





Project "Advanced Topics"

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March 13th, 2025

Instructions

This project will serve to evaluate students for the “Advanced Topics in Financial Modelling” class. Submit your work as a  Jupyter Notebook or an  Mark-down document. A significant part of the grade will be determined by the quality of presentation, including literary comments as well as nice \LaTeX equations, ideally directly in the notebook. This project can be done individually or by pair, but not more. [Submissions will not be accepted after April , 13th 2025.](#)

The brief

You have recently been hired as a quant analyst within the “client advisory” team of a Private Bank. In the past, the team has partnered with a US ETF provider called Direxion (<https://www.direxion.com/>) which claims to offer “Timely opportunities for tactical trades and precise investment exposure”. More specifically:

- In case of bear expectations, the team advises to invest into the Direxion Daily S&P 500® Bear (SPXS) 3X Shares which provides a 3 times leveraged short exposure on the S&P 500.
- In case of bull expectations, aside the traditional 3) “spider” SPDR S&P 500 ETF (SPY) which provides delta 1 exposure to the S&P 500 at a cheap price, the team advised 4) the Direxion Daily S&P 500® Bull (SPXL) 3X Shares which provides a 3 times leveraged long exposure on the S&P 500.

However, the team stopped advising those products because they did not behave as expected and even raised some suspicion among certain clients. A new head

of advisory has been appointed and asks to you review the performances of those ETFs and clarify certain points:

1. What should be told to clients to better manage their expectations ?
2. Should the team keep them on the advisory list ?

You are asked to write a comprehensive memo on the matter which will be presented to the Executive Committee, in April.

1 Data retrieval and descriptive analysis of S&P 500 returns

Question 1. Write a code which retrieves the last 5 years of Direxion data prices, thanks to the Yahoo API. This code should be written in such way that it could easily be executed later in the future.

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Question 2. The bank account B_t will be proxied by the SOFR Index. Retrieve the SOFR Index historical values thanks to the Federal Reserve of New York API.

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Question 3. Plot the historical plots of prices of the different products expressed in bank account units (i.e. V_t/B_t while rebasing them at 1 as their initial value.

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Question 4. Compute the following (annualized) descriptive statistics of daily log-returns: mean, volatility, skewness, kurtosis. Compute their correlations as well as their Sharpe ratio. Comment.

2 Power profiles and Gamma effects

In all the following, $V_t(l)$ denotes the value of the ETF with leverage l at date t and S_t denotes the value of the S&P 500. We also denote $\tilde{S}_t \equiv \frac{S_t}{B_t}$ and $\tilde{V}_t(l) \equiv \frac{V_t(l)}{B_t}$

Question 5. Explain why clients might believe that the following relationship should hold:

$$\frac{V_{T_2}(l)}{V_{T_1}(l)} \approx l \frac{S_{T_2}}{S_{T_1}} + (1-l)$$

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Question 6. Show that, in principle, between two dates T_1 and T_2 a better prediction is given by:

$$\frac{\tilde{V}_{T_2}(l)}{\tilde{V}_{T_1}(l)} \approx \left(\frac{\tilde{S}_{T_2}}{\tilde{S}_{T_1}} \right)^l \exp \left(-\frac{1}{2} (l^2 - l) \hat{\sigma}_{1,2}^2 (T_2 - T_1) \right)$$

where $\hat{\sigma}_{T_1, T_2}^2$ is the cumulated realised variance between T_1 and T_2 .

Question 7. Compute and plot the rolling 1 year estimated variance. Comment. What is the average trend due to leverage predicted by the formula for each ETF ? Comment

We want to test the formula on rolling yearly returns. Compute first $\hat{\sigma}_{t_i-1year, t_i}^2$, $X_{t_i} = \frac{S_{t_i}}{S_{t_i-1year}}$ and $R_{t_i} = \frac{B_{t_i}}{B_{t_i-1year}}$, $\tilde{X}_{t_i} = X_{t_i}/R_{t_i}$. Then, for each product with leverage l , for each date t_i in the dataset compute:

- $Y_{t_i}^{naive}(l) = lX_{t_i} + (1-l)$
- $Y_{t_i}^{predict}(l) = X_{t_i}^l \times R_{t_i}^{1-l} \exp \left(-\frac{1}{2} (l^2 - l) \hat{\sigma}_{t_i-1year, t_i}^2 \right)$
- $Y_{t_i}^{real}(l)$ the 1 year rolling price ratio of the ETF with leverage l .

Question 8. For each product:

- Compute the R^2 of the linear regressions of $Y_{t_i}^{real}(l)$ vs $Y_{t_i}^{predict}(l)$ as well as vs $Y_{t_i}^{naive}(l)$
- Plot $Y_{t_i}^{naive}$, $Y_{t_i}^{predict}$ and $Y_{t_i}^{real}$ as a function of X_{t_i} .

Question 9. Comment. Is the naive prediction good ? Is the theoretical description good ? How would you present this to a non-sophisticated investor ? Would you recommend these products ?

Question 10. Instead of buying a Direxion ETF, your bank considers to implement the leveraged strategy at home. They ask you to simulate the strategy with a leverage equal to 3. Compare the results with the ETF and with the prediction. Comment.