Homework Set 6: Extra Problem (Total 10 points)

a (1 point). Use the data in the files "Question1.txt" to fit an exponential distribution using the method of moments.

$$\theta = \frac{\sum_1^n x_i}{n}, \quad \lambda = \frac{n}{\sum_{i=1} x_i}$$

lambda = 1 / sample mean

= 1 / 29.72

= 0.0336

b. (2 points) Use the data in the files "Question1.txt" to fit an exponential distribution using the maximum likilihood method.

Since the MLE fution: $L(\theta|(x_1,\dots,x_2)) = \prod_{i=1}^{n} f(x_i|\theta)$

$L(\lambda; x_1,\dots,x_n) = \lambda^n \exp\left(-\lambda \sum_{j=1}^{n} x_j\right)$

$l(\lambda; x_1,\dots x_n) = n\ln(\lambda) - \lambda \sum_{i=1}^{n} x_j$

Since we get: $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$,

and we know $f(x) = \lambda e^{-\lambda x}$, $\lambda = 0.033b$

$f(x) = \begin{cases} 0.033b\, e^{-0.033bx} & x \geq 0 \\ 0 & x < 0 \end{cases}$

c. (1 point) Estimate a 90% credibility interval using your fitted exponential distribution.

Since $P = 1 - e^{-\lambda x}$

$e^{-\lambda x} = 1 - P$

$x = -\frac{\ln(1-P)}{\lambda}$

90% interval: $\left[-\frac{\ln(1-0.05)}{0.0336}, -\frac{\ln(1-0.95)}{0.0336}\right]$

$= [1.5244, 89.0332]$

d. (4 points) Perform a χ^2-goodness of fit test, using the equal bin-width method and the equal bin-probability method.

Completed on the excel sheet.

e. (2 points) Use MINITAB to draw a probability plot of the data. Estimate a 90% credibility interval using MINITAB by drawing it in the probability plot.

Attached the graph on the excel sheet.