Problem 1, Written (30 pts): Consider the application of Bayes rule to medical diagnosis.

$P(A|B) = \frac{P(A \wedge B)}{P(B)}$

Given:

- $P(T)$: probability of testing positive
- $P(D)$: probability of having the disease

then the your chance of having the disease given a positive test result is:

$$P(D|T) = \frac{P(T|D)P(D)}{P(T)}.$$

When $P(D) = 0.05$, how accurate should the test be ($P(T|D)$ = $P(\neg T|\neg D)$ = ?) so that $P(D|T) > 0.7$?

(1) Answer: $x = P(T|D) >$ .....

(2) Discuss the results in terms of the discrepancy between $P(T|D)$ and $P(D|T)$ and how extremely accurate the test should be to be useful.

$P(D) = .05$

$P(D|T) = \frac{P(T|D)P(D)}{P(T)} > .7$

$P(\neg D) = .95$

$P(T|\neg D) = \frac{P(\neg D|T)\,P(T)}{P(\neg D)}$

$P(T) = \frac{.95(1-x)}{P(\neg D|T)}$

- Note that if $P(T|D) = P(\neg T|\neg D) = x$ then $P(\neg T|D) = P(T|\neg D) = 1 - x$.
- You can plug in the above (x and $1 - x$) in the formula for $P(D|T)$.

$$P(D|T) = \frac{x \times 0.05}{x \times 0.05 + ...} > 0.7$$

$P(T|D) = P(\neg T|\neg D) = x$

$P(\neg T|D) = P(T|\neg D) = 1-x$

$P(D|T) = \frac{.05x}{P(T)} > .7$

$P(T) = \frac{.95x}{P(D|T)}$

$\frac{.95(1-x)}{P(\neg D|T)} = \frac{.95x}{P(D|T)}$

$\frac{P(D|T)}{P(\neg D|T)} = \frac{.05x}{.95(1-x)}$

$\frac{.7}{.3} = \frac{.95x}{.95(1-x)}$

$\frac{.7 \cdot .95}{.3 \cdot .05} = \frac{x}{1-x}$

$44\frac{1}{3} = \frac{x}{1-x}$

$\frac{133}{3} = \frac{x}{1-x}$ (calculator)

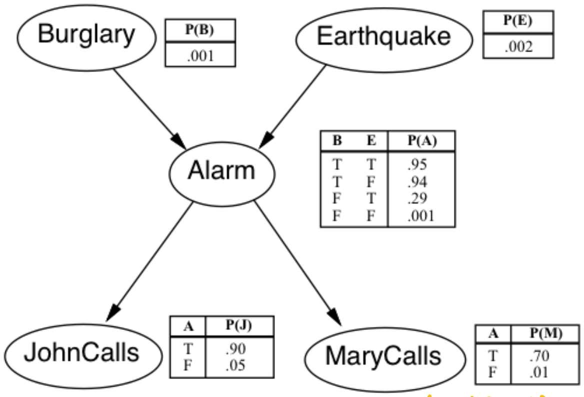
$x = \frac{133}{136} \approx 97.79\%$

* As x increases $P(D|T)$ increases*

1. $P(T|D) > \frac{133}{136}$
2. to acheive $P(D|T) > .7$ the $P(T|D) > .9779$

For an individual to reasonably trust the test's results, the test needs to be proven very accurate. The large difference between the different probabilities is due to the low probability $P(D) = .05$.

Problem 2, Written (20 pts): Given the belief network as shown below, calculate the joint probability $P(\neg JohnCalls, MaryCalls, \neg Alarm, Earthquake, \neg Burglary)$.



$P(\neg J | \neg A)$ $P(M | \neg A)$ $P(\neg A | E \wedge \neg B)$ $P(E)$ $P(\neg B)$

$P(\neg Jcall, Mcall, \neg Alarm, eq, \neg burglary)$

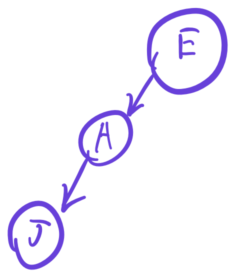
$.95 \cdot .01 \cdot .71 \cdot .002 \cdot .999$

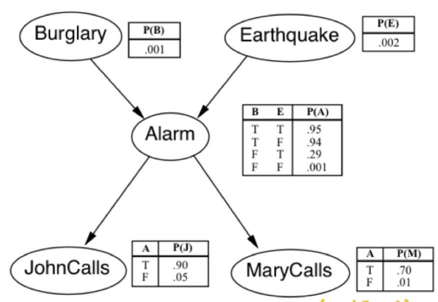
$= 1.348 \times 10^{-5}$

Problem 3, Written (20 pts): Given the domain above, consider constructing the belief network from scratch. First, you need the nodes and order them. Suppose we are only going to add three nodes.

(1) Which of the following order is the best?

- (a) $Earthquake, Alarm, JohnCalls$
- (b) $Alarm, JohnCalls, Earthquake$
- (c) $JohnCalls, Earthquake, Alarm.$





(2) Explain why. Explain in terms of cause and effect.

The probability of Alarm is dependant (effect) on the result of Earthquake (cause). P(JohnCalls) depends on P(Alarm). Since the result of one effects the other, the order matters.

Problem 4, Written (30 pts): With the domain above figure, give an example of intercausal inference. Think about common causes to an effect. Explain why.

$P(\neg J | \neg A), P(M | \neg A), \underline{P(\neg A | E \wedge \neg B)} \rightarrow (1 - .29)$

B	E	P(A)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

This is a burglar alarm that may occasionally be triggered by an earthquake.

Highly effective at detecting burglars.

This could be due to the categorization of our data. Small tremors qualify as an earthquake, but only higher magnitudes will trip alarm sensors.