6.7900: Machine Learning
Welcome!

Lecture start: Tues/Thurs 2:35pm

Who's speaking today? Prof. Tamara Broderick

Course website: gradml.mit.edu (includes, e.g., links to Piazza https://piazza.com/mit/fall2024/67900/)

Materials: Slides, video, etc linked from gradml.mit.edu after the lecture (but there is no livestream)

Today's Plan

I. Logistics

II. Machine learning: why & what

III. Getting started: can we solve all of supervised learning in the first lecture or two?

6.7900: Machine Learning

Staff

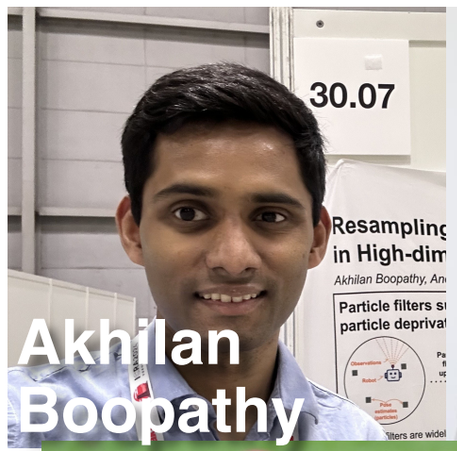
Instructors:

Leslie Kaelbling

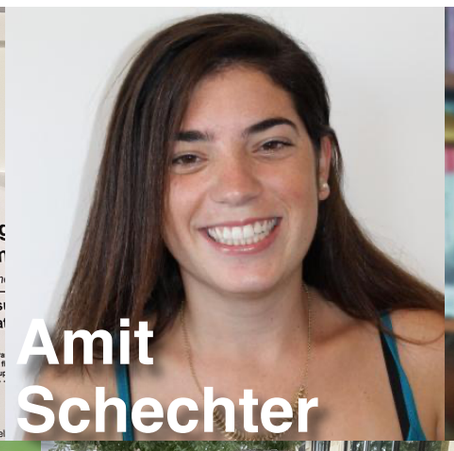
Tamara Broderick

Tommi Jaakkola

Teaching Assistants:

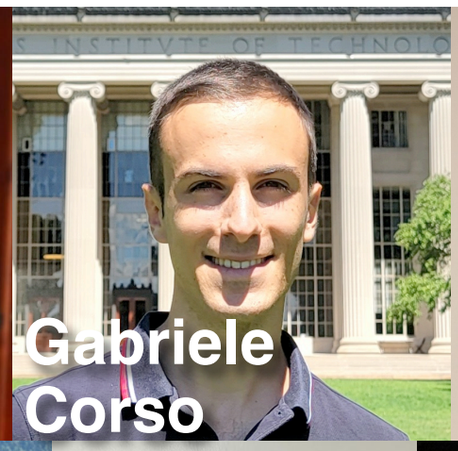


Akhilan Boopathy



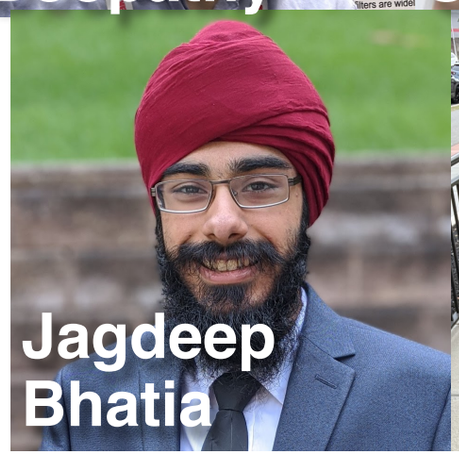
Amit Schechter

Feng Zhu

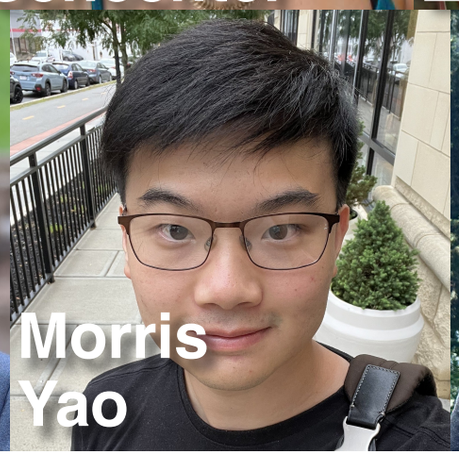


Gabriele Corso

Greg Pylypovych



Jagdeep Bhatia



Morris Yao

Ray Wang

Timmy Qian

Tom Wang

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Submit homework: Gradescope

Check grades: Canvas

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Introductory machine learning

- At the level of 6.390 or 6.370 or 6.C01

Linear algebra

- At the level of 18.06 or 18.C06

Probability

- At the level of 6.3700, 6.3800, or 18.600

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Homework 0 is a readiness assessment

- Released today (Thursday September 5)
- Due Tuesday September 10

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Exams: We’re using as evaluation tools

- **Midterm**: Th Oct 24, 7–9pm (conflict Fr Oct 25, 8–10am)
  - Contact us in advance if you plan to take the conflict exam
- **Final**: Scheduled by registrar. Make your plans now so that you can be physically present at MIT during finals period.

Machine learning (ML): why & what

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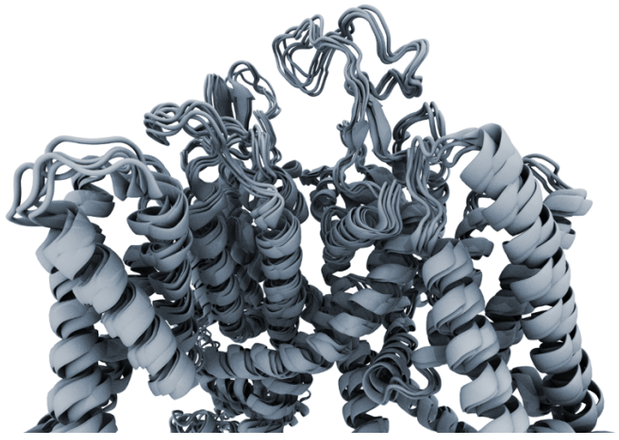

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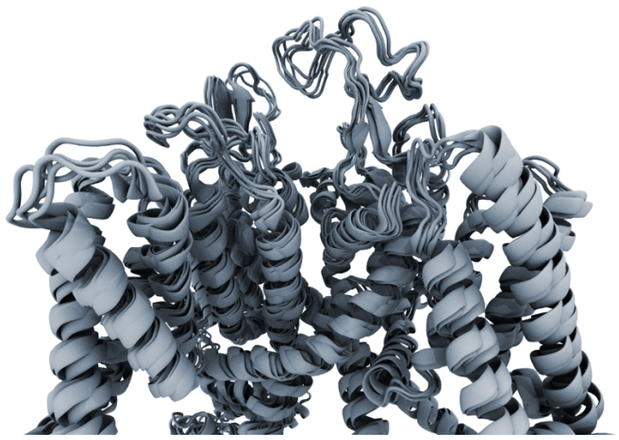

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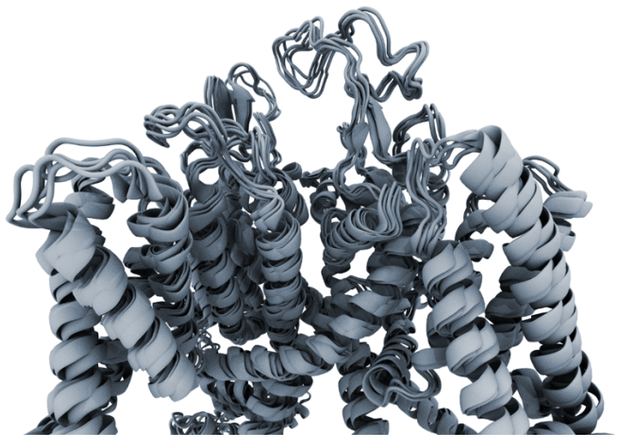

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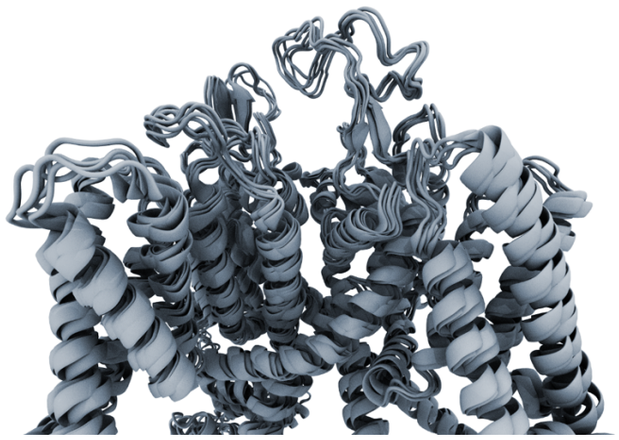

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The Economist

- Election predictions [Heidemanns et al 2020]

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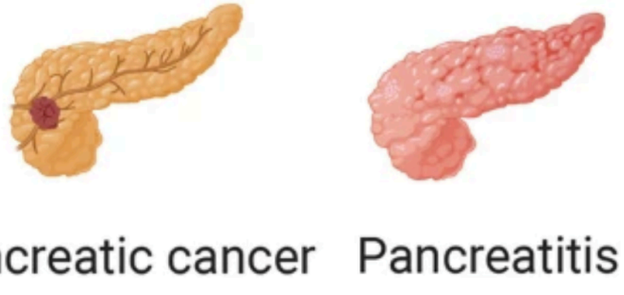

Pancreatic cancer Pancreatitis

- Predicting pancreatic cancer from serum samples

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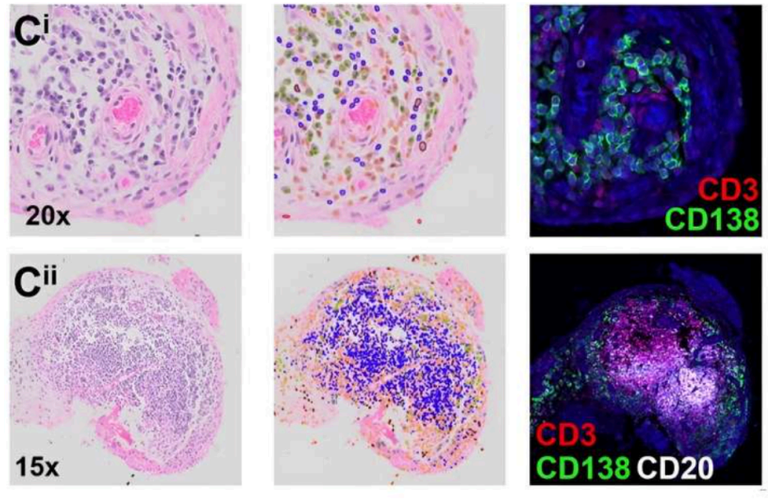

[Weill Cornell Medical College 2024;
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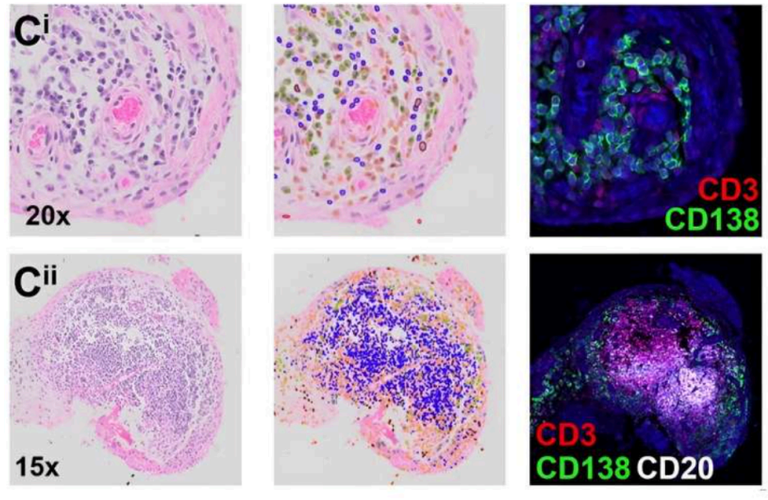

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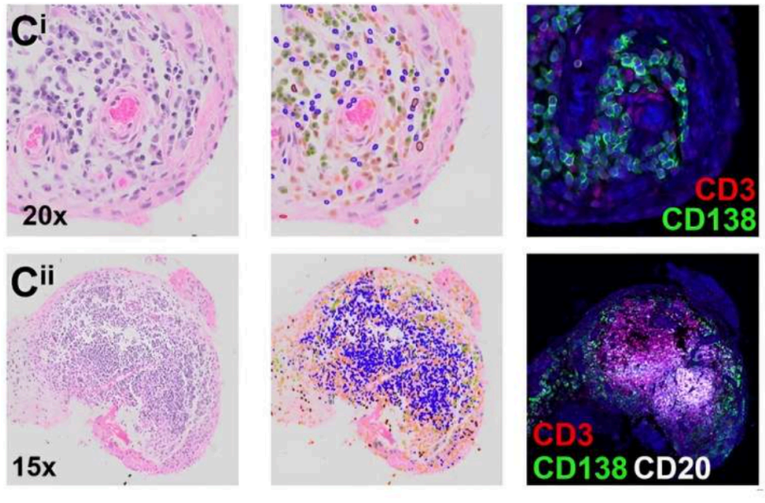

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- Predicting pancreatic cancer from serum samples
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- And many many more…
- **What is machine learning?** A set of methods for using data to become better at a task.
  - No quick summary will do full justice. See the rest of the course!

Machine learning (ML): why & what

- But also: "'[M]any scientists were never really trained properly to [apply ML] because the field is still relatively new,' says Casey Bennett at DePaul University […] 'I see a lot of common mistakes repeated over and over,' he says. For ML tools used in health research, he adds, 'it's like the Wild West right now.'" [Ball 2023]

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- **Why take this course?** To understand the mechanism behind machine learning methods
  - To responsibly apply them
  - To understand and evaluate them
  - To innovate
  - To ultimately advance our understanding of science, engineering, and social science

Getting started

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    - Not a decision rule with the example features above: $h(x) = 1$ if the string “free money” occurs, else 0

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- **Supervised learning**: learn a decision rule from features $x \in \mathcal{X}$ to labels $y \in \mathcal{Y}$ given training set $\{(x^{(n)}, y^{(n)})\}_{n=1}^{N}$

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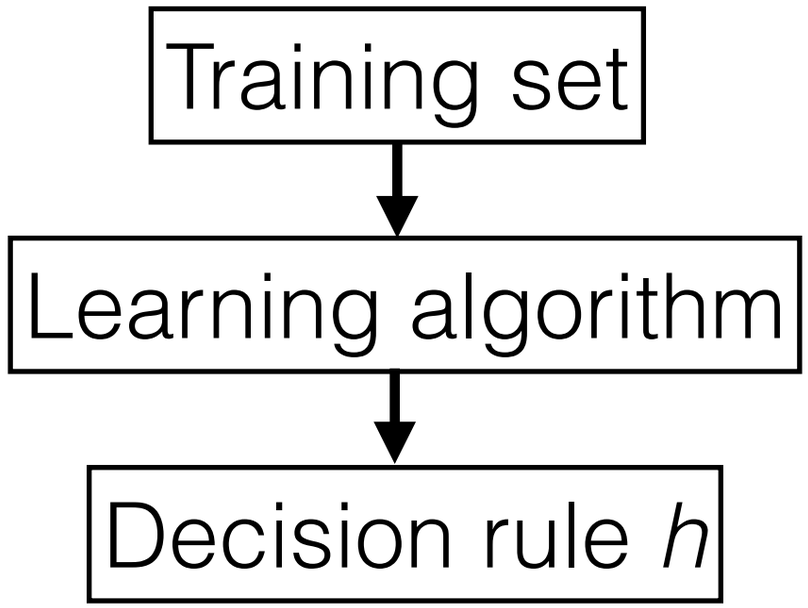

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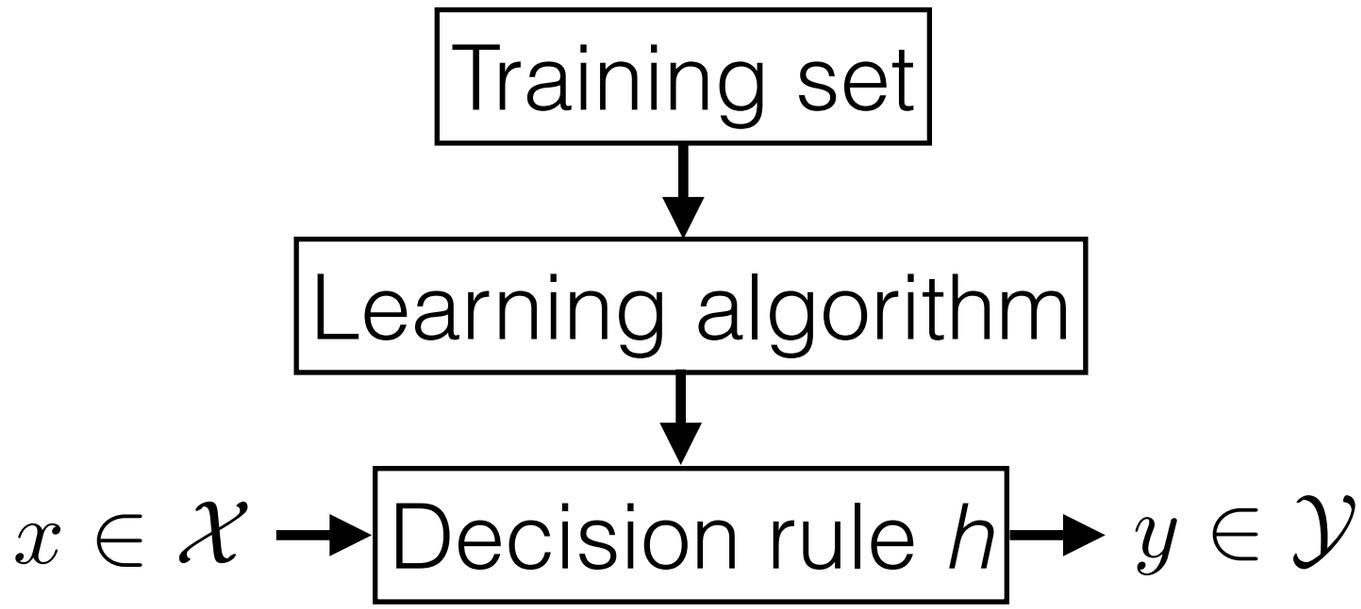

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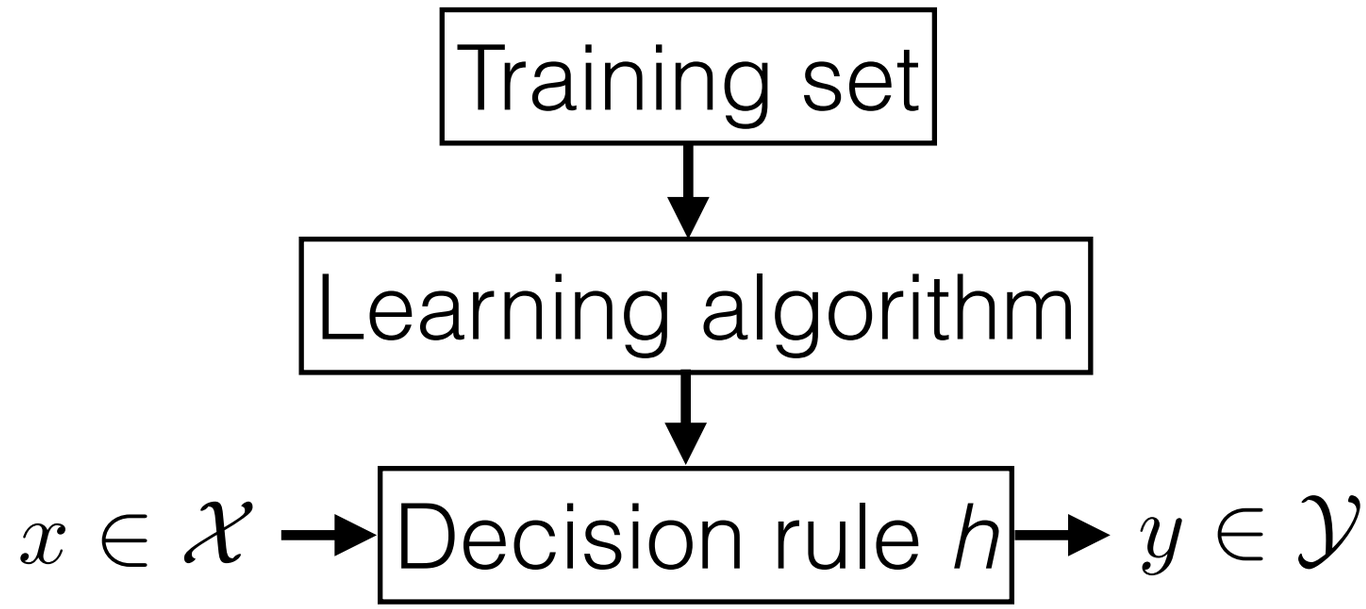

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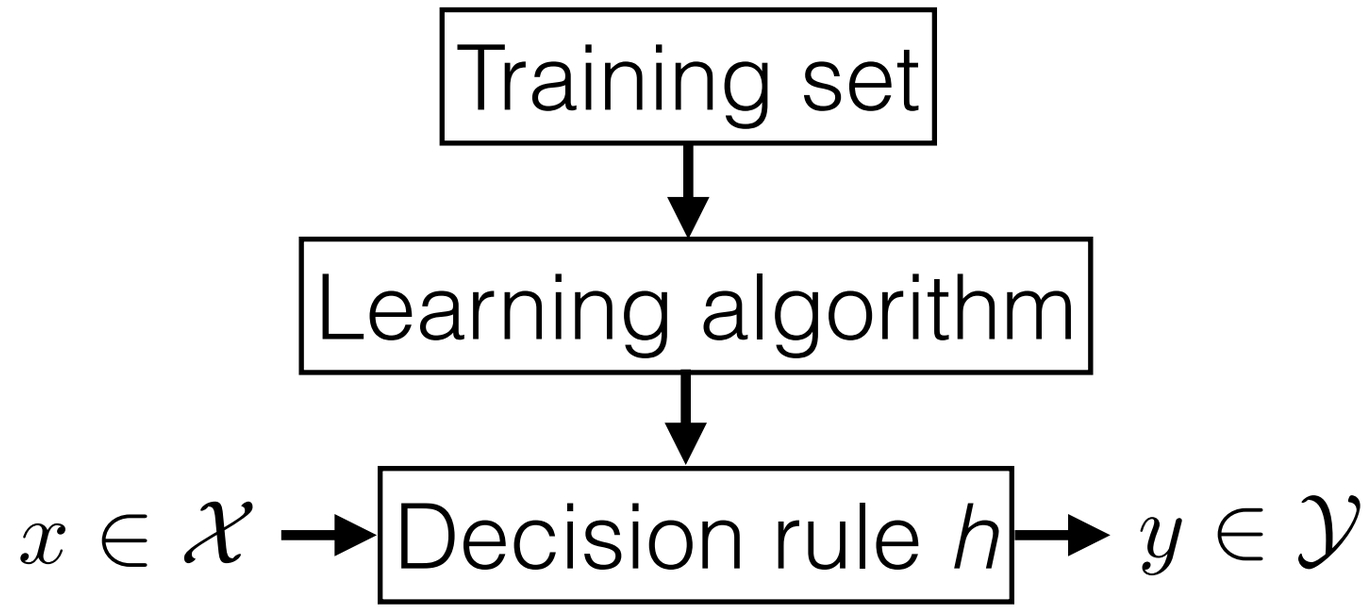

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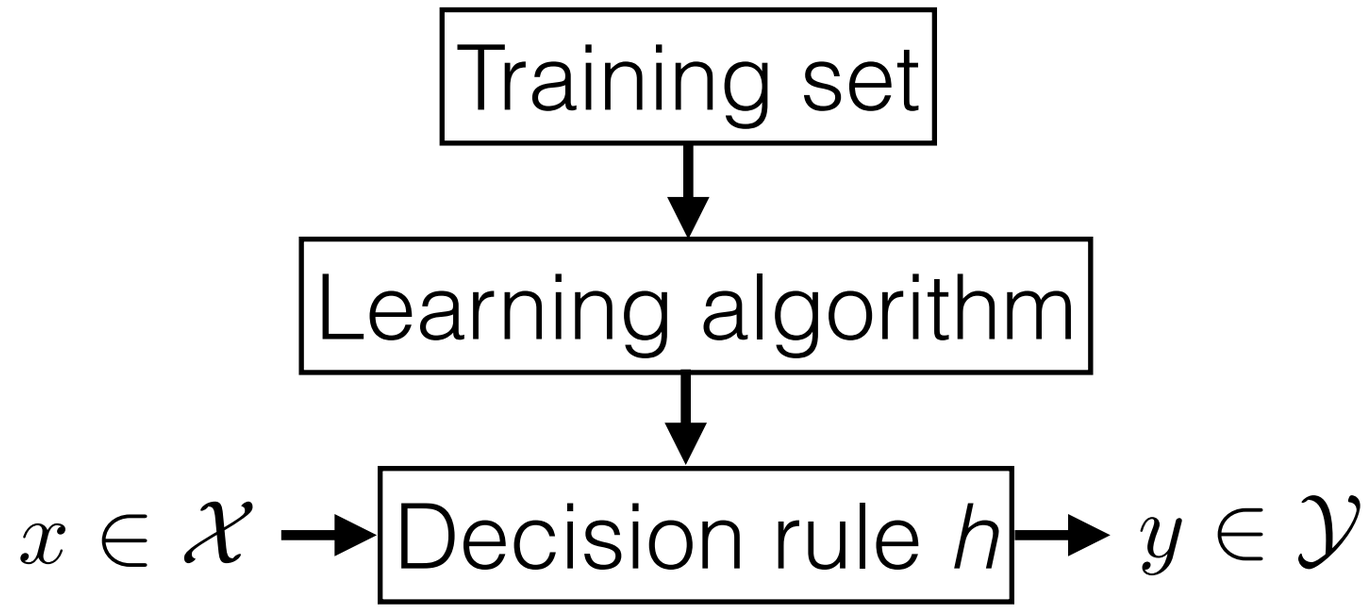

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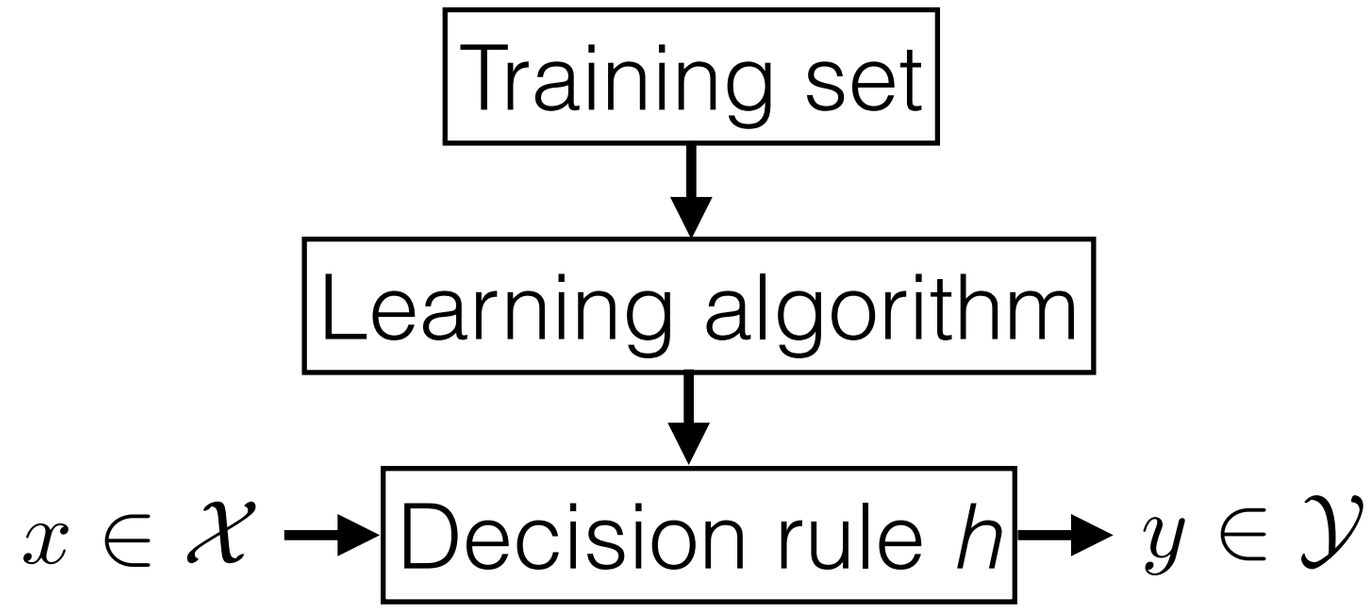

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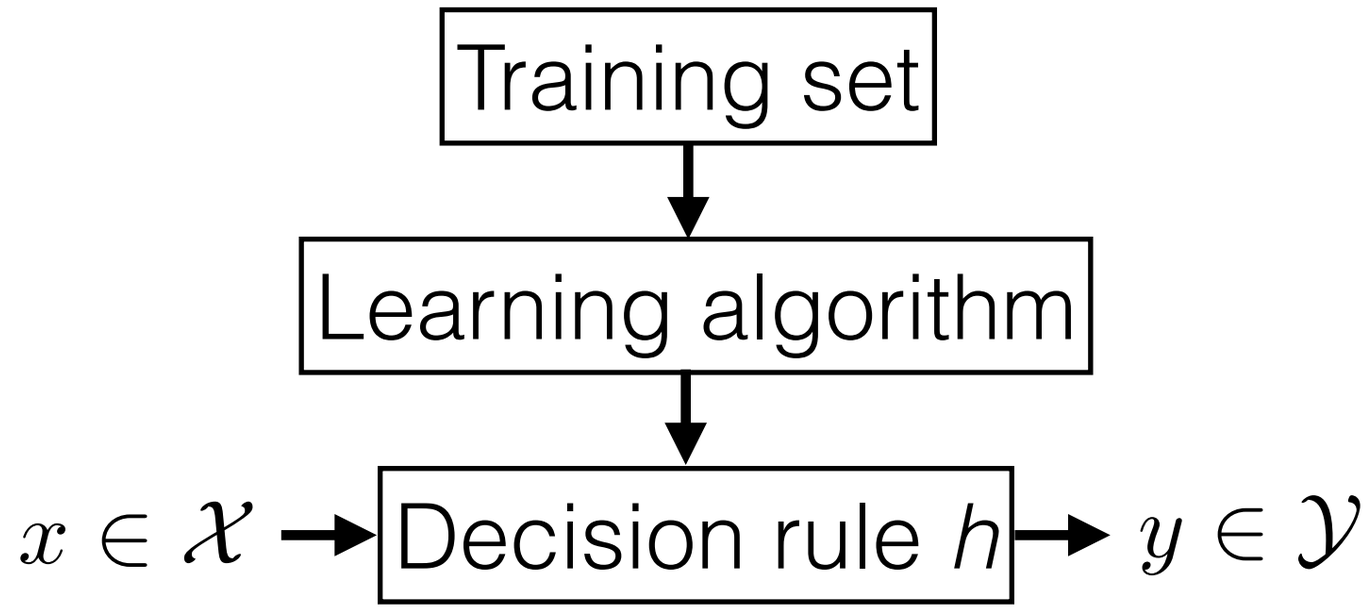

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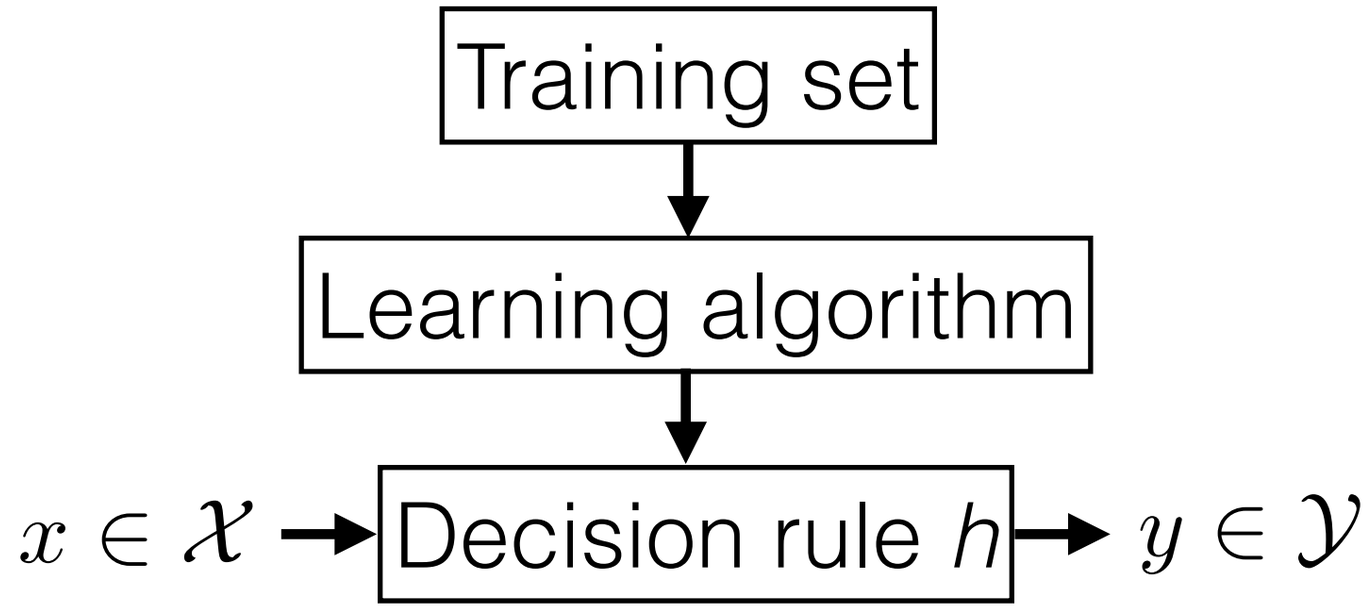

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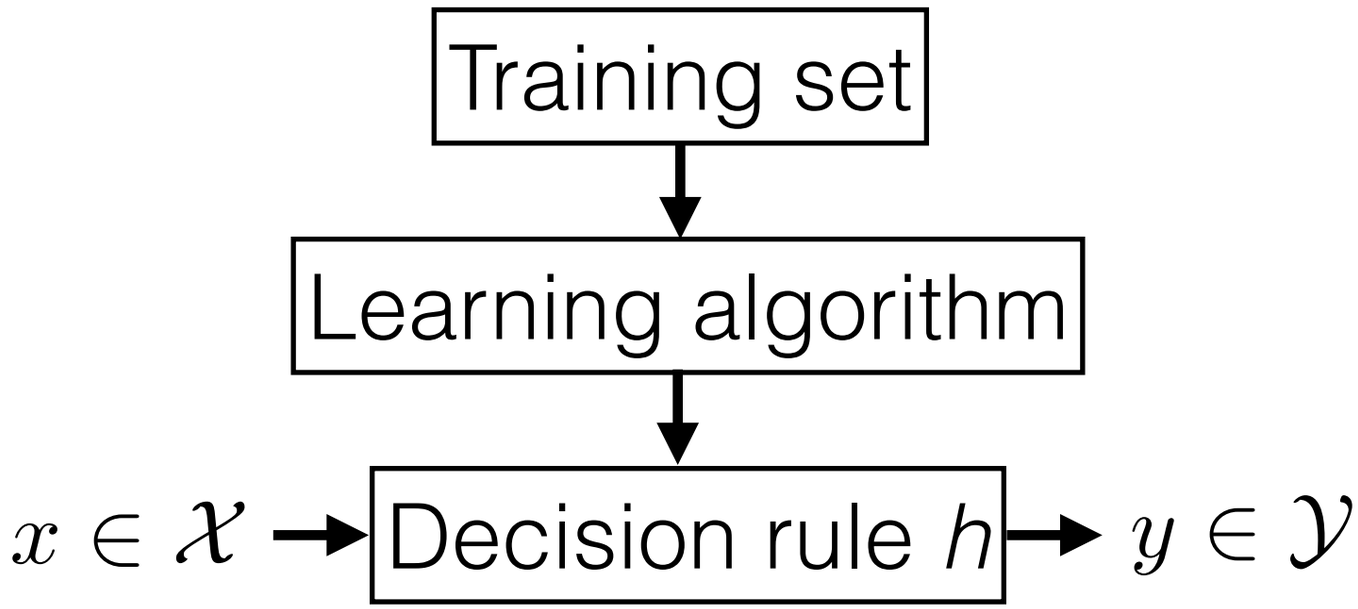

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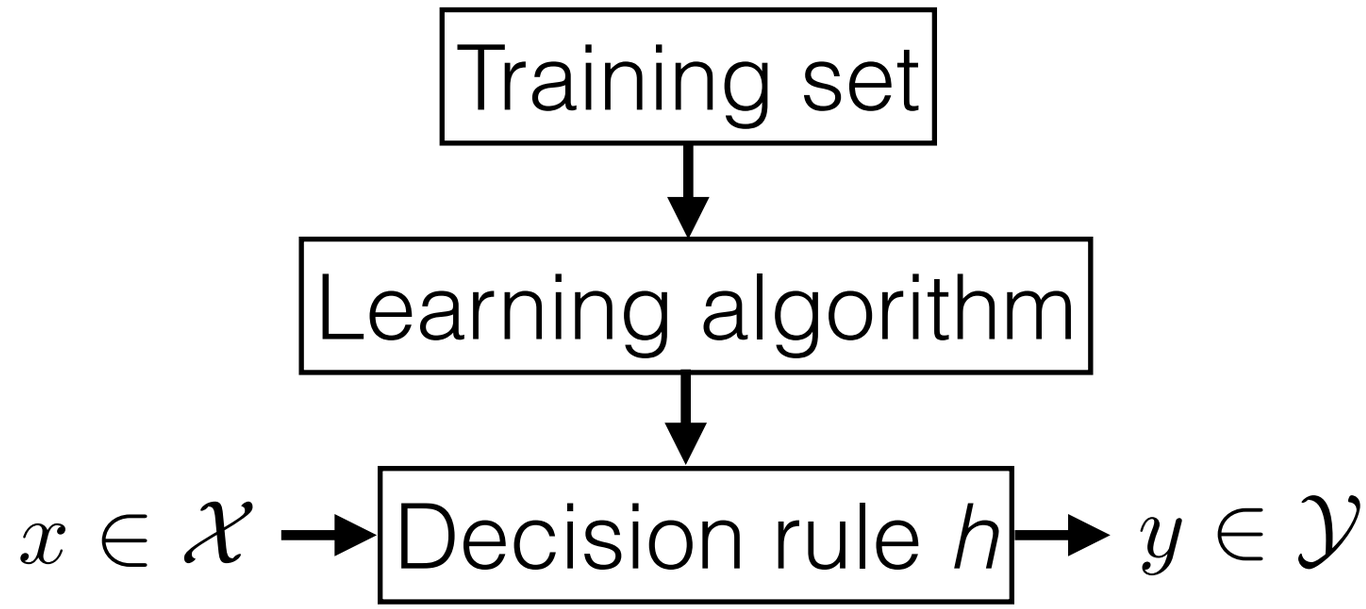

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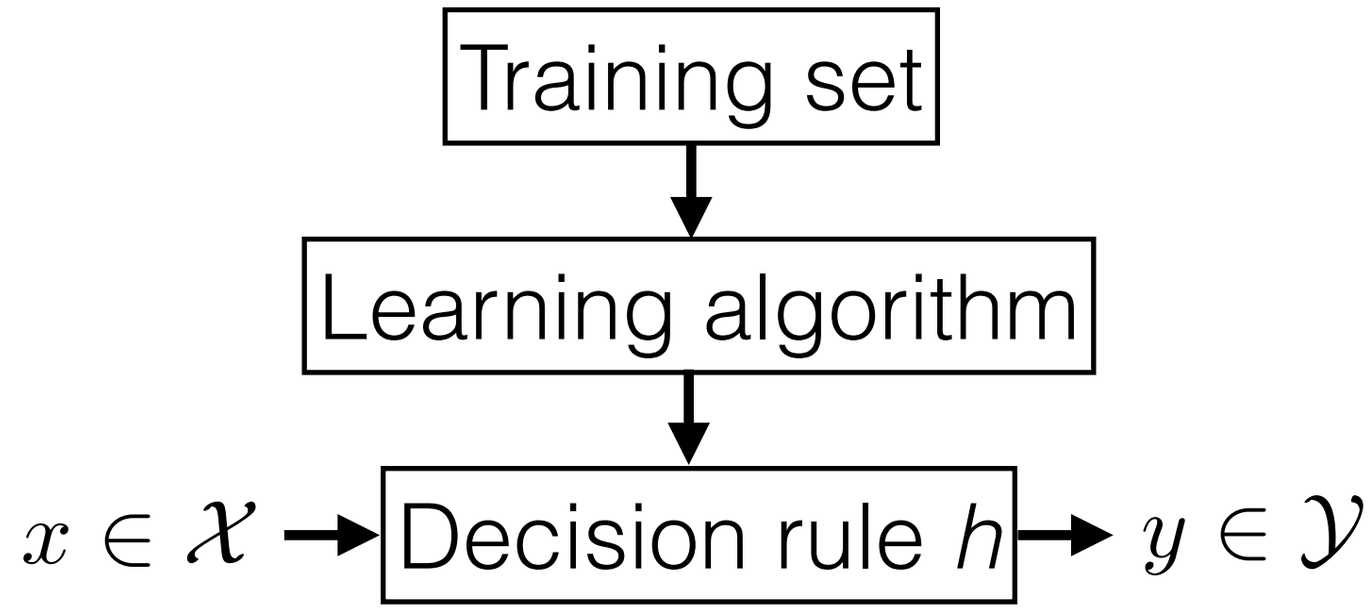

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- If 0-1 loss, choose *h*(*x*)=0 if *p*(*y*=1|*x*) < *p*(*y*=0|*x*)

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  - Hint: it might be useful to observe that

$$y - h(x) =$$
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[An Intro to Statistical Learning 2023, Fig 1.1]

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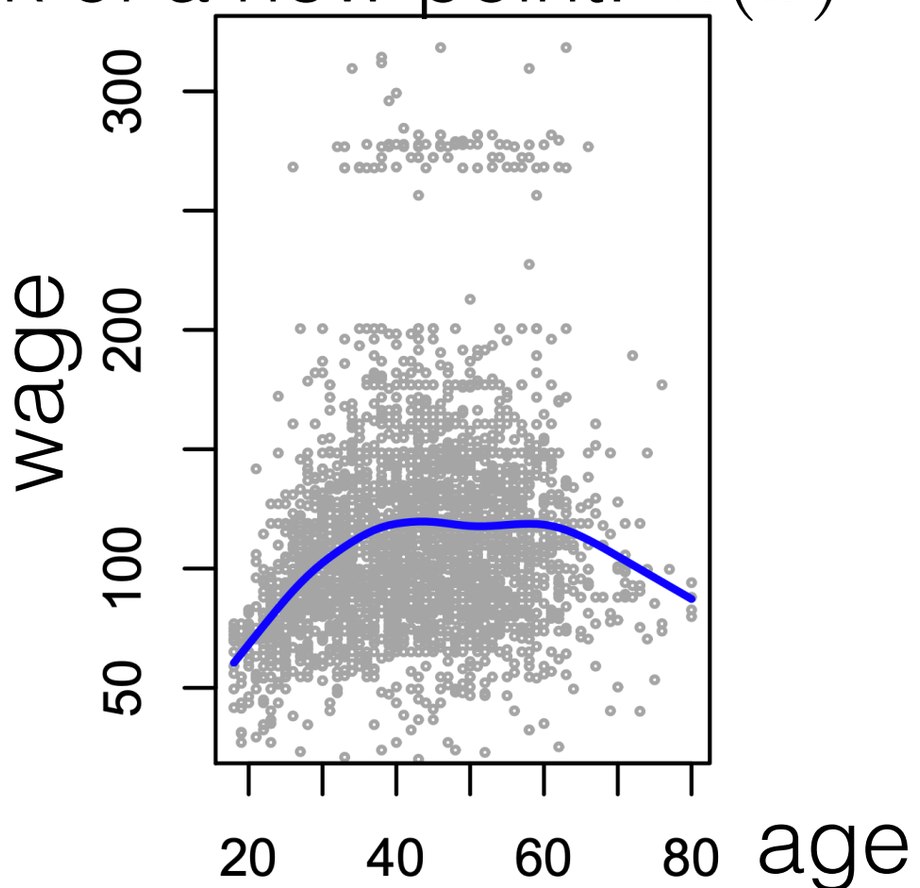

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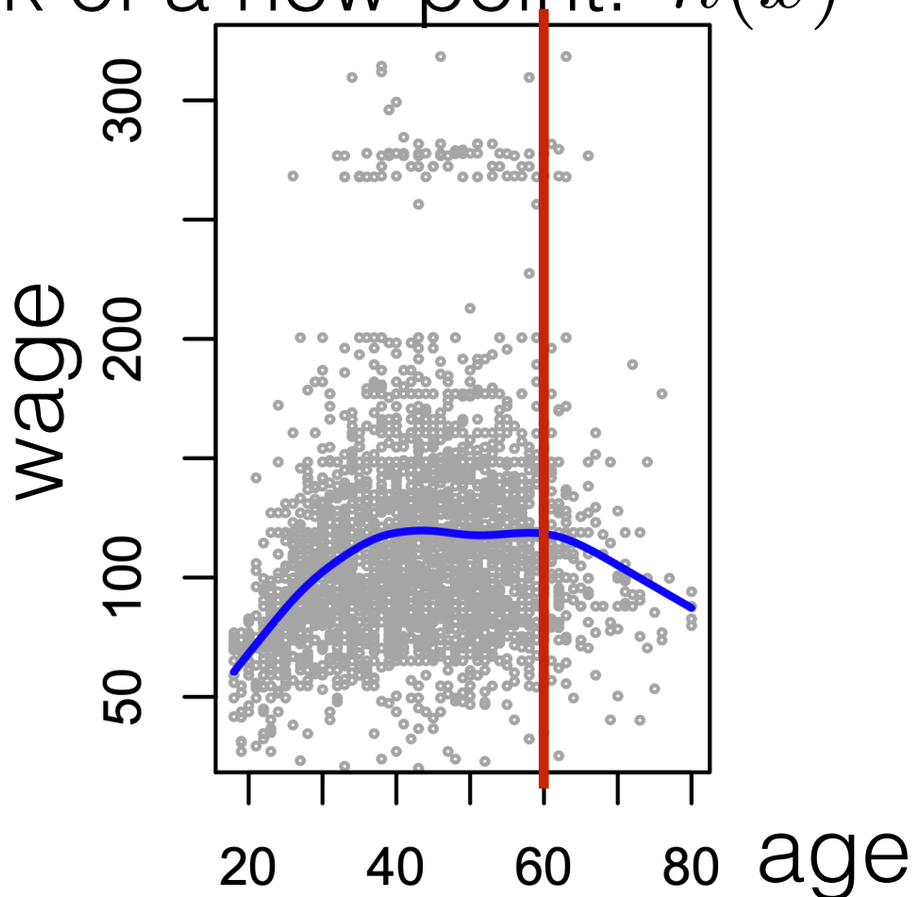

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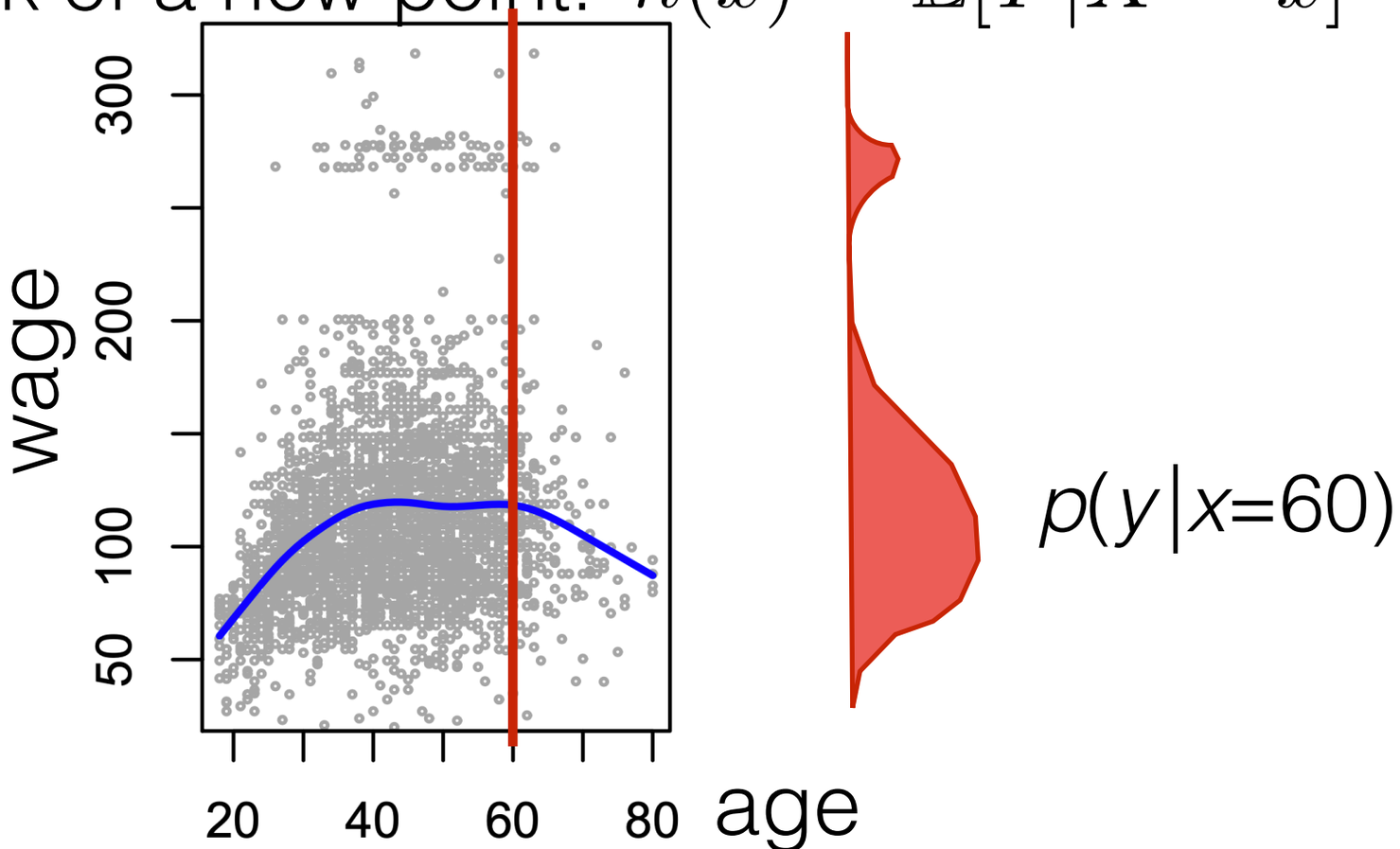

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- Additional idea: how about we replace the **risk** (of a new point) by the **empirical risk** over the training data?

$$\mathbb{E}[L(Y, h(X))] \approx \frac{1}{N} \sum_{n=1}^{N} L(y^{(n)}, h(x^{(n)}))$$

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