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CS 633 ML
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1 Gradients

(1)

$$f(x,y) = x^2 + ln(y) + xy + y^3$$

$$\frac{\partial f}{\partial x} = 2x + y$$

$$\frac{\partial f}{\partial y} = 3y^2 + \frac{1}{y} + x$$

$$\nabla f(10,-10) = [2 * 10 - 10, 3(-10)^2 - \frac{1}{10} + 10] = [10, 309.9]$$

(2)

$$f(x,y,z) = tanh(x^3y^3) + sin(z^2)$$

$$\frac{\partial f}{\partial x} = 3x^2y^3(1 - \tanh^2(x^3y^3))$$

$$\text{chain rule } (f(g(u)))' = f'(g(u))g'(u)$$

$$f = \tanh(g) \quad f' = 1 - \tanh^2(g)$$

$$g = x^3y^3 \quad g' = 3x^2y^3$$

$$(f(g(u)))' = 3x^2y^3(1 - \tanh^2(x^3y^3))$$

$$\frac{\partial f}{\partial y} = 3x^3y^2(1 - \tanh^2(x^3y^3))$$

$$\text{chain rule } (f(g(u)))' = f'(g(u))g'(u)$$

$$f = \tanh(g) \quad f' = 1 - \tanh^2(g)$$

$$g = x^3y^3 \quad g' = 3x^3y^2$$

$$(f(g(u)))' = 3x^3y^2(1 - \tanh^2(x^3y^3))$$

$$\frac{\partial f}{\partial z} = 2z \cos z^2$$

$$\text{chain rule } (f(g(u)))' = f'(g(u))g'(u)$$

$$f = \sin(g) \quad f' = \cos(g)$$

$$g = z^2 \quad g' = 2z$$

$$(f(g(u)))' = 2z \cos z^2$$

$$\nabla f(-1, 0, \frac{\pi}{2}) = [0, 0, \pi \cos(\frac{\pi^2}{4})] = [0, 0, -2.454]$$

2 Matrix Multiplication

(1)

$$\begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10*0 & 10*3 & 10*0 & 10*1 \\ -5*0 & -5*3 & -5*0 & -5*1 \\ 2*0 & 2*3 & 2*0 & 2*1 \\ 8*0 & 8*3 & 8*0 & 8*1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

(2)

$$\begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 6+0-18+21 & 2+1+0+28 & 0-1+24+49 \\ 54+0-24+3 & 18+0+0+4 & 0+0+32+7 \\ -48+0-6+9 & -16-1+0+12 & 0+1+8+21 \\ 60+0+0+3 & 20-4+0+4 & 0+4+0+7 \end{bmatrix} = \begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$