vaccinate below threshold without compromising elimination? Let us consider (Fig. 2a) a disease with $R_0 = 5$ (sometimes adequate for rubella) implying $p_c = 0.80$, in a community where group 2 represents 90% of the population ($f_2 = 0.9$). The foot $p^*_{2,c} \cong 0.89$ indicates that group 2 can achieve elimination "alone". However, the steepness of the CL ($m = -9$) indicates that small departures from this level make it necessary to vaccinate also in group 1. If coverage in group 2 is 85%, then elimination requires at least 35% coverage in group 1.

To sum up, the shape of the CL reflects both the difficulty to eliminate the disease, summarised by p_c (which in turns reflects transmissibility through the increasing relation $p_c = 1 - 1/R_0$), shifting upward when, other things being equal, p_c increases, and the relative size of the two groups. Fig. 2b illustrates the possible forms of the CL. These forms correspond to distinct socio-epidemiological "types": (a) the CL entirely lies within the vaccination square: this is characterized by the condition $p_c < \min\{f_1, f_2\}$ and implies the case of a "moderately" transmissible infection, i.e. such that $p_c < 1/2 \Leftrightarrow R_0 < 2$, with groups of not too dissimilar size ($p_c < f_1 < 1 - p_c$); (b) both the intercept and the foot of the CL lie outside the vaccination square: this corresponds to a "highly" transmissible infection ($p_c > 1/2 \Leftrightarrow R_0 < 2$) and groups of not too dissimilar size; (c) the intercept lies outside and the foot inside: this is expressed by the condition $f_1 < p_c < f_2$. This case is compatible with very large differences in size between the two groups when R_0 approaches 1 or when it is very large ($R_0 \gg 2$); (d) the intercept lies inside and the foot outside; this is just the symmetric of (c) when the roles of the two groups are interchanged (not reported in the figure).

Given our focus on the possibility to eliminate a highly transmissible disease in presence of a small group with low vaccine uptake, the "type" of major interest for our discussion is (c). It is important to remind that in this case group 2 is large enough to eliminate without cooperation from group 1, i.e. $p^*_{2,c} < 1$. This means that if for instance $R_0 = 15$, so that $p_c = 0.93$, the maximal "affordable" size of the anti-vaccine group is less than 7%.

3.2. The vaccination game

In a game-theoretic framework, unlike the individual optimization of Section 2, families ("players") make their choices by taking into considerations other families' decisions. We assume that families are subdivided into two groups of fixed (no migration between the groups is allowed) sizes f_1 and f_2. Families are characterised by homogeneous preferences within the group they belong to, but heterogeneous preferences between groups. This allows to *refer to a representative agent for each group*. Group 1 ("anti-vaccinators")[3] has a higher relative cost of VSE ($\beta_1 > \beta_2 \geq 0$) as in [8], and it is smaller ($f_1 < 1/2$). We focus on the case of *informed families*, but the extension to not-fully informed families is straightforward. The i*nformed representative family* of each group determines its vaccination propensity p_j ($j = 1, 2$) by solving the problem:

$$\min_{p_j} L_j = (\rho_I(p))^2 + \beta_j \alpha^2 p_j^2, \quad j = 1, 2;\ 0 \leq p_1, p_2 \leq 1 \tag{12}$$

where the perceived risk of infection is, as before, given by:

$$\rho_I(p) = \begin{cases} H(p_c - p), & p < p_c \\ 0, & \text{elsewhere} \end{cases} \tag{13}$$

In particular $p = p_1 f_1 + p_2 f_2$ (the constraint $0 \leq p_1 f_1 + p_2 f_2 \leq 1$ is always fulfilled as $0 \leq p_1, p_2 \leq 1, 0 \leq f_1, f_2 \leq 1$) represents the overall coverage that would follow from choices (p_1, p_2) of families of the two groups. In other words families know that the overall coverage depends not only on their choices (i.e. from people of their group) but also on the other group choices. Given that choices made by either group produces some effects also on the other group, solving (12) and (13) requires to make assumptions as to the type of strategic game families play. We will consider simultaneous games, non-simultaneous games (also termed Stackelberg games), and "social planner" games [25].

3.3. The basic strategic competition

In this case the two groups decide their action *simultaneously* by taking as given the action of the other group. The Nash solution to (12) and (13) uses the concept of vaccination *reaction function*. The reaction function $M_1(p_2)$ ($M_2(p_1)$) of players of group 1 (2) defines the optimal vaccination choice of player 1 (2) corresponding to any feasible vaccination choice $0 \leq p_2 \leq 1$ of players from group 2 (1). The points where the two reaction functions intersect are *Nash equilibria* [25]. The algebraic determination of the reaction functions, though elementary, requires a tedious computation to consider all the possible cases, depending on the "type" of the Critical Line, and on the values of the costs β_i. For sake of simplicity we just summarize the main graphic features of $M_1(p_2)$ (which hold mutatis mutandis for $M_2(p_1)$):

1. since the risk of infection is positive only in the region below the CL (see (13)), it is intuitive that the reaction functions will be positive only in such a region;
2. if group 2 is able to eliminate without cooperation from group 1 (f_2 large enough that $p^*_{2,c} < 1$), then every p_2 choice in excess of $p^*_{2,c}$ is *a feasible elimination choice*. In this case $M_1(p_2)$ is identically zero for $p^*_{2,c} \leq p_2 \leq 1$;
3. if group 2 is not large enough to eliminate alone ($p^*_{2,c} > 1$), then $M_1(p_2)$ is positive for all $0 \leq p_2 \leq 1$;
4. if the value p_1^{opt} determined in Section 2.3, which represents the intercept $M_1(0)$, exceeds 1 (this can happen if $p^*_{1,c} > 1$), a portion of $M_1(p_2)$ will lie "flat" on the upper boundary of the vaccination square;
5. the positive portion of $M_1(p_2)$, will be decreasing in p_2 to mirror that as group 2 increases its demand for vaccines, group 1 will find it convenient to vaccinate less (and symmetrically for $M_2(p_1)$). This is found by optimizing L_1 in the region below the CL, yielding:

$$p_1^{opt}(p_2) = \frac{f_1^2 H^2}{\beta_1 \alpha^2 + f_1^2 H^2}\left(p^*_{1,c} - \frac{f_2}{f_1} p_2\right) = \varepsilon_1 \left(p^*_{1,c} - \frac{f_2}{f_1} p_2\right) \tag{14}$$

with $0 < \varepsilon_1 < 1$. The line (14) is always flatter than the Critical Line, and it has the same foot. By finally relating the possible shapes of $M_1(p_2)$ to the underlying "types" of the CL (Fig. 2b) we obtain the forms represented in Fig. 3.

A special but important case occurs when families of group 2 assign a zero cost to vaccine side effects ($\beta_2 = 0$). In this case the internal portion of the reaction function $M_2(p_1)$ will always coincide with the Critical Line.

Finally, as for the case of *not-fully informed families*, it seems natural to assume that at least families of group 2 are not aware of herd immunity, and therefore obey (5) and (6). This will imply,

[3] We have made the distinction between "pro" and "anti"-vaccinators just for labeling the two groups. The only difference between the two types of agents lies in the different perceived costs following from their decisions (as a consequence perhaps of using distinct information sources), and neither imply distinct a priori propensities to vaccinate nor a larger a priori cooperative attitude. However we will see that these different costs imply that a posteriori one group will be more inclined to vaccinate than the other. This results from the competition within groups as a Nash strategy within group.

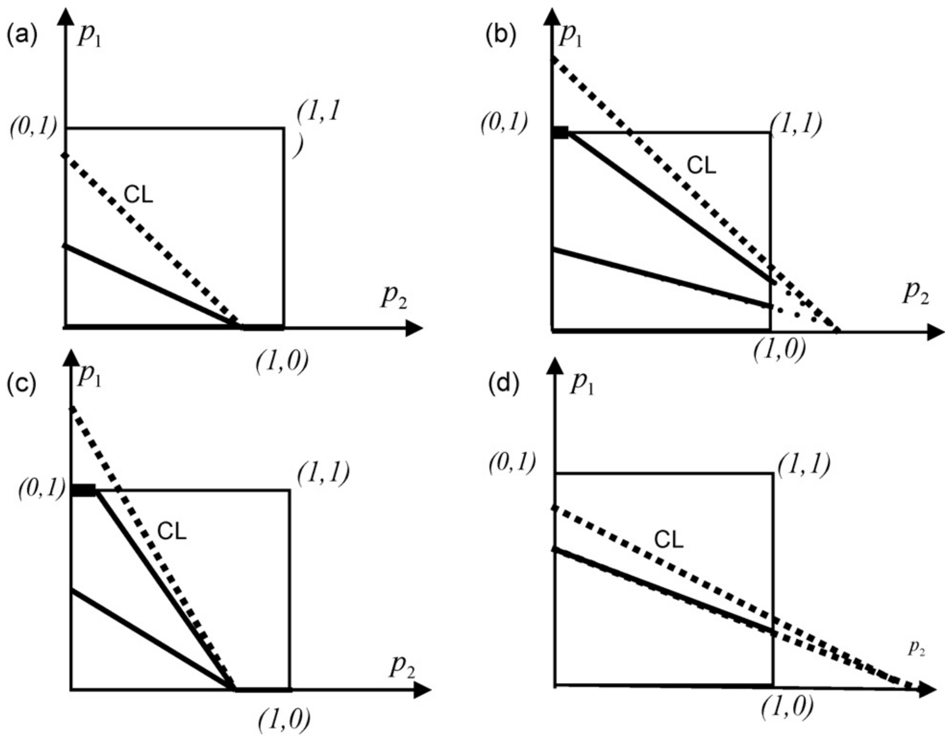

Fig. 3. Possible shapes of the reaction function $M_1(p_2)$ of group 1 in relation to the underlying "types" of the Critical Line.

other things being equal, a reaction function M_2 having a higher intercept and the same slope.

We can now collect our results that are summarised in Fig. 4 for a variety of different circumstances. For brevity we only discuss the case of type (c), that is the most important case for our purposes. For generality we consider two distinct possibilities as for the relative cost of VSE in group 1: $\beta_{1,low} = 1$, $\beta_{1,high} = 2.5$ and take $\beta_2 < \beta_{1,low}$. The case $\beta_{1,high}$ corresponds to a very "low" reaction function of group 1, which vaccinates little even if group 2 does not vaccinate, whereas in the case $\beta_{1,low}$ group 1 vaccinates 100% at very low vaccination choices of group 2. In particular Fig. 4a–c deal with the case of informed families, while Fig. 4d with the case

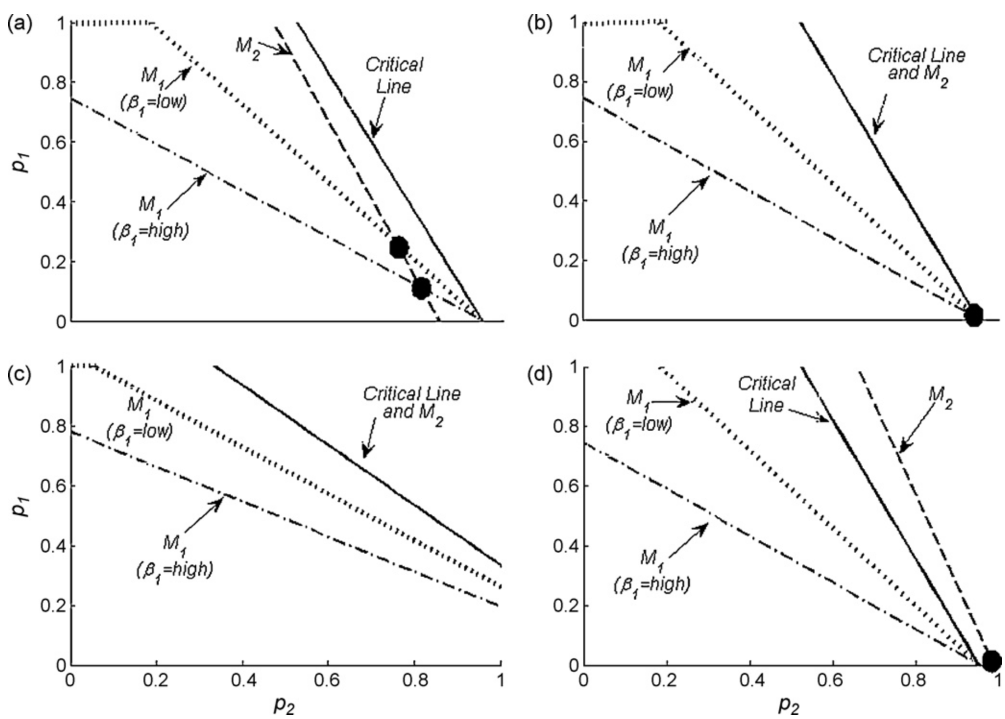

Fig. 4. Critical Line, reaction functions, and Nash equilibrium points (represented by •) under socio-epidemiological "type" c. Graphs (a–c) deal with informed families, (d) with non-fully informed families. H, R_0, p_c, $\alpha = 0.2$ are the same throughout the graphs: $H = 0.9$, $R_0 = 3$, $p_c = 0.66$, $\alpha = 0.2$. Moreover: (a) $f_2 = 0.7$ implying $p^*_{2c} = 0.952$, $\beta_2 = 0.75$; (b) $f_2 = 0.7$, $\beta_2 = 0$; (c) $f_2 = 0.55$, $\beta_2 = 0$; (d) $f_2 = 0.7$, $\beta_2 = 0.75$ as in (a).

of non-fully informed agents. If the relative cost of VSE in group 2 is positive, however small (Fig. 4a), then the Nash equilibrium, i.e. the intersection between the reaction functions, will lie strictly below the Critical Line, indicating that the families' behaviour is not compatible with elimination. When group 2 does not suffer costs from VSE ($\beta_2 = 0$) then (Fig. 4b) the internal portion of the reaction function of group 2 coincides with the CL, and the Nash equilibrium is located at the foot $p^*_{2,c}$ of the CL. In this case elimination is feasible because group 2 is large enough to support the whole elimination effort without any cooperation from group 1. Note however that if group 2 is not sufficiently large, then elimination becomes impossible even in the favourable case $\beta_2 = 0$ (Fig. 4c). Indeed a larger anti-vaccine group makes the CL (and M_2) flatter, so that the Nash equilibrium is "pushed" out of the vaccination square. If families of group 2 do not know herd immunity then, other things being equal, M_2 moves upward (Fig. 4d) and can therefore always lie above the CL. In this case the Nash equilibrium locates above the elimination threshold. Therefore lack of knowledge of the elimination threshold makes elimination possible even when this was not the case under fully informed agents. The previous results therefore prove the following:

Result 1 (*"Elimination is impossible under" informed families*). As long as both groups assign a positive value to the cost of VSE (β_1, $\beta_2 > 0$) elimination is never possible. Elimination is possible if pro-vaccinators do not suffer costs from VSE (i.e. $\beta_2 = 0$) and their group size is large enough to sustain elimination without the cooperation of the other group.

Summarizing, in presence of *informed families*, the forces operating in the Nash case are never able to promote coverage above the critical one. This provides the main rationale for compulsory vaccination as the rule to avoid non-cooperative Nash behaviour.

When families are not aware of herd immunity, then, as noted in Section 2, circumstances are more favourable to elimination (as in Fig. 4d). Nonetheless high costs of VSE and increases in the size of the anti-vaccine group can obviously lead to a community coverage below the critical threshold in this case as well.

We finally note that the coordinates of the Nash equilibrium can be computed explicitly:

$$p_1^{Nash} = \frac{H^2 f_1 \beta_2}{H^2(f_1^2\beta_2 + f_2^2\beta_1) + \alpha^2\beta_1\beta_2} p_c;$$

$$p_2^{Nash} = \frac{H^2 f_2 \beta_1}{H^2(f_1^2\beta_2 + f_2^2\beta_1) + \alpha^2\beta_1\beta_2} p_c \tag{15}$$

so that the corresponding overall coverage is:

$$p^{Nash} = p_1^{Nash} f_1 + p_2^{Nash} f_2 = \frac{H^2(f_1^2\beta_2 + f_2^2\beta_1)}{H^2(f_1^2\beta_2 + f_2^2\beta_1) + \alpha^2\beta_1\beta_2} p_c. \tag{16}$$

which immediately shows our main result: $p^{Nash} \le p_c$.

3.4. The Stackelberg case with anti-vaccinators leadership

In Stackelberg games[4] there is a "leader" who minimises its loss function by taking into account the reaction function of the "follower", rather than by taking it as given, as in the previous section. This implies therefore a behavioural asymmetry that might be consistent with the vaccination game in developed countries where anti-vaccinator groups tend to be very active in the information acquisition and decision processes. To model this anti-vaccinators leadership in our framework we assume that anti-vaccinators have an "information advantage", i.e. they know pro-vaccinators behaviour (i.e. their reaction function) and incorporate it into their own loss function.

We therefore assume that families from group 2 play as before, thereby computing their reaction function $M_2(p_1)$, whereas families from group 1 know $M_2(p_1)$, and incorporate it into their own loss function. Thus group 1 determines its optimal vaccination choice p_1^{Stack}, and group 2 will as a consequence "follow" by just recomputing its choice as $M_2(p_1^{Stack})$. In the case of *informed families* it holds:

$$p_1^{Stack} = \frac{f_1^2 H^2 (1-\varepsilon_2)^2}{f_1{}^2 H^2 (1-\varepsilon_2)^2 + \alpha^2 \beta_1} p_c. \tag{17}$$

$$\text{where} \quad \varepsilon_2 = \frac{f_2{}^2 H^2}{\alpha^2 \beta_2 + f_2^2 H^2}.$$

It is thus easy to see that when both groups have positive costs of VSE β_i, the "elimination impossible" result continues to hold. In addition it is possible to prove that in the Stackelberg equilibrium (p_1^{Stack}, $M_2(p_1^{Stack})$) the demand for vaccines by group 1 will be lower compared to the basic simultaneous game of Section 3.2, while that of group 2 will be consequently higher. The intuition for such a result is that when group 1 acts as Stackelberg leader it has the advantage to decide its optimal policy by considering the entire reaction of group 2 to its own behaviour. For instance group 1 knows that if it decides not to vaccinate, group 2 will entirely bear the burden of the (whole) community immunization (included the cost of VSE). No surprise, then, that in such a case the vaccination effort of group 1 will be lower than in the case in which the two groups play simultaneously.

3.5. The social planner case

The "elimination impossible" trap under Nash or Stackelberg rules suggests looking at the possibility that a social planner may attempt to find an "agreed" solution. To this end, we consider a social loss function $L = L(p_1, p_2)$ of a utilitarian type [25], defined as a weighted average of the loss functions of the two groups. The social planner will find a socially optimal solution by minimizing the social loss function and then will seek appropriate policy tools to actually implement the optimal solution among the two groups. He will solve:

$$\min_{p_1, p_2} xL_1 + (1-x)L_2, \quad 0 \le x \le 1 \tag{18}$$

where x is the weight that he assigns to the loss of group 1. This case requires some elaboration (appendix), but its results are easy to understand. We only consider the case of *fully informed families*. In this case no solution can lie above the CL ($p \ge p_c$): in fact the partial derivatives of the social loss function L are always non-negative and do not vanish simultaneously. Looking for solutions lying below the CL, standard computations yield the following optimum point, i.e. the so-called Nash equilibrium in the social planner case:

$$p_1^{Soc} = \begin{cases} \hat{p}_1, & \hat{p}_1 < 1 \\ 1, & \text{elsewhere} \end{cases};$$

$$p_2^{Soc} = \begin{cases} \hat{p}_2, & \hat{p}_1 < 1 \\ \dfrac{H^2 f_2^2 (p^*_{2,c} - f_1/f_2)}{H^2 f_2^2 + (1-x)\beta_2\alpha^2}, & \text{elsewhere} \end{cases} \tag{19}$$

[4] Stackelberg games are usually compared with Cournot games when distinguishing sequential and simultaneous play. Both types of games can have strategies satisfying Nash's criteria [33].

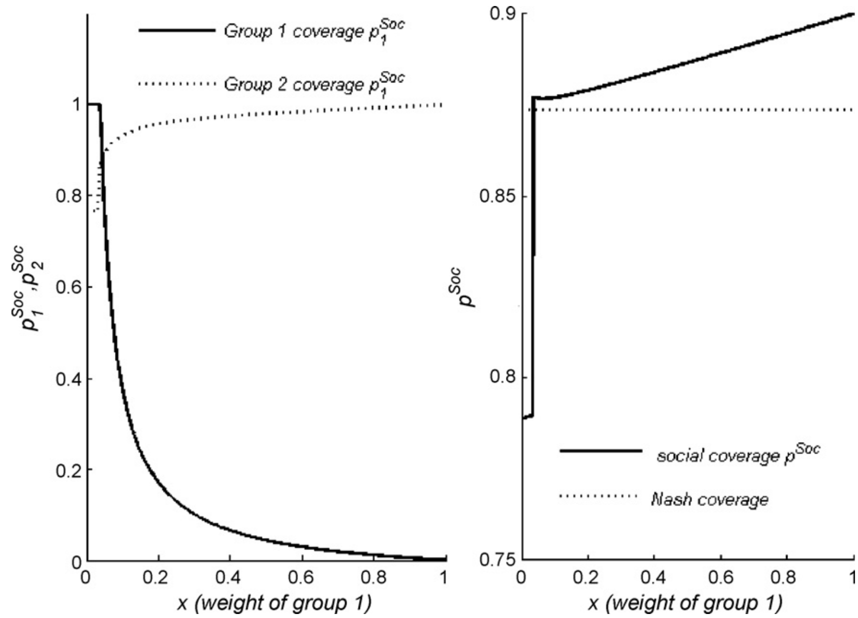

Fig. 5. The social planner case: (a) the optimal vaccine demand in groups 1 and 2 (p_1^{Soc} and p_2^{Soc}) as functions of the weight attributed to group 1 in the social loss function. (b) Overall vaccine demand (p^{Soc}) versus the corresponding quantity in the Nash case. Parameter values: $R_0 = 10$, $\beta_1 = 2.5$, $\beta_2 = 0.5$, $\alpha = 0.2$, $f_1 = 0.1$.

where

$$\hat{p}_1 = \frac{(1-x)f_1 H^2 \beta_2}{H^2((1-x)f_1^2\beta_2 + x\beta_1 f_2^2) + x(1-x)\alpha^2\beta_1\beta_2} p_c$$
$$\hat{p}_2 = \frac{x f_2 H^2 \beta_1}{H^2((1-x)f_1^2\beta_2 + x\beta_1 f_2^2) + x(1-x)\alpha^2\beta_1\beta_2} p_c \quad (20)$$

One thus immediately notes that for values of the weight x belonging to (0,1) it holds: $p^{Soc} = p_1^{Soc} f_1 + p_2^{Soc} f_2 < p_c$. This means that when both groups concur to form the social loss function, and in presence of informed families with positive costs from VSE, elimination is impossible even under social planning.[5] Nonetheless, the present case yields, in most situations, a higher coverage compared to both Stackelberg and Nash cases. This can be better understood from the limit cases where the social planner assigns a unit weight to either groups. These cases are discussed easily. Note instead that if $x = 1$ then:

$$p_1^{Soc} = \hat{p}_1 = 0; \qquad p_2^{Soc} = \hat{p}_2 = p_{2,c}^* \quad (21)$$

Therefore under "type" c, there exists an optimal social planning solution which allows elimination. On the other hand, if $x = 0$ then: $\hat{p}_1 = p_{1,c}^* > 1$ and $\hat{p}_2 = 0$. Therefore

$$p_1^{Soc} = 1; \qquad p_2^{Soc} = \frac{f_2^2 H^2}{f_2 H^2 + \beta_2\alpha^2}\left(p_{2,c}^* - \frac{f_1}{f_2}\right). \quad (22)$$

It is easy to see that this solution does not allow elimination since it yields an overall coverage lower than p_c:

$$p^{Soc} = p_1^{Soc} f_1 + p_2^{Soc} f_2$$
$$= f_1\left(1 - \frac{f_2 H^2}{f_2 H^2 + \beta_2\alpha^2}\right) + p_c \frac{f_2 H^2}{f_2 H^2 + \beta_2\alpha^2} < p_c. \quad (23)$$

To sum up, for $x = 1$ social planning yields a feasible elimination policy if group 2 is large enough to eliminate "alone".

The overall implications of different weights given by the social planner to the two groups in the social loss functions are illustrated (Fig. 5) for a highly transmissible disease with $R_0 = 10$ ($p_c = 0.9$) and $f_2 = 0.9$. This means $p_{c2}^* = 1.0$, i.e. a vaccine demand of 100% in group 2 would be required to achieve elimination when group 1 does not vaccinate. In addition, we assume that group 2 has a cost of VSE smaller than the cost of infection ($\beta_2 = 0.75$) whereas for group 1 this cost is much higher ($\beta_2 = 2.5$). It happens (Fig. 5a) that the vaccine demand in group 1 is constant at 100% ($p_1^{Soc} = 1$) for very small x values, but as x exceeds a threshold value it starts to decline fast up to zero. Similarly, the vaccine demand in group 2 is initially constant at a level around 76%, but when the vaccine demand of group 1 starts declining it starts increasing and reaches universal coverage for $x = 1$. As regards the overall coverage (Fig. 5b) this is initially constant around the level of 78%, but when coverage in group 2 starts increasing it starts increasing as well, achieving the elimination threshold $p_c = 0.9$ for $x = 1$. All this is consistent with our theoretical findings. It is interesting to note that social planning does not necessarily yields an improvement of the Nash solution (the flat line in Fig. 5b): this happens only when the vaccine demand in group 2 becomes sufficiently high, which does not occur if x is close to zero.[6] Note also that if the social planner assigns to group 1 a weight proportional to its demographic frequency ($x = 0.1$), which "a priori" could appear a reasonable choice, an overall coverage of 87.7% follows, which is better than the Nash coverage.

The results for $x = 1$ and $x = 0$ lead to interesting remarks. Note that for $x = 1$ elimination does not require that $\beta_2 = 0$, as in the Nash and Stackelberg cases. The explanation is that for $x = 1$ the preferences (i.e. the costs) of group 2 are not taken into consideration by the social planner (whereas those of group 1 fully are), so that group 1 achieves its optimum (not vaccinating at all), and then group 2 needs to supply residually the amount of vaccination needed to minimize the social loss function. It happens that this amount is, consistently with the frequencies of the two groups, the one required to eliminate the disease. In more concrete terms, for $x = 1$ the social planner gives whole weight to the preferences of anti-vaccinators, and no weight to the preferences of vaccina-

[5] It is possible to envisage other social planner functions, based on very different perspectives, that might lead to elimination.

[6] We note that the Nash solution yields a larger coverage than the Stackelberg one (not reported in the figure). Moreover under *not-fully informed families* an increase in the overall coverage will follow, in line with our previous findings.

tors, on the assumption that this is a good way to achieve the social optimum i.e. disease elimination. This means that "vaccinators" contribute fully altruistically to the social optimum, since they entirely supply the elimination effort. It is also interesting to note that the opposite case ($x=0$), i.e. "full weight" to the pro-vaccinators preferences, is the one yielding the worse result in terms of coverage, even worse of the non-cooperative Nash outcome.

4. Discussion: can we get off the no-elimination trap?

A variety of models of vaccination choice, both without and with strategic behaviour, have been investigated, under both informed, and not-fully informed agents.

The analysis of the non-strategic choice indicates that *informed* families knowing the herd immunity principle will always vaccinate below the critical threshold as soon as they suffer any, however small, cost of VSE, while a high collective coverage is much more likely to be achieved if families are not aware of herd immunity. The analysis of the strategic case, carried out on the assumption of the existence of a vast (fixed) majority of agents having a small perceived cost of VSE (labeled as "pro-vaccinators") versus a minority having a high perceived cost of VSE ("anti-vaccinators"), confirms that in presence of *fully informed* families disease elimination continues to be impossible unless pro-vaccinators do not associate any cost to vaccine side effects, and moreover their frequency is in excess of the critical elimination coverage. In a Stackelberg game in which anti-vaccinators lead the game, the outcome in terms of coverage will be worse compared to the Nash case. Even if the State aims at favouring "cooperative" behaviour through social planning, elimination is possible only when the State takes into account in the social loss functions the preferences of anti-vaccinators only.[7] This scenario seems to be representative of the current Italian situation, where the possibility to switch from a compulsory to a voluntary vaccination system is actively debated, and one region, Veneto, has recently "made the step" [15]. In this context, under the pressure of anti-vaccinators, the Italian government has gradually accepted an increasing number of right claims by anti-vaccinators, for example it has removed the parents' duty to vaccinate their children in order to enroll them to compulsory school grades. This can be interpreted as a massive increase of the weight of anti-vaccinators in the social loss function. Our results indicate that a social planning policy of this sort, i.e. paying more weight to anti-vaccinators, seems to be "technically" correct (in fact in the opposite case, where the State takes into account the preferences of pro-vaccinators only, dramatically low coverage would follow, even worse than the Stackelberg outcome). However such static results figure out the possible dynamic unsustainability of such a policy. Indeed, how to prevent migrations towards the anti-vaccine choice, if this starts to be more generally perceived as more protective of children's health? Though the consideration of migration between groups would require a dynamic model (as [5]), we note that global socio-economic trends are not favourable in this sense: empirical studies indicate that often those vaccinating less are the more educated, or the richer, ones [6,35]. One can certainly invoke the economic argument that no free-rider groups can expand above a certain threshold. In epidemiological terms this means that any further expansion in the anti-vaccinator group will decrease the degree of herd immunity and will therefore be stopped by the necessary re-emergence of the disease. The public health dangerousness of such a situation, think for instance to slow declines in herd immunity due to slow migrations toward the anti-vaccinator group, call for a careful monitoring of such processes.

The case of *not-fully informed* families produces, as a rule, better outcomes. However it also raises substantive questions. Which information mechanisms cause real communities to polarise into a majority who usually vaccinates universally, and a very active minority who is strongly reluctant to vaccinate? Our model static is not capable of explaining polarization (it just postulates it) it strongly suggests that universal vaccination in the majority follows from: (a) a very small cost associated to VSE, (b) the lack of knowledge of the critical threshold, i.e. parents believing that a propensity to vaccinate of 100% is necessary to fully protect children against disease. This wrong perception (mathematics tells indeed that the critical threshold for disease elimination is always less, sometimes substantially less, than 100%) is the likely cause of the asymmetry of the "real world" vaccination game. Asymmetry stems from the fact that some groups of agents might instead have a correct knowledge of herd immunity. But this asymmetry is potentially dangerous for the public health system since it implies the uneven situation where someone takes the risks of vaccine side effects to protect all, as a consequence of limited information. This further enhances the danger that people improve their information set, and as a consequence rationally "migrate" towards the anti-vaccinator group.

In conclusion, do we need to fear RE? Our results, in line with [4,5,8,9,11,29], show that RE can threaten the implementation of immunization programmes. This is especially true when rumours, e.g. the MMR scare [31], contribute to disproportionately increase perceived risks of vaccine-associated side effects compared to the perceived risk of infection [35]. As regards remedies against RE, first economic papers on the subject [7,20] suggested that the free-rider problem might partly be overcome through e.g. taxes and subsidies. A recent modelling effort [27] has shown that the free-rider effect may be amplified by the assumption of homogeneous mixing, and that the problem can be largely reduced when more realistic contact networks are considered. Be things as they may, the actual impact of vaccination free riding is hard to predict: RE is human behaviour, which, as recently pointed out, is a "missing factor" of our epidemiological explanation [14].

However, better mathematical models could help to improve our understanding of such phenomena, and to design more informative studies of vaccination behaviour, possibly aimed to also capture "strategic" parameters. On the other hand, we believe that for those public health systems that have already initiated a "roadmap" towards voluntary vaccination, as in Italy is the case of Veneto region [15], investment in education to the social role of vaccination will be an unavoidable task in the future.

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Appendix A.

A.1. Elementary features of the reaction functions. We prove properties 2–4 of Section 3.2

To prove property 2 let us consider the optimal choice of group 1 when $p_2 = p^*_{2,c} < 1$. Since in this case $\rho_I = H(p_c - p_2 f_2) = 0$ the loss function L_1 becomes: $L_1 = (H^2 f_1^2 + \beta_1 \alpha^2) p_1^2$, which is minimised for

[7] Clearly this result relies upon the adopted form of the social loss function. This does not exclude that other social planner functions, based on different perspectives, for example including the Government loss, could allow to achieve elimination under weaker conditions. We thank an anonymous referee for this remark.

$p_1 = 0$. In words, no one will vaccinate in group 1 if group 2 sustains the whole elimination effort alone. It follows that $M_1(p_2) = 0$ for all $p^*_{2,c} \le p_2 \le 1$.

To prove property 3 assume that $p^*_{2,c} = p_c/(1-f) > 1$ and compute the value of $M_1(p_2)$ at $p_2 = 1$. It immediately holds $M_1(1) = ((H^2 f_1^2)/(H^2 f_1^2 + \beta_1\alpha^2))(f_1/f_2)(p^*_{2c} - 1) > 0$.

4. If $p_1^{opt} > 1$ (which represents the intercept $M_1(p_2 = 0)$), group 1 will make the "feasible" choice $p^{opt}_{1,F} = 1$. If we consider, instead of $p_2 = 0$, the case $p_2 = \varepsilon$, where ε is "small", for reasons of continuity we expect also $M_1(\varepsilon) > 1$, and therefore that the feasible choice $M_1(\varepsilon) = 1$ is still made. Therefore a portion of $M_1(p_2)$ necessarily lies on the boundary of the vaccination square.

A.2. The Nash game: details on the socio-epidemiological "type" c

We solve the vaccination game for informed families sub "type" c under the assumption $\beta_1 > \beta_2 \ge 0$. We first determine the reaction functions for the representative player of each group by solving the optimization problem (12) and (13) and then compute the Nash equilibria by intersecting them.

We recall first that "type" c is described by $f_1 < p_c < f_2$. Let us denote the abscissa of the intersection between the CL and the horizontal line $p_1 = 1$ by:

$$\bar{p}_2(1) = \frac{R_0 f_2 - 1}{R_0 f_2}$$

Moreover, let $\arg\min_{p_j} L_j(p_1, p_2)$ denote the minimum point of the function $L_j(p_1,p_2)$ with respect to p_j. Due to the position of the CL, in order to compute the reaction function for the representative player of group 1 we need to distinguish three cases, depending on whether p_2 belongs (1) to the region $[0, \bar{p}_2(1)]$ where the CL lies outside the vaccination square and the loss function is given by $L_1(p_1, p_2) = H^2(p_c - p_1 f_1 - p_2 f_2)^2 + \beta_1\alpha^2 p_1^2$; (2) to the region $[\bar{p}_2(1), p^*_{2,c}]$ where the CL lies inside and the risk of infection can be either positive or null (that is, the loss function L_1 is defined by $H^2(p_c - p_1 f_1 - p_2 f_2)^2 + \beta_1\alpha^2 p_1^2$ or by $\beta_1\alpha^2 p_1^2$ according to whether $p_1 < p^*_{1,c} - (f_2/f_1)p_2$ or $p_1 \ge p^*_{1,c} - (f_2/f_1)p_2$); (3) to the region $[p^*_{2,c}, 1]$ where the risk of infection vanishes and the loss function reduces to $L_1(p_1, p_2) = \beta_1\alpha^2 p_1^2$.

As regards the first two cases, we solve the optimization problem (12) and (13) by studying the monotonicity of the loss function L_1 with respect to p_1 for every relevant p_2. This is done by looking at the sign of the partial derivative $\partial L_1/\partial p_1$. For the non-trivial law, we get:

$$\frac{\partial L_1}{\partial p_1} = 2p_1[\beta_1\alpha^2 + H^2 {f_1}^2] - 2H^2 f_1(p_c - p_2 f_2)$$

which is positive if and only if:

$$p_1 > p_1^{opt}(p_2) = \frac{{f_1}^2 H^2}{\beta_1\alpha^2 + {f_1}^2 H^2}\left(p^*_{1,c} - \frac{f_2}{f_1}p_2\right)$$

It is easily checked that $p_1^{opt}(p_2) > 0$ for every $p_2 \in [0, p^*_{2,c}]$ and that $p_1^{opt}(p_2) \le 1$ if and only if $p_2 \ge \hat{p}_2$, where:

$$\hat{p}_2 = 1 - \frac{H^2 f_1 + \beta_1\alpha^2}{H^2 f_1 f_2}$$

Note also that $\hat{p}_2$ is always smaller than $\bar{p}_2(1)$. Bearing in mind this, we have:

(1) $p_2 \in [0, \bar{p}_2(1)]$. In this case it is important noting further that $\hat{p}_2$ can be greater or lesser than zero depending on the parameters $\beta_1 f_1, R_0$ (in the former case, in particular, $p_1^{opt}(p_2)$ lies outside the vaccination square whenever $p_2 \in [0, \hat{p}_2]$). Thus, we can conclude that:
- If $\hat{p}_2 \le 0$, then $0 < p_1^{opt}(p_2) \le 1$ and therefore $\arg\min_{p_1} L_1(p_1, p_2) = p_1^{opt}(p_2)$, i.e. the reaction function of player 1 is given by the straight line $p_1^{opt}(p_2)$.
- If $\hat{p}_2 > 0$ and $p_2 \in [\hat{p}_2, \bar{p}_2(1)]$, then $0 < p_1^{opt}(p_2) \le 1$ and again $\arg\min_{p_1} L_1(p_1, p_2) = p_1^{opt}(p_2)$.
- If $\hat{p}_2 > 0$ and $p_2 \in [0, \hat{p}_2]$, then $p_1^{opt}(p_2) > 1$ and therefore $\arg\min_{p_1} L_1(p_1, p_2) = 1$, i.e. the reaction function of player 1 is given by the corresponding portion of the vaccination square. This corresponds to the case $\beta_{1,low}$ in Fig. 3.

(2) $p_2 \in [\bar{p}_2(1), p^*_{2,c}]$. In this case it always holds that $p_1^{opt}(p_2)$ lies below the CL. Since the function L_1 is continuous with respect to p_1 and it is strictly decreasing on the left of $p_1^{opt}(p_2)$ and strictly increasing on its right, we conclude that $\arg\min_{p_1} L_1(p_1, p_2) = p_1^{opt}(p_2)$

(3) $p_2 \in [p^*_{2,c}, 1]$. In this case the loss function for player 1 is always increasing with respect to p_1 and therefore $\arg\min_{p_1} L_1(p_1, p_2) = 0$.

This analysis confirms that the possible shapes for the reaction function of group 1 are only those represented in Fig. 3.

To sum up, according to whether $\hat{p}_2 \le 0$ or $\hat{p}_2 > 0$, the reaction function M_1 of players of group 1 is given respectively by:

$$M_1(p_2) = \begin{cases} p_1^{opt}(p_2), & p_2 \in [0, p^*_{2,c}) \\ 0, & p_2 \in [p^*_{2,c}, 1] \end{cases}$$

and

$$M_1(p_2) = \begin{cases} 1, & p_2 \in [0, \hat{p}_2) \\ p_1^{opt}(p_2), & p_2 \in [\hat{p}_2, p^*_{2,c}) \\ 0, & p_2 \in [p^*_{2,c}, 1] \end{cases}$$

As regards the representative player of group 2, we note from (12) to (13) that for every $p_1 \in [0,1]$ his loss function is given by:

$$L_2(p_1, p_2) = \begin{cases} H^2(p_c - p_1 f_1 - p_2 f_2)^2 + \beta_2\alpha^2 p_2^2, & p_1 f_1 + p_2 f_2 < p_c \\ \beta_2\alpha^2 p_2^2, & \text{elsewhere} \end{cases}$$

This function is continuous with respect to p_2; moreover, it is strictly increasing for $p_1 f_1 + p_2 f_2 \ge p_c$ whenever $\beta_2 > 0$. For $p_1 f_1 + p_2 f_2 < p_c$, the partial derivative with respect to p_2 is given by:

$$\frac{\partial L_2}{\partial p_2} = 2(\beta_2\alpha^2 + H^2 f_2^2)p_2 - 2H^2 f_2(p_c - p_1 f_1)$$

and it is positive if and only if:

$$p_2 > p_2^{opt}(p_1) = \frac{{f_2}^2\mu^2 R_0^2}{\beta_2\alpha^2 + {f_2}^2\mu^2 R_0^2}\left(p^*_{2,c} - \frac{f_1}{f_2}p_1\right)$$

Since $(f_2^2\mu^2 R_0^2/(\beta_2\alpha^2 + f_2^2\mu^2 R_0^2)) < 1$, it holds that $p_2^{opt}(p_1)$ lies below the CL $p_2 = p^*_{2,c} - (f_1/f_2)p_1$, for every $p_1 \in [0,1]$; therefore, $\arg\min_{p_2} L_2(p_1, p_2) = p_2^{opt}(p_1)$. The same results trivially holds for $\beta_2 = 0$. Thus, the reaction function M_2 for player 2 is given by the straight line $p_2^{opt}(p_1)$ for every $p_1 \in [0,1]$.

Finally, in order to find Nash equilibria as in Fig. 4, we need to plot the reaction functions in the correct reciprocal position, and then to look at their intersections. It is worthwhile noting that:

- if $\hat{p}_2 \le 0$, then: $M_1(0) \le 1$; $M_1(p^*_{2,c}) = 0$; $M_2(0) < p^*_{2,c}$; $M_2(1) > 0$;
- if $\hat{p}_2 > 0$ and $\beta_2 > 0$, then: $\hat{p}_2 < M_2(0) < p^*_{2,c}$; $M_2(1) = \hat{p}_2$;
- if $\hat{p}_2 > 0$ and $\beta_2 > 0$, then[8] $M_2(0) = p^*_{2,c}$; $M_2(1) = \bar{p}_2(1)$, that is, the reaction function for player 2 just coincides with the CL.

which describe the forms of Fig. 4a,b.

A.3. The "social planner case"

We solve problem (18) in the case of informed families. This means to solve:

$$\min_{p_1,p_2} L(p_1, p_2) \quad \text{sub} \quad p_1, p_2 \in [0, 1]$$

where

$$L(p_1, p_2) = \begin{cases} x\beta_1\alpha^2 p_1^2 + (1-x)\beta_2\alpha^2 p_2^2, & p \ge p_c \\ H^2(p_c - p)^2 + x\beta_1\alpha^2 p_1^2 + (1-x)\beta_2\alpha^2 p_2^2, & p < p_c \end{cases}$$

with $p = f_1 p_1 + f_2 p_2, \alpha > 0$; $\beta_1 > \beta_2 \ge 0$; $0 < f_1, f_2 < 1, f_1 + f_2 = 1$; $0 \le x \le 1$. We will only treat the case of "type c": $f_1 < p_c < f_2$, and distinguish the two cases $\beta_2 > 0$ and $\beta_2 = 0$. We assume $x \in [0,1]$. A final remark is devoted to the special cases $x = 0$ and $x = 1$. Suppose initially $\beta_2 > 0$. We will prove that no solutions of (18) can lie on the CL. To this aim, note preliminarily that L is continuous on the closed and bounded set $[0,1] \times [0,1]$, hence a minimum exists. The partial derivatives of L for $p \ne p_c$ are

$$\frac{\partial L}{\partial p_1}(p_1, p_2) = 2 \begin{cases} x\beta_1\alpha^2 p_1, & p > p_c \\ -f_1 H^2(p_c - p) + x\beta_1\alpha^2 p_1, & p < p_c \end{cases}$$

$$\frac{\partial L}{\partial p_2}(p_1, p_2) = 2 \begin{cases} (1-x)\beta_2\alpha^2 p_2, & p > p_c \\ -f_2 H^2(p_c - p) + (1-x)\beta_2\alpha^2 p_2, & p < p_c \end{cases}$$

By continuity, L has partial derivatives also on the CL ($p = p_c$) given by

$$\frac{\partial L}{\partial p_1}(p_1, p_2) = 2x\beta_1\alpha^2 p_1; \qquad \frac{\partial L}{\partial p_2}(p_1, p_2) = 2(1-x)\beta_2\alpha^2 p_2.$$

Thus above and on the CL ($p \ge p_c$) the partial derivatives of L are non-negative and do not vanish simultaneously; as a consequence the minimum is attained below the CL. To find the minimum, note that for $p < p_c$ the partial derivatives of L are zero in a unique point $(\hat{p}_1, \hat{p}_2)$ with

$$\hat{p}_1 = (1-x)\beta_2 G^{-1} p_c; \qquad \hat{p}_2 = x\beta_1 \frac{f_2}{f_1} G^{-1} p_c$$

having set for simplicity

$$G = x\beta_1 \frac{f_2^2}{f_1} + (1-x)\beta_2 f_1 \left(1 + \frac{x\beta_1\alpha^2}{R_0^2 f_1^2 \mu^2}\right).$$

Obviously $\hat{p}_1, \hat{p}_2$ are positive, and

$$\hat{p}_1 f_1 + \hat{p}_2 f_2 = G^{-1}\left((1-x)\beta_2 f_1 + x\beta_1 \frac{f_2^2}{f_1}\right) p_c < p_c$$

and

$$\hat{p}_1 \overset{\ge}{\underset{\le}{=}} 1 \Leftrightarrow (1-x)\beta_2 \frac{f_1}{f_2}\left(p^*_{2,c} - \frac{f_1}{f_2}\left(1 + \frac{x\beta_1\alpha^2}{H^2 f_1^2}\right)\right) \overset{\ge}{\underset{\le}{=}} x\beta_1$$

where $p^*_{2,c} = p_c/f_2$. Hence:

- if $\hat{p}_1 \ge 1$ then, by the previous analysis

$$p_1^{Soc} = 1; \qquad p_2^{Soc} = \frac{R_0^2 f_2^2 \mu^2 (p^*_{2,c} - (f_1/f_2))}{\mu^2 f_2^2 R_0^2 + (1-x)\beta_2\alpha^2}$$

- if $0 < \hat{p}_1 < 1$ then

$$p_1^{Soc} = \hat{p}_1; \qquad p_2^{Soc} = \hat{p}_2.$$

For $\beta_2 = 0$ the social loss function for $p \ge p_c$ is

$$L(p_1, p_2) = x\beta_1\alpha^2 p_1^2.$$

Its positive level sets are vertical lines of equation $p_1 = \sqrt{\eta / x\beta_1\alpha^2}$. Hence the minimum is attained in each point $(0,p_2)$ for $p_2 \in [p^*_{2,c}, 1]$ with $L(0,p_2) = 0$. They represent the unique global minimum points, since for $p < p_c$ the social loss function is strictly positive:

$$L(p_1, p_2) = \mu^2(R_0(1-p) - 1)^2 + x\beta_1\alpha^2 p_1^2 > x\beta_1\alpha^2 p_1^2 \ge 0.$$

We finally consider the two limit cases $x = 0$ and $x = 1$. If $x = 0$ (or $x \to 0$), then by (*) it follows $\hat{p}_1 > 1$ and the global minimum point of L is

$$p_1^{Soc} = 1; \qquad p_2^{Soc} = \frac{f_2^2 H^2}{f_2 H^2 + \beta_2\alpha^2}\left(p^*_{2,c} - \frac{f_1}{f_2}\right).$$

If $x = 1$ (or $x \to 1$), then $\hat{p}_1 < 1$ and the global minimum point of L is

$$p_1^{Soc} = 0; \qquad p_2^{Soc} = p^*_{2,c}.$$

[8] Note that in the proof that $M_2(1) > \hat{p}_2$ and $\hat{p}_2 < M_2(0)$ both the assumptions $f_1 \le 1/2$ and $\beta_2 < \beta_1$ have been used.

References

[1] Anderson RM, May RM. Infectious diseases of humans: dynamics and control. Oxford: Oxford University Press; 1991.
[2] Asch DA, Baron J, Hershey JC, Kunreuther H, Meszaros J, Ritov I, et al. Omission bias and pertussis vaccination. Medical Decision Making 1994;14(2):118–23.
[3] Barrett S. Global disease eradication. Journal of the European Economic Association 2003;1(2–3):591–600.
[4] Bauch CT, Earn DJD. Vaccination and the theory of games. Proceedings of the National Academy of Science of the United States of America 2004;101:13391–4.
[5] Bauch CT. Imitation dynamics predict vaccinating behaviour. Proceedings of the Royal Society of London Series B 2005;27:1669–75.
[6] Bedford H, Lansley M. More vaccines for children? Parents' views. Vaccine 2007;25:7818–23.
[7] Brito DL, Sheshinski E, Intriligator MD. Externalities and compulsory vaccinations. Journal of Public Economics 1991;45:69–90.
[8] Cojocaru MG, Bauch CT, Johnston MD. Dynamics of vaccination strategies via projected dynamical systems. Bulletin of Mathematical Biology 2007;69:1453–76.
[9] D'Onofrio A, Manfredi P, Salinelli E. Vaccinating behaviour, information, and the dynamics of SIR vaccine preventable diseases. Theoretical Population Biology 2007;71:301–17.
[11] D'Onofrio A, Manfredi P, Salinelli E. Fatal SIR diseases and rational exemption to vaccination. Mathematical Medicine and Biology 2009;25:337–57, doi:10.1093/imammb/dqn019.
[12] D'Onofrio A, Manfredi P. Information-related changes in contact patterns may trigger oscillations in the endemic prevalence of infectious diseases. Journal of Theoretical Biology 2008;259:473–8.
[13] Farchy D. A common good: whither the global eradication of the measles virus, Working paper World Bank, 2006.
[14] Ferguson N. Capturing human behaviour. Nature 2007;446:733.
[15] Ferro A, Cinquetti S, Menegon T, Napoletano G, Bertoncello L, Valsecchi M. Overcoming mandatory vaccination policy: first steps. Annali di Igiene 2008;20:3–8.
[16] Fine PEM, Clarkson JA. Individual versus public priorities in the determination of optimal vaccination policies. American Journal of Epidemiology 1986;124:1012–20.
[17] Friederichs V, Cameron JC, Robertson C. Impact of adverse publicity on MMR vaccine uptake: a population based analysis of vaccine uptake records for one million children, born 1987–2004. Achieves of Diseases in Childhood 2006;91(6):465–8.

[18] Galvani AP, Reluga TC, Gretchen B. Chapman long-standing influenza vaccination policy is in accord with individual self-interest but not with the utilitarian optimum. PNAS 2007;104(13):5692–7.

[19] Gangarosa EJ, Galazka AM, Wolfe CR, Phillips LM, Gangarosa RE, Miller E, et al. Impact of anti-vaccine movements on pertussis control: the untold story. Lancet 1998;351:356–61.

[20] Geoffard PY, Philipson T. Disease eradication: private versus public vaccination. American Economic Review 1997;87:222–30.

[21] Logulloa P, Barbosa de Carvalho H, Saconi R, Massad E. Fear of injections is a reason for not to vaccinate their children, say caretakers. Vaccine 2008;26:141–3.

[22] Luman ET, Fiore AE, Strine TW, Barker LE. Impact of thimerosal-related changes in hepatitis B vaccine, birth-dose recommendations. Journal of American Medical Association 2004;291:2351–8.

[24] Maayan-Metzger A, Kedem-Friedrich P, Kuint J. To vaccinate or not to vaccinate—that is the question: why are some mothers opposed to giving their infants hepatitis B vaccine? Vaccine 2005;23:1941–8.

[25] Mas Colell A, Whinston MD. Microeconomic theory. Oxford: Oxford University Press; 1995.

[26] Paulussen TGW, Hoekstra F, Lanting CI, Buijs GB, Hirasing RA. Determinants of Dutch parents' decisions to vaccinate their child. Vaccine 2006;24(5):644–51.

[27] Perisic A, Bauch CT. Social contact networks and disease eradicability under voluntary vaccination. PLoS Computational Biology 2009;5(2):e1000280.

[28] Plotkin S. Lessons learned concerning vaccine safety. Vaccine 2002;20(Suppl. 1):S16–9.

[29] Reluga TC, Bauch CT, Galvani AP. Evolving public perceptions and stability in vaccine uptake. Mathematical Biosciences 2006;204(2):185–98.

[30] Reluga TC. An sis game with two subpopulations. Journal of Biological Dynamics 2009;3:515–31.

[31] Roberts RJ, Sandifer QD, Evans MR, Nolan-Farrell MZ, Davis PM. Reasons for non-uptake of measles, mumps and rubella catch up immunisation in a measles epidemic and side effects of the vaccine. British Medical Journal 1995;310:1629–39.

[32] Salmon DA, Teret SP, MacIntyre CR, Salisbury D, Burgess MA, Halsey NA. Compulsory vaccination and conscientious or philosophical exemptions: past, present and future. Lancet 2006;367:436–42.

[33] Schotter A. Microeconomics: a modern approach. New York: Harper Collins College Publishers; 1994.

[34] Smailbegovic MS, Laing GJ, Bedford H. Why do parents decide against immunization? The effect of health beliefs and health professionals. Child Care Health and Development 2003;29:303–11.

[35] Smith A, Yarwood J, Salisbury DM. Tracking mothers' attitudes to MMR immunisation 1996–2006. Vaccine 2007;25:3996–4002.

[36] Stiglitz JP. Economics of the public sector. New York: W.W. Norton & Company; 2000.

[37] Vardavas R, Breban R, Blower S. Can influenza epidemics be prevented by voluntary vaccination? PLoS Computational Biology 2007;3(5):e85.

[38] Vynnycky E, Gay NJ, Cutts FT. The predicted impact of private sector MMR vaccination on the burden of Congenital Rubella Syndrome. Vaccine 2003;21:2708–19.

[39] Wright JA, Polack C. Understanding variation in measles–mumps–rubella immunization coverage—a population-based study. European Journal of Public Health 2005;16(2):137–42.

[40] Wroe AL, Bhan A, Salkovskis P, Bedford H. Feeling bad about immunizing our children. Vaccine 2005;23:1428–33.