2) x and $b \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$

$f(x) = b^T x + x^T A x$

a) $\nabla f = b + Ax$

$H = A$

b) $f(\vec{0}) = 0$, $\nabla f(\vec{0}) = b$, $H(\vec{0}) = A$

i) $f(x) \approx 0 + b^T x$ not exact

ii) $f(x) = 0 + b^T x + x^T A x$ exact

c) For A to be positive definite, its eigenvalues must all be positive.

d) For A to have full rank, its determinant must be nonzero.

e) If $y \in \mathbb{R}^n$ and $y \neq 0$ such that $A^T y = 0$, then $Ax = b$ has a solution for x if $b \perp y$.

3) Minimize $Cost(X) = C^T X$
subject to $A^T X \leq b$

where $\vec{X} = [X_1\ X_2 \dots X_N]^T$

$$\vec{C} = [C_1\ C_2 \dots C_N]^T$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1M} \\ a_{21} & \ddots & & a_{2M} \\ \vdots & & & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NM} \end{bmatrix}$$

$$\vec{b} = [b_1\ b_2 \dots b_M]^T$$