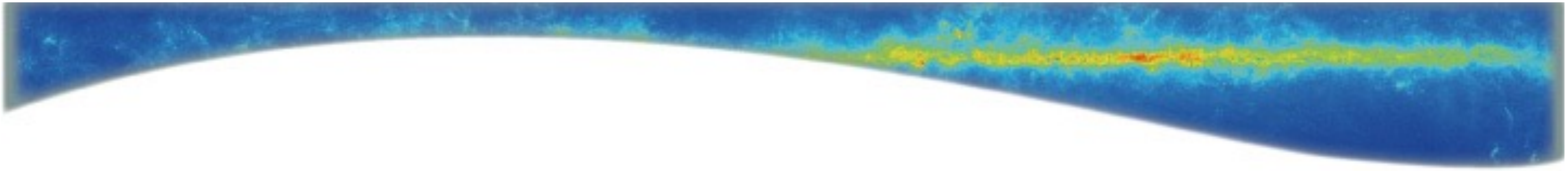

Measuring Visual Extinction by Star Count

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(1) Measurements of the Interstellar Dust

●Except for dark matter, ~99% of the mass in the Galaxy, which can be traced by the electromagnetic wave, is stars, and ~1 % is due to the interstellar matter consisting of gas and dust.

●Atomic gas (H, He) and molecular gas (H_2) occupy ~99% of the interstellar matter, and only ~1% is due to dust.

●Dust occupies a small fraction of the total mass, but it plays an important role for molecular formation and molecular cloud formation, which eventually leads to star formation therein.

●What is the origin? Dust contains elements heavier than He, which originates in nuclear fusion in stars.

●Methods to Measure Dust

Emission in the Far-Infrared (FIR) wavelengths

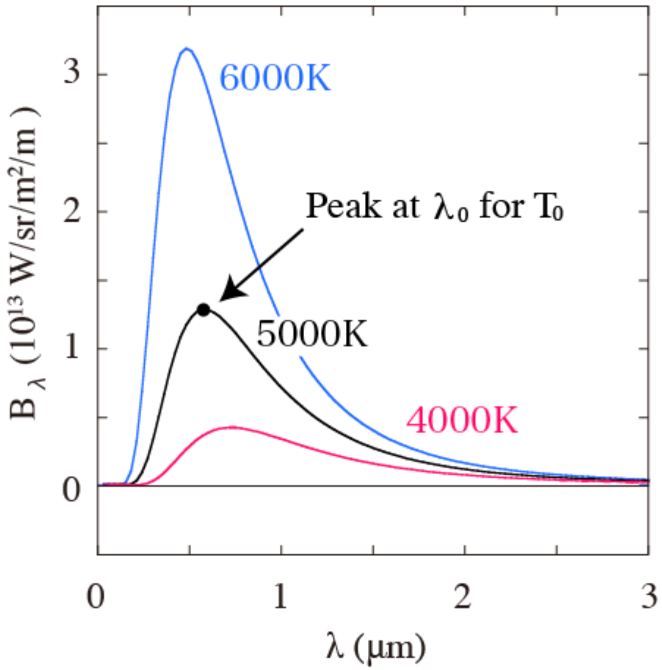
Extinction in the Visible (VIS) to Near-Infrared (NIR) wavelengths

There are some merits and demerits for each of them, and they are complementary.

●Emission

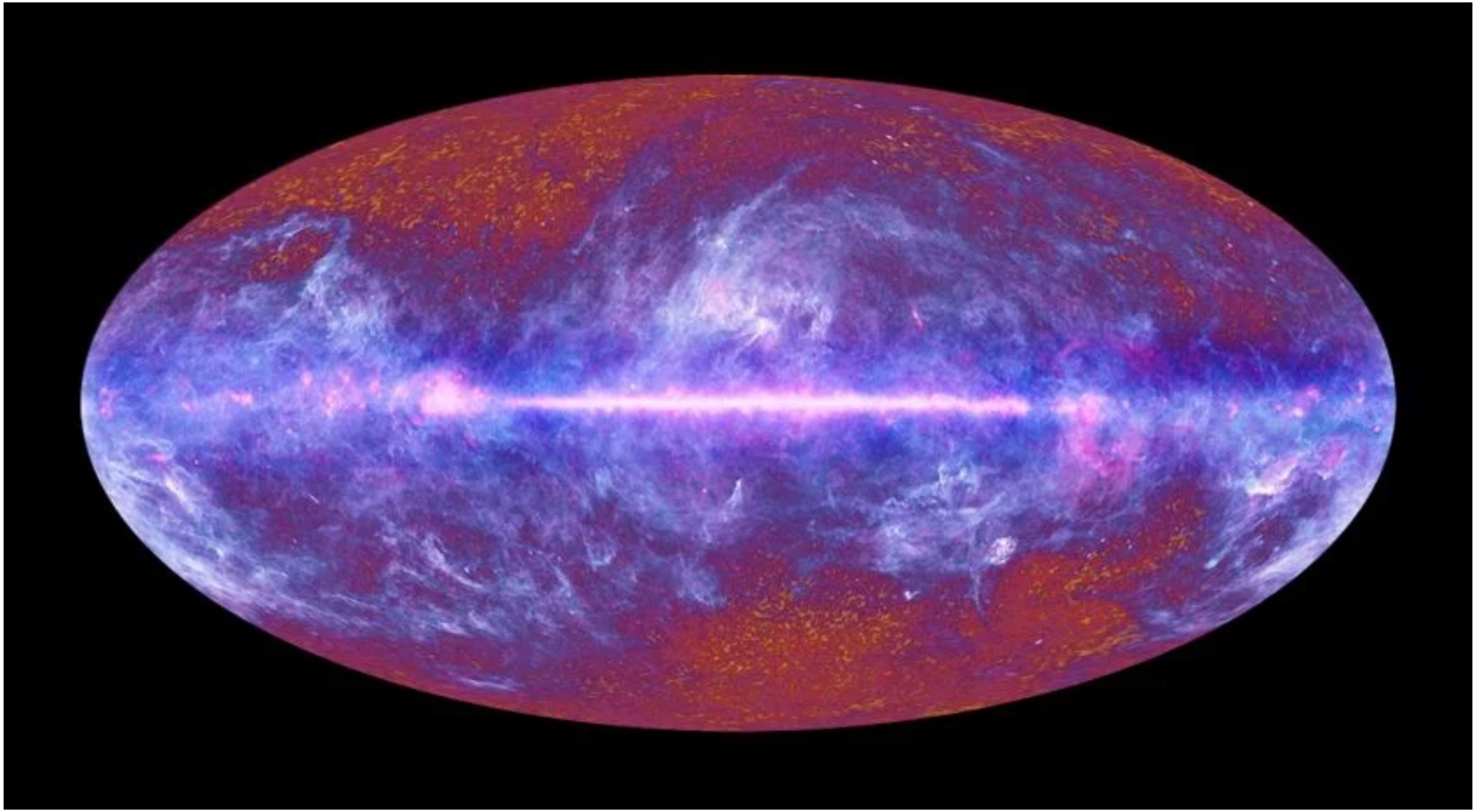
This is the black body radiation from dust grains. The emission is in the FIR because dust temperature is very low (10–20K), and the wavelengths cannot be observed from the ground. Therefore, satellites such as IRAS, Spitzer, Akari, Planck, and Herschel are used.

Q1: Blackbody radiation from the Sun whose surface temperature is ~6000K peaks at ~0.5 μm. Calculate the peak wavelengths of the emission from the Earth (~300K) and interstellar dust in dark clouds (~10K).



Wien's displacement law

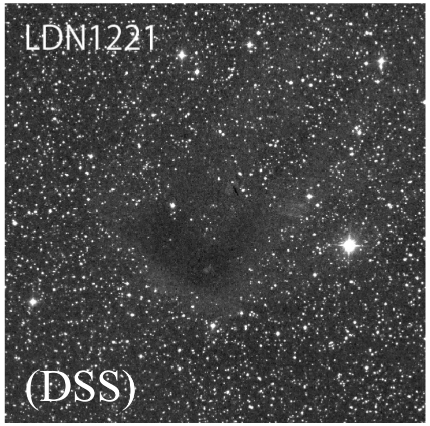
$$\lambda_0 T_0 = \text{constant } (=2900\ \mu\text{m K})$$



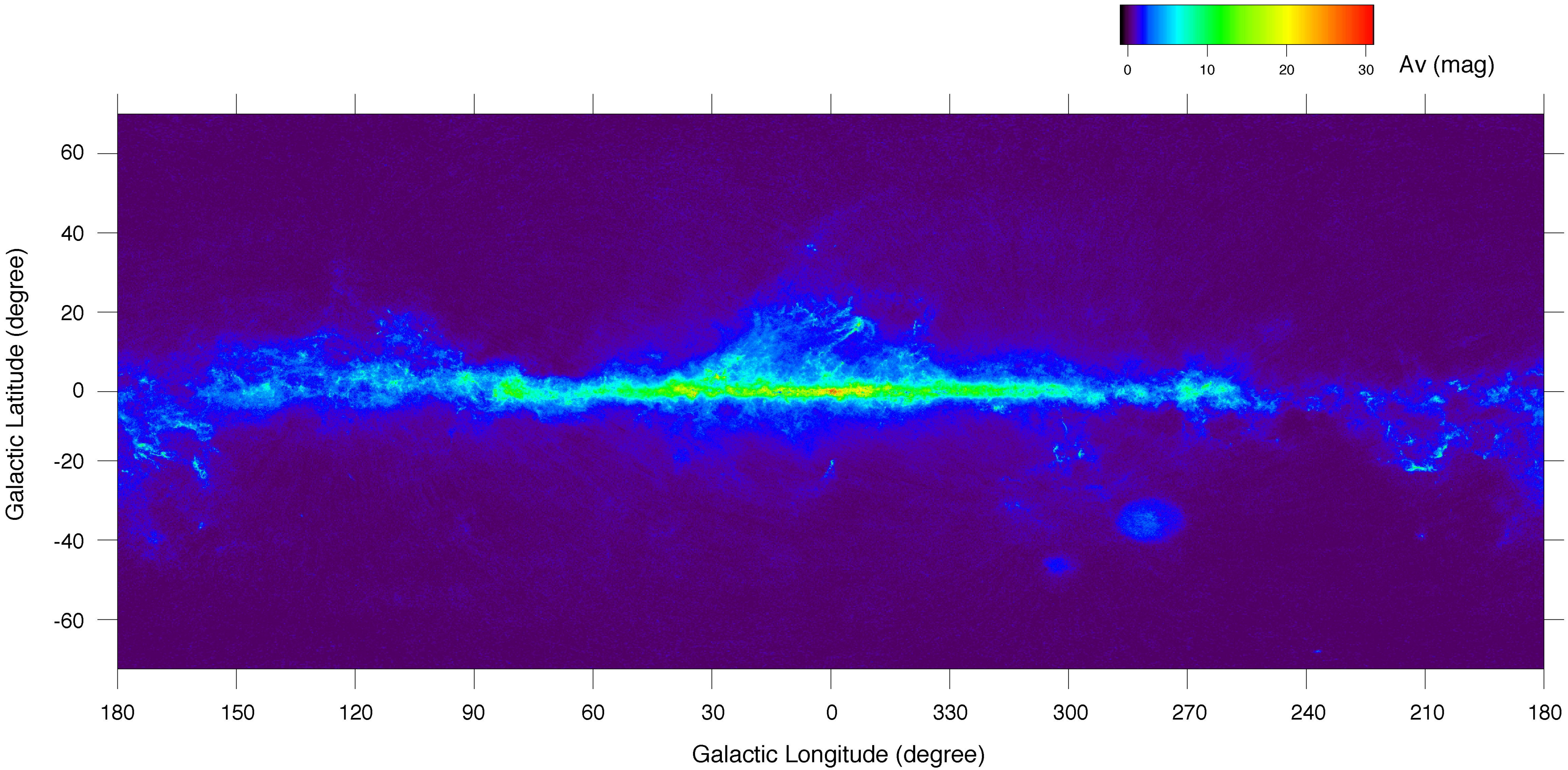
Distribution of dust revealed by Planck (ESA)

●Extinction

In a photograph of the Milky Way, you easily find “dark clouds” . They appear dark because starlight from the background is absorbed and scattered. The extinction is often measured in VIS an NIR.

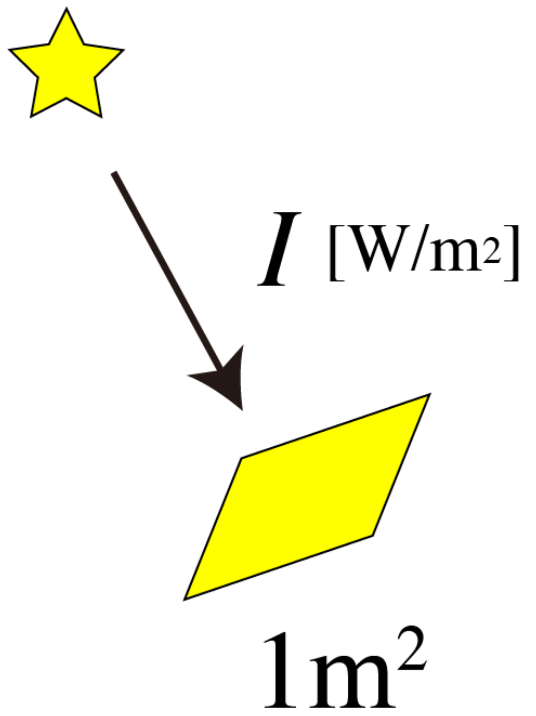


If we can quantify how much the star light is absorbed, it is possible to make a map of dust distribution, which is called "extinction map". This is what we will study today.



Dobashi et al. (2005, 2011)

(2) Pogson's Equation



● Definition of "magnitude" of stars

I : Flux density (or intensity) of star light that we observe.

m : Magnitude of the star

Pogson's Equation

$$m = -2.5 \log I + C \qquad [1]$$

where C is a constant that can be defined depending on the instruments. C is designed so that m for A0V type stars (such as Vega) is ~0 mag.

When the magnitude and intensity of star "A" are m_A and I_A, and those of star "B" are m_B and I_B, the Pogson's equation yields

$$m_A = -2.5\log I_A + C \qquad [2]$$

$$m_B = -2.5\log I_B + C \qquad [3]$$

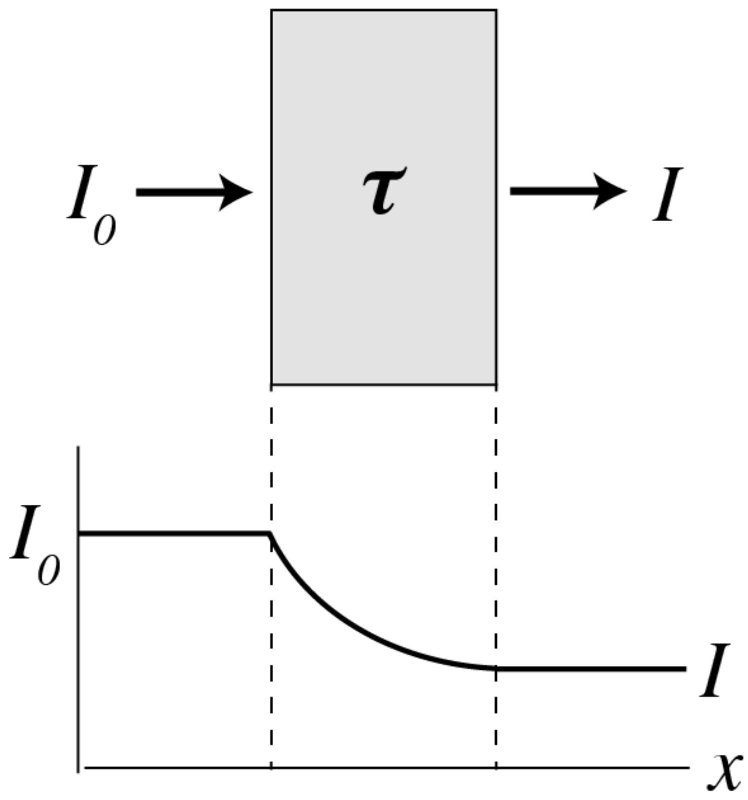
and thus

$$\frac{I_B}{I_A} = 100^{\frac{m_A - m_B}{5}} . \qquad [4]$$

Q2: Calculate I_B/I_A for stars with m_A=16 mag and m_B= 6 mag.

Q3: Suppose that there are 2 distinct stars closely located and that each of them has m=10 mag. What would be the magnitude if you regard them as a single star? (hint: log2 = 0.3)

(3) Optical Depth and Extinction

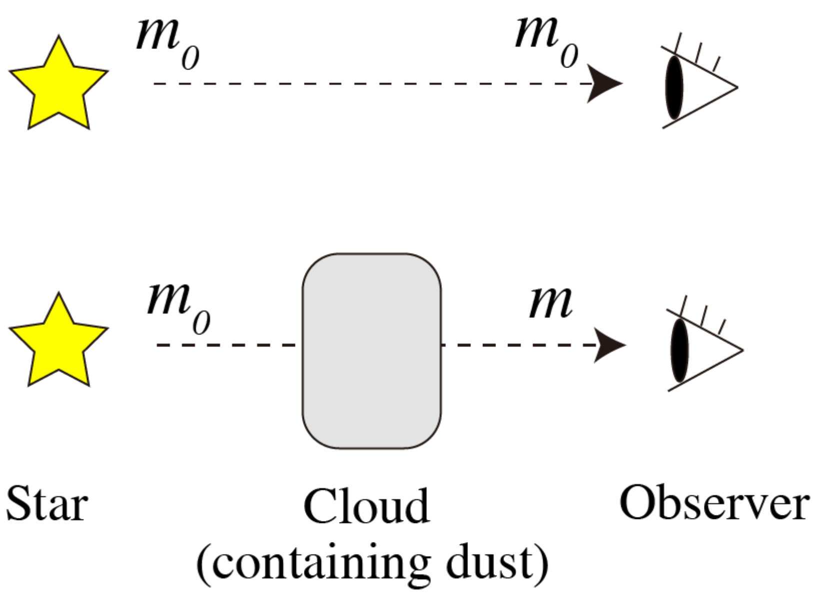


● Optical Depth

The “optical depth”, τ, is proportional to the total number of particles which absorb and scatter the light along the path. Input and output intensities of light are related by τ as,

$$I = I_0 e^{-\tau}. \qquad [5]$$

Extinction in magnitude



We can express the relationship I_0 and I by substituting [5] to Eq.[1].

$$-2.5\log I_0 + C = -2.5\log I + C + (2.5\log e)\tau$$

$$\downarrow$$

$$m = m_0 + 1.0857\tau$$

$$\swarrow$$

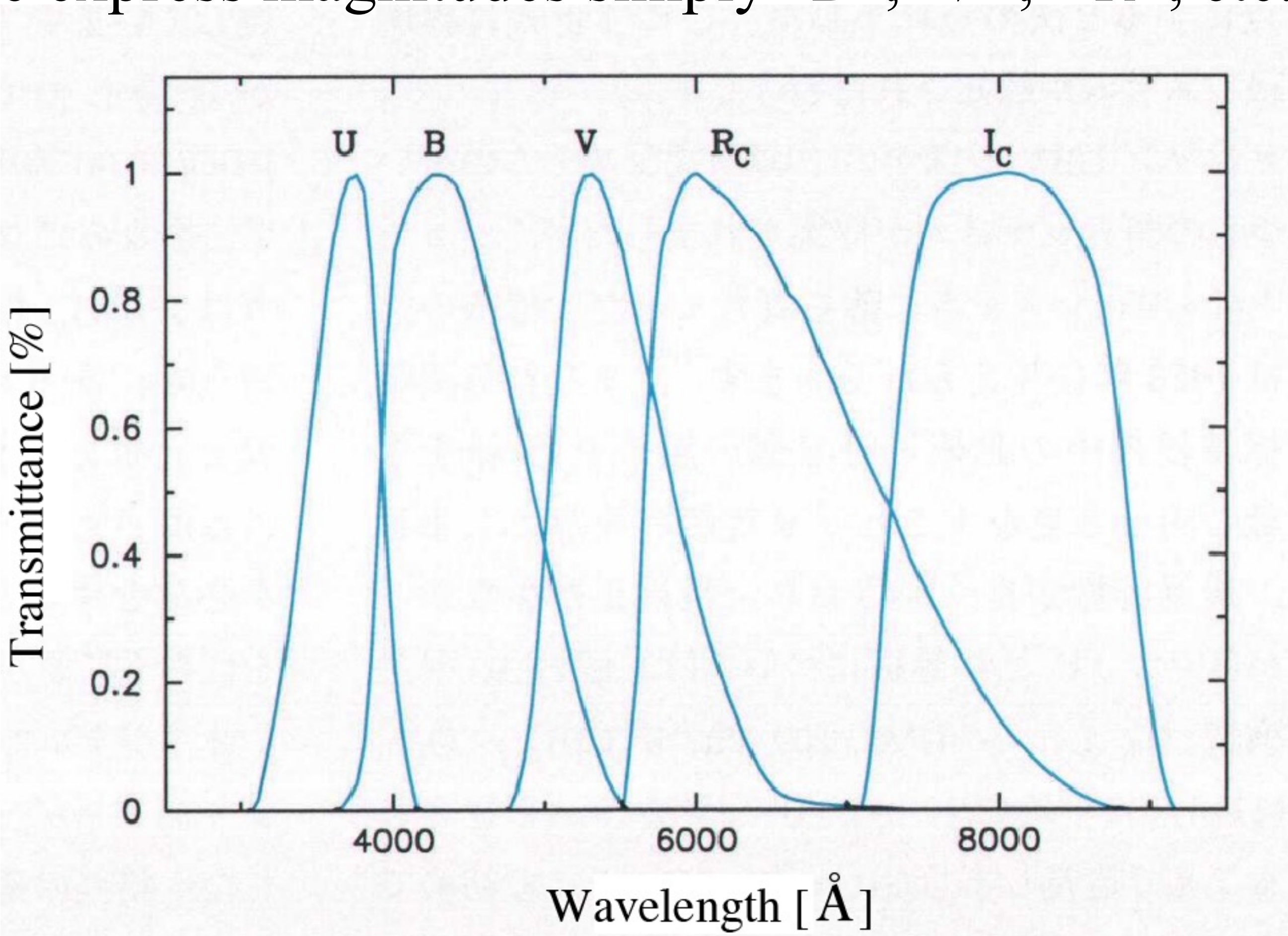
$$A = m - m_0 (= 1.0857\tau) \qquad [6]$$

A, the difference of m and m_0, is the "extinction" in units of "mag", and it is equivalent to the optical depth τ.

Q4: Magnitude of a star with the intrinsic magnitude $m_0 = 5$ mag is observed to be $m = 7$ mag due to a dusty cloud in the foreground. Answer what is A (extinction) by the cloud.

(4) Photometric System

We measure the magnitude of stars using a certain set of filters. There are some standard sets of filters such as the "Johnson-Cousins sytem". We express magnitudes simply "*B*", "*V*", "*R*", etc.

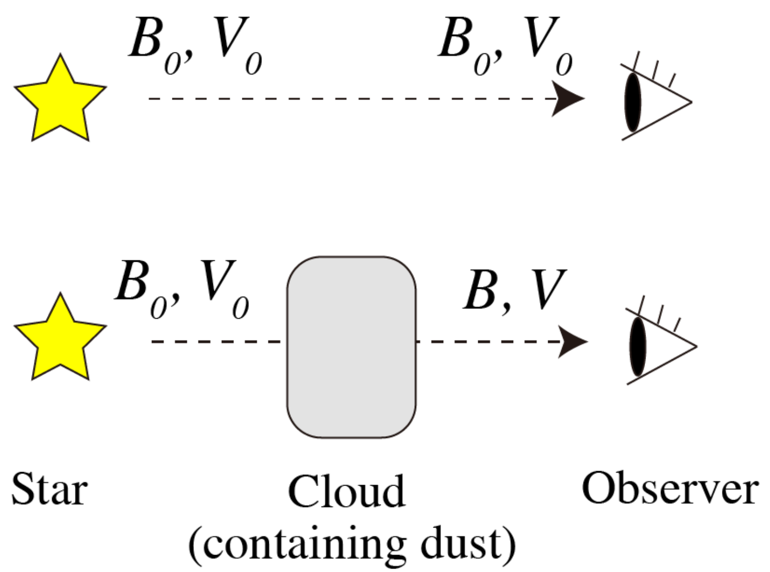


U: 0.35μm
B: 0.44μm
V: 0.55μm
R: 0.66μm

(5) Color, Color Excess

● Color and Color Excess

Difference of magnitudes measured at two different bands, such as $B - V$ and $V - R$, is called "color" of the star.



Suppose that there is a star whose original/intrinsic magnitudes are B_0 and V_0, and we observe it through a cloud and measure its magnitudes to be B and V. Difference of the observed and intrinsic color is called "color excess", and is expressed as $E(B - V)$.

● Color Excess

Definition of the color excess is

$$E(B-V) = (B-V) - (B_0 - V_0)\ , \qquad [7]$$

which can be transformed to

$$E(B-V) = (B - B_0) - (V - V_0)$$

$$= A_B - A_V\ . \qquad [8]$$

Thus, the color excess is equal to the difference of extinction in the 2 bands.

Q5: Calculate A_B, A_V, and $E(B-V)$ for the case B_0=5, V_0=4, B=8, and V=6 mag.

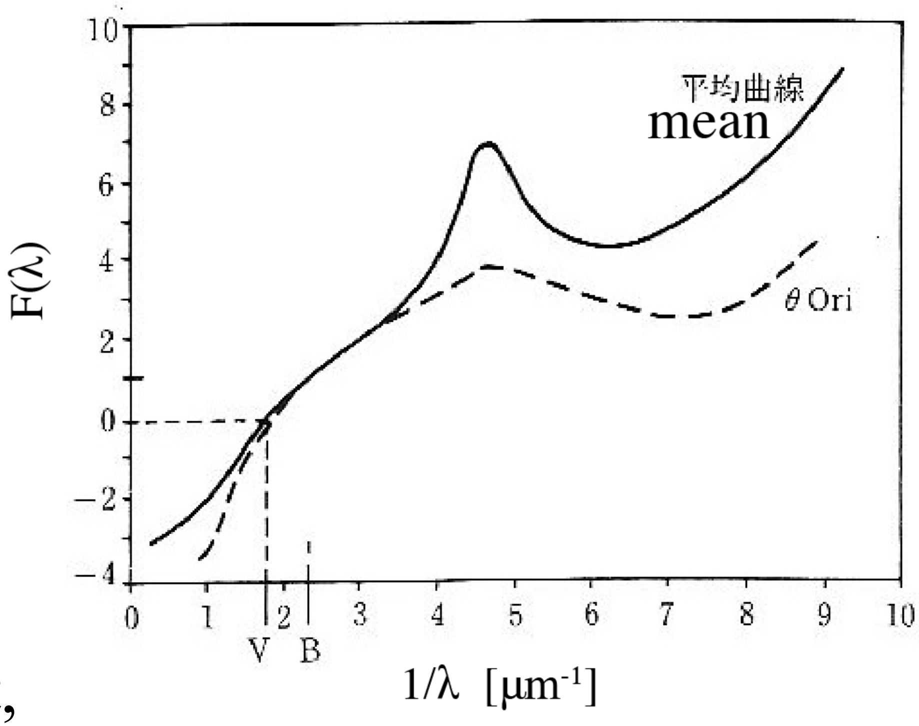
(6) Reddening Law

We define $F(\lambda)$ which is called "reddening law" or "extinction curve" as

$$F(\lambda) = \frac{E(\lambda - V)}{E(B - V)} \qquad [9]$$

which can be transformed to

$$F(\lambda) = \frac{A_\lambda - A_V}{E(B - V)} \; . \qquad [10]$$



Scheffler and Elsasser (1988)

$F(\lambda)$ varies depending on the regions observed. The intercept, $F(\lambda)$ at $1/\lambda = 0$ in the right figure, is called "total-to-selective extinction ratio" and is expressed as "R_V".

Because $A_\lambda \to 0$ for $\lambda \to \infty$ (or $1/\lambda \to 0$) ,

Equation [10] leads to

$$-R_V = \frac{-A_V}{E(B-V)} \quad , \qquad [11]$$

and thus

$$A_V = R_V E(B-V) \quad . \qquad [12]$$

The average value in the solar neighborhood is known to be

$$R_V \approx 3.1 \quad . \qquad [13]$$

R_V varies, depending on some dust properties such as the size-distributions and components (i.e., graphite and silicate) of dust grains.

Equations [9] –[13] indicate that the values of extinction A_λ for various λ are proportional to each other, and so are the color excess $E(\lambda_1 – \lambda_2)$ for a constant value of R_V .

Q6: Calculate A_B/A_V for $R_V = 3$ and $R_V = 6$.

表 10.1 平均の星間減光量(Scheffler and Elsässer 1988, 基本文献)

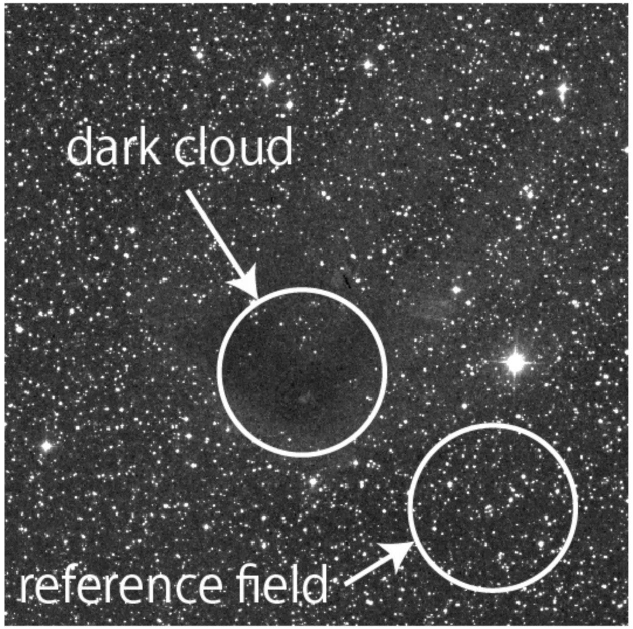
λ (μm)	A(λ)/Av	λ (μm)	A(λ)/Av
0.1000	4.60	0.5000	1.16
0.1250	3.10	0.5480(V)	1.00
0.1375	2.73	0.5840	0.91
0.1590	2.57	0.6050	0.88
0.1830	2.53	0.6436	0.83
0.2000	2.80	0.7100	0.71
0.2080	2.98	0.7550	0.68
0.2140	3.09	0.8090	0.62
0.2190	3.10	0.8446	0.58
0.2230	3.07	0.871	0.54
0.2360	2.68	0.970	0.47
0.2500	2.39	1.061	0.40
0.2740	2.03	1.087	0.38
0.3200	1.74	1.25	0.30
0.3400	1.64	2.2	0.15
0.3636	1.56	3.4	0.10
0.4036	1.43	4.9	0.05
0.4255	1.35	8.7	0.01
0.4366(B)	1.32	10.0	0.01
0.4566	1.26	∞	0.00

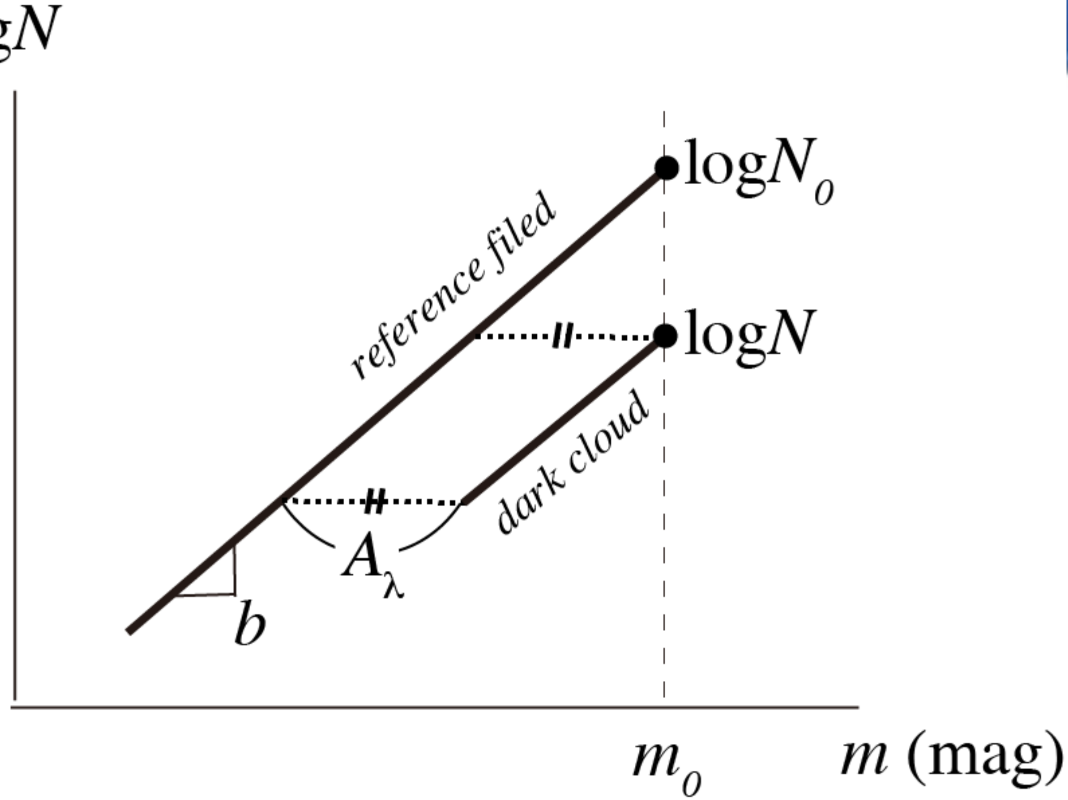
(7) Star-count

A dark cloud can be recognized as a region where we don't see many stars compared with surrounding regions.
This is because the cloud is located in the distribution of stars and star light from the background is absorbed and scattered by dust in the cloud.

R band image (DSS)

Wolf Diagram





$$A_\lambda = \frac{1}{b}(\log N_0 - \log N) \qquad [14]$$

m_0 : Limiting magnitude of the image.

N_0 : Cumulative number of stars brighter than m_0 in the reference field.

N : Save as N_0 but in the cloud.

b : Slope of the Wolf diagram.

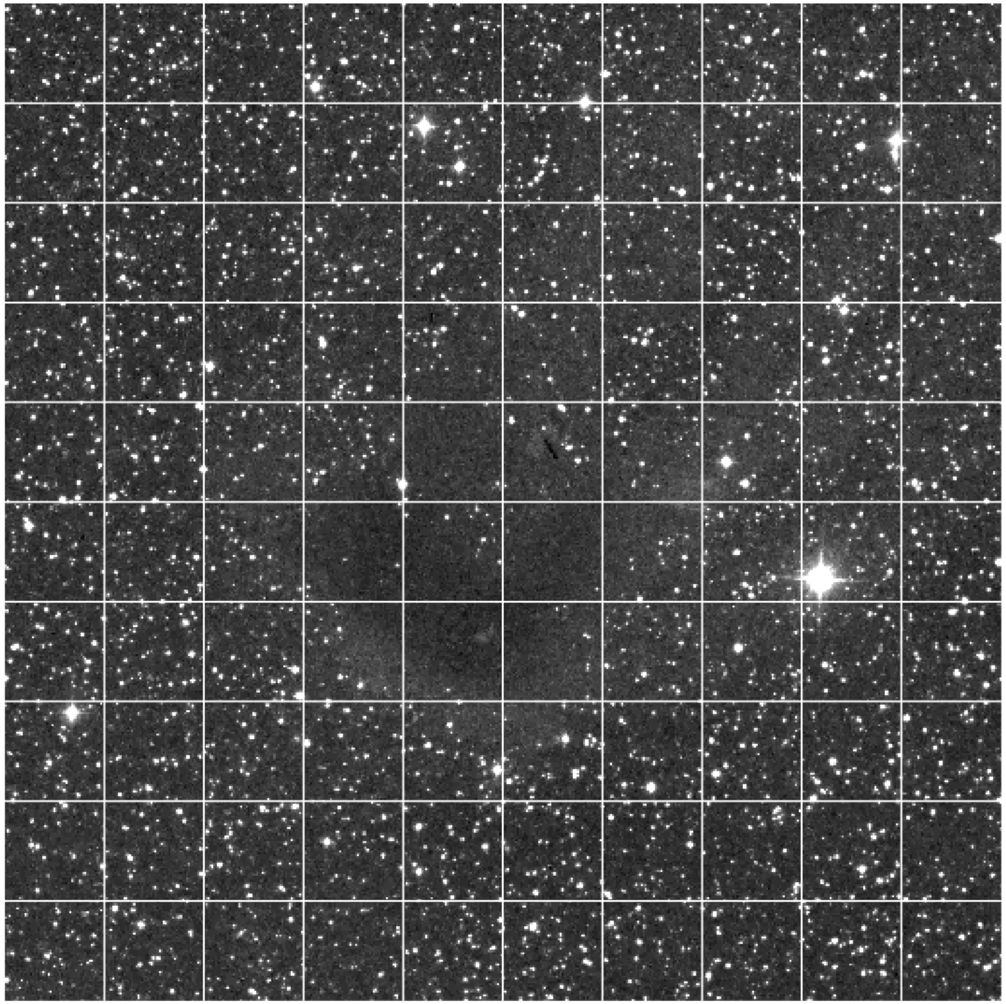
A_λ : Extinction by the dark cloud in λ band (λ=U, B, V, R, I, etc).

In general, the slope of the Wolf diagram b often takes a value of ~0.3.

Q7: Estimate the value of b if all of the stars have the same absolute magnitude and if they are distributed uniformly.

● Star-count

$$A_\lambda = \frac{1}{b}(\log N_0 - \log N) \qquad [14]$$



1. Set mesh on an image.

2. Count stars in each cell.

   → distribution of N

3. Define the reference field, and calculate the mean star numbers.

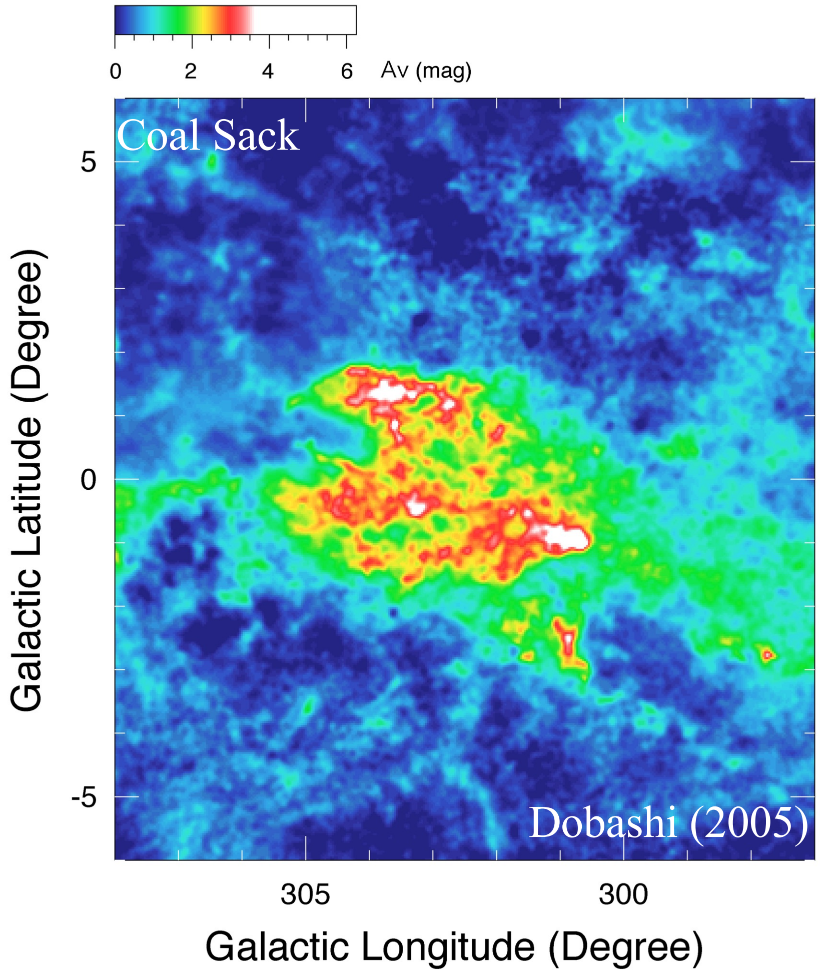
   → a value of N_0

4. Find a value of b, and derive A_λ for each cell.

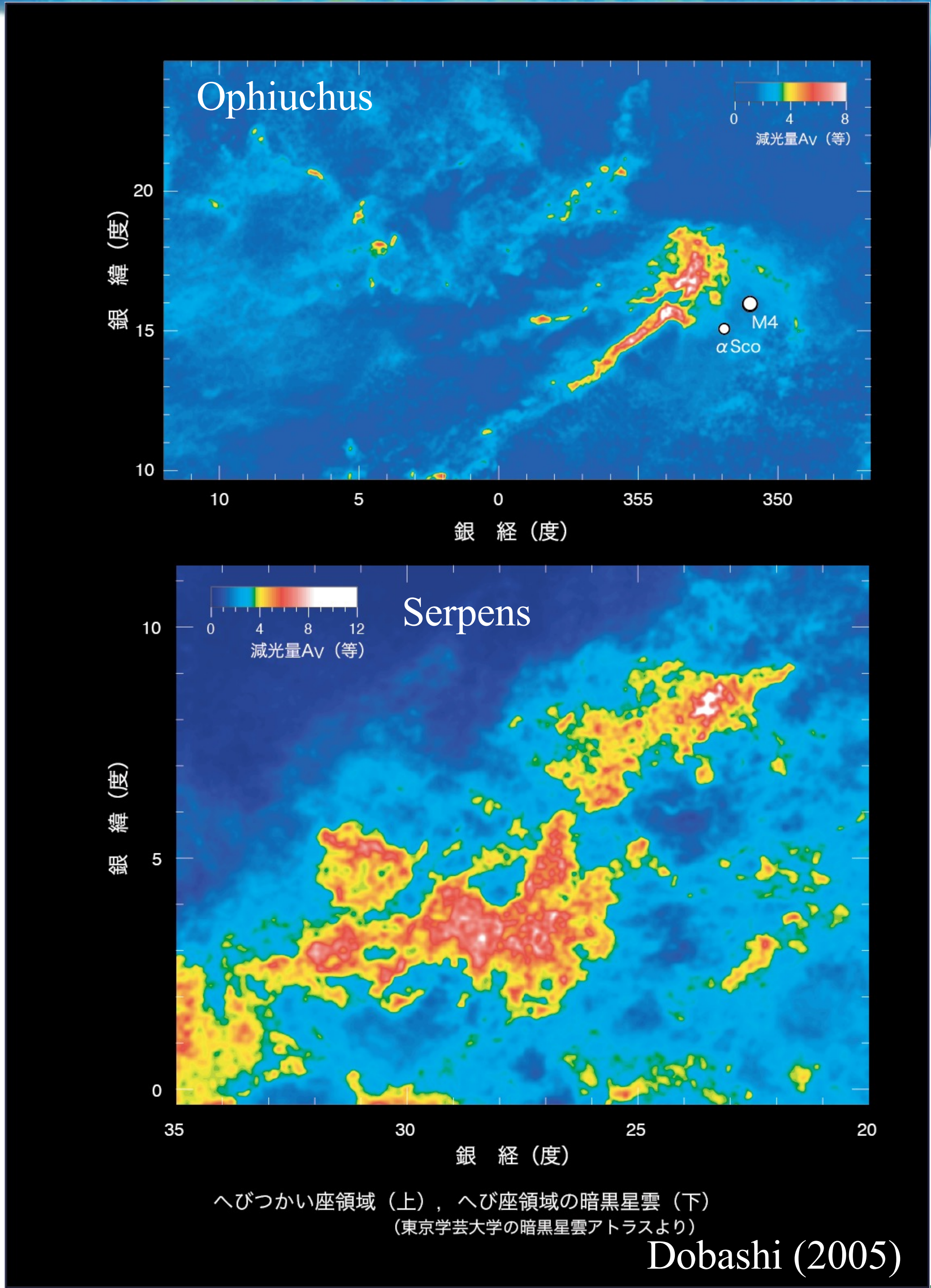
   → distribution of A_λ

5. Convert A_λ to A_V if necessary.

Examples



Nowdays, people can do 3D extinction maps using database of stars such as GAIA.
https://www.youtube.com/watch?v=aOevXQqLRgE



Error

The major error in the extinction maps is the counting uncertainty of star number, N. We often take its uncertainty to be $\Delta N = \sqrt{N}$. Of course, there are other smaller errors arising from b, N_0, etc.

$$\Delta A_\lambda = \sqrt{\left(\frac{\partial A_\lambda}{\partial b}\right)^2 \Delta b^2 + \left(\frac{\partial A_\lambda}{\partial N_0}\right)^2 \Delta {N_0}^2 + \left(\frac{\partial A_\lambda}{\partial N}\right)^2 \Delta N^2}$$

$$\approx \left|\frac{\partial A_\lambda}{\partial N}\right| \Delta N = \frac{1}{\ln 10}\frac{1}{bN}\sqrt{N} \approx \frac{1}{2.3b\sqrt{N}} \qquad [15]$$

Q8: When you have $b = 0.3$, $N_0 = 20$, and $N = 10$, calculate A_λ and its error ΔA_λ mainly arising from N.

(8) Exercise

Let's make an extinction map of the dark cloud LDN 1221 using the star-count technique, and estimate the total mass of the cloud.

Step1

Please look at the image of LDN 1221 on a separate sheet. Rectangular cells with a size of $3' \times 3'$ are drawn. Count stars in each cell, and make distribution map of N.

Step2

Determine the reference field, and measure the background Star number N_0. The image is an R band image, and the slope of the Wolf diagram is $b = 0.26$. A_R is related with A_V as $A_V = 1.21\ A_R$. Derive an extinction map of A_V.

Step3

Molecular hydrogen is the main component of dark cloud, but it is difficult to quantify the molecular mass directly.
According to an empirical relation (Bholin et al. 1978), column density of molecular hydrogen $N(\mathrm{H_2})$ in units of cm^{-2} and A_V in units of mag are related as

$$N(\mathrm{H_2}) = 1.25 \times 10^{21} A_V \quad . \qquad [16]$$

Using the above relation and A_V map derived in Step 2, estimate the total mass of LDN 1221 in units of solar mass (Mo).
Distance to LDN 1221 is 200 pc, and mass of hydrogen atom is 1.6735×10^{-24}g. 1 Mo is 2×10^{33} g. 1 pc is 3.08×10^{18}cm, and 1 au is 1.5×10^{13} cm.