Imperial College London

BSc/MSci EXAMINATION June 2016

This paper is also taken for the relevant Examination for the Associateship

ELECTRICITY & MAGNETISM AND RELATIVITY

For 1st-Year Physics Students

Wednesday, 8th June 2016: 14:00 to 16:00

Answer all questions
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the FOUR answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in FOUR answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

1. (i) A charge of value $+Q$ is placed at the origin and a charge of value $-Q$ is placed at $(x = d, y = 0, z = 0)$. What is the magnitude of the electrostatic force between the charges?

The negative charge is now shaped into a uniformly charged thin rod of length l. The rod is aligned along the x-axis with the end nearest the positive charge at $x = d$. Show that the magnitude of the force between the charges is now given by

$$\left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q^2}{d\,(d+l)}.$$

[5 marks]

(ii) An insulating solid sphere of radius R has charge density of zero at its centre which increases linearly as a function of its radius to a value of $+\rho_0\,\mathrm{Cm}^{-3}$ at its surface. Find an expression for the electric field strength as a function of distance from the centre of the sphere for the range $0 \rightarrow R$.

[5 marks]

(iii) A 250 nF parallel plate capacitor is connected to a 5.0 V battery. A material of dielectric constant $\kappa \approx 4.0$ is pushed between the plates with the battery still connected. For this arrangement what is the change in

(a) the capacitance,

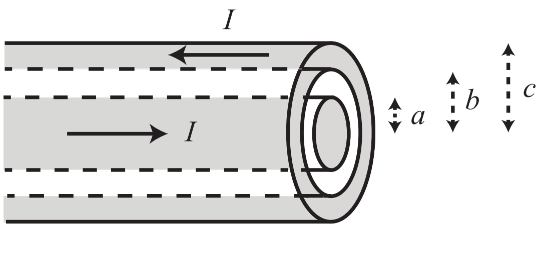
(b) the potential difference between the plates,

(c) the electrostatic energy stored, and,

(d) the charge on each plate?

[5 marks]

(iv) An infinite coaxial cable consists of a solid cylindrical conductor of radius a and a concentric thick hollow cylinder of inner radius b and outer radius c. A short section of the cable is shown in the figure.



The region between the two conductors (i.e. $a < r < b$) is a vacuum. A uniform current I flows along the inner cylinder, whilst a uniform current I flows in the opposite direction along the outer cylinder. Calculate B, the magnitude of the magnetic field strength in the regions (a) $0 < r \leq a$, and (b) $b < r \leq c$.

[5 marks]

(v) Show that the magnitude of the smallest magnetic moment possible for an electron (mass m_e) in a classical circular orbit around a nucleus is given by $\mu_B = |e|\hbar/2m_e$, where e is the electronic charge. (You may assume that the magnitude of the orbital angular momentum is quantised in units of $\hbar$.) This quantity (called the Bohr magneton) is also the minimum value possible for the electron's *intrinsic* angular momentum (called spin), a property shared by nuclear particles. Why does the nucleus of an atom usually make a negligible contribution to the magnetic properties of materials? [5 marks]

[Total 25 marks]

2. (i) A futuristic space ship is being shown off on a space race course. Travelling at its top speed of $\beta = 0.6$, it crosses the start line of the race course at $t = t' = 0$ in the rest frames of the race course and the space ship respectively. The ship maintains its top speed along the entire race course.

(a) The race course is linear and 5 million km long in its rest frame. Calculate the course length L' in the rest frame of the space ship. At what time t'_{finish} does the space ship cross the finish line in this frame? (The size of the spaceship is negligible compared the length of the racecourse)

(b) Interplanetary fireworks are being set off 5 million km further down the course from the finish line. The fireworks are timed so that the first firework goes off at the pre-calculated moment that the space ship crosses the finish line in the race course reference frame. Draw and carefully annotate a space time diagram of the race from start to finish: include all the relevant world lines and events with their coordinates.

(c) Calculate the time $t'_{\text{fireworks}}$ in the space ship frame at which the first firework goes off and indicate $t'_{\text{fireworks}}$ in your space time diagram. Compare $t'_{\text{fireworks}}$ with t'_{finish} from question (a) and comment on the result.
You may use the following Lorentz transformations:

$$\begin{aligned} t' &= \gamma(t - vx/c^2), \\ x' &= \gamma(x - vt), \\ y' &= y, \\ z' &= z, \end{aligned}$$

where all the symbols have their usual meaning.

[12 marks]

(ii) In a particle physics lab, an electron e^- and a positron e^+ collide, annihilate, and produce a W^+ boson and a W^- boson. Just before the collision, the electron and positron have a total energy of $E_T = 100$ GeV each, with velocities pointing along the +x-axis and -x-axis respectively. In the questions below you may use:

$$m_{e^-} = m_{e^+} = 0.511\,\text{MeV}/c^2, \quad m_{W^-} = m_{W^+} = 80.385\,\text{GeV}/c^2,$$

$$E_T = \gamma mc^2, \quad E_T^2 = (pc)^2 + (mc^2)^2.$$

(a) The velocity (as a fraction of c) of particles travelling at extremely relativistic speed can be written as:

$$\beta = \frac{v}{c} = 1 - \epsilon,$$

where $\epsilon << 1$. By considering the total energy, or otherwise, calculate ϵ for the electron in the lab.

(b) What is the momentum p of each of the W bosons after the collision? [Hint: use both conservation of momentum and conservation of energy.]

(c) The W^+ boson's mean lifetime is 3.0×10^{-25} s. Assuming it decays exactly at its mean lifetime, how long after it was created does it decay in the lab frame?

[18 marks]

[Total 30 marks]

3. This question concerns two point charges of values +3 C and −1 C. The two charges are fixed in position with a separation of exactly 1 metre. Consider the positive charge to be placed at the origin and the negative charge to be placed at (x = 1 m, y = 0, z = 0) with the location given in Cartesian coordinates.

Surd format and multiples of π and ϵ_0 are acceptable forms for the final answers. SI units must be included.

(i) Find the following:

(a) the mutual force of attraction between the charges,

(b) the binding energy of the charges,

(c) the electrostatic potential energy shared between the charges,

(d) the single position in Cartesian coordinates where the electric field strength due to the charges is exactly zero,

(e) the work required to bring an electron in from a large distance away from the charges to a final position midway between them if it is (i) brought in straight along the perpendicular bisector between the charges i.e. along the line $x = \frac{1}{2}$ m (ii) brought in following a spiral shape of decreasing distance from the charges until it reaches the midpoint.

[11 marks]

(ii) Show that the equipotential surface for the two charges where the potential is exactly zero is a sphere of radius $\frac{3}{8}$ m centred at $\left(x = \frac{9}{8}\text{ m}, y = 0, z = 0\right)$. What is the electrostatic flux through this sphere?

Hint: the equation for a circle in the xy plane in Cartesian coordinates is given by $(x - x_0)^2 + (y - y_0)^2 = r^2$ where r is the radius of the circle and (x_0, y_0) is the centre of the circle.

[7 marks]

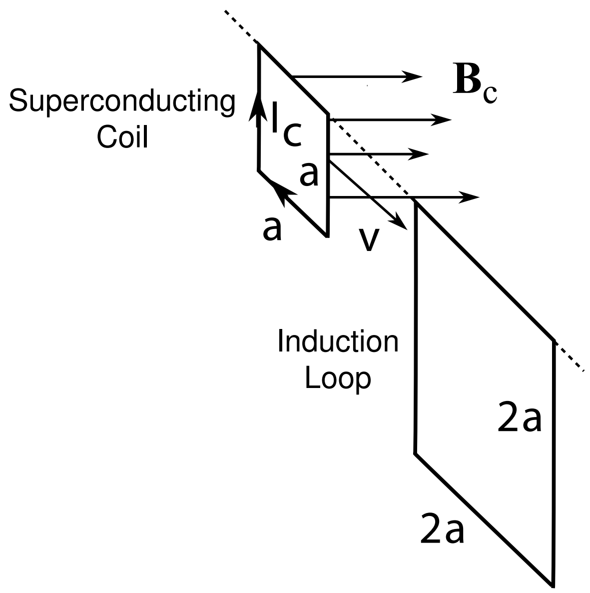
(iii) Sketch a 2D representation in the xy plane of the electric field lines and equipotential lines due to the two charges. Both sets of lines should appear on the same diagram.

[7 marks]

[Total 25 marks]

4. Throughout this problem, assume that the magnetic field B_s due to a current I_s flowing in a wire in the shape of a square of side s is uniform within the area bounded by the wire and perpendicular to the plane of the square, with magnitude $B_s = \alpha\mu_0 I_s/s$, where α is a dimensionless number.

Consider a square loop (called the induction loop) of wire of side $2a$ that is held in a vertical position, past which a superconducting coil is moving at a speed v, with the top edge of the coil aligned with the top of the loop, as sketched in the figure. The coil has a square cross-section of side a and carries a constant current I_c which gives rise to a magnetic field $\mathbf{B}_c$.



(i) The induction loop has negligible resistance and a self-inductance $L_\ell = 2\alpha\mu_0 a$. At $t = 0$ the front edge of the superconducting coil begins to overlap the area of the loop. Show that while the overlap region is increasing, the current I_ℓ induced in the loop obeys

$$L_\ell \frac{dI_\ell}{dt} = B_c a v \ .$$

Hence show that the current I_ℓ increases linearly with time until the coil completely overlaps the loop according to

$$I_\ell = \left(\frac{B_c v}{2\alpha\mu_0}\right) t \ .$$

Copy the sketch and indicate on it the direction of I_ℓ. [8 marks]

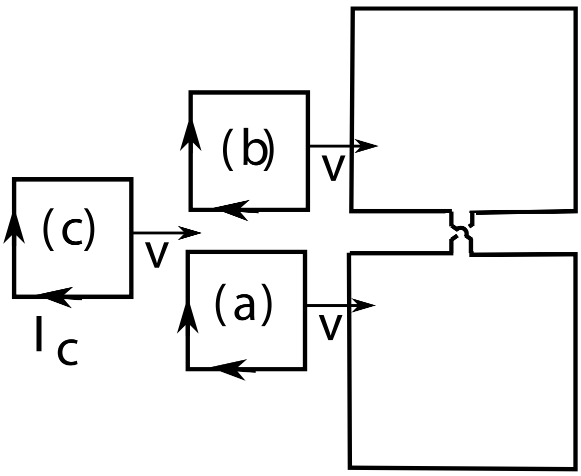
The superconducting coil will experience a force due to the magnetic field induced by the current I_ℓ flowing in the loop.

(ii) Find the magnetic field B_ℓ generated by the current I_ℓ in the induction loop. Hence show that the force on the coil during the initial phase analysed in part (i) includes an upward component given by

$$F = \frac{(B_c vt)^2}{4\alpha\mu_0} \; .$$

[You should assume that the top of the coil and the top of the induction loop are aligned with one another, so that the magnetic field there is identically zero.]
[6 marks]

The set up considered above forms the basis of magnetic levitation employed in Maglev trains, where the superconducting coil resides in the undercarriage of the train, and a set of induction loops are embedded along the track. In real Maglev systems, each induction loop is extended upwards and twisted, to form a figure-of-eight configuration as sketched below.



(iii) Indicate on suitable sketches the direction of any currents I_ℓ induced by the superconducting coils passing the induction loop for the three cases (a), (b), and (c) shown in the above sketch. By considering the resulting vertical forces in these three cases, explain briefly how this figure-of-eight configuration acts to stabilise the vertical position of the coil. [6 marks]

[Total 20 marks]