Image Compression

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Hierarchical Clustering

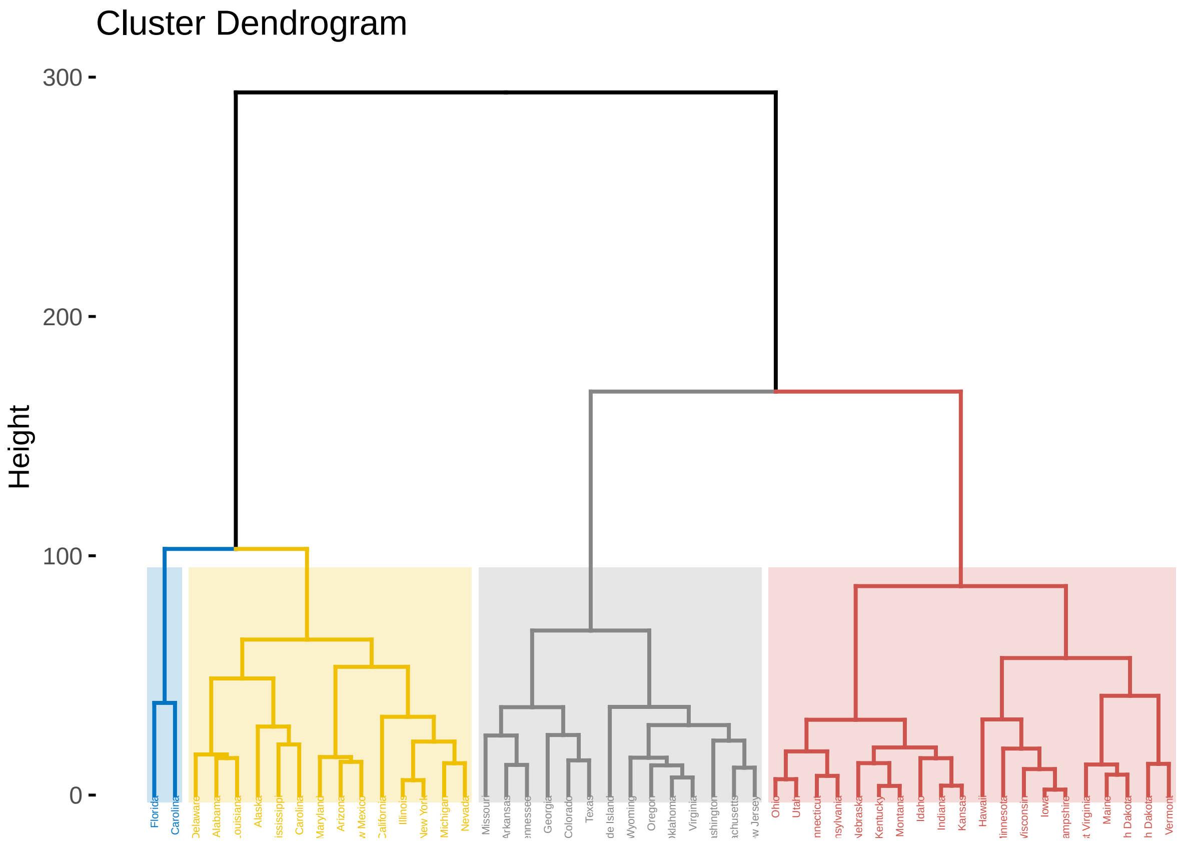
```
#load data
data("USArrests")

data = USArrests
```

a) Cluster states using complete linkage and Euclidean distance

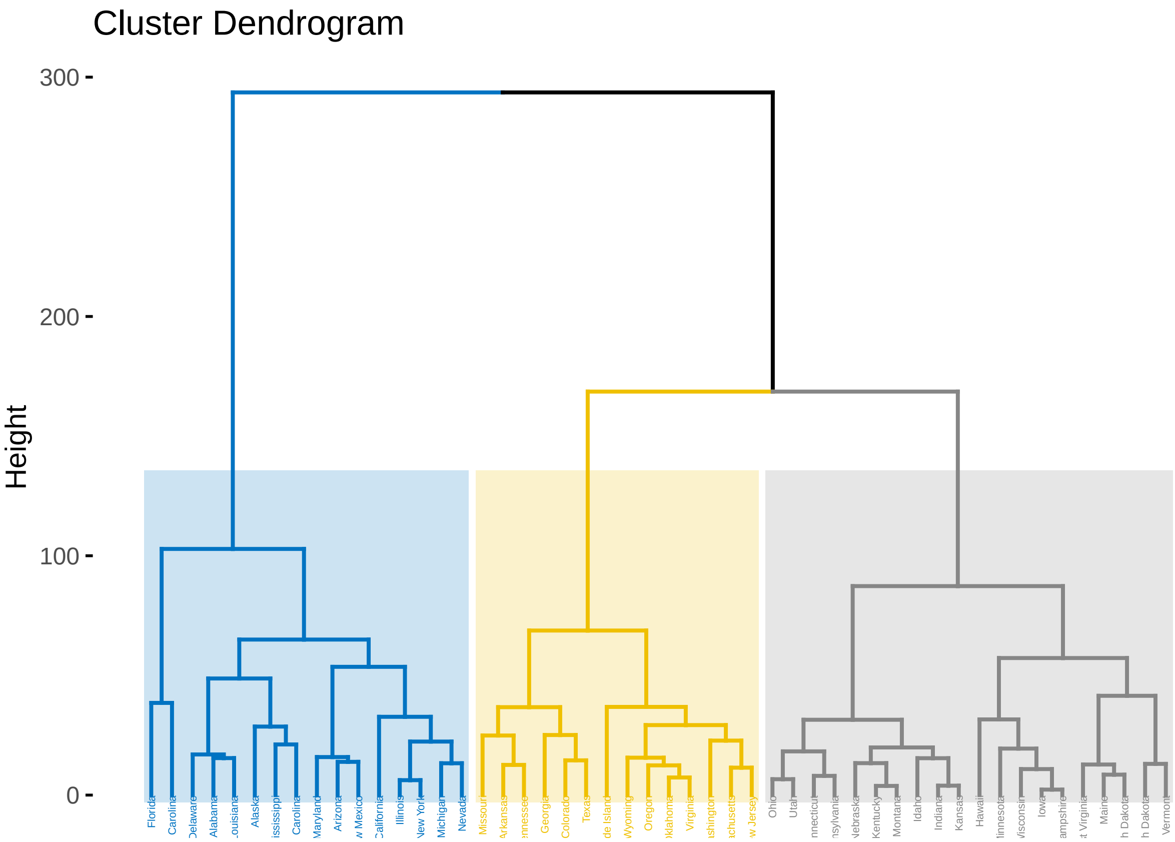
```
hc.complete <-hclust(dist(data), method = "complete")

fviz_dend(hc.complete, k = 4,
          cex = 0.3,
          palette = "jco",
          color_labels_by_k = TRUE,
          rect = TRUE, rect_fill = TRUE,
          rect_border = "jco",labels_track_height = 2.5)
```



b) Cut the dendrogram at height that results in 3 distinct clusters

```
fviz_dend(hc.complete, k = 3,
          cex = 0.3,
          palette = "jco",
          color_labels_by_k = TRUE,
          rect = TRUE, rect_fill = TRUE,
          rect_border = "jco",labels_track_height = 2.5)
```



Look at the states that are in each clusters

```
ind3.complete <-cutree(hc.complete, 3)

#cluster 1
data[ind3.complete==1,]
```

```
##                Murder Assault UrbanPop Rape
## Alabama          13.2     236       58 21.2
## Alaska           10.0     263       48 44.5
## Arizona           8.1     294       80 31.0
## California        9.0     276       91 40.6
## Delaware          5.9     238       72 15.8
## Florida          15.4     335       80 31.9
## Illinois         10.4     249       83 24.0
## Louisiana        15.4     249       66 22.2
## Maryland         11.3     300       67 27.8
## Michigan         12.1     255       74 35.1
## Mississippi      16.1     259       44 17.1
## Nevada           12.2     252       81 46.0
## New Mexico       11.4     285       70 32.1
## New York         11.1     254       86 26.1
## North Carolina   13.0     337       45 16.1
## South Carolina   14.4     279       48 22.5
```

```
#cluster 2
data[ind3.complete==2,]
```

```
##               Murder Assault UrbanPop Rape
## Arkansas         8.8     190       50 19.5
## Colorado         7.9     204       78 38.7
## Georgia         17.4     211       60 25.8
## Massachusetts    4.4     149       85 16.3
## Missouri         9.0     178       70 28.2
## New Jersey       7.4     159       89 18.8
## Oklahoma         6.6     151       68 20.0
## Oregon           4.9     159       67 29.3
## Rhode Island     3.4     174       87  8.3
## Tennessee       13.2     188       59 26.9
## Texas           12.7     201       80 25.5
## Virginia         8.5     156       63 20.7
## Washington       4.0     145       73 26.2
## Wyoming          6.8     161       60 15.6
```

```
#cluster 3
data[ind3.complete==3,]
```

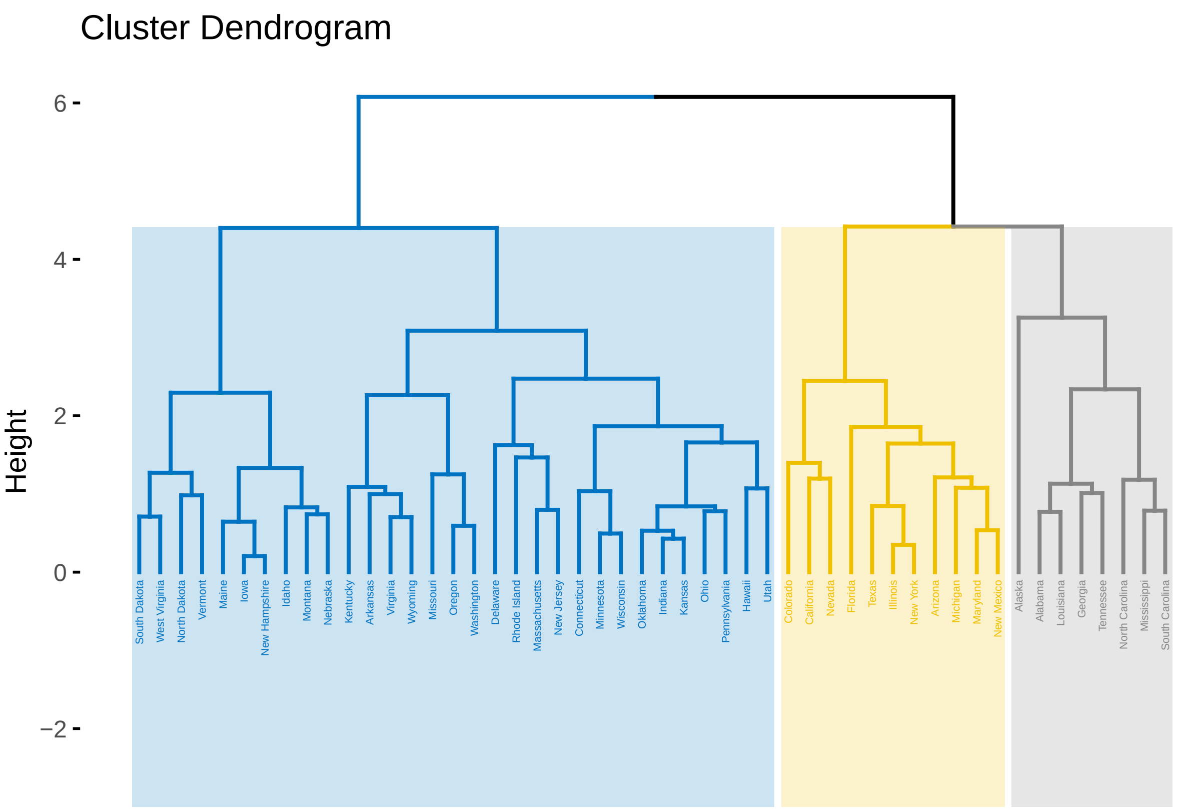
```
##                Murder Assault UrbanPop Rape
## Connecticut      3.3     110       77 11.1
## Hawaii           5.3      46       83 20.2
## Idaho            2.6     120       54 14.2
## Indiana          7.2     113       65 21.0
## Iowa             2.2      56       57 11.3
## Kansas           6.0     115       66 18.0
## Kentucky         9.7     109       52 16.3
## Maine            2.1      83       51  7.8
## Minnesota        2.7      72       66 14.9
## Montana          6.0     109       53 16.4
## Nebraska         4.3     102       62 16.5
## New Hampshire    2.1      57       56  9.5
## North Dakota     0.8      45       44  7.3
## Ohio             7.3     120       75 21.4
## Pennsylvania     6.3     106       72 14.9
## South Dakota     3.8      86       45 12.8
## Utah             3.2     120       80 22.9
## Vermont          2.2      48       32 11.2
## West Virginia    5.7      81       39  9.3
## Wisconsin        2.6      53       66 10.8
```

c) Hierarchically cluster the states using complete linkage and Euclidean distance after scaling the variables to have SD = 1

```
dat1 <-scale(data)
```

```
hc.complete.scale <-hclust(dist(dat1), method = "complete")

fviz_dend(hc.complete.scale, k = 3,
          cex = 0.3,
          palette = "jco",
          color_labels_by_k = TRUE,
          rect = TRUE, rect_fill = TRUE,
          rect_border = "jco",labels_track_height = 2.5)
```


Cluster Dendrogram

```
ind3.complete.scale <-cutree(hc.complete.scale, 3)

#cluster 1
dat1[ind3.complete.scale==1,]
```

```
##                   Murder  Assault    UrbanPop         Rape
## Alabama        1.2425641 0.7828393 -0.52090661 -0.003416473
## Alaska         0.5078625 1.1068225 -1.21176419  2.484202941
## Georgia        2.2068599 0.4828549 -0.38273510  0.487701523
## Louisiana      1.7476714 0.9388312  0.03177945  0.103348309
## Mississippi    1.9083874 1.0588250 -1.48810723 -0.441152078
## North Carolina 1.1966452 1.9947764 -1.41902147 -0.547916860
## South Carolina 1.5180772 1.2988126 -1.21176419  0.135377743
## Tennessee      1.2425641 0.2068693 -0.45182086  0.605142783
```

```
#cluster 2
dat1[ind3.complete.scale==2,]
```

```
##                Murder   Assault  UrbanPop      Rape
## Arizona    0.07163341 1.4788032 0.9989801 1.0428784
## California 0.27826823 1.2628144 1.7589234 2.0678203
## Colorado   0.02571456 0.3988593 0.8608085 1.8649672
## Florida    1.74767144 1.9707777 0.9989801 1.1389667
## Illinois   0.59970018 0.9388312 1.2062373 0.2955249
## Maryland   0.80633501 1.5507995 0.1008652 0.7012311
## Michigan   0.99001041 1.0108275 0.5844655 1.4806140
## Nevada     1.01296983 0.9748294 1.0680658 2.6443501
## New Mexico 0.82929443 1.3708088 0.3081225 1.1603196
## New York   0.76041616 0.9988281 1.4134946 0.5197310
## Texas      1.12776696 0.3628612 0.9989801 0.4556721
```

```
#cluster 3
dat1[ind3.complete.scale==3,]
```

```
##                    Murder     Assault    UrbanPop        Rape
## Arkansas        0.23234938  0.23086801 -1.07359268 -0.18491660
## Connecticut    -1.03041900 -0.72908214  0.79172279 -1.08174077
## Delaware       -0.43347395  0.80683810  0.44629400 -0.57994629
## Hawaii         -0.57123050 -1.49704226  1.20623733 -0.11018125
## Idaho          -1.19113497 -0.60908837 -0.79724965 -0.75076995
## Indiana        -0.13500142 -0.69308401 -0.03730631 -0.02476943
## Iowa           -1.28297267 -1.37704849 -0.58999237 -1.06038781
## Kansas         -0.41051452 -0.66908525  0.03177945 -0.34506377
## Kentucky        0.43898421 -0.74108152 -0.93542116 -0.52656390
## Maine          -1.30593210 -1.05306531 -1.00450692 -1.43406455
## Massachusetts -0.77786532 -0.26110644  1.34440885 -0.52656390
## Minnesota      -1.16817555 -1.18505846  0.03177945 -0.67603460
## Missouri        0.27826823  0.08687549  0.30812248  0.74393700
## Montana        -0.41051452 -0.74108152 -0.86633540 -0.51588743
## Nebraska       -0.80082475 -0.82507715 -0.24456358 -0.50521095
## New Hampshire -1.30593210 -1.36504911 -0.65907813 -1.25256442
## New Jersey     -0.08908257 -0.14111267  1.62075188 -0.25965195
## North Dakota  -1.60440462 -1.50904164 -1.48810723 -1.48744694
## Ohio           -0.11204199 -0.60908837  0.65355127  0.01793648
## Oklahoma       -0.27275797 -0.23710769  0.16995096 -0.13153421
## Oregon         -0.66306820 -0.14111267  0.10086521  0.86137826
## Pennsylvania  -0.34163624 -0.77707965  0.44629400 -0.67603460
## Rhode Island  -1.00745957  0.03887798  1.48258036 -1.38068216
## South Dakota  -0.91562187 -1.01706718 -1.41902147 -0.90024064
## Utah           -1.05337842 -0.60908837  0.99898006  0.17808366
## Vermont        -1.28297267 -1.47304350 -2.31713632 -1.07106429
## Virginia        0.16347111 -0.17711080 -0.17547783 -0.05679886
## Washington     -0.86970302 -0.30910395  0.51537975  0.53040744
## West Virginia -0.47939280 -1.07706407 -1.83353601 -1.27391738
## Wisconsin      -1.19113497 -1.41304662  0.03177945 -1.11377020
## Wyoming        -0.22683912 -0.11711392 -0.38273510 -0.60129925
```

d) We observe that the cluster memberships changed after scaling the variables. The variables should be scaled before the inter-observation dissimilarities are computed in order to ensure equal weights given to every variable in **X** despite their different scales.

PCA

```
img <- readJPEG('piggy.jpg')

dim(img)
```

```
## [1] 233 233   3
```

```
r <- img[,,1]
g <- img[,,2]
b <- img[,,3]

img.r.pca <- prcomp(r, center = FALSE)
```

```r
img.g.pca <- prcomp(g, center = FALSE)
img.b.pca <- prcomp(b, center = FALSE)

rgb.pca <- list(img.r.pca, img.g.pca, img.b.pca)

# Approximate X with XV_kV_k^T
compress <- function(pr, k)
{
  compressed.img <- pr$x[,1:k] %*% t(pr$rotation[,1:k])
  compressed.img
}

# Using first 20 PCs
pca20 <- sapply(rgb.pca, compress, k = 20, simplify = "array")

writeJPEG(pca20, "pca20.jpeg")

# Try to increase the number of PCs!
pca50 <- sapply(rgb.pca, compress, k = 50, simplify = "array")

writeJPEG(pca50, "pca50.jpeg")

pca100 <- sapply(rgb.pca, compress, k = 100, simplify = "array")

writeJPEG(pca100, "pca100.jpeg")

pca200 <- sapply(rgb.pca, compress, k = 200, simplify = "array")

writeJPEG(pca200, "pca200.jpeg")
```