I. PROAKIS

3.25 Determine all possible signals that can have the following z-transforms.

(a) $X(z)=\dfrac{1}{1-1.5z^{-1}+0.5z^{-2}}$

(b) $X(z)=\dfrac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$

Giải:

a) $X(z)=\dfrac{1}{\left(1-z^{-1}\right)\left(1-0.5z^{-1}\right)}=\dfrac{2}{1-z^{-1}}-\dfrac{1}{1-0.5z^{-1}}$ → Biện luận theo $|z|=0,5,\ \ |z|=1$

b) Ta có:

$$a^{n}\sin\left(\omega_0 n\right)u(n)\longrightarrow\frac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}\qquad (*)$$

$$\Leftrightarrow a^{n+1}\sin\left(\omega_0\left(n+1\right)\right)u\left(n+1\right)\longrightarrow\frac{a\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$$

$$\Leftrightarrow a^{n+1}\sin\left(\omega_0\left(n+1\right)\right)u\left(n\right)\longrightarrow\frac{a\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}\qquad (**)$$

+ Đồng nhất mẫu số ta có: $\begin{cases}a^2=\dfrac{1}{4}\Rightarrow a=\dfrac{1}{2}\\ \cos\omega_0=\dfrac{1}{2}\Rightarrow\omega_0=\dfrac{\pi}{3}\end{cases}$

$$\Leftrightarrow\left(\frac{1}{2}\right)^{n+1}\sin\left(\frac{\pi}{3}\left(n+1\right)\right)u\left(n\right)\longrightarrow\frac{\sqrt{3}/4}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}\Rightarrow x\left(n\right)=\frac{4}{\sqrt{3}}\left(\frac{1}{2}\right)^{n}\sin\left(\frac{\pi}{3}\left(n+1\right)\right)u\left(n\right)$$

3.26 Determine the signal $x(n)$ with z-transform

$$X(z)=\frac{3}{1-\frac{10}{3}z^{-1}+z^{-2}}$$

if $X(z)$ converges on the unit circle.

Giải: $X(z)=\dfrac{3}{\left(1-3z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)},\quad |z|<1$ → làm tiếp

3.32 Show that the following systems are equivalent.

(a) $y(n) = 0.2y(n-1) + x(n) - 0.3x(n-1) + 0.02x(n-2)$

(b) $y(n) = x(n) - 0.1x(n-1)$

Giải:

$$+\ H_1(z) = \frac{1 - 0.3z^{-1} + 0.02z^{-2}}{1 - 0.2z^{-1}} = \frac{\left(1 - 0.2z^{-1}\right)\left(1 - 0.1z^{-1}\right)}{1 - 0.2z^{-1}} = 1 - 0.1z^{-1}$$

$$+\ H_2(z) = 1 - 0.1z^{-1}$$

Vì $H_1(z) = H_2(z)$ → **Hai hệ thống đã cho là tương đương.**

3.38 Determine the impulse response and the step response of the following causal systems. Plot the pole–zero patterns and determine which of the systems are stable.

(a) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n)$

(b) $y(n) = y(n-1) - 0.5y(n-2) + x(n) + x(n-1)$

(c) $H(z) = \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$

(d) $y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

(e) $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$

Giải:

a) $$H(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1}{\left(1 - 0.5z^{-1}\right)\left(1 - 0.25z^{-1}\right)}$$

b) $$H(z) = \frac{1 + z^{-1}}{1 - z^{-1} + 0.5z^{-2}} = \frac{1}{1 - z^{-1} + 0.5z^{-2}} + \frac{z^{-1}}{1 - z^{-1} + 0.5z^{-2}} = H_1(z) + H_2(z)$$

$$+\ H_1(z) = \frac{1}{1 - z^{-1} + 0.5z^{-2}}$$

Áp dụng: $\Leftrightarrow a^{n+1}\sin\left(\omega_0(n+1)\right)u(n) \longrightarrow \frac{a\sin\omega_0}{1 - 2az^{-1}\cos\omega_0 + a^2z^{-2}}$

+ Đồng nhất mẫu số ta có: $$\begin{cases} a^2 = 0.5 \Rightarrow a = \frac{\sqrt{2}}{2} \\ \cos\omega_0 = \frac{\sqrt{2}}{2} \Rightarrow \omega_0 = \frac{\pi}{4} \end{cases}$$

$$\Leftrightarrow \left(\frac{\sqrt{2}}{2}\right)^{n+1}\sin\left(\frac{\pi}{4}(n+1)\right)u(n) \longrightarrow \frac{1/2}{1 - z^{-1} + 0.5z^{-2}} \Rightarrow h_1(n) = 2\left(\frac{\sqrt{2}}{2}\right)^{n+1}\sin\left(\frac{\pi}{4}(n+1)\right)u(n)$$

$$+\ H_2(z) = \frac{z^{-1}}{1 - z^{-1} + 0.5z^{-2}}$$

Áp dụng $a^n\sin(\omega_0 n)u(n) \longrightarrow \frac{az^{-1}\sin\omega_0}{1 - 2az^{-1}\cos\omega_0 + a^2z^{-2}}$

$$\Leftrightarrow \left(\frac{\sqrt{2}}{2}\right)^{n}\sin\left(\frac{\pi}{4}n\right)u(n) \longrightarrow \frac{1/2z^{-1}}{1 - z^{-1} + 0.5z^{-2}} \Rightarrow h_2(n) = 2\left(\frac{\sqrt{2}}{2}\right)^{n}\sin\left(\frac{\pi}{4}n\right)u(n)$$

$$\Rightarrow h(n) = h_1(n) + h_2(n)$$

c)

❖ **Tìm đáp ứng xung: Sửa đề đi, lỡ giải rồi.**

$$H(z) = \frac{z^{-1}(1+z^{-1})}{1-z^{-3}} = \frac{z^{-1}+z^{-2}}{(1-z^{-1})(1+z^{-1}+z^{-2})} = \frac{A}{1-z^{-1}} + \frac{Bz^{-1}+C}{1+z^{-1}+z^{-2}}$$

$$A = \lim_{z^{-1}\to 1} \frac{z^{-1}+z^{-2}}{1+z^{-1}+z^{-2}} = \frac{2}{3}, \qquad \begin{cases} z^{-1} = 2 \Rightarrow -\frac{6}{7} = -\frac{2}{3} + \frac{2}{7}B + \frac{1}{7}C \\ z^{-1} = 0 \Rightarrow 0 = \frac{2}{3} + C \end{cases} \Rightarrow \begin{cases} B = -\frac{1}{3} \\ C = -\frac{2}{3} \end{cases}$$

$$\Rightarrow H(z) = \frac{2}{3}.\frac{1}{1-z^{-1}} - \frac{1}{3}.\frac{z^{-1}}{1+z^{-1}+z^{-2}} - \frac{2}{3}.\frac{1}{1+z^{-1}+z^{-2}} = H_1(z) - \frac{1}{3}H_2(z) - \frac{2}{3}H_3(z)$$

$$+\ H_1(z) = \frac{2}{3}.\frac{1}{1-z^{-1}} \Rightarrow h_1(n) = \frac{2}{3}u(n)$$

$$+\ H_3(z) = \frac{1}{1+z^{-1}+z^{-2}}$$

$$\boxed{\textbf{AD: } a^{n+1}\sin(\omega_0(n+1))u(n) \longrightarrow \frac{a\sin\omega_0}{1-2az^{-1}\cos\omega_0 + a^2z^{-2}}}$$

+ Đồng nhất mẫu số ta có: $\begin{cases} a^2 = 1 \Rightarrow a = 1 \\ \cos\omega_0 = -\frac{1}{2} \Rightarrow \omega_0 = \frac{2\pi}{3} \end{cases}$

$$\Leftrightarrow \sin\left(\frac{2\pi}{3}(n+1)\right)u(n) \longrightarrow \frac{\sqrt{3}/2}{1+z^{-1}+z^{-2}} \Rightarrow h_3(n) = \frac{2}{\sqrt{3}}\sin\left(\frac{2\pi}{3}(n+1)\right)u(n)$$

$$+\ H_2(z) = \frac{z^{-1}}{1+z^{-1}+z^{-2}} \Rightarrow x_2(n) = \frac{2}{\sqrt{3}}\sin\left(\frac{2\pi}{3}n\right)u(n)$$

$$\Rightarrow h(n) = h_1(n) - \frac{1}{3}h_2(n) - \frac{2}{3}h_3(n)$$

❖ **Tìm đáp ứng bậc:**

$$S(z) = H(z)U(z) = \frac{z^{-1}+z^{-2}}{(1-z^{-1})^2(1+z^{-1}+z^{-2})} = \frac{A}{(1-z^{-1})^2} + \frac{B}{1-z^{-1}} + \frac{Cz^{-1}+D}{1+z^{-1}+z^{-2}}$$

d) $H(z) = \dfrac{1}{1-0.6z^{-1}+0.08z^{-2}} = \dfrac{1}{(1-0.4z^{-1})(1-0.2z^{-1})}$

e) $H(z) = \dfrac{2-z^{-2}}{1-0.7z^{-1}+0.1z^{-2}} = \dfrac{2-z^{-2}}{(1-0.5z^{-1})(1-0.2z^{-1})} = -10 + \dfrac{A}{1-0.5z^{-1}} + \dfrac{B}{1-0.2z^{-1}}$

3.36 Consider the system

$$H(z) = \frac{1 - 2z^{-1} + 2z^{-2} - z^{-3}}{(1 - z^{-1})(1 - 0.5z^{-1})(1 - 0.2z^{-1})}, \qquad \text{ROC: } 0.5|z| > 1$$

(a) Sketch the pole–zero pattern. Is the system stable?

(b) Determine the impulse response of the system.

Giải:

$$H(z) = \frac{\left(1 - z^{-1}\right)\left(1 - z^{-1} + z^{-2}\right)}{\left(1 - z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{5}z^{-1}\right)} = \frac{1 - z^{-1} + z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{5}z^{-1}\right)}$$

3.37 Compute the response of the system

$$y(n) = 0.7y(n-1) - 0.12y(n-2) + x(n-1) + x(n-2)$$

to the input $x(n) = nu(n)$. Is the system stable?

Giải:

$$H(z) = \frac{z^{-1} + z^{-2}}{1 - 0.7z^{-1} + 0.12z^{-2}} = \frac{z+1}{z^2 - 0.7z + 0.12} = \frac{z+1}{(z-0.4)(z-0.3)}$$

$$X(z) = \frac{z^{-1}}{\left(1 - z^{-1}\right)^2} \Rightarrow Y(z) = X(z)H(z) = \frac{z^{-2} + z^{-3}}{\left(1 - 0.4z^{-1}\right)\left(1 - 0.3z^{-1}\right)\left(1 - z^{-1}\right)^2}$$