Black-Litterman Example

The assets

- We are considering a portfolio of 7 countries' equity markets.
- Australia, Canada, France, Germany, Japan, UK, USA

Country	Equity Index Volatility (%)	Equilibrium Portfolio Weight (%)
Australia	16.0	1.6
Canada	20.3	2.2
France	24.8	5.2
Germany	27.1	5.5
Japan	21.0	11.6
UK	20.0	12.4
USA	18.7	61.5

The variance-covariance matrix Σ:

	AUS	CAN	FRA	GER	JAP	UK	USA
AUS	0.0256	0.0159	0.0190	0.0223	0.0148	0.0164	0.0147
CAN	0.0159	0.0412	0.0334	0.0360	0.0132	0.0247	0.0296
FRA	0.0190	0.0334	0.0615	0.0579	0.0185	0.0388	0.0310
GER	0.0223	0.0360	0.0579	0.0734	0.0201	0.0421	0.0331
JAP	0.0148	0.0132	0.0185	0.0201	0.0441	0.0170	0.0120
UK	0.0164	0.0247	0.0388	0.0421	0.0170	0.0400	0.0244
USA	0.0147	0.0296	0.0310	0.0331	0.0120	0.0244	0.0350

Investor preference

- Let μ be expected excess returns and Σ be the covariance matrix of returns.
- The investor is assumed to maximize $w'\mu - 0.5\delta w'\Sigma w$, where δ is risk aversion. (No constraint; treat cash is residual.)
- The solution is $\mu = \delta\Sigma w^*$, or $w^* = (\delta\Sigma)^{-1}\mu$.
- If all investors solve the same problems, then the equilibrium weight must be equal to the market cap weight, i.e., $\mu_{eq} = \delta\Sigma w_{eq}$. In this case, the cash position is zero, as borrowing and lending add up to zero.
- We now consider deviation from this market-cap-weighted portfolio.

Equilibrium risk premium

- Using risk aversion = 2.5, the covariance matrix, and the market weights, we get the equilibrium risk premium, or "neutral view" risk premium.
- It can be different from historical returns.

Country	Equity Index Volatility (%)	Equilibrium Portfolio Weight (%)	Equilibrium Expected Returns (%)
Australia	16.0	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9.0
Japan	21.0	11.6	4.3
UK	20.0	12.4	6.8
USA	18.7	61.5	7.6

Adding a view

- Suppose the view is that Germany will overperform the rest of Europe (France and UK) by 5%.
- By market cap, we can thus set $P' = [0, 0, -0.3, 1, 0, -0.7, 0]$ and $Q = 5\%$.
- Suppose the view has a variance of $\Omega = 0.2^2$.
- The new expected excess return is $\mu^{BL} = [(\tau\Sigma)^{-1} + P\Omega^{-1}P']^{-1}[(\tau\Sigma)^{-1}\pi + P\Omega^{-1}Q]$
- And the new covariance matrix of returns is $[(\tau\Sigma)^{-1} + P\Omega^{-1}P']^{-1}$.

New risk premium and portfolio weights under the view

- New risk premium is $[(\tau\Sigma)^{-1} + P\Omega^{-1}P']^{-1}[(\tau\Sigma)^{-1}\pi + P\Omega^{-1}Q]$.
- We calculate the new portfolio weight $w^{BL} = (\delta\Sigma)^{-1}\mu^{BL}$, using original covariance $\tau\Sigma$.
- We can also use the updated covariance $[(\tau\Sigma)^{-1} + P\Omega^{-1}P']^{-1}$.
- The following numbers assume $\tau = 1$.

	Eqm risk premium	BL risk premium
Australia	3.9%	4.2%
Canada	6.9%	7.4%
France	8.4%	9.0%
Germany	9.0%	10.4%
Japan	4.3%	4.4%
UK	6.8%	6.9%
USA	7.6%	7.9%

	Market weight	BL weight w/ original cov matrix	BL weight w/ updated cov matrix
Australia	1.6%	1.6%	1.6%
Canada	2.2%	2.2%	2.2%
France	5.2%	-1.1%	-9.8%
Germany	5.5%	26.5%	55.5%
Japan	11.6%	11.6%	11.6%
UK	12.4%	-2.3%	-22.6%
USA	61.5%	61.5%	61.5%

Reference

- He and Litterman, 1999. “The Intuition Behind Black-Litterman Model Portfolios”, Goldman Sachs Investment Management.