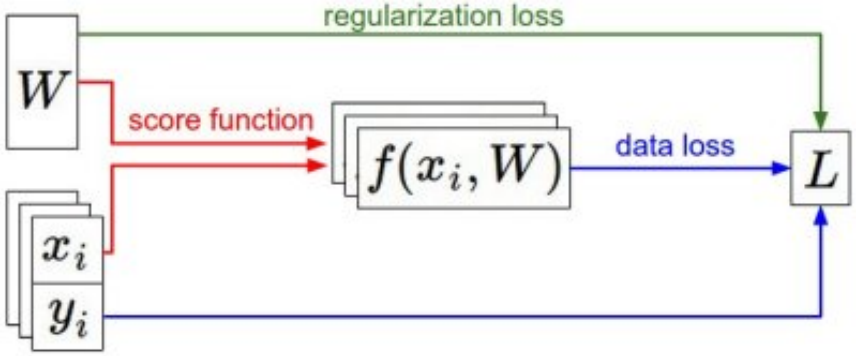
Administrative

- **A1** is due Today (midnight). You can use up to 3 late days
- **A2** will be up this Friday, it's due next next Wednesday (Feb 4)
- **Project Proposal** is due next Friday at midnight (~one paragraph (200-400 words), send as email)

Lecture 5:

Backprop and intro to Neural Nets

Linear Classification



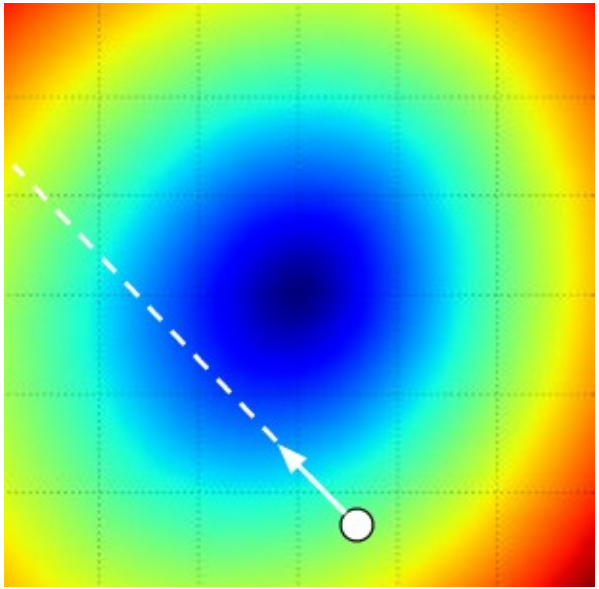
SVM:

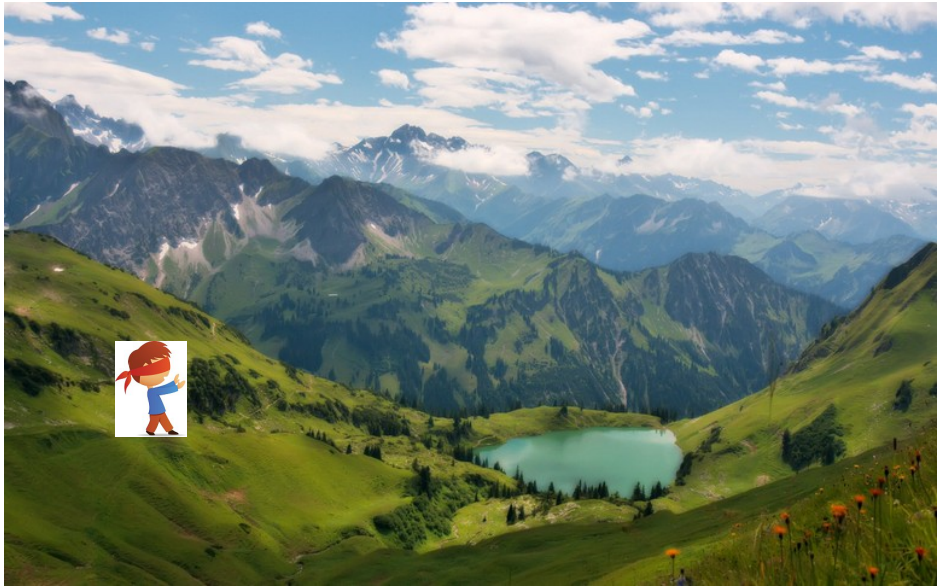
$$L = \frac{1}{N}\sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta)\right] + \lambda \sum_k \sum_l W_{k,l}^2$$

Softmax:

$$L = \frac{1}{N}\sum_i -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}\right) + \lambda \sum_k \sum_l W_{k,l}^2$$

Optimization Landscape





Gradient Descent

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Numerical gradient: slow :(, approximate :(, easy to write :)
Analytic gradient: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

This class:

Becoming a backprop ninja

$$f(x,y) = xy \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h \frac{df(x)}{dx}$$

$$f(x,y) = xy \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h \frac{df(x)}{dx}$$

Example: x = 4, y = -3. => f(x,y) = -12

$$\frac{\partial f}{\partial x} = -3$$

$$\frac{\partial f}{\partial y} = 4$$

partial derivatives

$$\nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]$$

gradient

$$f(x,y) = xy \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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$$\frac{\partial f}{\partial x} = -3$$

$$\frac{\partial f}{\partial y} = 4$$

partial derivatives

$$\nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]$$

gradient

Question: If I increase x by h, how would the output of f change?

Compound expressions: $f(x, y, z) = (x + y)z$

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

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Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

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Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

```
# set some inputs
x = -2; y = 5; z = -4

# perform the forward pass
q = x + y # q becomes 3
f = q * z # f becomes -12

# perform the backward pass (backpropagation) in reverse order:
# first backprop through f = q * z
dfdz = q # df/fz = q, so gradient on z becomes 3
dfdq = z # df/dq = z, so gradient on q becomes -4
# now backprop through q = x + y
dfdx = 1.0 * dfdq # dq/dx = 1. And the multiplication here is the chain rule!
dfdy = 1.0 * dfdq # dq/dy = 1
```

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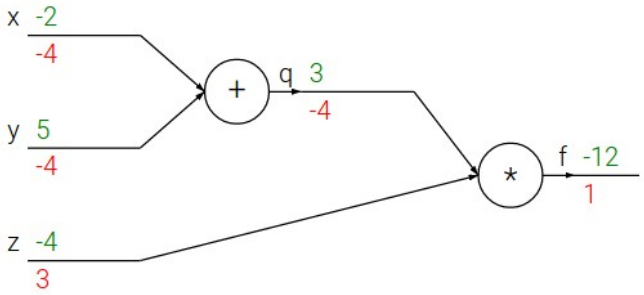


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dfdx = 1.0 * dfdq # dq/dx = 1. And the multiplication here is the chain rule!
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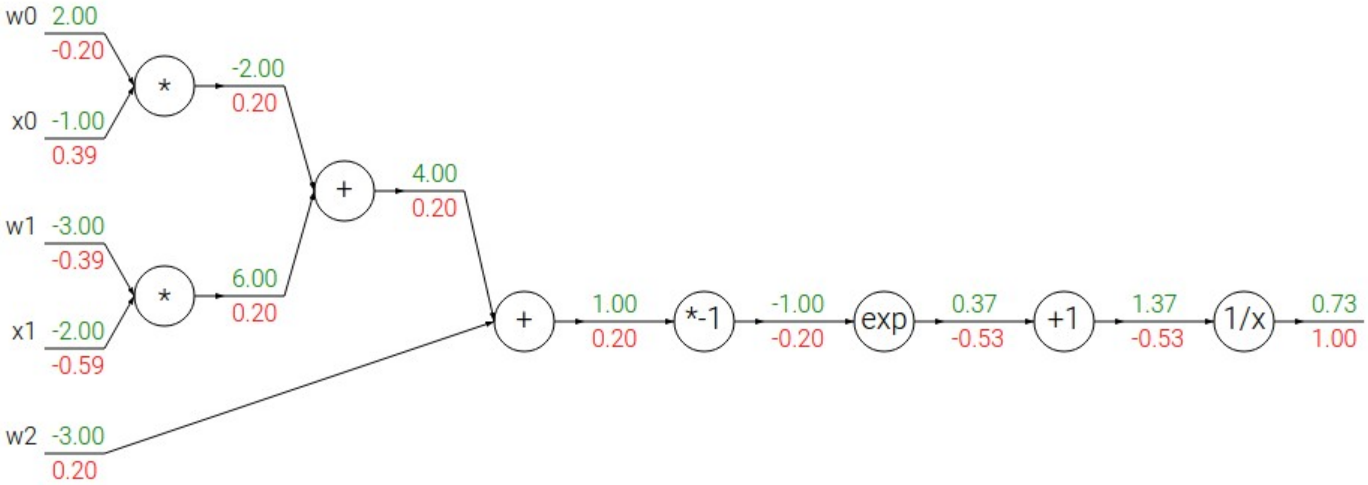
```

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$$

$$f_a(x) = ax \rightarrow \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$$

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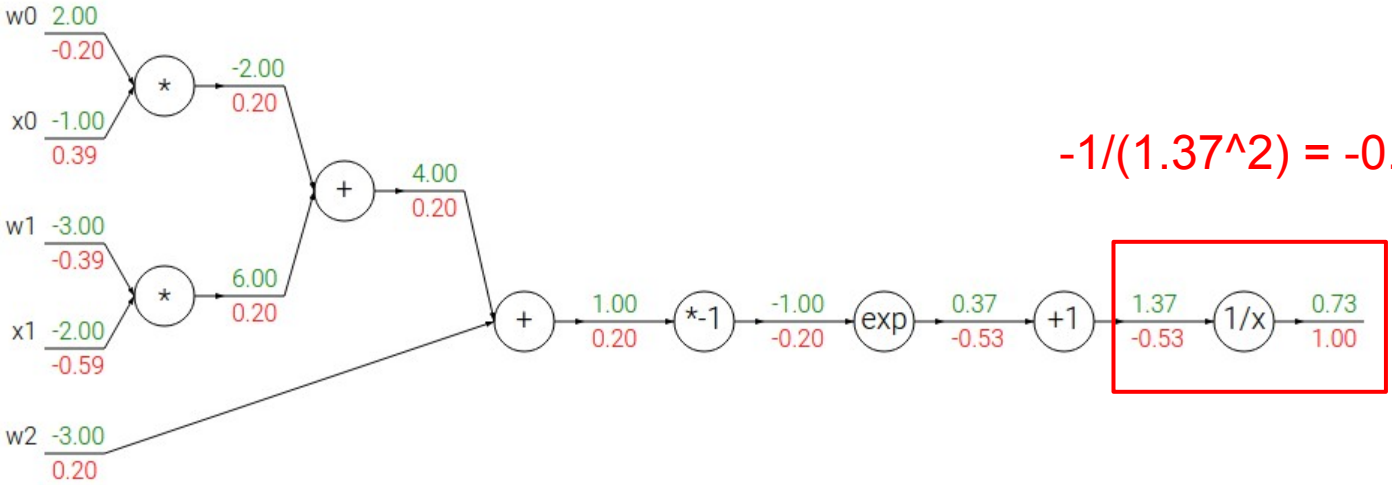

-1/(1.37^2) = -0.53

$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$

$f_a(x) = ax \rightarrow \frac{df}{dx} = a$

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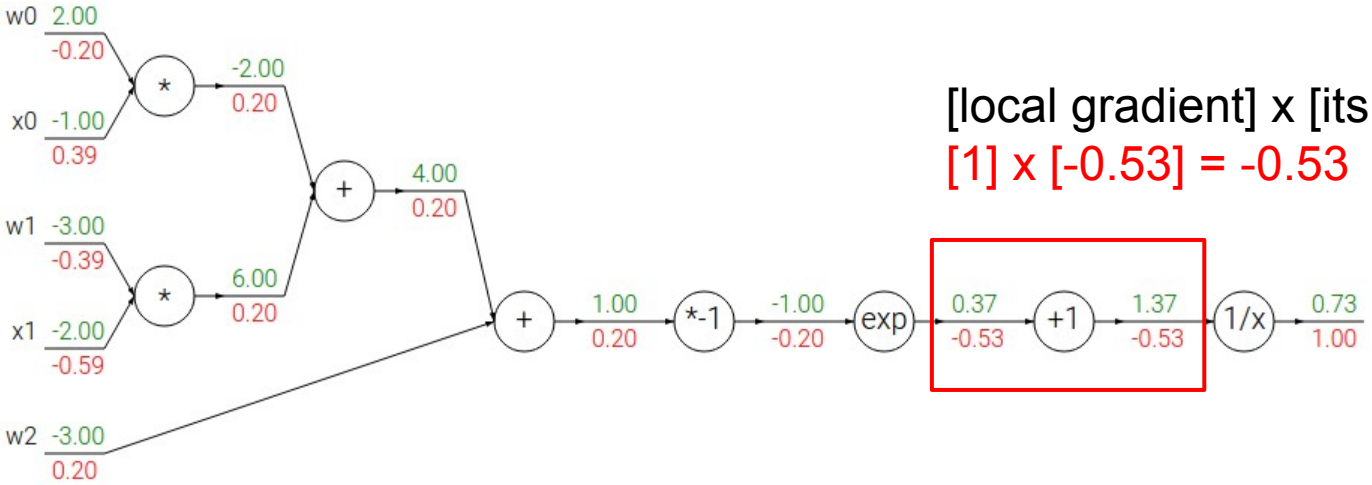

[local gradient] x [its gradient]
[1] x [-0.53] = -0.53

$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$$

$$f_a(x) = ax \rightarrow \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$$

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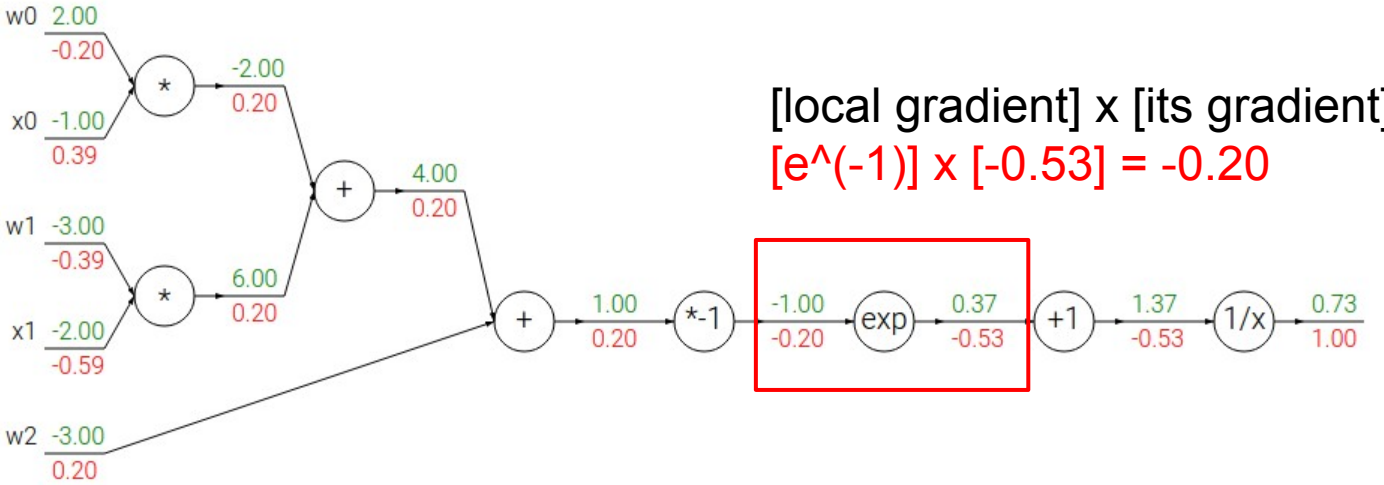

[local gradient] x [its gradient]
[e^(-1)] x [-0.53] = -0.20

$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$$

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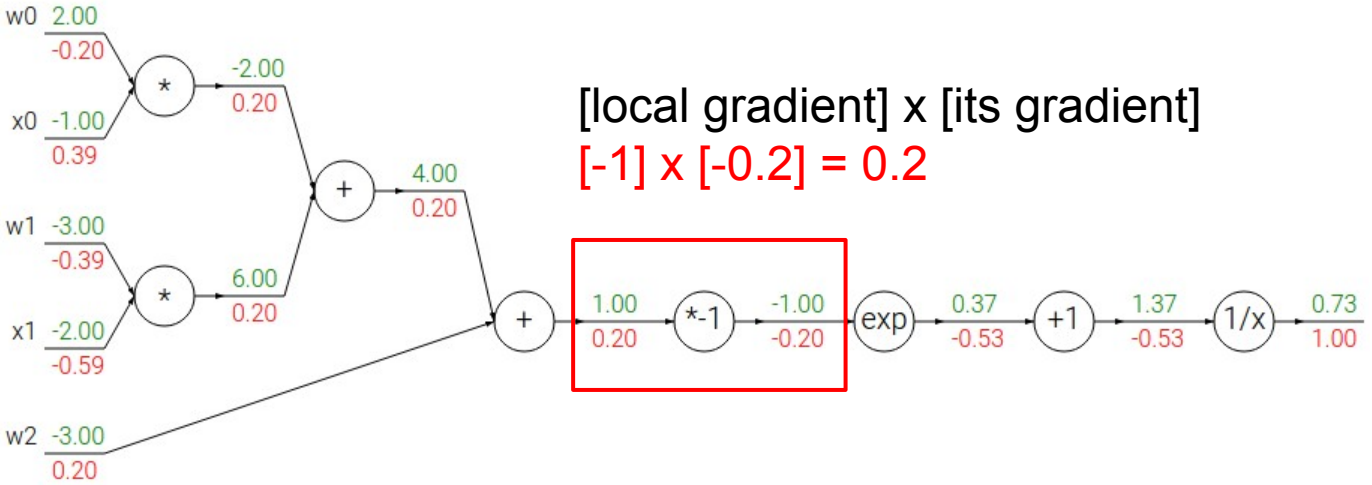

[local gradient] x [its gradient]

[-1] x [-0.2] = 0.2

$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$$

$$f_a(x) = ax \rightarrow \frac{df}{dx} = a$$

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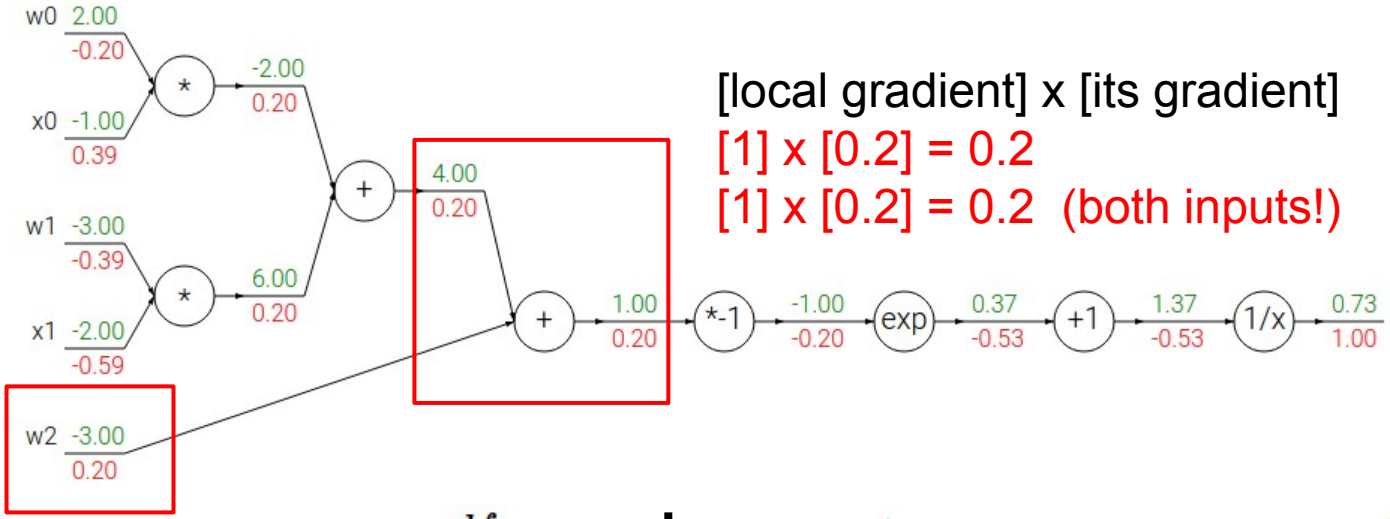

[local gradient] x [its gradient]

[1] x [0.2] = 0.2

[1] x [0.2] = 0.2 (both inputs!)

$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$$

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$$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$$

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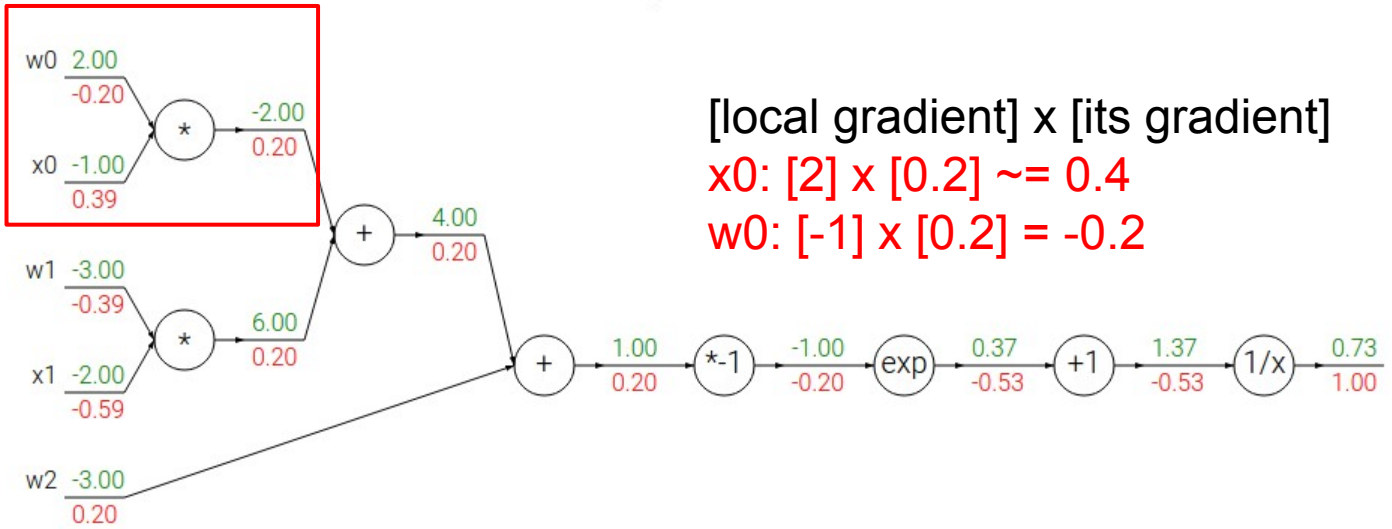

[local gradient] x [its gradient]
x0: [2] x [0.2] ~= 0.4
w0: [-1] x [0.2] = -0.2

$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$

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$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$

$f_c(x) = c + x \rightarrow \frac{df}{dx} = 1$

Every gate during backprop computes, for all its inputs:

[LOCAL GRADIENT] x [GATE GRADIENT]

Can be computed right away, even during forward pass

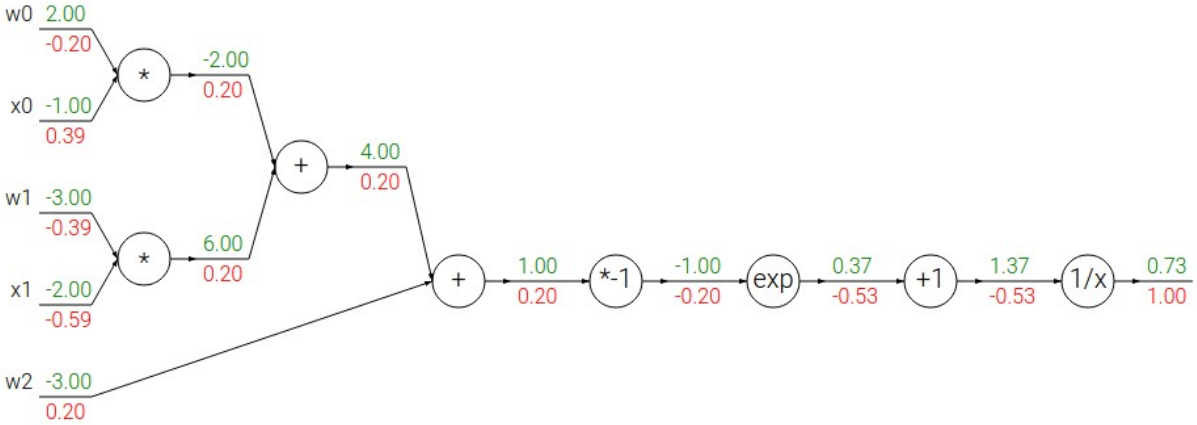
The gate receives this during backpropagation

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$



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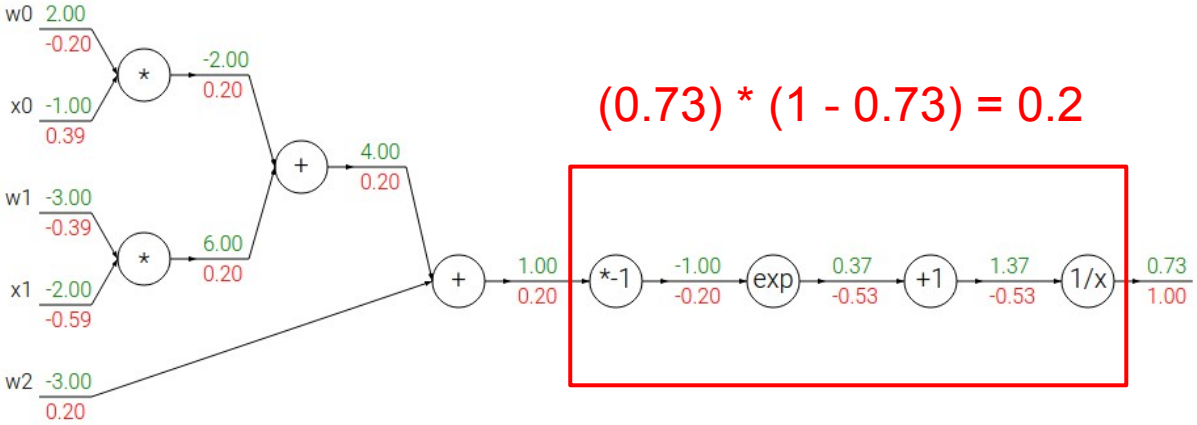

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$

```
w = [2,-3,-3] # assume some random weights and data
x = [-1, -2]

# forward pass
dot = w[0]*x[0] + w[1]*x[1] + w[2]
f = 1.0 / (1 + math.exp(-dot)) # sigmoid function
```

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$ sigmoid function

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# forward pass
dot = w[0]*x[0] + w[1]*x[1] + w[2]
f = 1.0 / (1 + math.exp(-dot)) # sigmoid function

# backward pass through the neuron (backpropagation)
ddot = (1 - f) * f # gradient on dot variable, using the sigmoid gradient derivation
dx = [w[0] * ddot, w[1] * ddot] # backprop into x
dw = [x[0] * ddot, x[1] * ddot, 1.0 * ddot] # backprop into w
# we're done! we have the gradients on the inputs to the circuit
```

We are ready: $$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

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$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

```
x = 3 # example values
y = -4

# forward pass
sigy = 1.0 / (1 + math.exp(-y)) # sigmoid in numerator   #(1)
num = x + sigy # numerator                               #(2)
sigx = 1.0 / (1 + math.exp(-x)) # sigmoid in denominator #(3)
xpy = x + y                                              #(4)
xpysqr = xpy**2                                          #(5)
den = sigx + xpysqr # denominator                        #(6)
invden = 1.0 / den                                       #(7)
f = num * invden # done!                                 #(8)
```

```
# backprop f = num * invden
dnum = invden # gradient on numerator                        #(8)
dinvden = num                                                #(8)
```

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

forward pass was:

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x = 3 # example values
y = -4

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sigy = 1.0 / (1 + math.exp(-y)) #
num = x + sigy # numerator
sigx = 1.0 / (1 + math.exp(-x)) #
xpy = x + y
xpysqr = xpy**2
den = sigx + xpysqr # denominato
invden = 1.0 / den
f = num * invden # done!
```

```
# backprop f = num * invden
dnum = invden # gradient on numerator                    #(8)
dinvden = num                                            #(8)
# backprop invden = 1.0 / den
dden = (-1.0 / (den**2)) * dinvden                       #(7)
```

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

forward pass was:

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sigy = 1.0 / (1 + math.exp(-y)) #
num = x + sigy # numerator
sigx = 1.0 / (1 + math.exp(-x)) #
xpy = x + y
xpysqr = xpy**2
den = sigx + xpysqr # denominato
invden = 1.0 / den
f = num * invden # done!
```

```
# backprop f = num * invden
dnum = invden # gradient on numerator                    #(8)
dinvden = num                                            #(8)
# backprop invden = 1.0 / den
dden = (-1.0 / (den**2)) * dinvden                       #(7)
# backprop den = sigx + xpysqr
dsigx = (1) * dden                                       #(6)
dxpysqr = (1) * dden                                     #(6)
```

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

forward pass was:

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x = 3 # example values
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dinvden = num                                            #(8)
# backprop invden = 1.0 / den
dden = (-1.0 / (den**2)) * dinvden                       #(7)
# backprop den = sigx + xpysqr
dsigx = (1) * dden                                       #(6)
dxpysqr = (1) * dden                                     #(6)
# backprop xpysqr = xpy**2
dxpy = (2 * xpy) * dxpysqr                               #(5)
```

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

forward pass was:

```
x = 3 # example values
y = -4

# forward pass
sigy = 1.0 / (1 + math.exp(-y)) #
num = x + sigy # numerator
sigx = 1.0 / (1 + math.exp(-x)) #
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f = num * invden # done!
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# backprop f = num * invden
dnum = invden # gradient on numerator                    #(8)
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# backprop den = sigx + xpysqr
dsigx = (1) * dden                                       #(6)
dxpysqr = (1) * dden                                     #(6)
# backprop xpysqr = xpy**2
dxpy = (2 * xpy) * dxpysqr                               #(5)
# backprop xpy = x + y
dx = (1) * dxpy                                          #(4)
dy = (1) * dxpy                                          #(4)
```

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

forward pass was:

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sigx = 1.0 / (1 + math.exp(-x)) #
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den = sigx + xpysqr # denominato
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f = num * invden # done!
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# backprop f = num * invden
dnum = invden # gradient on numerator                        #(8)
dinvden = num                                                #(8)
# backprop invden = 1.0 / den
dden = (-1.0 / (den**2)) * dinvden                           #(7)
# backprop den = sigx + xpysqr
dsigx = (1) * dden                                           #(6)
dxpysqr = (1) * dden                                         #(6)
# backprop xpysqr = xpy**2
dxpy = (2 * xpy) * dxpysqr                                   #(5)
# backprop xpy = x + y
dx = (1) * dxpy                                              #(4)
dy = (1) * dxpy                                              #(4)
# backprop sigx = 1.0 / (1 + math.exp(-x))
dx += ((1 - sigx) * sigx) * dsigx # Notice += !! See notes below  #(3)
```

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

forward pass was:

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y = -4

# forward pass
sigy = 1.0 / (1 + math.exp(-y)) #
num = x + sigy # numerator
sigx = 1.0 / (1 + math.exp(-x)) #
xpy = x + y
xpysqr = xpy**2
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invden = 1.0 / den
f = num * invden # done!
```

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# backprop f = num * invden
dnum = invden # gradient on numerator                             #(8)
dinvden = num                                                     #(8)
# backprop invden = 1.0 / den
dden = (-1.0 / (den**2)) * dinvden                                #(7)
# backprop den = sigx + xpysqr
dsigx = (1) * dden                                                #(6)
dxpysqr = (1) * dden                                              #(6)
# backprop xpysqr = xpy**2
dxpy = (2 * xpy) * dxpysqr                                        #(5)
# backprop xpy = x + y
dx = (1) * dxpy                                                   #(4)
dy = (1) * dxpy                                                   #(4)
# backprop sigx = 1.0 / (1 + math.exp(-x))
dx += ((1 - sigx) * sigx) * dsigx # Notice += !! See notes below  #(3)
# backprop num = x + sigy
dx += (1) * dnum                                                  #(2)
dsigy = (1) * dnum                                                #(2)
```

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

forward pass was:

```
x = 3 # example values
y = -4

# forward pass
sigy = 1.0 / (1 + math.exp(-y)) #
num = x + sigy # numerator
sigx = 1.0 / (1 + math.exp(-x)) #
xpy = x + y
xpysqr = xpy**2
den = sigx + xpysqr # denominato
invden = 1.0 / den
f = num * invden # done!
```

```
# backprop f = num * invden
dnum = invden # gradient on numerator                             #(8)
dinvden = num                                                     #(8)
# backprop invden = 1.0 / den
dden = (-1.0 / (den**2)) * dinvden                                #(7)
# backprop den = sigx + xpysqr
dsigx = (1) * dden                                                #(6)
dxpysqr = (1) * dden                                              #(6)
# backprop xpysqr = xpy**2
dxpy = (2 * xpy) * dxpysqr                                        #(5)
# backprop xpy = x + y
dx = (1) * dxpy                                                   #(4)
dy = (1) * dxpy                                                   #(4)
# backprop sigx = 1.0 / (1 + math.exp(-x))
dx += ((1 - sigx) * sigx) * dsigx # Notice += !! See notes below  #(3)
# backprop num = x + sigy
dx += (1) * dnum                                                  #(2)
dsigy = (1) * dnum                                                #(2)
# backprop sigy = 1.0 / (1 + math.exp(-y))
dy += ((1 - sigy) * sigy) * dsigy                                 #(1)
# done! phew
```

$$f(x,y) = \frac{x + \sigma(y)}{\sigma(x) + (x+y)^2}$$

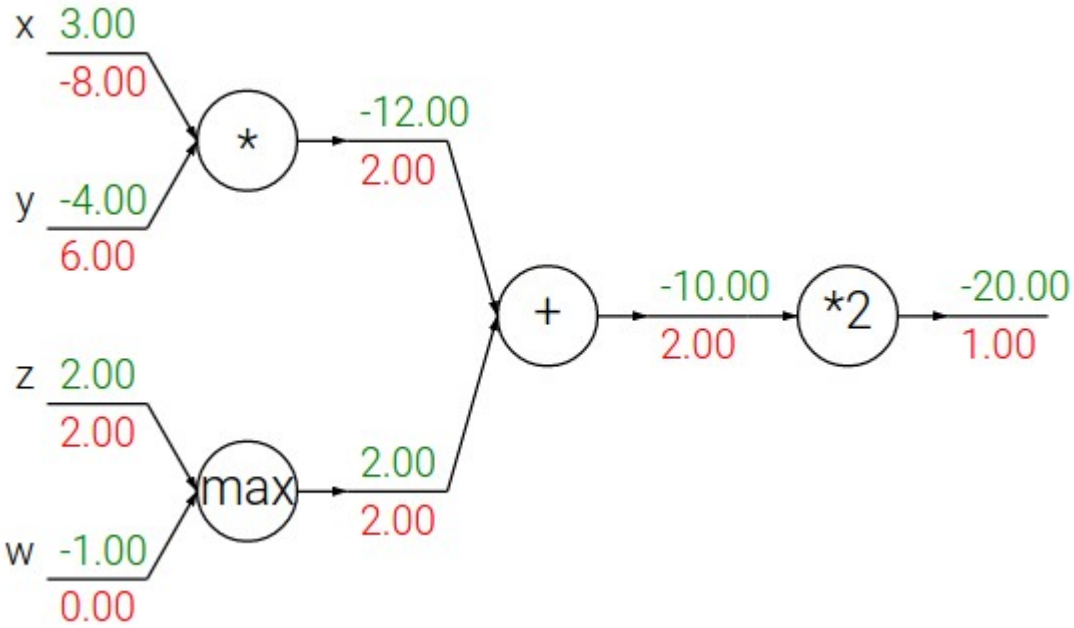
forward pass was:

```
x = 3 # example values
y = -4

# forward pass
sigy = 1.0 / (1 + math.exp(-y)) #
num = x + sigy # numerator
sigx = 1.0 / (1 + math.exp(-x)) #
xpy = x + y
xpysqr = xpy**2
den = sigx + xpysqr # denominato
invden = 1.0 / den
f = num * invden # done!
```

Patterns in backward flow

add gate: gradient distributor
max gate: gradient router
mul gate: gradient… “switcher”?



Gradients for vectorized code

```
# forward pass
W = np.random.randn(5, 10)
X = np.random.randn(10, 3)
D = W.dot(X)

# now suppose we had the gradient on D from above in the circuit
dD = np.random.randn(*D.shape) # same shape as D
```

Gradients for vectorized code

```
# forward pass
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D = W.dot(X)

# now suppose we had the gradient on D from above in the circuit
dD = np.random.randn(*D.shape) # same shape as D
```
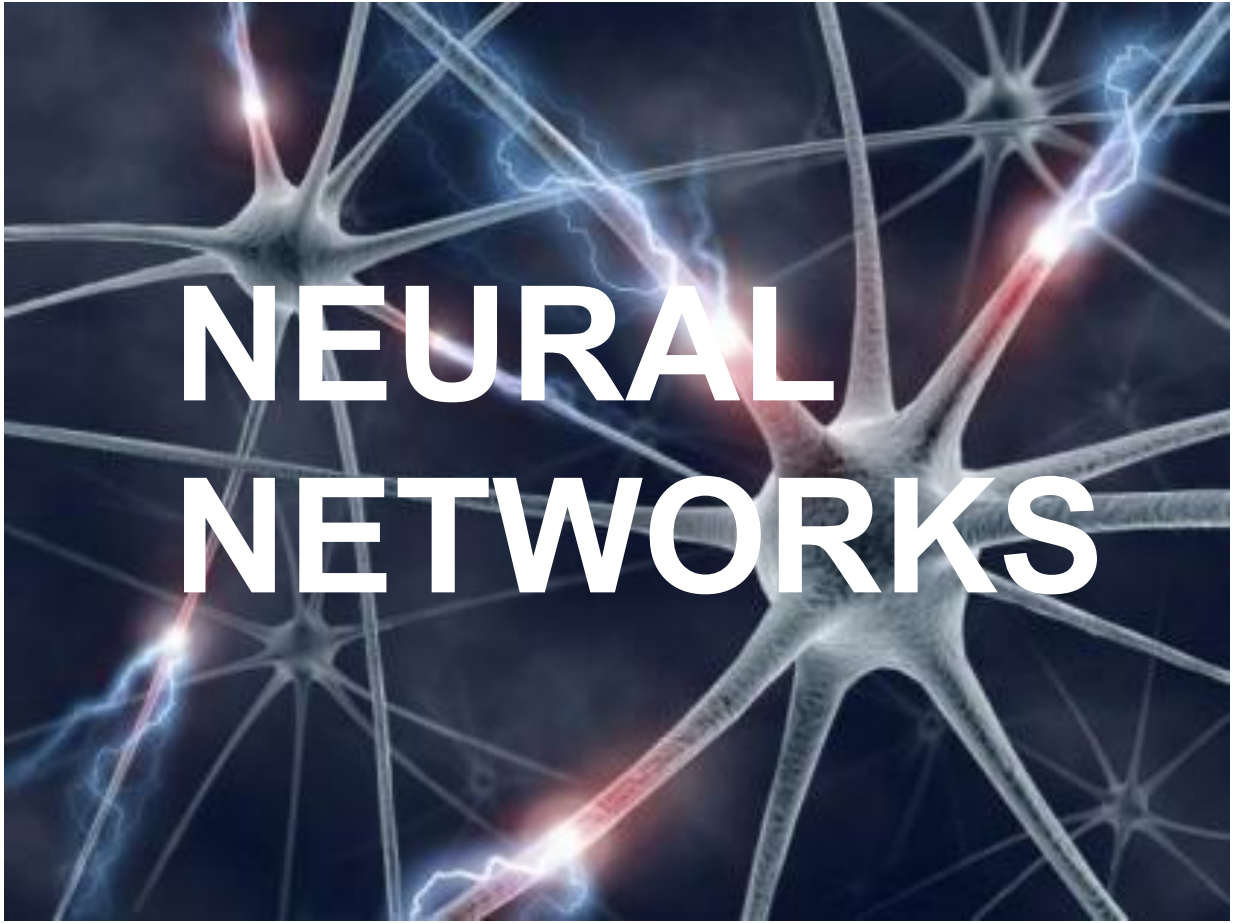

X is [10 x 3], dD is [5 x 3]
dW must be [5 x 10]
dX must be [10 x 3]

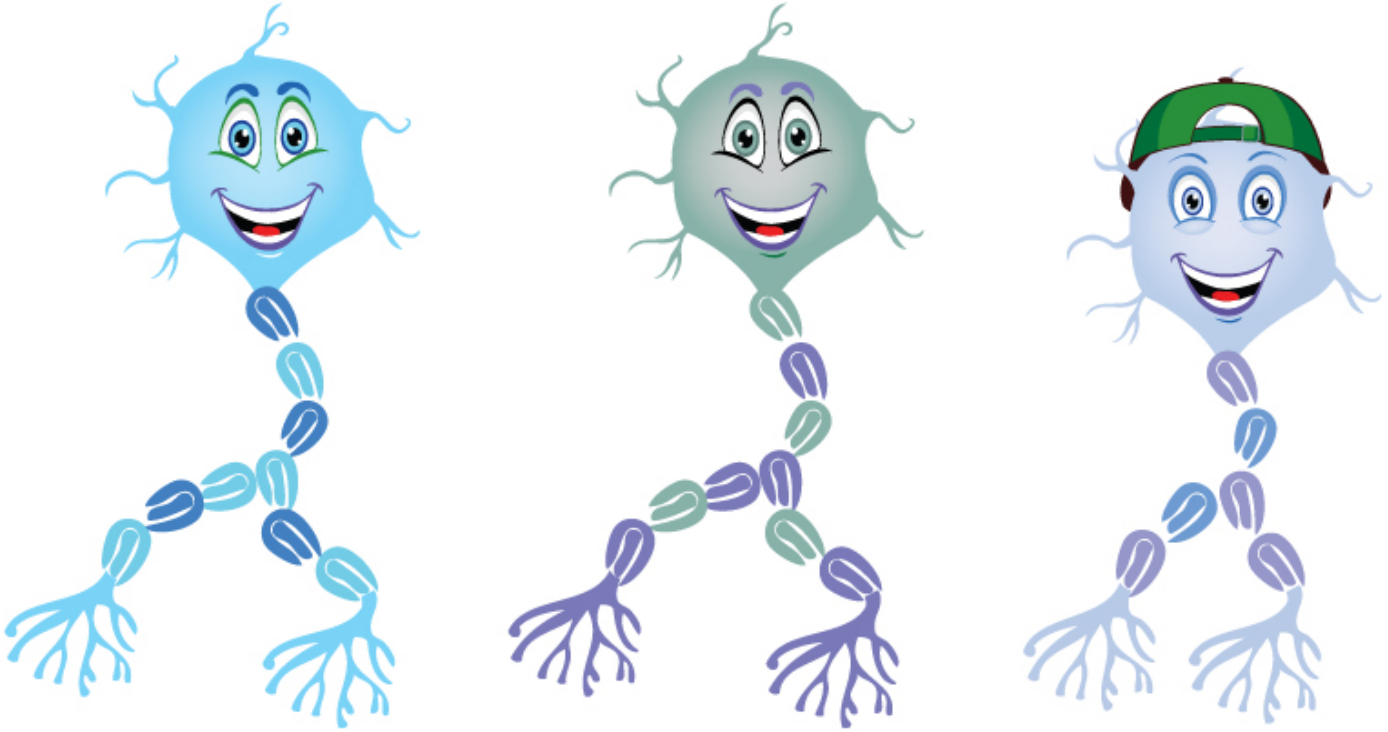
Gradients for vectorized code

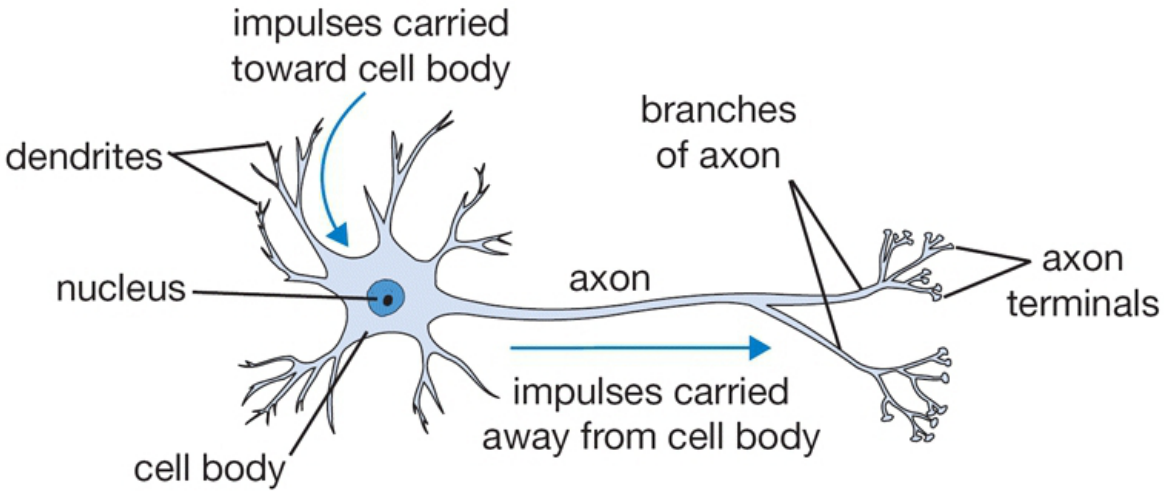
```
# forward pass
W = np.random.randn(5, 10)
X = np.random.randn(10, 3)
D = W.dot(X)

# now suppose we had the gradient on D from above in the circuit
dD = np.random.randn(*D.shape) # same shape as D
dW = dD.dot(X.T) #.T gives the transpose of the matrix
dX = W.T.dot(dD)
```

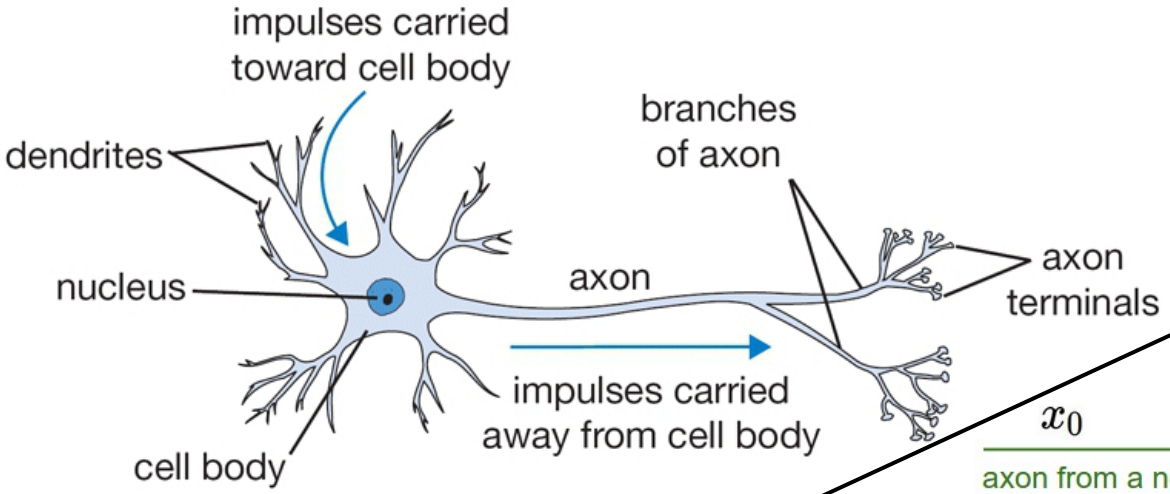
In summary

- in practice it is rarely needed to derive long gradients of variables on pen and paper
- structured your code in **stages** (layers), where you can derive the local gradients, then chain the gradients during **backprop**.
- caveat: sometimes gradients simplify (e.g. for sigmoid, also softmax). Group these.


NEURAL
NETWORKS




impulses carried
toward cell body
dendrites
branches
of axon
nucleus
axon
axon
terminals
impulses carried
away from cell body
cell body


impulses carried
toward cell body
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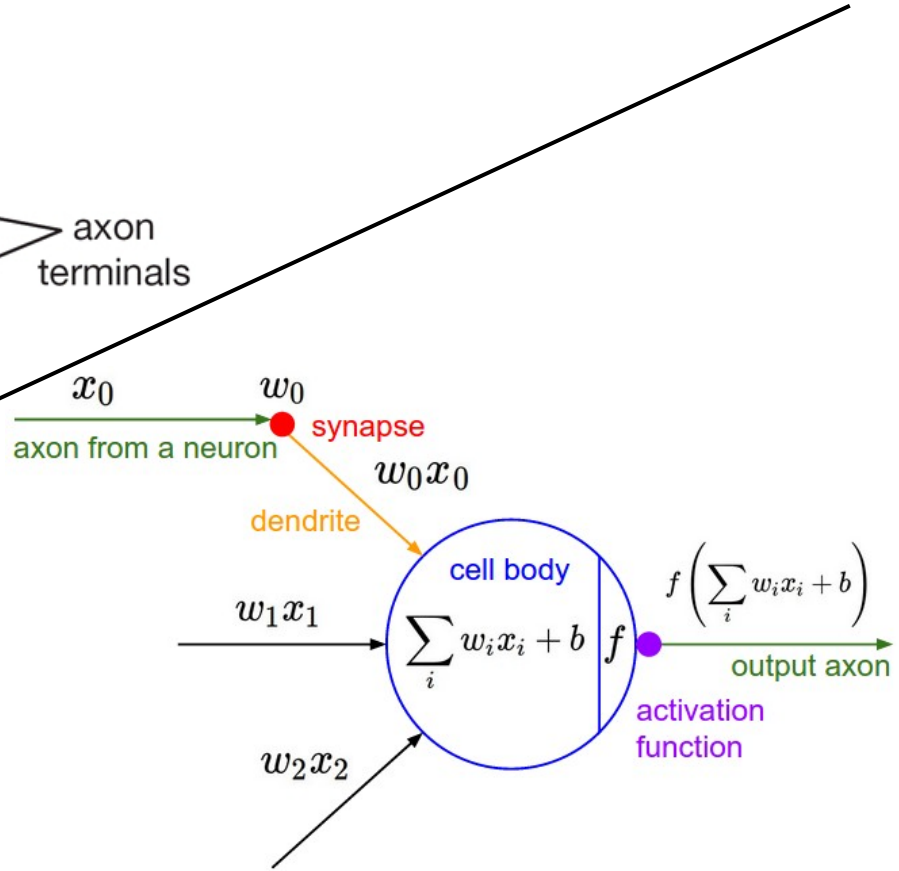
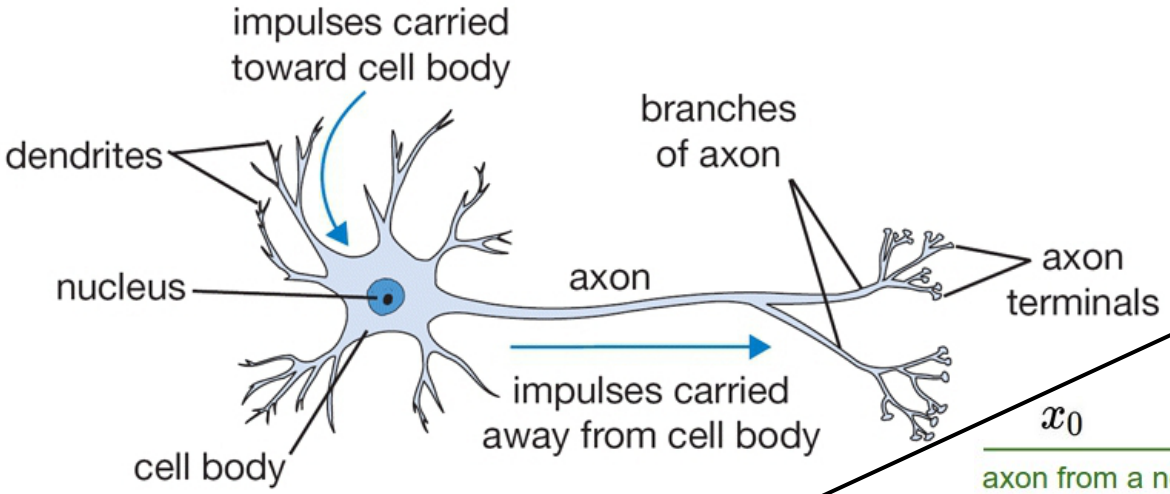
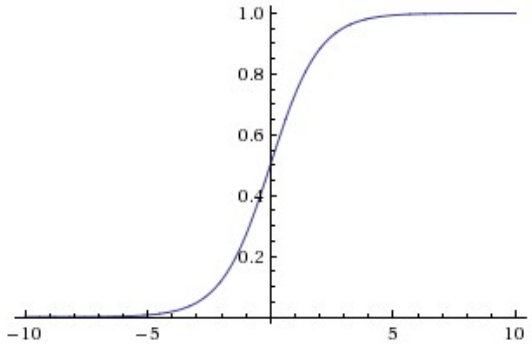
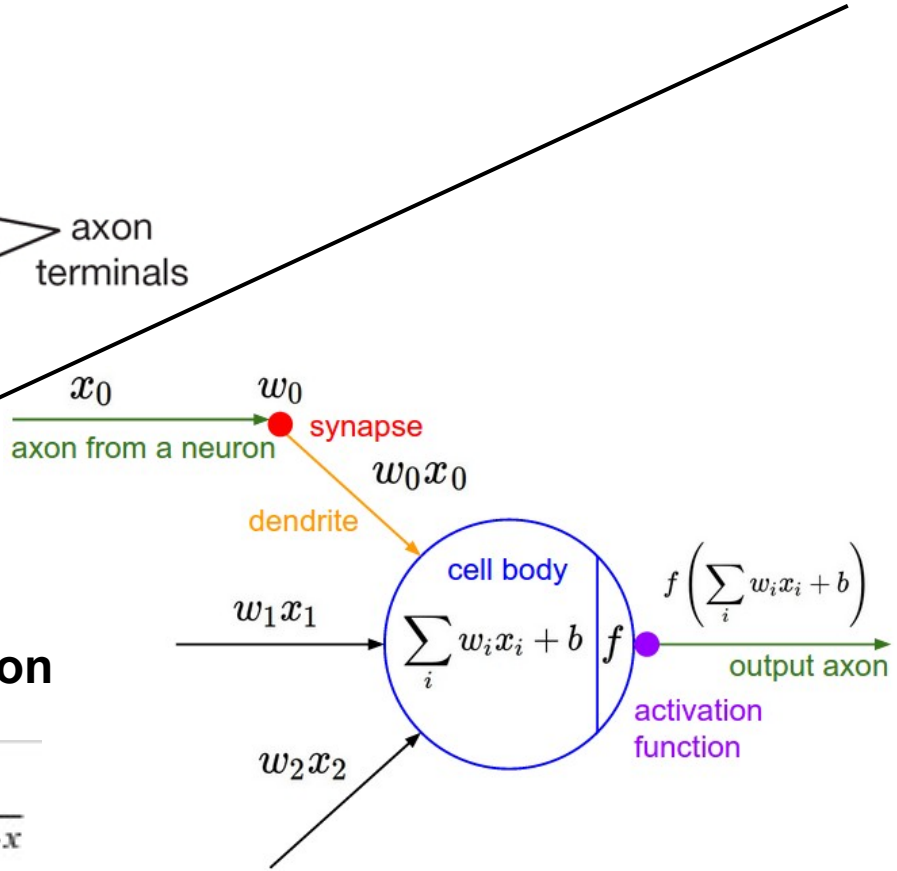

x_0
w_0
axon from a neuron
synapse
$w_0 x_0$
dendrite
cell body
$w_1 x_1$
$\sum_i w_i x_i + b$
f
$f\left(\sum_i w_i x_i + b\right)$
output axon
activation
function
$w_2 x_2$

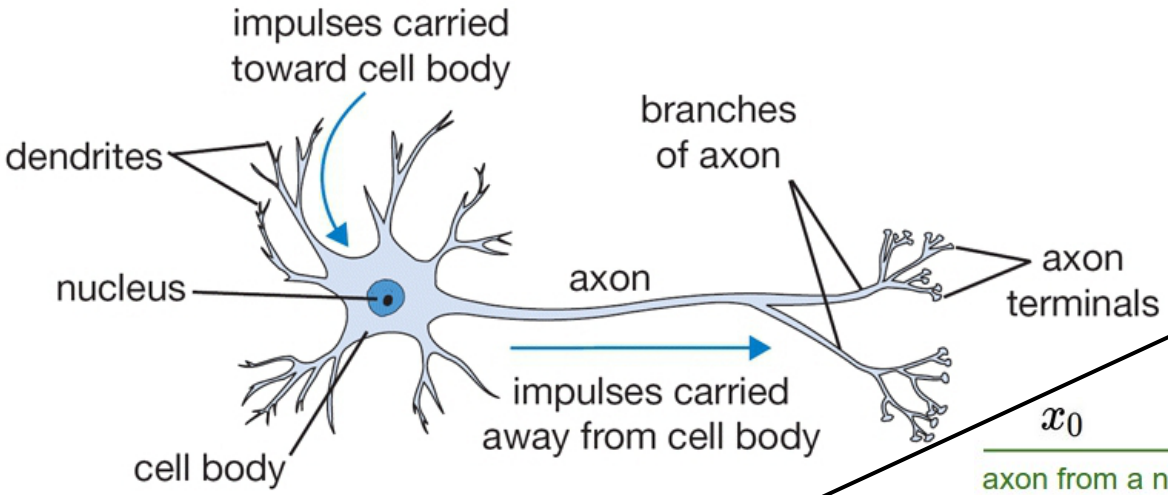


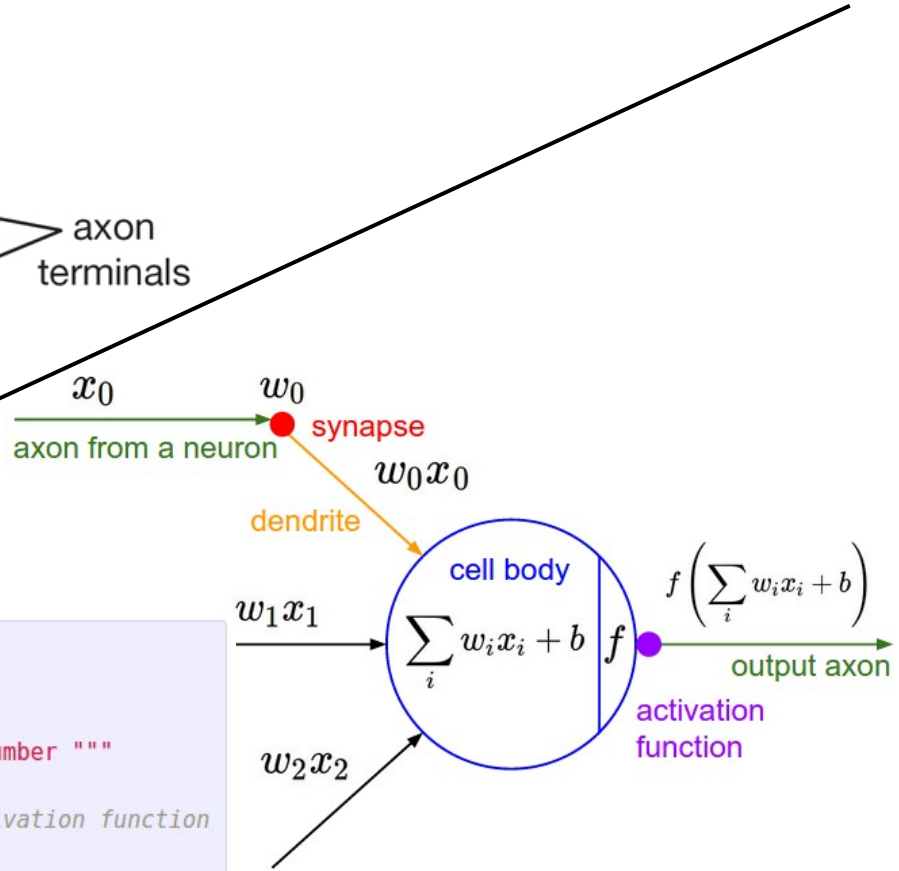


sigmoid activation function

$$\frac{1}{1+e^{-x}}$$

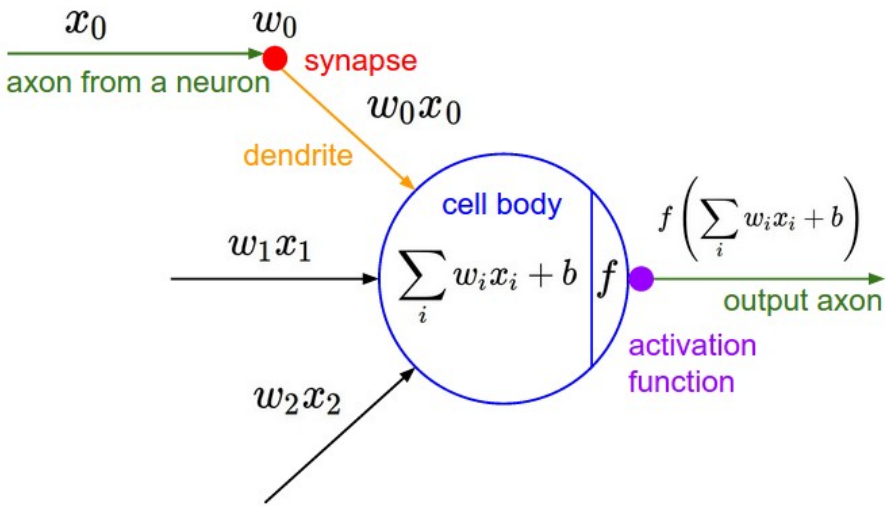







```
class Neuron:
  # ...
  def neuron_tick(inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```

A Single Neuron can be used as a binary linear classifier

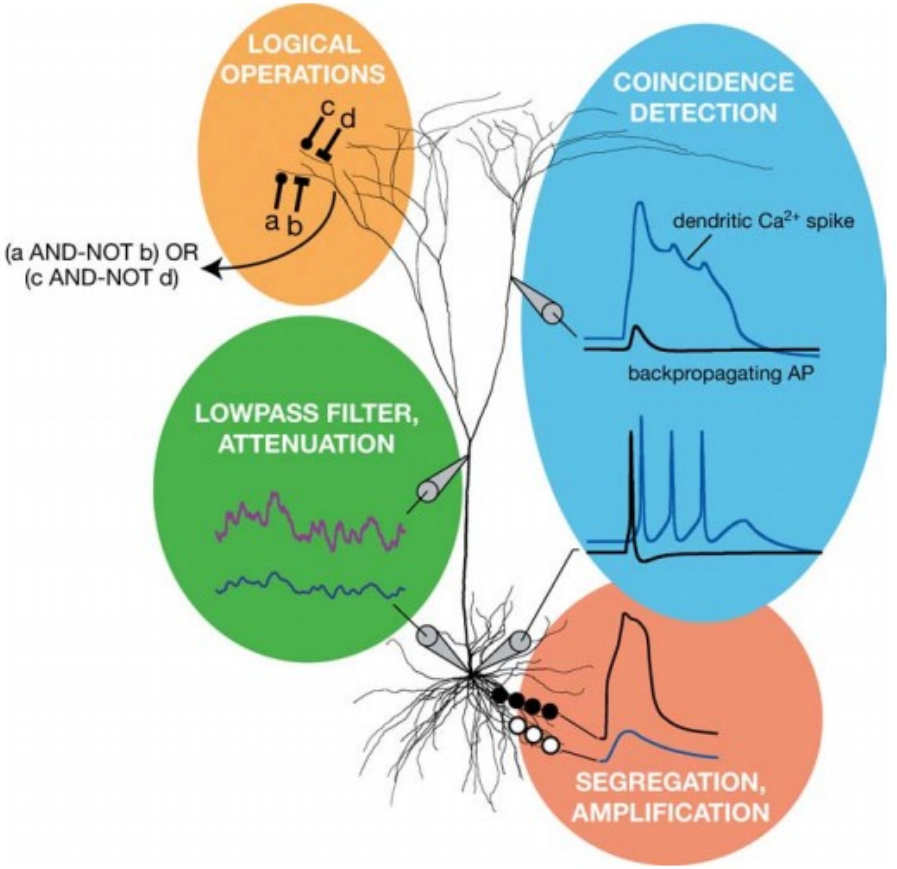


Regularization has the interpretation of "gradual forgetting"

Be very careful with your Brain analogies:

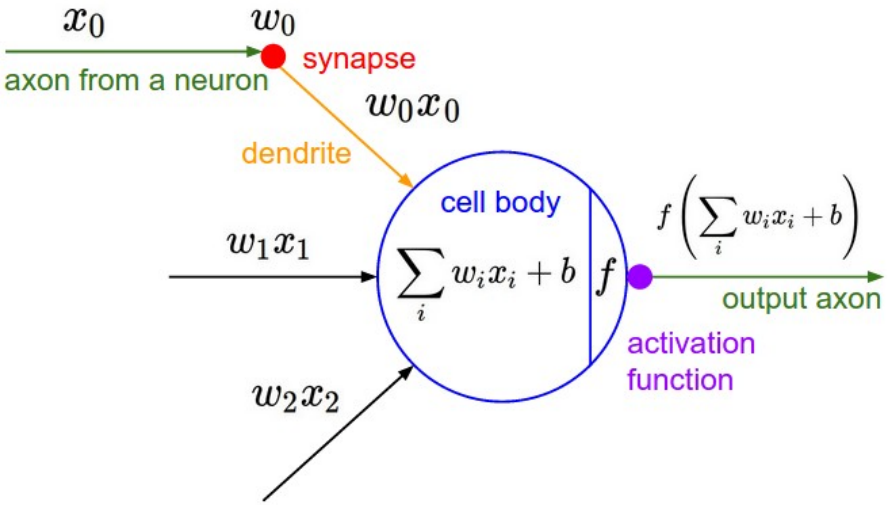
Biological Neurons:

- Many different types
- Dendrites can perform complex non-linear computations
- Synapses are not a single weight but a complex non-linear dynamical system
- Rate code may not be adequate



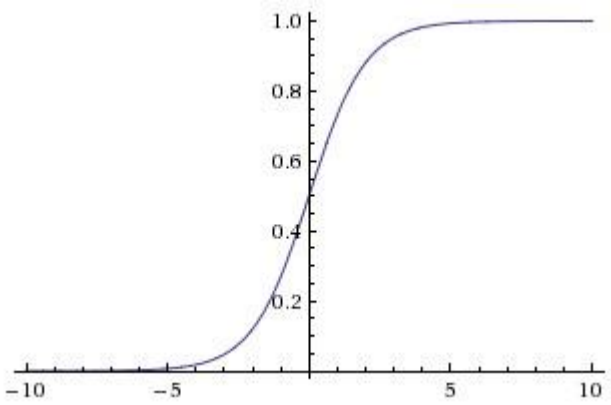
[Dendritic Computation. London and Hausser]

Activation Functions



Activation Functions

$$\sigma(x) = 1/(1+e^{-x})$$



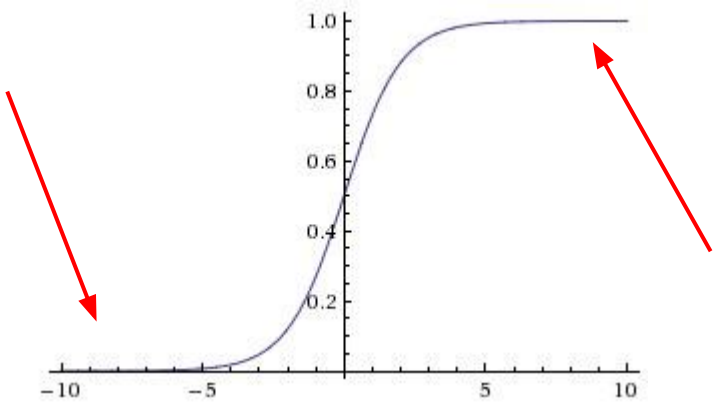
Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

2 BIG problems:

Activation Functions

$$\sigma(x) = 1/(1+e^{-x})$$



Sigmoid

- Squashes numbers to range [0,1]
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2 BIG problems:

1. Saturated neurons “kill” the gradients

Activation Functions

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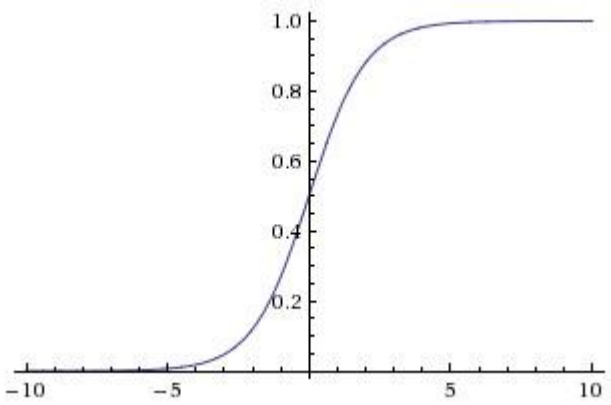


Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

2 BIG problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

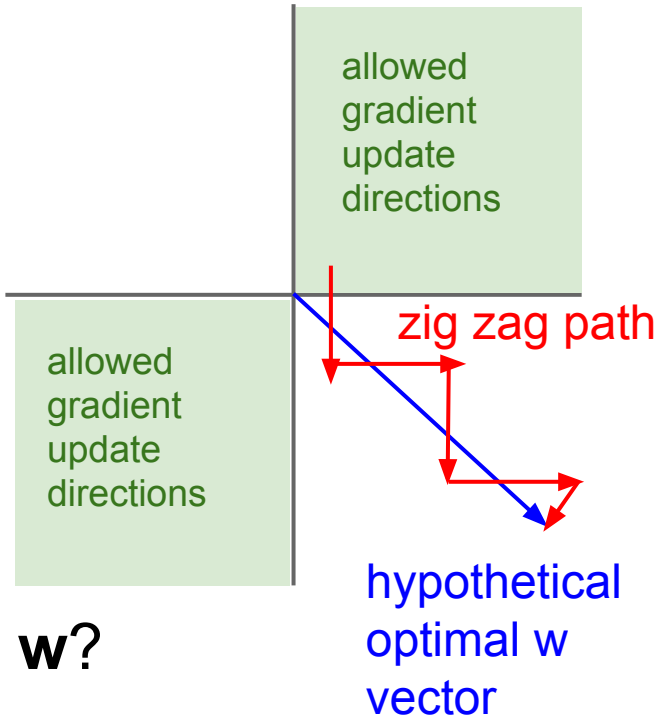
Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

What can we say about the gradients on **w**?

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

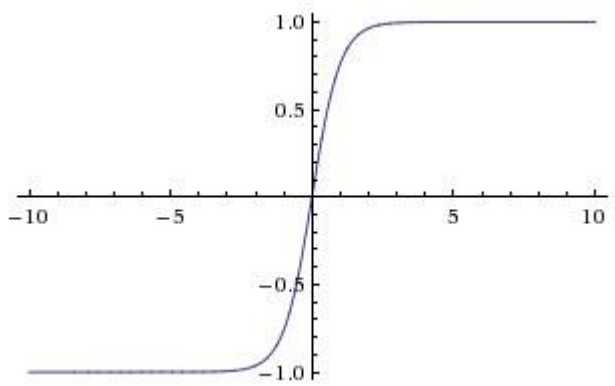


What can we say about the gradients on **w**?

Always all positive or all negative :(

(this is also why you want zero-mean data!)

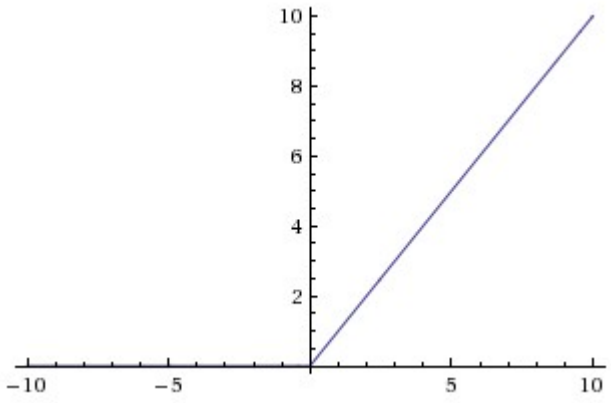
Activation Functions



tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

Activation Functions



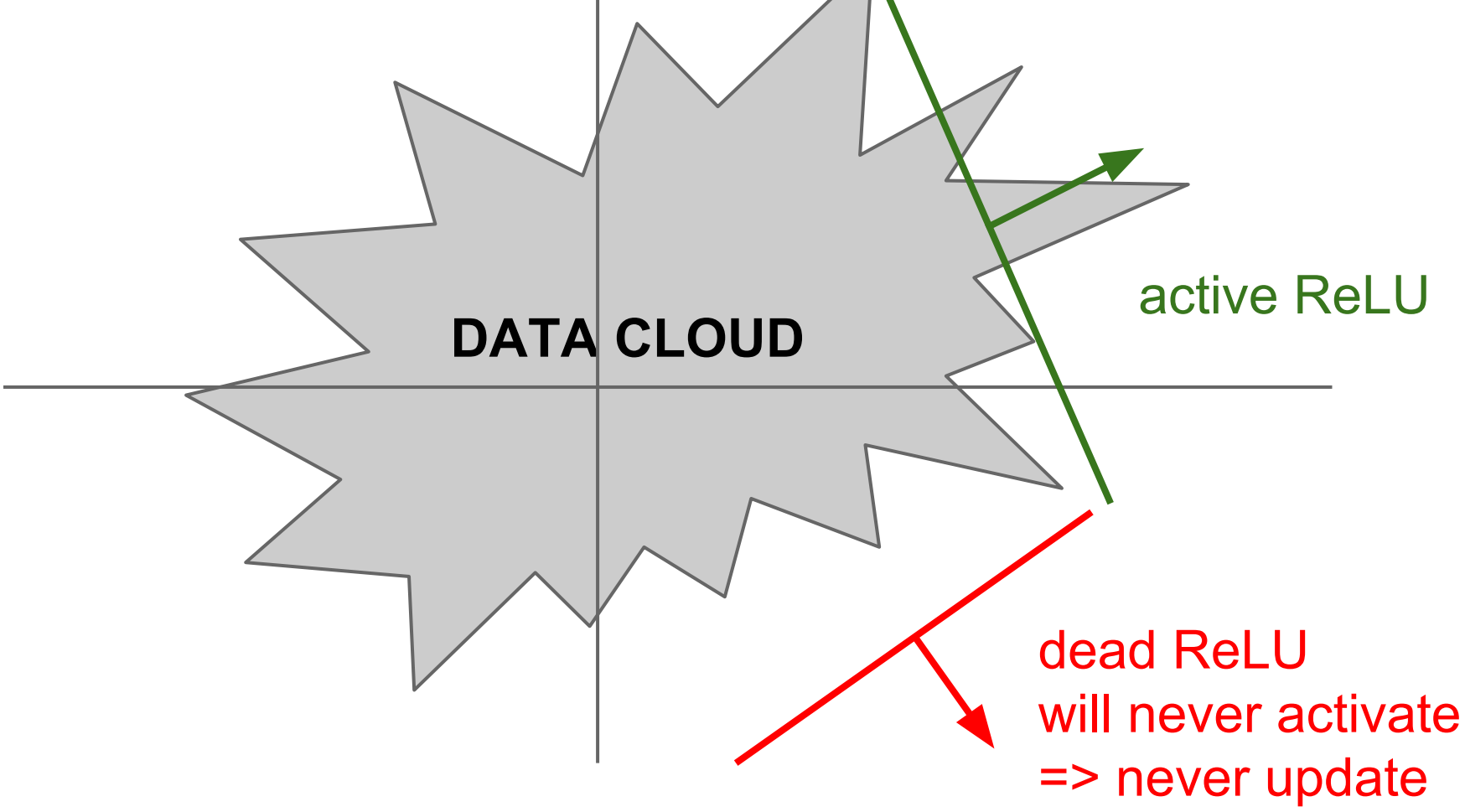
ReLU

- Computes f(x) = max(0,x)

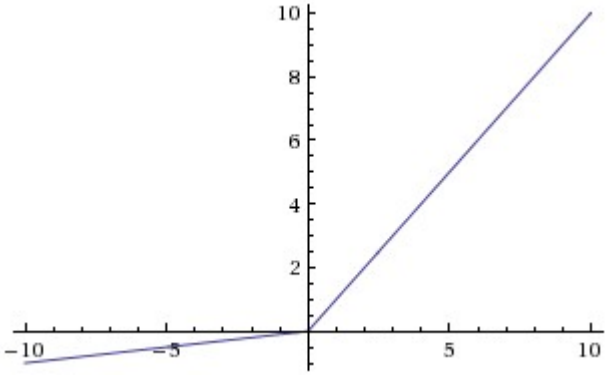
- Does not saturate
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)

- Just one annoying problem…

hint: what is the gradient when x < 0?


DATA CLOUD
active ReLU
dead ReLU
will never activate
=> never update

Activation Functions



Leaky ReLU

- Does not saturate
- computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not “die”.

Maxout “Neuron”

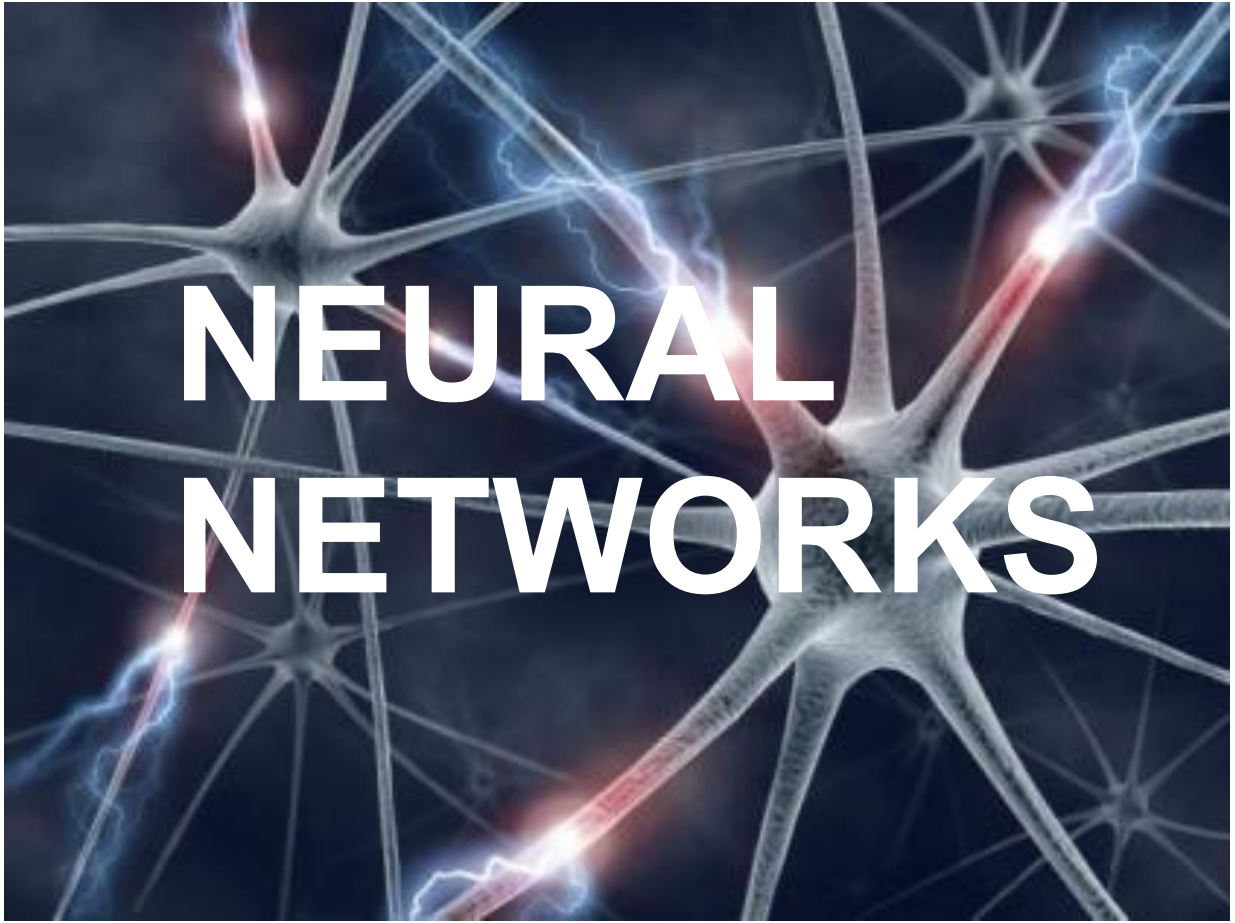
- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

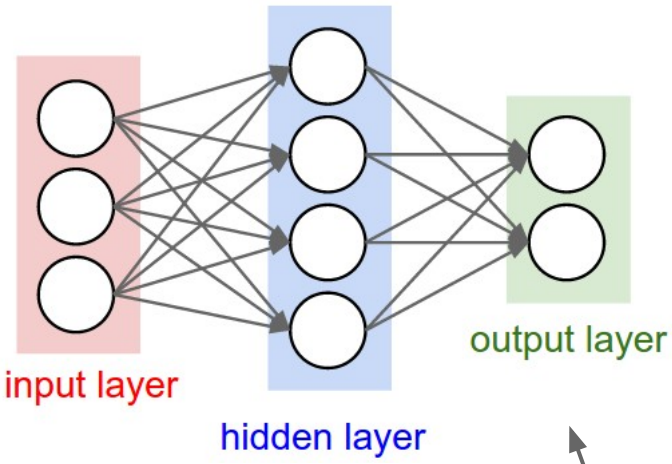
Problem: doubles the number of parameters :(

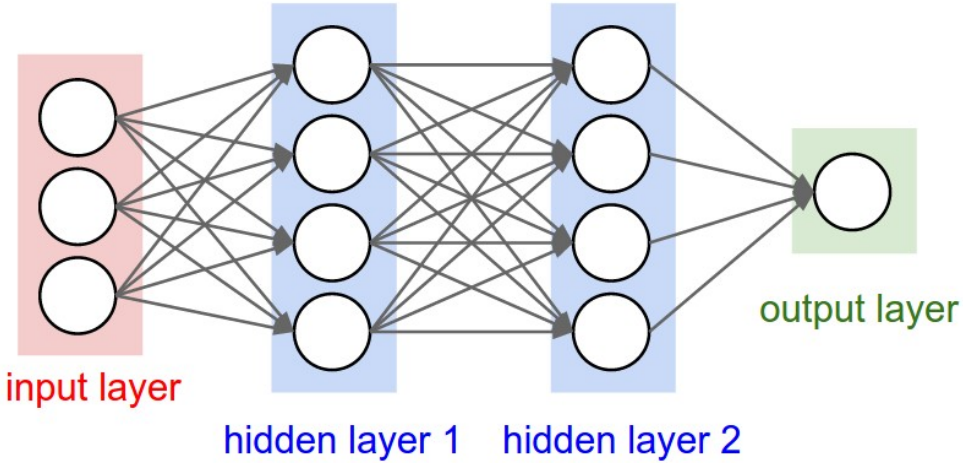
TLDR: In practice:

- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout
- Try out tanh but don’t expect much
- Never use sigmoid


NEURAL
NETWORKS

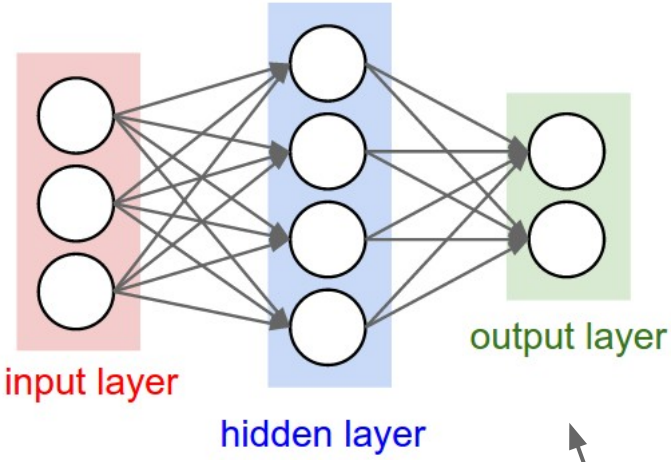
Neural Networks: Architectures

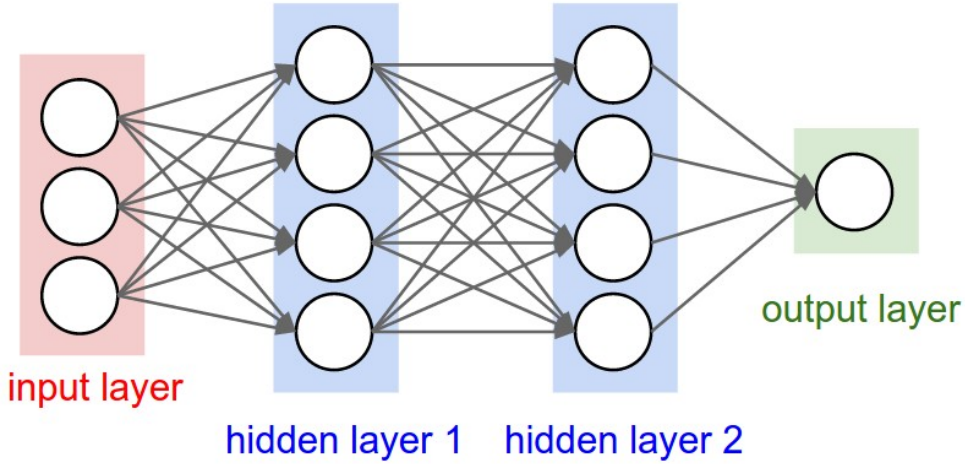




“Fully-connected” layers

Neural Networks: Architectures



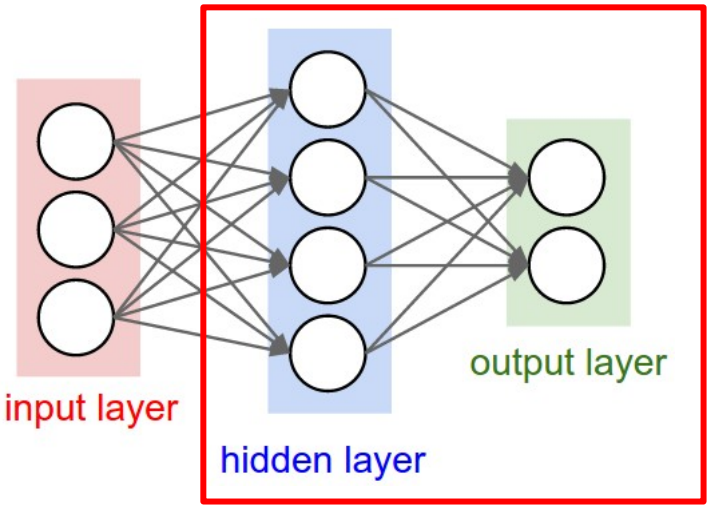


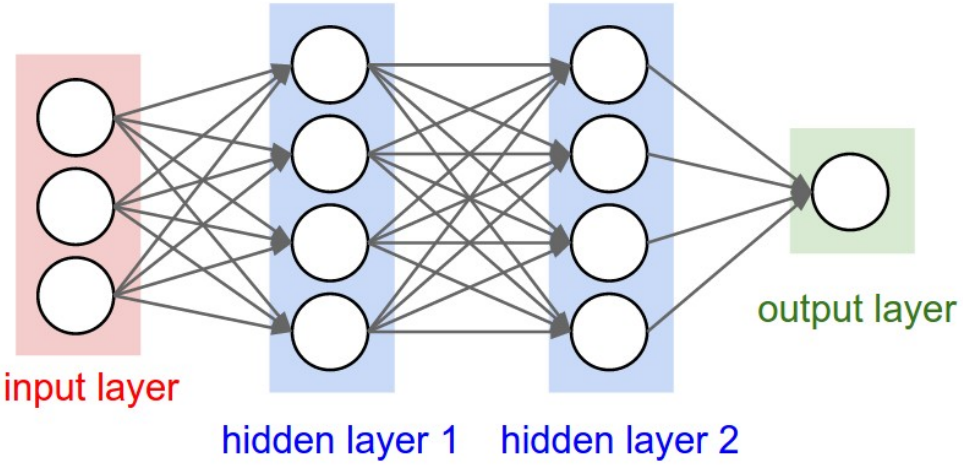
"2-layer Neural Net", or
"1-hidden-layer Neural Net"

"3-layer Neural Net", or
"2-hidden-layer Neural Net"

"Fully-connected" layers

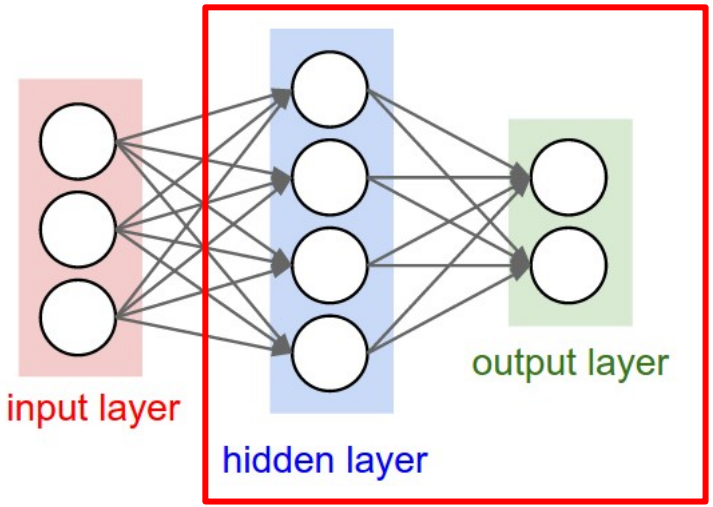
Neural Networks: Architectures

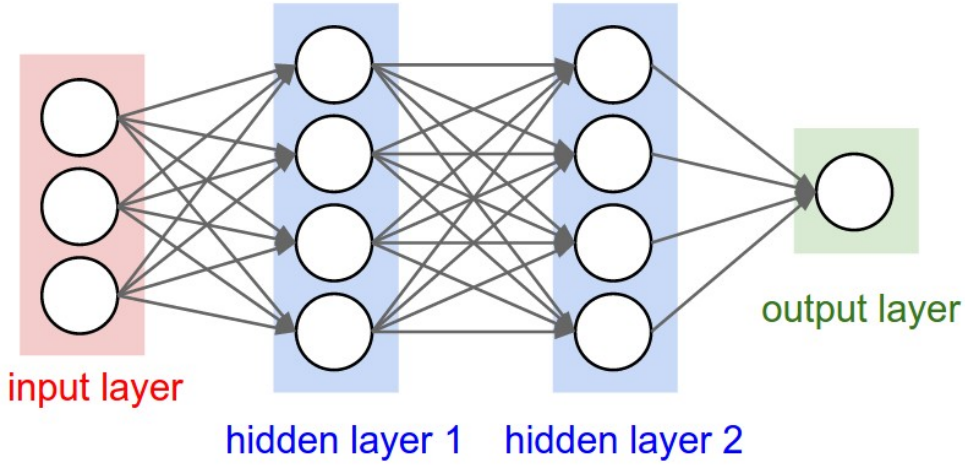




Number of Neurons: ?
Number of Weights: ?
Number of Parameters: ?

Neural Networks: Architectures

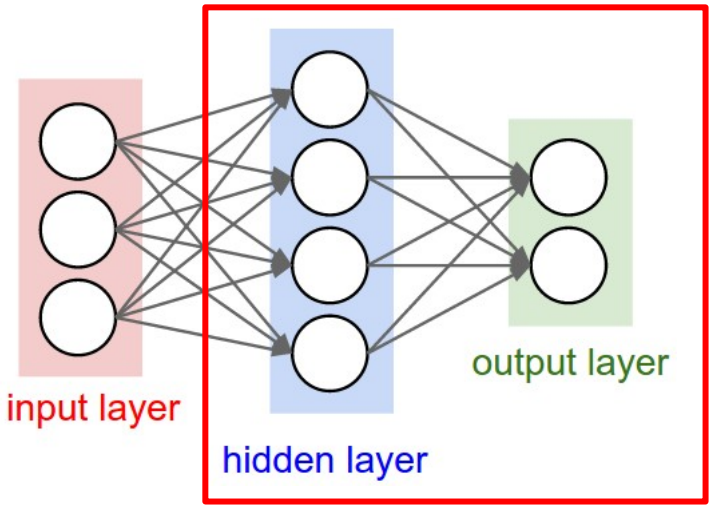


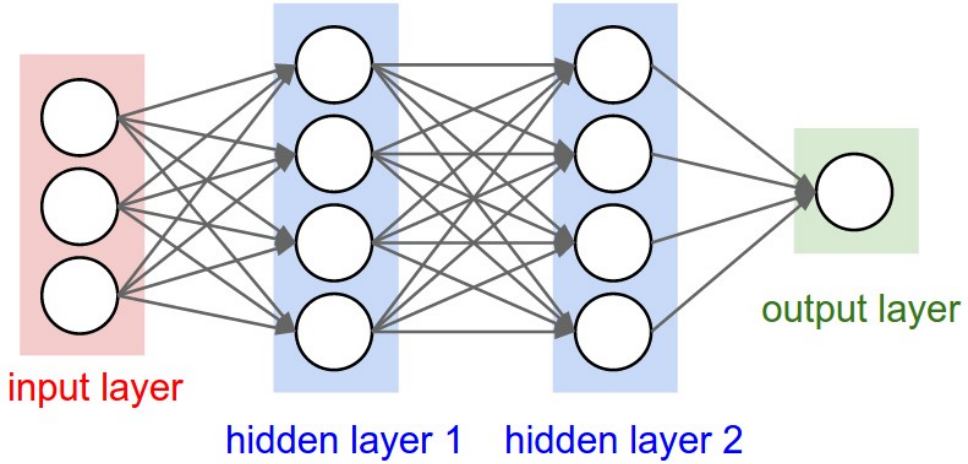


Number of Neurons: 4+2 = 6
Number of Weights: [4x3 + 2x4] = 20
Number of Parameters: 20 + 6 = 26 (biases!)

Number of Neurons: ?
Number of Weights: ?
Number of Parameters: ?

Neural Networks: Architectures

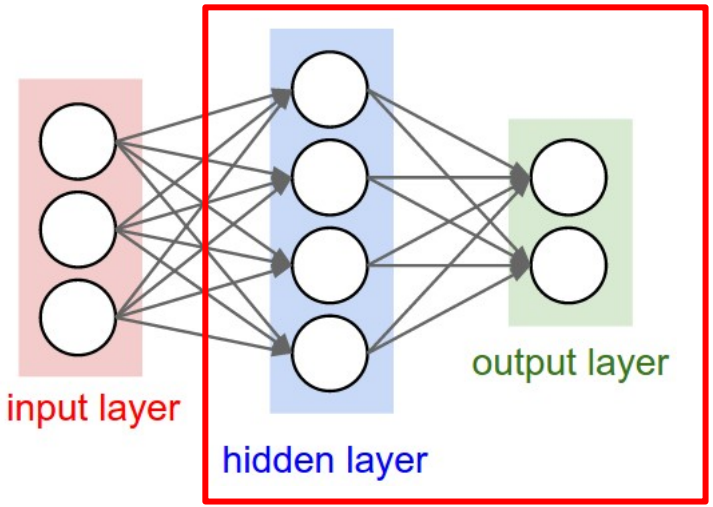


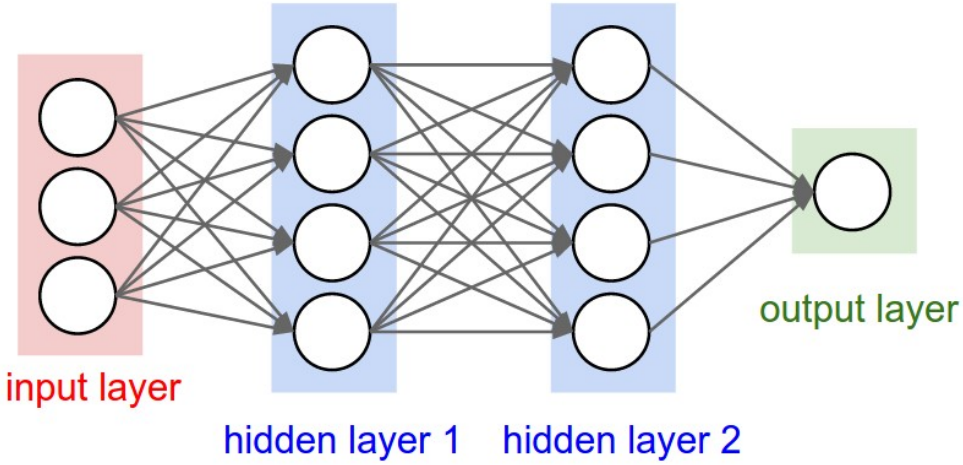


Number of Neurons: 4+2 = 6
Number of Weights: [4x3 + 2x4] = 20
Number of Parameters: 20 + 6 = 26 (biases!)

Number of Neurons: 4 + 4 + 1 = 9
Number of Weights: [4x3+4x4+1x4]=32
Number of Parameters: 32+9 = 41

Neural Networks: Architectures





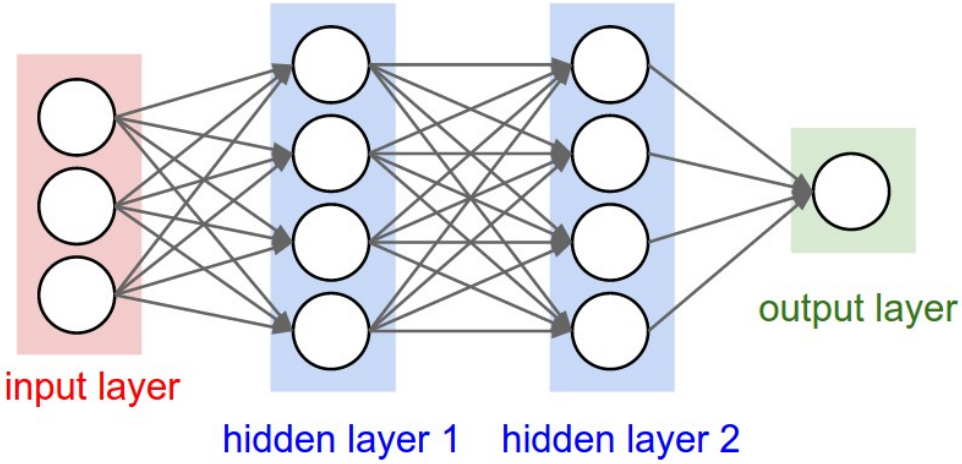
Modern CNNs: ~10 million neurons
Human visual cortex: ~5 billion neurons

Example Feed-forward computation of a Neural Network

```
class Neuron:
  # ...
  def neuron_tick(inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```

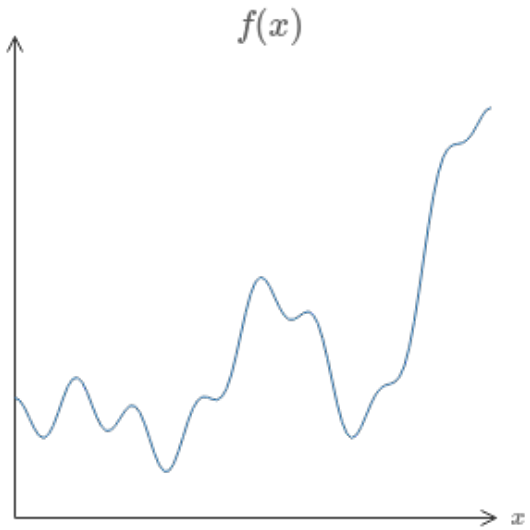
We can efficiently evaluate an entire layer of neurons.

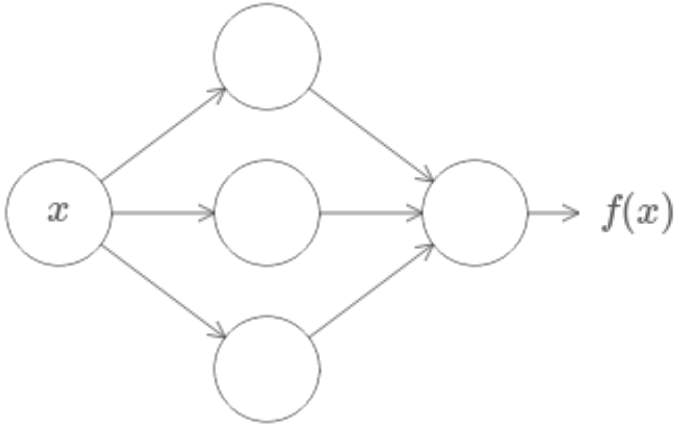
Example Feed-forward computation of a Neural Network



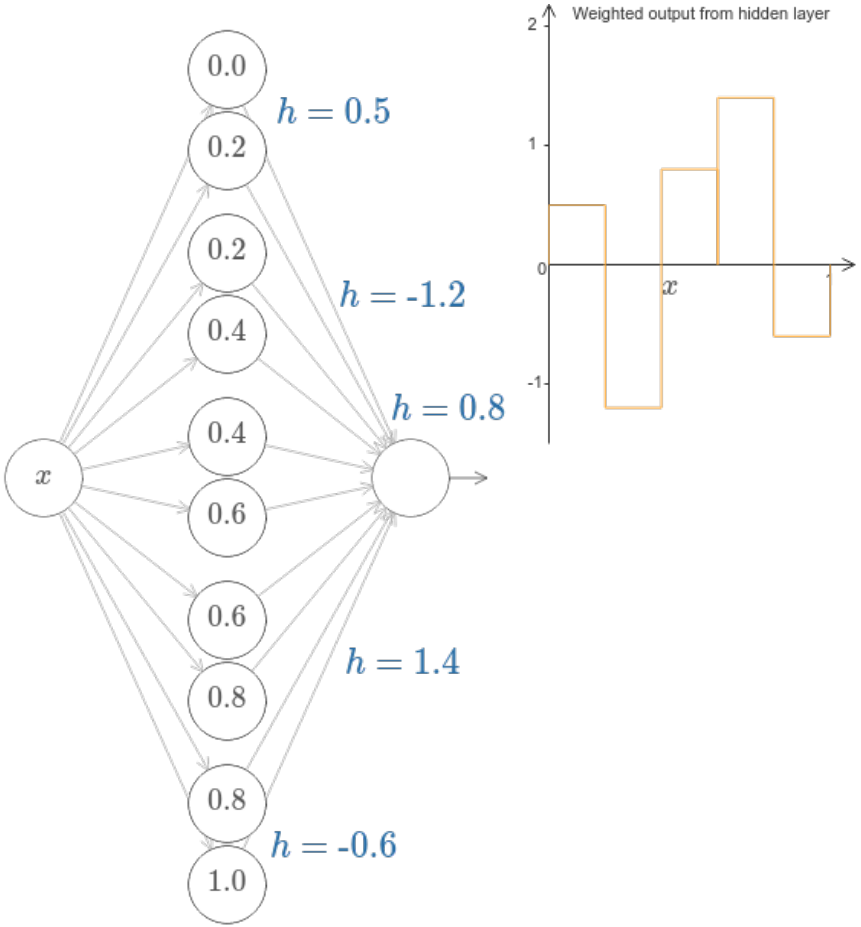

```
# forward-pass of a 3-layer neural network:
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)
out = f(np.dot(W3, h2) + b3) # output neuron (1x1)
```

What kinds of functions can a Neural Network represent?

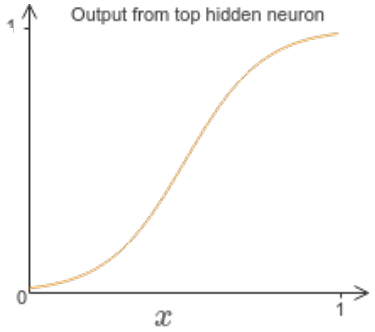




[http://neuralnetworksanddeeplearning.com/chap4.html]

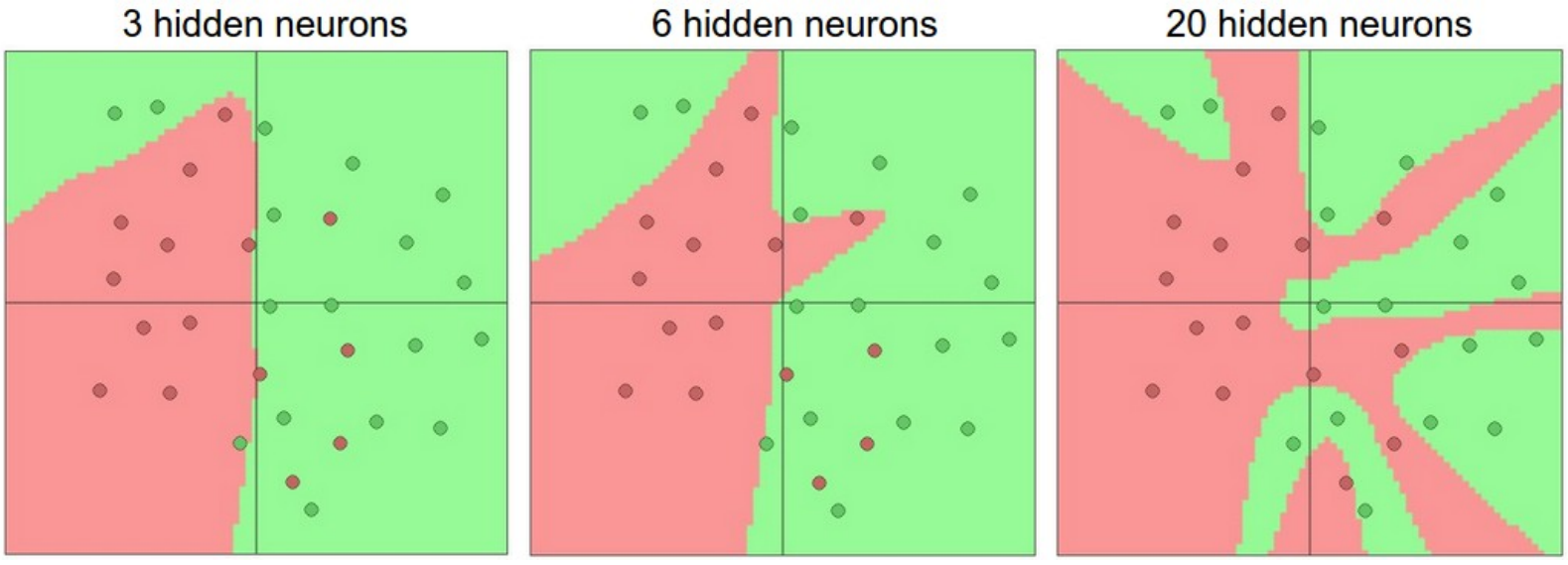


What kinds of functions can a Neural Network represent?



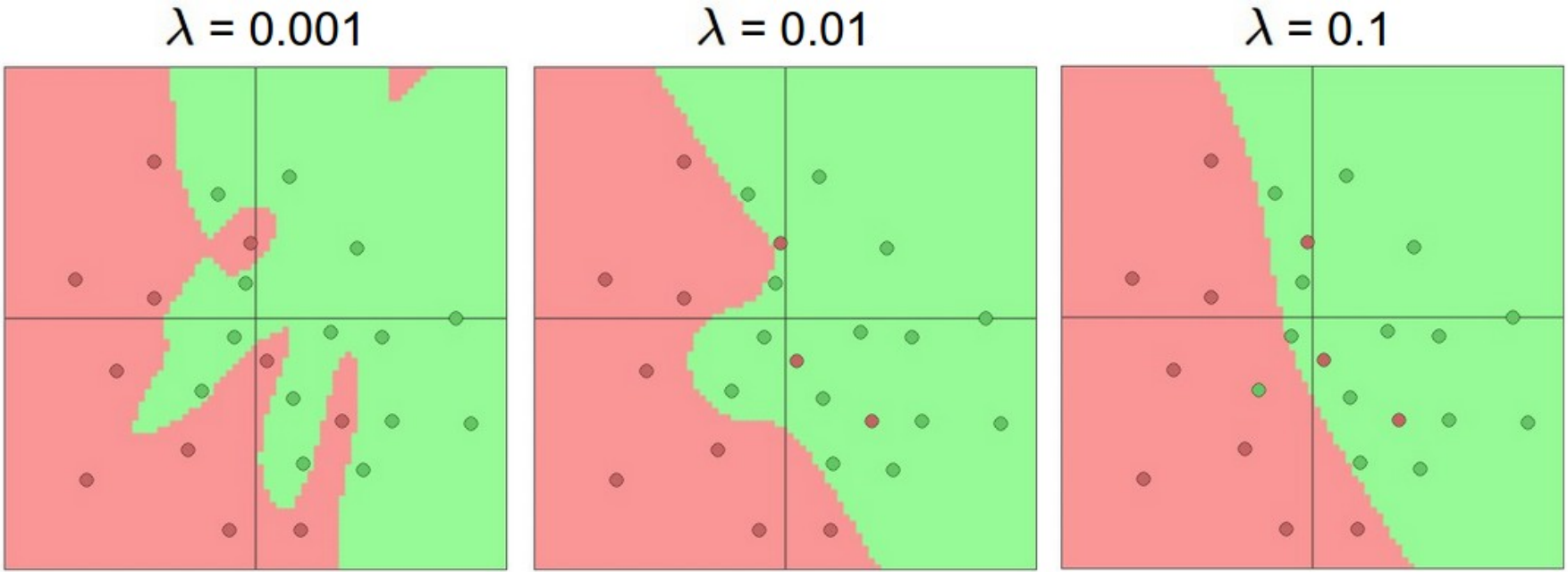
[http://neuralnetworksanddeeplearning.com/chap4.html]

Setting the number of layers and their sizes



more neurons = more capacity

Do not use size of neural network as a regularizer. Use stronger regularization instead:



(you can play with this demo over at ConvNetJS: http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html)

Summary

- we arrange neurons into fully-connected layers
- the abstraction of a layer has a nice property in that it allows us to use efficient vectorized code (matrix multiplies)
- neural networks are universal function approximators but this doesn't mean much.
- neural networks are not *neural*
- neural networks: bigger = better (but might have to regularize more strongly)

Next Lecture:

More than you ever wanted to know about Neural Networks and how to train them