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3.2 Determine the causal impulse response $h(n)$ for $n \geq 0$ of the LTI systems described by the following I/O difference equations:

a. $y(n) = 3x(n) - 2x(n-1) + 4x(n-3)$

b. $y(n) = 4x(n) + x(n-1) - 3x(n-3)$

c. $y(n) = x(n) - x(n-3)$

Giải:

a) Thay $x(n) = \delta(n) \Rightarrow y(n) = h(n)$ *ta được:*

$$h(n) = 3\delta(n) - 2\delta(n-1) + 4\delta(n-3) = \{3, -2, 0, 4\}$$

b) Thay $x(n) = \delta(n) \Rightarrow y(n) = h(n)$ *ta được:*

$$h(n) = 4\delta(n) + \delta(n-1) - 3\delta(n-3) = \{4, 1, 0, -3\}$$

c) Thay $x(n) = \delta(n) \Rightarrow y(n) = h(n)$ *ta được:*

$$h(n) = \delta(n) - \delta(n-3) = \{1, 0, 0, -1\}$$

3.3 Determine the causal impulse response $h(n)$ for $n \geq 0$ of the LTI systems described by the following I/O difference equations:

a. $y(n) = -0.9y(n-1) + x(n)$

b. $y(n) = 0.9y(n-1) + x(n)$

c. $y(n) = 0.64y(n-2) + x(n)$

d. $y(n) = -0.81y(n-2) + x(n)$

e. $y(n) = 0.5y(n-1) + 4x(n) + x(n-1)$

a) Thay $x(n) = \delta(n) \Rightarrow y(n) = h(n)$ *ta được:*

$$h(n) = -0.9h(n-1) + \delta(n)$$

- $n = 0 \Rightarrow h(0) = -0.9h(-1) + \delta(0) = 1$
- $n = 1 \Rightarrow h(1) = -0.9h(0) + \delta(1) = (-0.9)^1$
- $n = 2 \Rightarrow h(2) = -0.9h(1) + \delta(2) = (-0.9)^2$

$$\Rightarrow h(n) = (-0.9)^n u(n) = \begin{cases} (-0.9)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

b) Tương tự câu a, ta có:

$$\Rightarrow h(n) = 0.9^n u(n) = \begin{cases} 0.9^n, & n \ge 0 \\ 0 & , n < 0 \end{cases}$$

c) Thay $x(n) = \delta(n) \Rightarrow y(n) = h(n)$ *ta được:*

$$h(n) = 0.64h(n-2) + \delta(n)$$

- $n = 0 \Rightarrow h(0) = 0.64h(-2) + \delta(0) = 1$
- $n = 1 \Rightarrow h(1) = 0.64h(-1) + \delta(1) = 0$
- $n = 2 \Rightarrow h(2) = 0.64h(0) + \delta(2) = 0.64^1$
- $n = 3 \Rightarrow h(3) = 0.64h(1) + \delta(3) = 0$
- $n = 4 \Rightarrow h(4) = 0.64h(2) + \delta 4 = 0.64^2$
- $n = 5 \Rightarrow h(5) = 0.64h(3) + \delta(5) = 0$
- $n = 6 \Rightarrow h(6) = 0.64h(4) + \delta(6) = 0.64^3$

$$\Rightarrow h(n) = \begin{cases} (0.64)^{\frac{n}{2}}, & n \quad chan \\ 0 & , n \quad le \end{cases} \quad \text{hay} \quad h(n) = \begin{cases} 0.8^n, & n \quad chan \\ 0 & , n \quad le \end{cases}$$

d) Tương tự câu c, ta có:

$$\Rightarrow h(n) = \begin{cases} (-0.81)^{\frac{n}{2}}, & n \quad chan \\ 0 & , n \quad le \end{cases} \quad \text{hay} \quad h(n) = \begin{cases} -0.9^n & , n \quad chan \\ 0 & , n \quad le \end{cases}$$

d) Thay $x(n) = \delta(n) \Rightarrow y(n) = h(n)$ *ta được:*

$$h(n) = 0.5h(n-1) + 4\delta(n) + \delta(n-1)$$

- $n = 0 \Rightarrow h(0) = 0.5h(-1) + 4\delta(0) + \delta(-1) = 4$
- $n = 1 \Rightarrow h(1) = 0.5h(0) + 4\delta(1) + \delta(0) = 0,5.4 + 1 = 3 = 3.\left(\frac{1}{2}\right)^0$
- $n = 2 \Rightarrow h(2) = 0.5h(1) + 4\delta(2) + \delta(1) = 0,5.3 = 3.\left(\frac{1}{2}\right)^1$
- $n = 3 \Rightarrow h(3) = 0.5h(2) + 4\delta(3) + \delta(2) = 0,5.0,5.3 = 3.\left(\frac{1}{2}\right)^2$

$$\Rightarrow h(n) = 4\delta(n) + 3\left(\frac{1}{2}\right)^{n-1} u(n-1) = \begin{cases} 4, & n = 0 \\ 3.\left(\frac{1}{2}\right)^{n-1} & , n > 0 \\ 0, & n < 0 \end{cases}$$

3.4 Determine the I/O difference equations relating $x(n)$ and $y(n)$ for the LTI systems having the following impulse responses:

a. $h(n) = (0.9)^n u(n)$

b. $h(n) = (-0.6)^n u(n)$

c. $h(n) = (0.9)^n u(n) + (-0.9)^n u(n)$

d. $h(n) = (0.9j)^n u(n) + (-0.9j)^n u(n)$

a) Ta có:

$$y(n) = \sum_{m=0}^{\infty} h(m)x(n-m) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots$$

$$= x(n) + 0.9x(n-1) + 0.9^2 x(n-2) + 0.9^3 x(n-3) + \ldots$$

$$= x(n) + 0.9\left[x(n-1) + 0.9x(n-2) + 0.9^2 x(n-3)\right] + \ldots$$

$$= x(n) + 0.9\, y(n-1)$$

b) Làm như trên $\Rightarrow y(n) = x(n) - 0.6y(n-1)$

c) Ta có:

$$y(n) = \sum_{m=0}^{\infty} h(m)x(n-m) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots$$

$$= [1+1]x(n) + [0.9 - 0.9]x(n-1) + \left[0.9^2 + (-0.9)^2\right]x(n-2) + \left[0.9^3 - 0.9^3\right]x(n-3)$$
$$+ \left[0.9^4 + 0.9^4\right]x(n-4) + \ldots$$

$$\Leftrightarrow y(n) = 2x(n) + 2.0{,}9^2 x(n-2) + 2.0{,}9^4 x(n-4) + 2.0{,}9^6 x(n-6) + \ldots$$

$$= 2x(n) + 0.9^2\left[2x(n-2) + 2.0{,}9^2 x(n-4) + 2.0{,}9^4 x(n-6) + 2.0{,}9^6 x(n-8) + \ldots\right]$$

$$= 2x(n) + 0.9^2\, y(n-2)$$

$$= 2x(n) + 0.81y(n-2)$$

d) Ta có:

$$y(n) = \sum_{m=0}^{\infty} h(m)x(n-m) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots$$

$$= [1+1]x(n) + [0.9j - 0.9j]x(n-1) + \left[(0.9j)^2 + (-0.9j)^2\right]x(n-2) + \left[0.9j^3 - 0.9j^3\right]x(n-3)$$
$$+ \left[(0.9j)^4 + (0.9j)^4\right]x(n-4) + \ldots$$

$$= 2x(n) + 2.(0.9j)^2 x(n-2) + 2.(0.9j)^4 x(n-4) + 2.(0.9j)^6 x(n-6) + \ldots$$

$$= 2x(n) + (0.9j)^2\left[2x(n-2) + 2.(0.9j)^2 x(n-4) + 2.(0.9j)^4 x(n-6) + 2.(0.9j)^6 x(n-8) + \ldots\right]$$

$$= 2x(n) + (0.9j)^2 y(n-2) = 2x(n) - 0.81y(n-2)$$

3.5 A causal IIR filter has impulse response $h(n) = 4\delta(n) + 3(0.5)^{n-1}u(n-1)$. Working with the convolutional equation $y(n) = \sum_m h(m)x(n-m)$, derive the *difference equation* satisfied by $y(n)$.

Ta có đáp ứng xung: $h(n) = \{4,\ \ 3,\ \ 3.0,5,\ \ 3.0,5^2,\ \ 3.0,5^3,...\}$

$$y(n) = \sum_{m=0}^{\infty} h(m)x(n-m) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + ...$$

$$= 4x(n) + 3\left[x(n-1) + 0.5x(n-2) + 0.5^2x(n-3) + 0.5^3x(n-4)...\right]$$

$$\Leftrightarrow y(n) = 4x(n) + 3x(n-1) + \boxed{3\left[0.5x(n-2) + 0.5^2x(n-3) + 0.5^3x(n-4)...\right]} \quad (*)$$

Mặt khác:

$$y(n-1) = 4x(n-1) + \boxed{3\left[x(n-2) + 0.5x(n-3) + 0.5^2x(n-4) + ...\right]} \quad \rightarrow \textit{Đang thiếu 0.5}$$

$$\Leftrightarrow 0.5y(n-1) = 2x(n-1) + \boxed{3\left[0.5x(n-2) + 0.5^2x(n-3) + 0.5^3x(n-4) + ...\right]} (**)$$

Thêm 0.5 dô để cho xuất hiện cụm giống nhau, khi trừ nhau sẽ triệt tiêu ^^.

Lấy (*) – (**) → Phép màu là đây:

$$\Leftrightarrow y(n) - 0.5y(n-1) = 4x(n) + 3x(n-1) - 2x(n-1)$$

$$\Rightarrow y(n) = 4x(n) + x(n-1) + 0.5y(n-1)$$

3.6 A causal IIR filter has impulse response:

$$h(n) = \begin{cases} 5, & \text{if } n = 0 \\ 6(0.8)^{n-1}, & \text{if } n \ge 1 \end{cases}$$

Working with the convolutional filtering equation, derive the *difference equation* satisfied by $y(n)$.

Ta có đáp ứng xung: $h(n) = \{5,\ \ 6,\ \ 6.0,8,\ \ 6.0,8^2,...\}$

$$y(n) = \sum_{m=0}^{\infty} h(m)x(n-m) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + ...$$

$$= 5x(n) + 6\left[x(n-1) + 0.8x(n-2) + 0.8^2x(n-3) + 0.8^3x(n-4)...\right]$$

$$\Leftrightarrow y(n) = 5x(n) + 6x(n-1) + 6\left[0.8x(n-2) + 0.8^2x(n-3) + 0.8^3x(n-4)...\right] \quad (*)$$

Mặt khác:

$$y(n-1) = 5x(n-1) + 6\left[x(n-2) + 0.8x(n-3) + 0.8^2 x(n-4) + \ldots\right]$$

$$\Leftrightarrow 0.8y(n-1) = 4x(n-1) + 6\left[0.8x(n-2) + 0.8^2 x(n-3) + 0.8^3 x(n-4) + \ldots\right]$$ ***(**)***

Lấy (*) – (**) → Phép màu tiếp tục:

$$\Leftrightarrow y(n) - 0.8y(n-1) = 5x(n) + 6x(n-1) - 4x(n-1)$$

$$\Rightarrow y(n) = 5x(n) + 2x(n-1) + 0.8y(n-1)$$