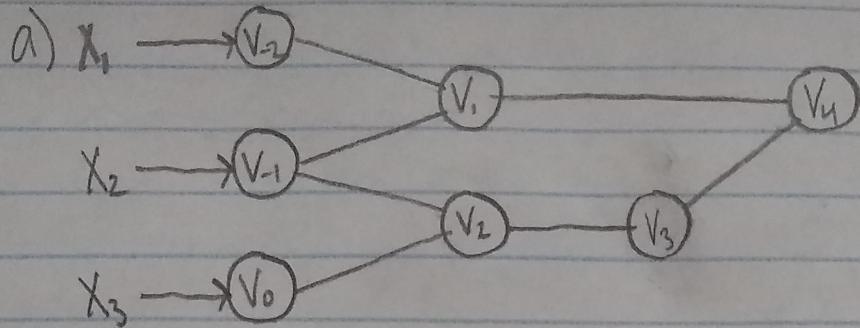


$$f(x_1, x_2, x_3) = x_1 x_2 \sin(x_2 x_3)$$



$$b) V_2 = x_1 = -1$$

$$V_1 = x_2 = 2$$

$$V_0 = x_3 = 3$$

$$V_4 = V_2 V_1 = -2$$

$$V_2 = V_1 V_0 = 6$$

$$V_3 = \sin(V_2) = -0.279$$

$$V_4 = V_1 V_3 = 0.559$$

$$\dot{V}_2 = \dot{x}_1 = 1$$

$$\dot{V}_1 = \dot{x}_2 = 0$$

$$\dot{V}_0 = \dot{x}_3 = 0$$

$$\dot{V}_4 = \dot{V}_2 V_1 + V_2 \dot{V}_1 = (1)(2) + (-1)(0) = 2$$

$$\dot{V}_2 = \dot{V}_1 V_0 + V_1 \dot{V}_0 = (0)(3) + (2)(0) = 0$$

$$\dot{V}_3 = \dot{V}_2 \cos(V_2) = (0)(1) = 0$$

$$\dot{V}_4 = \dot{V}_1 V_3 + V_1 \dot{V}_3 = (2)(-0.279) + (0)(-2) = \boxed{-0.559}$$

For \dot{x}_2

$$\dot{V}_2 = 0$$

$$\dot{V}_{-1} = 1$$

$$\dot{V}_0 = 0$$

$$\dot{V}_1 = (0)(2) + (-1)(1) = -1$$

$$\dot{V}_2 = (1)(3) + (2)(0) = 3$$

$$\dot{V}_3 = (3)\cos(6) = 2.88$$

$$\dot{V}_4 = (-1)(-0.279) + (2.88)(-2)$$

$$= \boxed{-5.48}$$

For \dot{x}_3

$$\dot{V}_2 = 0$$

$$\dot{V}_1 = 0$$

$$\dot{V}_0 = 1$$

$$\dot{V}_1 = (0)(2) + (1)(0) = 0$$

$$\dot{V}_2 = (0)(3) + (2)(1) = 2$$

$$\dot{V}_3 = (2)\cos(6) = 1.92$$

$$\dot{V}_4 = (0)(-0.279) + (1.92)(-2) = \boxed{-3.84}$$

$$\boxed{\frac{\partial f}{\partial x_1} = -0.559 \quad \frac{\partial f}{\partial x_2} = -5.48 \quad \frac{\partial f}{\partial x_3} = 3.84}$$

$$\begin{aligned}
 c) \quad \bar{V}_4 &= 1 \\
 \bar{V}_3 &= \bar{V}_4 \frac{\partial V_4}{\partial V_3} = \bar{V}_4 V_1 &= (1)(-2) = -2 \\
 \bar{V}_2 &= \bar{V}_3 \frac{\partial V_3}{\partial V_2} = \bar{V}_3 \cos(V_2) &= (-2)\cos(6) = -1.92 \\
 \bar{V}_1 &= \bar{V}_4 \frac{\partial V_4}{\partial V_1} = \bar{V}_4 V_3 &= (1)(-0.279) = -0.279 \\
 \bar{V}_0 &= \bar{V}_2 \frac{\partial V_2}{\partial V_0} = \bar{V}_2 V_{-1} &= (-1.92)(2) = -3.84 \quad \cancel{\text{X}} \\
 \bar{V}_{-1} &= \bar{V}_2 \frac{\partial V_2}{\partial V_{-1}} + \bar{V}_1 \frac{\partial V_1}{\partial V_{-1}} = \bar{V}_2 V_0 + \bar{V}_1 V_{-2} &= (-1.92)(3) + (-0.279)(-1) = -5.48 \quad \cancel{\text{X}} \\
 \bar{V}_{-2} &= \bar{V}_1 \frac{\partial V_1}{\partial V_{-2}} = \bar{V}_1 V_4 &= (-0.279)(2) = 0.559 \quad \cancel{\text{X}}
 \end{aligned}$$

$$\boxed{\frac{\partial f}{\partial x_1} = -0.559 \quad \frac{\partial f}{\partial x_2} = -5.48 \quad \frac{\partial f}{\partial x_3} = -3.84}$$

$$d) \frac{\partial f}{\partial x_1} \approx \frac{1}{0.002} \left\{ (-1+0.001)(2)\sin[(2)(3)] + (-1-0.001)(2)\sin[(2)(3)] \right\} \\
 = -0.559$$

$$\begin{aligned}
 \frac{\partial f}{\partial x_2} \approx \frac{1}{0.002} &\left\{ (-1)(2+0.001)\sin[(2+0.001)(3)] - \right. \\
 &\left. - (-1)(2-0.001)\sin[(2-0.001)(3)] \right\} \\
 = -5.48
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial f}{\partial x_3} \approx \frac{1}{0.002} &\left\{ (-1)(2)\sin[(2)(3+0.001)] - (-1)(2)\sin[(2)(3-0.001)] \right\} \\
 = -3.84
 \end{aligned}$$

Autograd

df/dx1: tensor([-0.5588])

df/dx2: tensor([-5.4816])

df/dx3: tensor([-3.8407])

Center Finite Difference

df/dx1: tensor([-0.5589], grad_fn=<DivBackward0>)

df/dx2: tensor([-5.4817], grad_fn=<DivBackward0>)

df/dx3: tensor([-3.8404], grad_fn=<DivBackward0>)