Universal gates for protected superconducting qubits using optimal control

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March 18, 2022

1. Quantum optimal-control theory
   - Revisiting superconducting qubits
   - Motivation
   - GRAPE in brief
   - Fidelity in open/closed-system

2. Optimized gates for fluxonium qubits
   - Physical properties
   - Flux noise
   - Results

3. Optimized gates for 0-π qubits
   - Physical properties
   - Offset charge sensitivity
   - Results

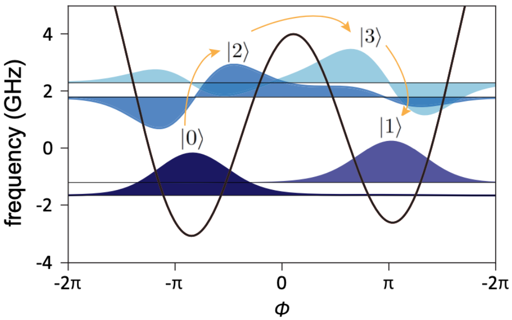
4. Conclusion

Table of Contents

1. Quantum optimal-control theory
   - Revisiting superconducting qubits
   - Motivation
   - GRAPE in brief
   - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
   - Physical properties
   - Flux noise
   - Results
3. Optimized gates for 0-π qubits
   - Physical properties
   - Offset charge sensitivity
   - Results
4. Conclusion

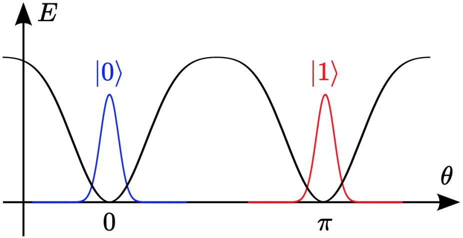
Quick review on superconducting qubits: Fluxonium

Figure: First four fluxonium wave functions. Auxiliary states $|2\rangle$ and $|3\rangle$ delocalize over both potential wells and serve as intermediate states. Note that the peak of frequency is slightly deviated from the flux sweet post $\Phi_{ext} = 0.45\Phi_0$.



Quick review on superconducting qubits: 0-π

Figure: Protected superconducting qubits, the 0-π circuit (arXiv:1302.4122v1). Energy as function of terminal phases θ.



Fabrication of superconducting circuit that intrinsically and physically protected from local pertubations is important in the context of fault-tolerant hardware.

1. Quantum optimal-control theory
   - Revisiting superconducting qubits
   - Motivation
   - GRAPE in brief
   - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
   - Physical properties
   - Flux noise
   - Results
3. Optimized gates for 0-π qubits
   - Physical properties
   - Offset charge sensitivity
   - Results
4. Conclusion

Why quantum optimal-control theory?

We apply optimal control (OC) theory to facilitate control over heavily noisy quantum system, since

1. we can maximize gate/state-transfer fidelity;
2. we can engineer better control pulses (amplitude, waveform, time resolution).

We use a technique called *automatic differentiation*. This is different from gradient-based technique, because we do not have to calculate much when adding new optimization targets.
Using such technique, we successfully optained a universal set of gates.

1. Quantum optimal-control theory
   - Revisiting superconducting qubits
   - Motivation
   - GRAPE in brief
   - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
   - Physical properties
   - Flux noise
   - Results
3. Optimized gates for 0-π qubits
   - Physical properties
   - Offset charge sensitivity
   - Results
4. Conclusion

In brief, QOC is about...

In essence, by optimizing control pulses $\{u_1(t), \ldots, u_M(t)\}$, which creates operators $\{\mathcal{H}_1, \ldots, \mathcal{H}_M\}$, we optimize the system dynamics. The resulting Hamiltonian read

$$H(t) = \mathcal{H}_0 + \mathcal{H}_c(t), \tag{1}$$

where $\mathcal{H}_0$ is localized, and $\mathcal{H}_c(t) = \sum_{k=1}^{M} u_k(t)\mathcal{H}_k$. Can we optimize $\mathcal{H}_0$? Yes, better fabrication perhaps. But our primary scope in OC remains minimizing the cost functional

$$C[\{u_k(t)\}] = 1 - \frac{1}{n^2} \left| \operatorname{Tr}\left(U_t^\dagger U_f \right) \right|^2, \tag{2}$$

which encodes the infidelity of the target process. Additional optimization goals can be reached readily.

Gradient ascent pulse engineering (GRAPE)

Minimizing cost function is commonly dealt by GRAPE, which relies on explicit analytical expressions for the gradients of C. Automatic-differentiation (TensorFlow) is ultilized for more efficient optimization factors scaling. Now we investigate the GRAPE scheme ultilized in this paper.

First, gate time is discreted $t_g \rightarrow N\delta t$. Control amplitudes at $t_j = j\delta$ is $u_{kj} = u_k(t_j)$ and can be conveniently grouped into a vector $\mathbf{u} = [u_{kj}] \in \mathbb{R}^{NM}$, which form the set of optimization parameters.

1. Quantum optimal-control theory
    - Revisiting superconducting qubits
    - Motivation
    - GRAPE in brief
    - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
    - Physical properties
    - Flux noise
    - Results
3. Optimized gates for 0-π qubits
    - Physical properties
    - Offset charge sensitivity
    - Results
4. Conclusion

Closed-system gate fidelities

The quantity of interest is gate fidelity

$$F_c(\mathbf{u}) = \frac{1}{n^2} \left| \operatorname{Tr}\left(U_t^\dagger U_f(\mathbf{u}) \right) \right|^2, \tag{3}$$

where $U_f(\mathbf{u})$ is the unitary realized by means of the control field. Decomposition of this unitary evolution

$$U_f = U_N U_{N-1} \cdots U_1 U_0 \tag{4}$$

where $U_j = \exp(-iH_j \delta t)$ describes the unitary evolution of the system from t_j to $t_j + \delta t$ and again, each Hamiltonian is well-defined at each instance,

$$H_j = \mathcal{H}_0 + \sum u_{kj} \mathcal{H}_k \tag{5}$$

C_1 and additional cost function C_2 and C_2

As mentioned, using automatic-differentiation allows us to avoid cumbersome calculation when adding new optimal goals. Thus, adding two more will not hurt,

$$C(\mathbf{u}) = C_1(\mathbf{u}) + \alpha_2 C_2(\mathbf{u}) + \alpha_3 C_3(\mathbf{u}), \tag{6}$$

where C_1 is the infidelity cost function $C_1(\mathbf{u}) = 1 - F_c(\mathbf{u})$, α_2 and α_3 are cost-function weights, respectively. The explicit expressions of C_2 and C_3 are

$$C_2 = \sum_{k,j} |u_{kj} - u_{kj-1}|^2 \quad \text{(1st control parameters)} \tag{7}$$

$$C_3 = \sum_{k,j} |u_{kj}|^2 \quad \text{(control pulse energy)} \tag{8}$$

Updating optimization parameters

Now we wish to update $\mathbf{u}$ by following the opposite direction of the gradient,

$$\mathbf{u}_{p+1} = \mathbf{u}_p - \eta_p \frac{\partial C(\mathbf{u}_p)}{\partial \mathbf{u}_p}, \tag{9}$$

where p denotes the p-th iteration and η_p the step size. Convegent rate can be accelerated using something called ADAM. Learning rate chosen is exponentially decaying

$$\eta_p = \eta_0 e^{-\beta p}, \tag{10}$$

where η_0 and β need to be intermittently adjusted.

1. Quantum optimal-control theory
    - Revisiting superconducting qubits
    - Motivation
    - GRAPE in brief
    - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
    - Physical properties
    - Flux noise
    - Results
3. Optimized gates for 0-π qubits
    - Physical properties
    - Offset charge sensitivity
    - Results
4. Conclusion

Gate fidelities with dissipation & dephasing

So far we have only considered ideal quantum system. For a weakly coupling quantum system, we employ a Lindblad master equation for a good description of $1 - F_c(\mathbf{u})$ in the context of decoherence,

$$\frac{d\rho}{dt} = -i\,[H(t), \rho] + \sum_l \gamma_l \left[c_l \rho c_l^\dagger - \frac{1}{2}\{c_l^\dagger c_l, \rho\} \right], \tag{11}$$

To *master* the master equation, we must first vectorize the matrix,

$$\rho = \sum \rho_{ij} |i\rangle\langle j| \longrightarrow \tilde{\rho} \sum \rho_{ij} |i\rangle |j\rangle, \tag{12}$$

and thus the evolution of the density matrix is then

$$|\tilde{\rho}(t)\rangle = L|\tilde{\rho}(0)\rangle \tag{13}$$

Gate fidelities with dissipation & dephasing

The measure of fidelity for a superoperator L is given by

$$F_0 = \frac{1}{n^2}\,\mathrm{Tr}\left(L_t^\dagger L_f\right), \tag{14}$$

where L_t is the target superoperator and L_f is the final achieved superoperator. A few note:

- For quantum open-system, a complete description must be given by density matrices; such matrices can be stacked into $n^2 \times 1$ vectors;
- The metric F_0 correctly reduces to F_c in the case of the closed-system.

1. Quantum optimal-control theory
   - Revisiting superconducting qubits
   - Motivation
   - GRAPE in brief
   - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
   - Physical properties
   - Flux noise
   - Results
3. Optimized gates for 0-π qubits
   - Physical properties
   - Offset charge sensitivity
   - Results
4. Conclusion

Table of Contents

1 Quantum optimal-control theory
Revisiting superconducting qubits
Motivation
GRAPE in brief
Fidelity in open/closed-system

2 Optimized gates for fluxonium qubits
Physical properties
Flux noise
Results

3 Optimized gates for 0-π qubits
Physical properties
Offset charge sensitivity
Results

4 Conclusion

Physical properties

Fluxonium's hardware architect complicates physical realization of universal gate operators (by means of MW-field). This problem is overcame by optimal control protocols.
Typically in circuit QED, the Jaynes-Cummings Hamiltonian for a drive pulse is

$$
\begin{aligned}
H_{JC} = &\sum \epsilon_l |l\rangle\langle l| + \omega_r a^\dagger a + \sum g_{ll'} |l\rangle\langle l'|(a^\dagger + a) \\
&+ U(t)(a^\dagger + a),
\end{aligned}
\tag{15}
$$

where ω_r is the resonator frequency; ϵ_l, $|l\rangle$ are the fluxonium eigenenergies and eigenstates, respectively. They are governed by the Hamiltonian $\mathcal{H}_0 = 4E_c n_\phi^2 + 1/2 E_L \phi^2 - E_f \cos(\phi + 2\pi\Phi_{ext}/\Phi_0)$

Physical properties

Throughout this work, we focus on dispersive control of the qubit, in which the drive tone $u(t)$ steers dynamics within the qubit subsystem, but leaves the resonator state essentially unchanged.

Using closed-system optimal control, we optimize the control pulse $u(t)$ to realize three different single-qubit gates: X, H, and T, thus universal set of single-qubit gates.

Optimization requirements

The process of optimization requires

- Gate time t_g as short as possible to minimize dissipation and dephasing;
- Maximum pulse amplitude $|u(t)|$ must remain small enough to avoid population of the resonator with unwanted photons

We find that t_g on the order of $40 - 70$ nanoseconds satisfy these conditions, while also producing high-fidelity gates.

In addition to C_1, C_2 and C_3 was employed to limit the time derivatives and $|u(t)|$. This ensures *occupation of the resonator with spurious photons is minimized* and smooth rendered pulses.

1. Quantum optimal-control theory
    - Revisiting superconducting qubits
    - Motivation
    - GRAPE in brief
    - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
    - Physical properties
    - Flux noise
    - Results
3. Optimized gates for 0-π qubits
    - Physical properties
    - Offset charge sensitivity
    - Results
4. Conclusion

Sensitivity to $1/f$ flux noise

Operating the fluxonium qubit away from $0.45\Phi_0$ makes the gate fidelity more vulnerable to $1/f$ flux noise. A hybrid approach was used in which dephasing rates due to classical $1/f$ noise was evaluated, and then incorporated into Lindblad master equation,

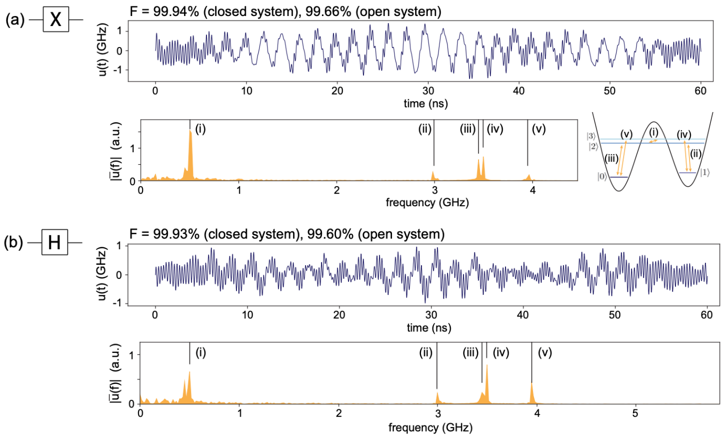
$$\frac{d\rho(t)}{dt} = -[H(t), \rho(t)] + \left(\mathbb{D}[c_0] + \sum_{l<l'} \mathbb{D}[c_{ll'}]\right)\rho(t), \tag{16}$$

where dephasing due to flux noise represented by $c_0 = \sum_l \sqrt{(\gamma_\phi)_{l0}}|l\rangle\langle l|$ and depolarization due to dielectric loss by $c_{ll'} = \sqrt{\gamma_{ll'}}|l\rangle\langle l'|$.

1. Quantum optimal-control theory
   - Revisiting superconducting qubits
   - Motivation
   - GRAPE in brief
   - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
   - Physical properties
   - Flux noise
   - Results
3. Optimized gates for 0-π qubits
   - Physical properties
   - Offset charge sensitivity
   - Results
4. Conclusion

Obtained pulses

Figure: Optimized pulse shapes $u(t)$ and their discrete Fourier transform for Pauli-X gate and Hadamard.



Time evolution

Figure: Time evolution of state population for 60 ns high-fidelity single-qubit gates. (a) X gate necessitate delocalized, auxiliary states for transition between computational basis. (b) H gate exihibits a near-perfect equal superposition.

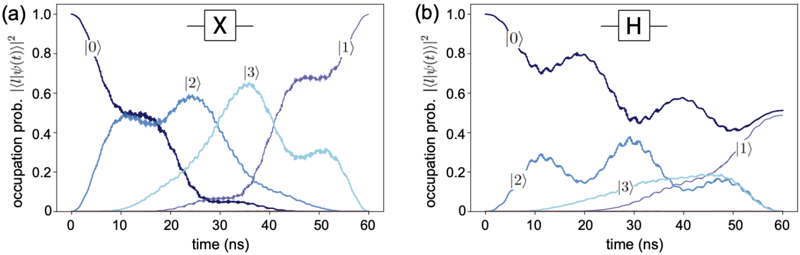


Table of Contents

1 Quantum optimal-control theory
   Revisiting superconducting qubits
   Motivation
   GRAPE in brief
   Fidelity in open/closed-system

2 Optimized gates for fluxonium qubits
   Physical properties
   Flux noise
   Results

3 Optimized gates for 0-π qubits
   Physical properties
   Offset charge sensitivity
   Results

4 Conclusion

Physical properties

The ideal 0-π Hamiltonian reads

$$\mathcal{H}_0 = \frac{(q_\theta - n_g)^2}{2C_\theta} + \frac{q_\phi^2}{2C_\phi} + E_L\phi^2 - 2E_j \cos(\theta)\cos(\phi - \pi\Phi_{ext}/\Phi_0), \tag{17}$$

where n_g is the offset charge and $q_\theta = 2en_\theta$, $q_\phi = 2en_\phi$ are the charge operators canonically conjugate. The effective capacitances associated with these two variables are

$$C_\theta = 2(C + C_J) + C_g \tag{18}$$

$$C_\phi = 2C_J + C_g. \tag{19}$$

The external magnetic flux through the circuit loop is denoted Φ_{ext}.

Physical properties

Circuit parameters above must satisfy several conditions so that the qubit is protected:

1. Localization about the θ axis, i.e. $C_\theta \ll C_\phi$;
2. Delocalization along the ϕ asxis, i.e. $E_L \ll E_J$;
3. Deep potential wells to hold localized states, $e^2/2C_\theta \ll E_J$.

Obtaining these conditions in this NISQ era is experimentally challeging.

In this paper, a range of optimistic parameters was chosen providing an appropriate amount of qubit protection.

Physical properties

For direct transitions between quantum states, we dispersively couple the 0-π qubit to a resonator via n_θ and drive with a microwave pulse. The drift and control Hamiltonian is

$$H(t) = \mathcal{H}_0(n_g) + \mathcal{H}_c(t), \tag{20}$$

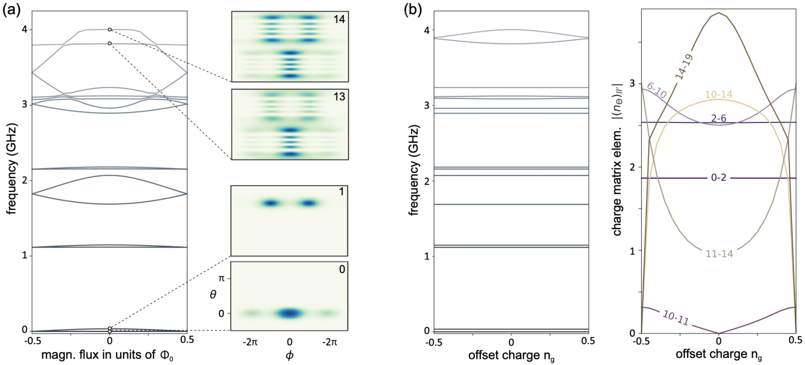
from which oc is ultilized to find the appropriate $u(t)$. Another challenge being the increasing dependence of the charge matrix elements n_θ on the offset charge n_g, namely

$$(n_\theta)_{jj'} = \langle j|n_\theta|j'\rangle\,, \tag{21}$$

At low-lying, θ-localized states, n_θ is insentitive to n_g. By constrast, θ-delocalized states, $|13\rangle$ and $|14\rangle$ for example, weakly dependent on offset charge. The problem is further exacerbated since n_g is not well-controlled.

Dependence of n_θ on offset charge

Figure: Spectrum and matrix elements of the 0-π qubit. (a) The first 14 eigenenergies vs magnetic flux Φ; offset charge $n_g = 0.25$. (b) Eigenenergies and n_θ charge matrix elements vs offset charge n_g.



Overcoming $n_\theta(n_g)$ at higher states

Our strategy is thus to steer optimizer towards control solutions that are maximally insensitive to offset-charge fluctuations. This is obtained by modifying optimal-control code (Khani *et al.*) to allow for the drift and control Hamiltonian $H(t)$ now vary from iteration to iteration, allowing us to randomly choose values of offset charge $0 \leq n_g < 1$ for each individual iteration of the optimizer.

$$H(t) \longrightarrow H(t, n_g). \tag{22}$$

Directly applying the gradients from iteraction results in a Markovian-gradient-descent process. With careful tuning cost-function weights, this process converges to an average F_c balancing all possible values of n_g.

Pauli X and Hadamard H

Figure: Optimized pulses for 0-π single qubit gates at $\Phi_{ext} = 0$.

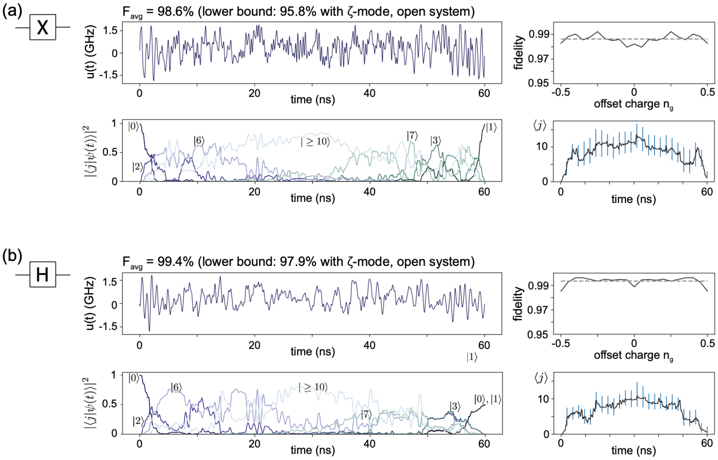


Table of Contents

1. Quantum optimal-control theory
   - Revisiting superconducting qubits
   - Motivation
   - GRAPE in brief
   - Fidelity in open/closed-system
2. Optimized gates for fluxonium qubits
   - Physical properties
   - Flux noise
   - Results
3. Optimized gates for 0-π qubits
   - Physical properties
   - Offset charge sensitivity
   - Results
4. Conclusion

Conclusion

For superconducting qubits whose computational states are practically disjoint, an optimal control pulses can be employed with careful modification. For noncoupling system,

- X, H, T fidelities obtained $> 99.9\%$ for heavy-fluxonium;
- X (98.6%), H (99.4%) for 0-π qubit;

Weakly coupling system fidelity loss can be assessed by the incoroporation of master equation. For heavy-fluxonium generations, gate fidelities obtained were $> 99\%$. For 0-π, they range from 95.8% to 99.7%.