

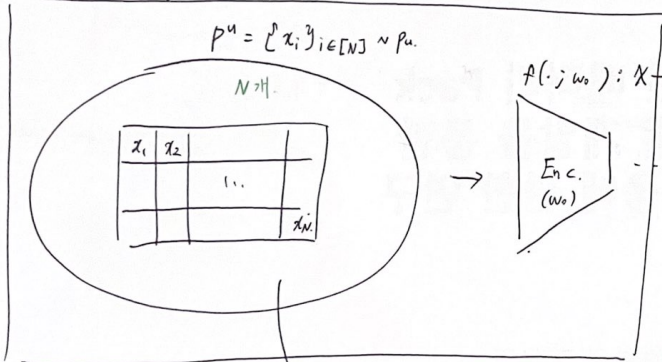
- Active FT.

unsupervised pretraining + supervised finetuning

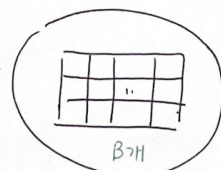
active finetuning

parametric model optimization \equiv continuous space optimization

- unsupervised pretraining

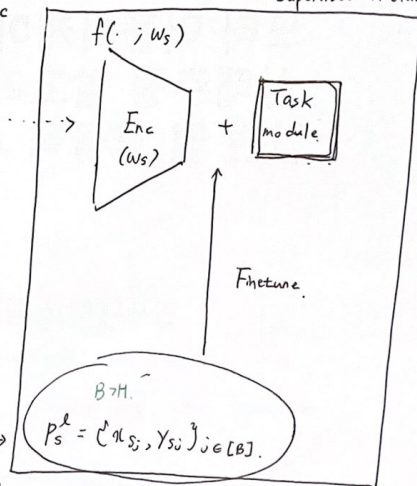


$S = \{s_j \in [N]\}_{j \in [B]}$ data selection.



$P_s^u = \{x_{s_j}\}_{j \in [B]} \subset P^u$

- supervised finetuning



- active finetuning $\equiv \frac{\sum \mathcal{L}}{\mathcal{L}}$

$= \mathcal{J}_{opt.}$

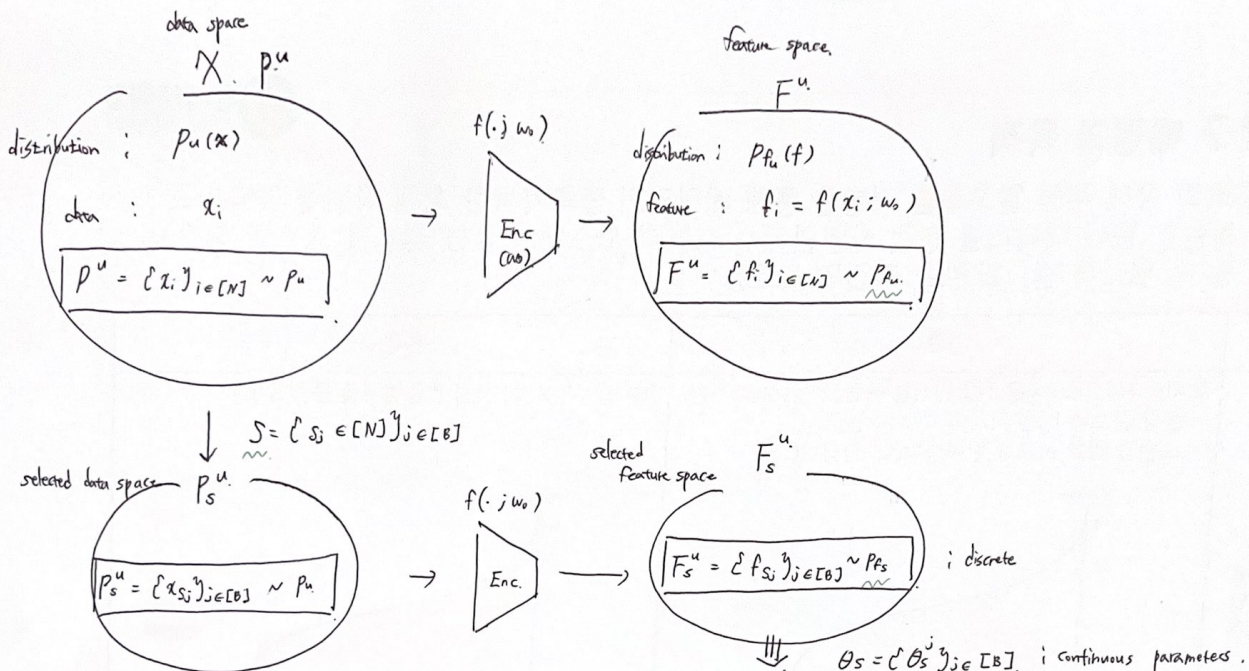
$\mathcal{J}_{opt.}$

$= \argmin_S E[\text{error}(f(x; w_s), y)]$

$\frac{\sum \mathcal{L}}{\mathcal{L}} \approx |P_s^d| \approx |P^u| \approx 10\%$ data ratio.

X : data space

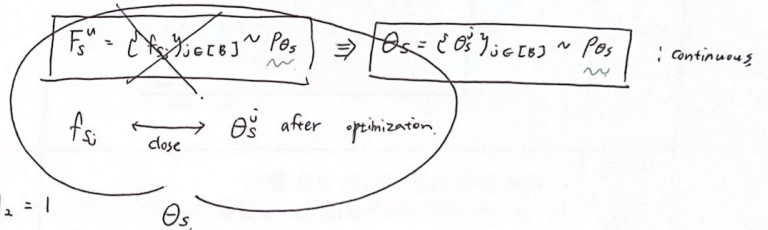
Y : label space.



- active finetuning $\Rightarrow \frac{\partial \mathcal{L}}{\partial \theta} = S_{opt}$.

$$S_{opt} = \underset{S}{\operatorname{argmin}} D(p_{f_u}, p_{f_s}) - \lambda R(F_s^u)$$

$$\Rightarrow \theta_{S, opt} = \underset{\theta_s}{\operatorname{argmin}} D(p_{f_u}, p_{\theta_s}) - \lambda R(\theta_s), \quad \|\theta_s^j\|_2 = 1$$



— parametric model p_{θ_s}

$$p_{\theta_s}(f) = \sum_{j=1}^B \phi_j \cdot p(f | \theta_s^j), \quad \sum_{j=1}^B \phi_j = 1, \quad j \in [B]$$

$$= \phi_1 \cdot p(f | \theta_s^1) + \phi_2 \cdot p(f | \theta_s^2) + \dots + \phi_B \cdot p(f | \theta_s^B)$$

$$p(f | \theta_s^j) = \frac{\exp(\text{sim}(f, \theta_s^j) / \tau)}{Z_j}, \quad \text{sim}(f_1, f_2) = f_1^T f_2, \quad \|f_1\|_2 = \|f_2\|_2 = 1.$$

* $f_i \in F^u$ 이 때에, f_i 와 가장 가까운 θ_s^i 이, optimization 진행 중 C_i 가 되는 것임/예.

$$C_i = \underset{j \in [B]}{\text{argmax}} \text{sim}(f_i, \theta_s^j), \quad i \in [N]$$

N 개 있는데
중요한 B 개.

F^u N 개
 θ_s B 개

— Assumption 1.

$$\forall i \in [N], j \in [B], \tau \text{ is small}, \quad \exp(\text{sim}(f_i, \theta_s^{C_i}) / \tau) \gg \exp(\text{sim}(f_i, \theta_s^j) / \tau), \quad j \neq C_i$$

$$p(f_i | \theta_s^{C_i}) \gg p(f_i | \theta_s^j), \quad j \neq C_i, \quad j \in [B].$$

- parametric model for $f_i \in F^u$.

$$p_{\theta_s}(f_i) \approx \phi_{c_i} \cdot p(f_i | \theta_s^{c_i}).$$

$$= \frac{\exp(\sin(f_i | \theta_s^{c_i}) / \tau)}{Z_{c_i} / \phi_{c_i}}$$

$$= \frac{\exp(\sin(f_i, \theta_s^{c_i}) / \tau)}{\tilde{Z}_{c_i}}, \quad \tilde{Z}_{c_i} = Z_{c_i} / \phi_{c_i}$$

$$\therefore p_{\theta_s}(f_i) \propto \exp(\sin(f_i, \theta_s^{c_i}) / \tau)$$

p_{f_u} 와 p_{θ_s} 는 KL-divergence 는 $\frac{1}{2} \sum_{i=1}^n \log \frac{p_{f_u}(f_i)}{p_{\theta_s}(f_i)}$ 가라하길 $\frac{1}{2}$ 는

$$KL(p_{f_u} | p_{\theta_s}) = \sum_{f_i \in F^u} p_{f_u}(f_i) \cdot \log \frac{p_{f_u}(f_i)}{p_{\theta_s}(f_i)}$$

$$= \underbrace{E_{f_i \in F^u} [\log p_{f_u}(f_i)]}_{: f_{S_j}^k} - E_{f_i \in F^u} [\log p_{\theta_s}(f_i)]$$

$$\therefore \text{Minimize KL-divergence } KL(p_{f_u} | p_{\theta_s}) \hat{=} \text{Maximize } E_{f_i \in F^u} [\log p_{\theta_s}(f_i)]$$

$$\hat{=} \text{Maximize } E_{f_i \in F^u} [\log \exp(\sin(f_i, \theta_s^{c_i}) / \tau)] \hat{=} \text{Maximize } E_{f_i \in F^u} [\sin(f_i, \theta_s^{c_i}) / \tau]$$

$$\therefore D(p_{f_u}, p_{\theta_s}) = - E_{f_i \in F^u} [\sin(f_i, \theta_s^{c_i}) / \tau]$$

- extra regularization term to ensure diversity of selected subset

$$R(\theta_s) = - E_{j \in [B]} \left[\log \sum_{k \neq j, k \in [B]} \exp(\sin(\theta_s^j, \theta_s^k) / \tau) \right]$$

$$\therefore L = D(p_{f_u}, p_{\theta_s}) - \lambda \cdot R(\theta_s)$$

$$= - E_{f_i \in F^u} [\sin(f_i, \theta_s^{c_i}) / \tau]$$

$$+ E_{j \in [B]} \left[\log \sum_{k \neq j, k \in [B]} \exp(\sin(\theta_s^j, \theta_s^k) / \tau) \right]$$

- 결론

optimization $\frac{1}{2} \sum_{i=1}^n \log \frac{p_{f_u}(f_i)}{p_{\theta_s}(f_i)}$,

θ_s^j 와 θ_s^k 가 θ_s^j 와 θ_s^k ($f_{S_j}^k$) $j \in [B]$ $k \neq j$ 이

$$f_{S_j}^k = \arg \max_{f_k \in F^u} \sin(f_k, \theta_s^j)$$

2가지 2개의 θ_s^j 와 θ_s^k ($f_{S_j}^k$) $j \in [B]$ $k \neq j$ 이

$$p_{S^u}^u \text{ 가 } S = \{S_j\}_{j \in [B]}$$

optimization $\frac{1}{N}$, $\{f_{s_i}\}_{i \in [B]}$.

$$p_{f_s}(f_{s_i}) = \frac{|G|}{N}, \quad G = \{f_i \mid c_i = j\}, \quad f_{s_i} \in F_s^u$$

$$p_{f_u}(f_i) = \frac{1}{N}, \quad f_i \in F^u$$

$$EMD(p_{f_u}, p_{f_s}) = \inf_{\gamma \in \Pi(p_{f_u}, p_{f_s})} E[\|f_i - f_{s_i}\|_2]$$

$$\gamma_{f_u, f_s}(f_i, f_{s_i}) = \begin{cases} \frac{1}{N}, & f_i \in F^u, f_{s_i} \in F_s^u, c_i = j \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore EMD(p_{f_u}, p_{f_s}) = \frac{1}{N} \sum_{i=1}^N [\sqrt{2 - 2 \sin(f_i, f_{s_{c_i}})}]$$