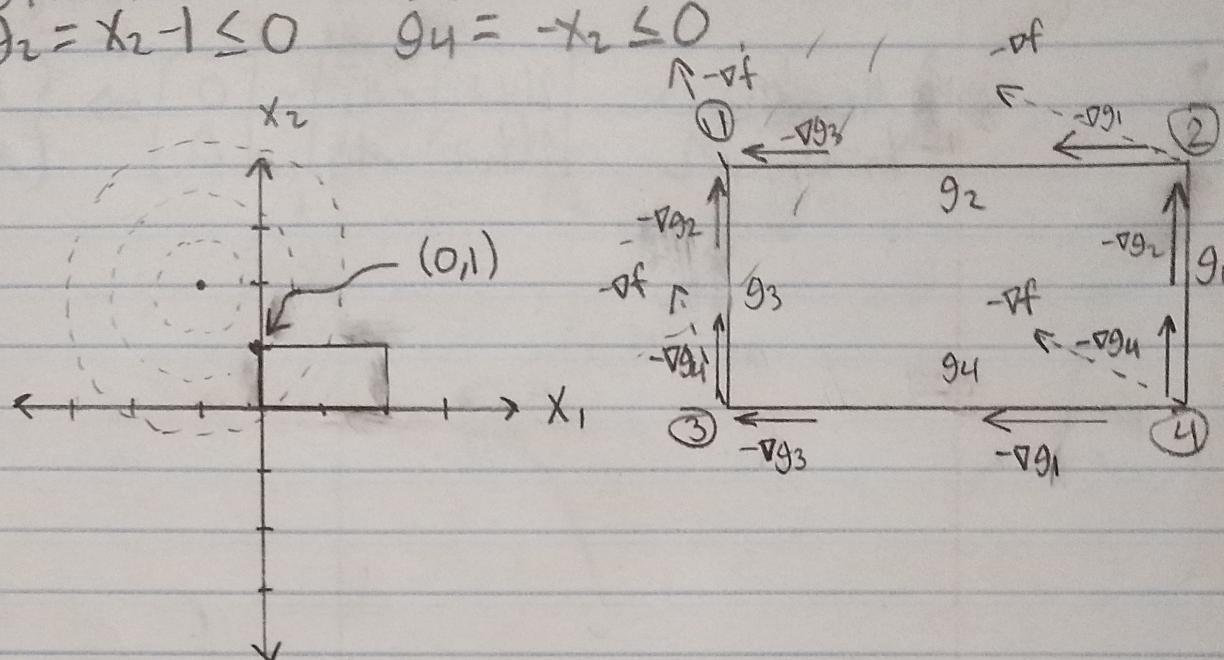


$$1) \min f(x) = (x_1+1)^2 + (x_2-2)^2$$

$$\text{s.t. } g_1 = x_1 - 2 \leq 0, \quad g_3 = -x_1 \leq 0$$

$$g_2 = x_2 - 1 \leq 0, \quad g_4 = -x_2 \leq 0$$



$$L = (x_1+1)^2 + (x_2-2)^2 + M_1(x_1-2) + M_2(x_2-1) + M_3(-x_1) + M_4(-x_2)$$

$$\nabla L = \begin{bmatrix} 2(x_1+1) + M_1 - M_3 \\ 2(x_2-2) + M_2 - M_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

if $x_1 - 2 = 0$ then $M_1 > 0$; if $x_1 - 2 < 0$ then $M_1 = 0$

if $x_2 - 1 = 0$ then $M_2 > 0$; if $x_2 - 1 < 0$ then $M_2 = 0$

if $-x_1 = 0$ then $M_3 > 0$; if $-x_1 < 0$ then $M_3 = 0$

if $-x_2 = 0$ then $M_4 > 0$; if $-x_2 < 0$ then $M_4 = 0$

$$1) M_1 = M_3 = 0 \quad \begin{bmatrix} 2(x_1+1) - M_3 \\ 2(x_2-2) + M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} M_3 = 2 \\ M_2 = 2 \end{cases} \quad \checkmark$$

$$2) M_3 = M_4 = 0 \quad \begin{bmatrix} 2(x_1+1) + M_1 \\ 2(x_2-2) + M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} M_1 = -6 \\ M_2 = 2 \end{cases} \quad \times$$

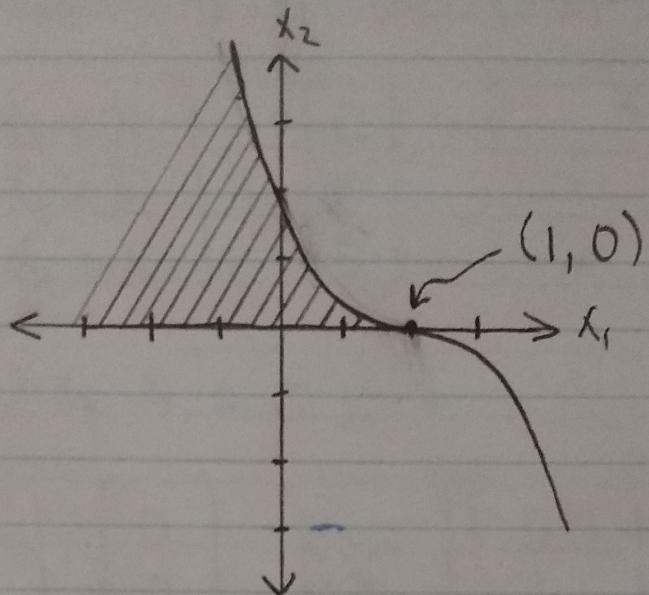
$$3) M_1 = M_2 = 0 \quad \begin{bmatrix} 2(x_1+1) - M_3 \\ 2(x_2-2) - M_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} M_3 = 2 \\ M_4 = -4 \end{cases} \quad \checkmark$$

$$4) M_3 = M_4 = 0 \quad \begin{bmatrix} 2(x_1+1) + M_1 \\ 2(x_2-2) - M_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} M_1 = -6 \\ M_4 = -4 \end{cases} \quad \times$$

$$2) \min f = -x_1$$

$$\text{s.t. } g_1 = x_2 - (1-x_1)^3 \leq 0$$

$$g_2 = -x_2 \leq 0$$



$$L = -x_1 + M_1(x_2 - (1-x_1)^3) + M_2(-x_2)$$

$$\nabla_L = \begin{bmatrix} -1 + 3M_1(1-x_1)^2 \\ M_1 - M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

if $x_2 - (1-x_1)^3 = 0$ then $M_1 > 0$; if $x_2 - (1-x_1)^3 < 0$ then $M_1 = 0$
 if $-x_2 = 0$ then $M_2 > 0$; if $-x_2 < 0$ then $M_2 = 0$

$$1) x_2 - (1-x_1)^3 = 0, M_1 > 0 \Rightarrow x_1 = 1 \begin{bmatrix} -1 \\ M_1 - M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} X$$

$$-x_2 = 0, M_1 > 0$$

$$2) x_2 - (1-x_1)^3 = 0, M_1 > 0 \Rightarrow M_1 - 0 > 0 X$$

$$-x_2 < 0, M_2 = 0$$

$$3) x_2 - (1-x_1)^3 < 0, M_1 > 0 \Rightarrow -1 > 0 X$$

$$-x_2 < 0, M_2 = 0$$

$$4) x_2 - (1-x_3)^3 < 0, \quad m_1 = 0 \Rightarrow m_1 - m_2 < 0 \quad X$$
$$-x_2 = 0, \quad m_2 > 0$$

A solution cannot be found based on optimality conditions because the gradients of g_1 and g_2 do not span the gradient of the objective f .

$$3) \max f = x_1x_2 + x_2x_3 + x_1x_3$$

$$\text{s.t. } h = x_1 + x_2 + x_3 - 3 = 0$$

$$L = x_1x_2 + x_2x_3 + x_1x_3 + \lambda_1(x_1 + x_2 + x_3 - 3)$$

$$\nabla_x L = \begin{bmatrix} x_2 + x_3 + \lambda_1 \\ x_1 + x_3 + \lambda_1 \\ x_2 + x_1 + \lambda_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\nabla_\lambda L = x_1 + x_2 + x_3 - 3 = 0$$

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 3 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 3 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 3 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & 3 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & -1 & -2 & 3 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 3 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 1 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right] \boxed{\begin{array}{l} x_1 = 1 \\ x_2 = 1 \\ x_3 = 1 \\ \lambda_1 = -2 \end{array}}$$

5) Garbage truck routing problem: N nodes to visit, where cost of moving from node i to j is c_{ij} , or ∞ if there is no direct path. Site 0 is the beginning and end. Minimize the total cost.

$$x(t) = [x_1, x_2, \dots, x_N]^T \quad x_i = 0 \text{ or } 1 \quad \text{State tensor}$$

$$L(x(t), x(t-1)) = c_{ij} \text{ or } \infty \quad \text{Loss function}$$

$$F = \sum_{t=1}^T L(x(t), x(t-1)) \quad \text{Objective}$$

$$\min_x F(x)$$

$$\text{s.t. } \sum_{t=1}^T x(t) \geq 1_{nx1} \quad \text{All nodes visited at least once}$$

$$x_1(0) + x_1(T) \geq 2 \quad \text{First and last node are } i=1.$$