Exercise: Mass Estimation of Molecular Clouds

Let's calculate the mass of the molecular cloud S134. For the calculation, use the following constants:

Planck's constant $h = 6.626\times10^{-34}$ J·s, The frequency of ^{13}CO $\nu = 1.1020137\times10^{11}$Hz,
Boltzmann constant $k = 1.380649\times10^{-23}$ J/K,
1 au = 1.496×10^{13} cm, Distance to the S134 molecular cloud = 900 pc
Mass of a proton $m_p = 1.6735\times10^{-24}$ g, Mean molecular weight $\mu = 2.4$, $1M_\odot = 1.989 \times 10^{33}$ g

Figures 1 and 2 show the ^{12}CO and ^{13}CO molecular emission line spectra obtained at the peak position of the integrated intensity map (indicated by the black dot in Figure 3).

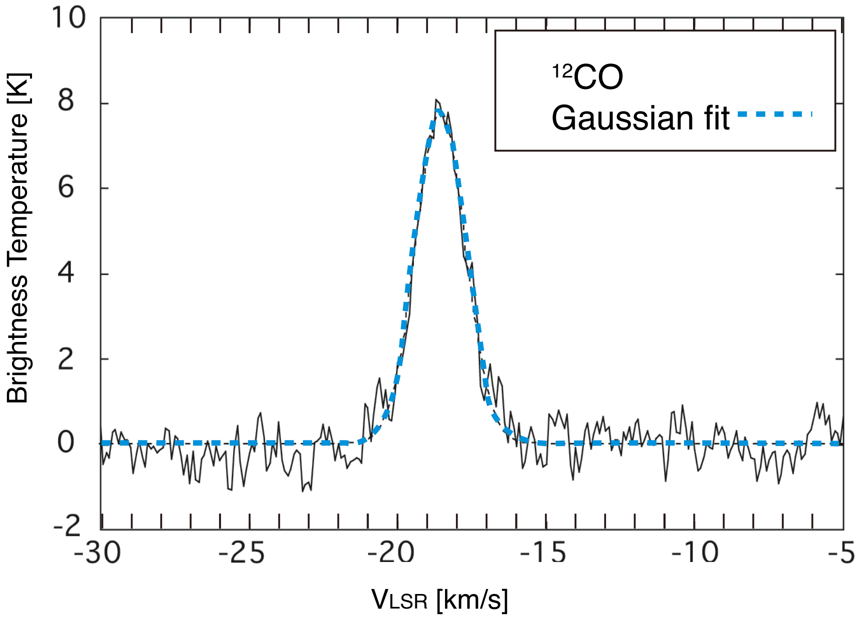

Figure 1: Spectrum of ^{12}CO. The dashed line represents the result of the Gaussian fit to the data.

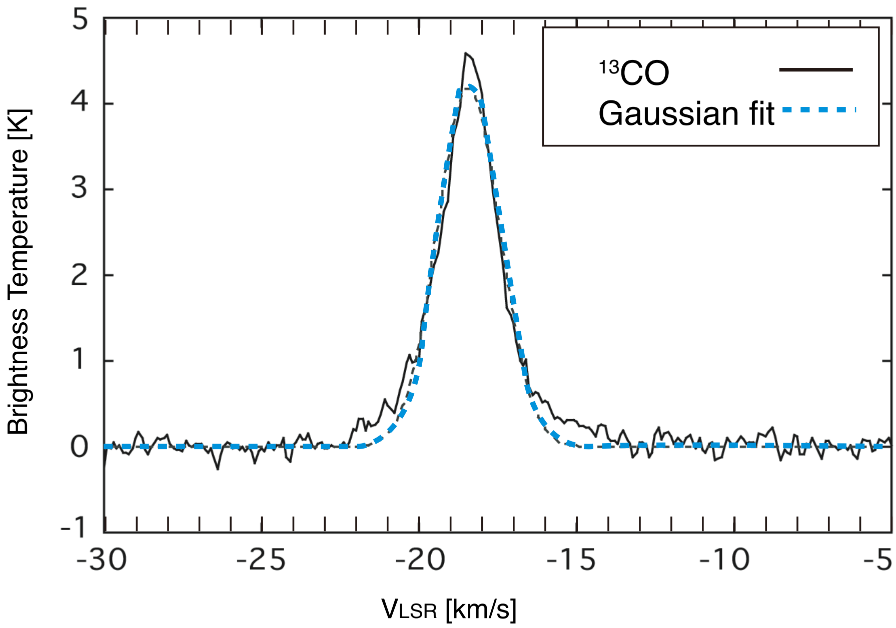

Figure 2: Spectrum of ^{13}CO. The dashed line represents the result of the Gaussian fit to the data.

1. From these spectra, we will extract the Gaussian parameters. Determine the values of $T(^{12}\mathrm{CO})$, $T(^{13}\mathrm{CO})$, and $\Delta V(^{13}\mathrm{CO})$, and record them in the table below. Note that $T(^{12}\mathrm{CO})$ and $T(^{13}\mathrm{CO})$ should be measured in K, and $\Delta V(^{13}\mathrm{CO})$ in $\mathrm{km/s}$.

$T(^{12}\mathrm{CO})$	$T(^{13}\mathrm{CO})$	$\Delta V(^{13}\mathrm{CO})$

2. Calculate the excitation temperature, T_{ex}, in units of K.

$$T_{\mathrm{ex}} = \frac{5.53}{\ln\left[1 + \frac{5.53}{T(^{12}\mathrm{CO}) + 0.819}\right]}$$

3. Calculate the optical depth of ^{13}CO, $\tau(^{13}\mathrm{CO})$.

$$\tau(^{13}\mathrm{CO}) = -\ln\left\{1 - \frac{T(^{13}\mathrm{CO})}{J(T_{\mathrm{ex}}) - 0.868}\right\} \qquad J(T_{\mathrm{ex}}) = \frac{h\nu/k}{\exp(h\nu/kT_{\mathrm{ex}}) - 1}$$

4. Calculate the column density of ^{13}CO, $N(^{13}\mathrm{CO})$, in units of cm^{-2}.

$$N(^{13}\mathrm{CO}) = \frac{2.52\times10^{14}\tau(^{13}\mathrm{CO})\,\Delta V(^{13}\mathrm{CO})\,T_{\mathrm{ex}}}{1 - \exp(-5.29/T_{\mathrm{ex}})}$$

5. Calculate the column density of $\mathrm{H_2}$, $N(\mathrm{H_2})$, in units of cm^{-2}.

$$N(\mathrm{H_2}) = 5.0\times10^{5} N(^{13}\mathrm{CO})$$

6. Measure the area S of the molecular cloud at half the maximum integrated intensity (within the thick black line) in the ^{13}CO integrated intensity map shown in Figure 3. Express this measurement in cm^2.

$$S \sim \pi(d\times\theta\times\{1\ \mathrm{au}\})^2$$

7. Calculate the mass of the molecular cloud in $M_\odot$.

$$M = \frac{\mu\, \mathrm{m_p} N(\mathrm{H_2}) S}{\ln 2}\,\frac{1}{M_\odot}$$

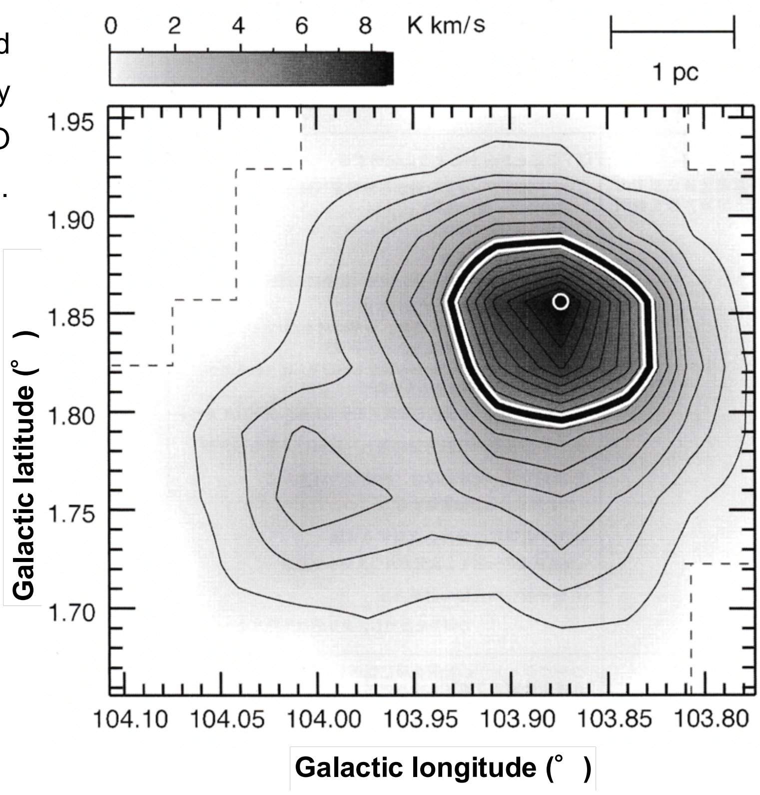

Figure 3: Integrated intensity map of ^{13}CO. The map has a minimum contour level of 0.5 K km/s, with contour intervals of 0.5 K km/s.