Assume P, Q, R, S, T are atoms (propositions).

1 Inference rule

Problem 1 (written: 15 pts): Using a truth table, show that the resolution inference rule is valid (if the premises are true, the conclusion is also true, or, $((P \vee R) \wedge (Q \vee \neg R)) \rightarrow (P \vee Q)$ is valid). Note: valid means "true under all interpretations".

$$\frac{P \vee R,\ \ Q \vee \neg R}{P \vee Q}$$

T → T
F → —

P	Q	R	$(P \vee R)$	$(Q \vee \neg R)$	$(P \vee R) \wedge (Q \vee \neg R)$	$(P \vee Q)$	$((P \vee R) \wedge (Q \vee \neg R)) \rightarrow (P \vee Q)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	F	F	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	F	T	F	F	T valid.

$(P \vee R) \wedge (Q \vee \neg R) \rightarrow P \vee Q$

2 Normal forms

In all of the problems in this section, show each step of the derivation and indicate which axioms (or other rules) you used: For example, *distributive law, by definition, etc*.

Problem 2 (written: 5 pts): Convert $(\neg P \wedge R) \vee S \vee (Q \wedge R)$ into conjunctive normal form.

distributive

$$((S \vee \neg P) \wedge (S \vee R)) \vee (Q \wedge R)$$

distributive

$$((Q \wedge R) \vee (S \vee \neg P)) \wedge ((Q \wedge R) \vee (S \vee R))$$

2 clause CNF

associative

$$\underbrace{((Q \wedge R) \vee (S \vee \neg P))}_{C1} \wedge \underbrace{((Q \wedge R) \vee S \vee R)}_{C2}$$

Problem 3 (written: 5 pts): Convert $\neg(P \vee \neg Q) \wedge (S \to R)$ into disjunctive normal form.

implication

$$\neg(P \vee \neg Q) \wedge (\neg S \vee R)$$

demorgan + double negative Q

$$(\neg P \wedge Q) \wedge (\neg S \vee R)$$

associative

$$\neg P \wedge Q \wedge (\neg S \vee R)$$

distributive

$$\underbrace{(\neg P \wedge Q \wedge \neg S)}_{T1} \vee \underbrace{(\neg P \wedge Q \wedge R)}_{T2}$$

2 term DNF

3 Proof by Resolution

Given:

1. $S \vee \neg P \vee Q$
2. $\neg R \vee \neg Q$
3. $P \vee T \vee \neg R$

to prove $R \to S \vee T$, prove $\neg(R \to S \vee T)$ inconsistent

show that $R \to (S \vee T)$ is a logical consequence of the above using **resolution**. Note: T is an atom (a proposition), not **True**.

Precisely follow the steps below.

Problem 4 (written: 15 pts): Transform the above problem into a set of clauses (premises and the conclusion), suitable for resolution-based theorem proving.

- Turn each axiom in the list of premises above into conjunctive normal form.
  - One premise may result in multiple clauses.
  - For example, a premise $\neg((P \wedge \neg R) \vee S)$ will convert to CNF as $(\neg P \vee R) \wedge \neg S$, which results in two clauses:
    Clause 1: $\neg P \vee R$
    Clause 2: $\neg S$

- Don't forget to negate the conclusion ($R \to (S \vee T)$), before adding to the clause list. Multiple clauses may (or may not) result from the negated conclusion.

$\neg(R \to S \vee T) = R \wedge \neg(S \vee T) = R \wedge \neg S \wedge \neg T = 4.\ 5.\ 6.$

Problem 5 (written: 20 pts): With the resulting resolution problem from the above, prove the theorem using resolution. Show every step.

1. $S \vee \neg P \vee Q$	7. (3,4)	$P \vee T$
2. $\neg R \vee \neg Q$	8. (6,7)	P
3. $P \vee T \vee \neg R$	9. (2,4)	$\neg Q$
4. R	10. (1,8)	$S \vee Q$
5. $\neg S$	11. (9,10)	S
6. $\neg T$	12. (5,11)	False