PC1222 cheat sheet. 1 year = 365.25 days

Electrostatic attraction

Electric charge
- like charges repels, unlike charges attract
- conservation of charge
- quantization of charge

Principle of superposition
$$\vec{F} = \vec{F_1} + \vec{F_2} + \cdots \vec{F_N}$$

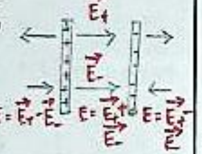
Coulomb's law:
$$F = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{k}\frac{|q_1||q_2|}{r^2}$$
- only to point charges
- only to electric forces between static charges

Electric fields
$$\vec{E} = \frac{\vec{F}_{on\ q_0}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

For spherical shell: $E = 0$, $E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$

For infinite plate: $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$

Between 2 parallel plates: $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$



Electric potential

Work done
$$W_{a\to b} = -\Delta U = U_a - U_b = Fd\cos\theta$$

Electric potential energy
$$U = q_0 Ey = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q_2}{r}$$
$$U = \sum_{i<j}\frac{k q_i q_j}{r_{ij}}$$

Electric potential
$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$
$$V = \sum_i \frac{k q_i}{r_i}$$

Conservation of energy
$$K_i + qV_i = K_f + qV_f$$
K = kinetic energy ($\frac{1}{2}mv^2$)

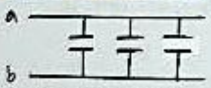
qV / qEy = electric potential energy

Equipotential surface
$$E = \frac{|\Delta V|}{d}$$

Electric potential difference

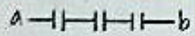
$\Delta V = Ed$

Capacitance
$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$
$$C = \frac{k\varepsilon_0 A}{d}$$
k = dielectric constant

Parallel capacitor


$C = C_1 + C_2 + C_3$

Energy stored
$$U = \frac{1}{2}QV = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}\varepsilon_0 E^2(Ad)$$
energy density: $u = \frac{1}{2}\varepsilon_0 E^2$

Series capacitor


$$C = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

Dielectric strength (V/m)
- Maximum electric field a material can withstand without breakdown

Current and Resistance
$$I = \frac{\Delta Q}{\Delta t} = \frac{\Delta V}{R} = |q|nAV_d$$

Electromotive force
$$\mathcal{E} = I(R + r)$$
R = load resistance, r = internal resistance.

Resistance & resistivity
$$R = \frac{\rho L}{A}$$
ρ = resistivity ($\Omega\cdot m$)

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$
$$R = R_0[1 + \alpha(T - T_0)]$$
T_0 usually = 20°C

α = temperature coefficient of resistivity (°C)$^{-1}$

Electrical power
$$P = \frac{\Delta Q \Delta V}{\Delta t} = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

Direct current circuit

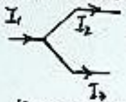
Resistor Network
- series

$R_{eq} = R_1 + R_2 + R_3$

- Parallel

$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

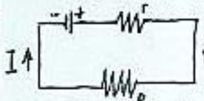
Kirchhoff's rules
- Kirchhoff's junction rule



$I_1 = I_2 + I_3$

↳ conservation of charge

- Kirchhoff's loop rule



$\mathcal{E} - Ir - IR = 0$

↳ conservation of energy

RC circuit
- Time constant (s)

$\tau = RC$

- Charging capacitor

$I_0 = \frac{\mathcal{E}}{R}$, $q_\infty = \mathcal{E}C$

$q(t) = q_\infty(1 - e^{-t/RC})$, $I(t) = I_0 e^{-\frac{t}{RC}}$

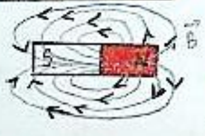
- Discharging capacitor

$I_0 = -\frac{q_0}{RC}$, $q_\infty = 0$

$q(t) = q_0 e^{-\frac{t}{RC}}$, $I(t) = I_0 e^{-\frac{t}{RC}}$

* if τ is small, capacitor charged and discharged very quickly.

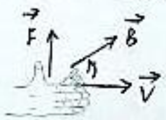
Magnetism

Magnetic field lines.


Magnetic force
$F = |q|vB\sin\theta$ Direction: right hand rule.

Radius of circular orbit

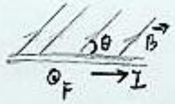
$r = \frac{mv}{|q|B}$



Current and Magnetism
⊙ out ⊗ In

Magnetic force of many charges
$F = ILB\sin\theta$

direction: Right hand rule



Sources of magnetic field


Thumb: I
Fingers: $\vec{B}$

$$B = \frac{\mu_0 I}{2\pi r}$$
r: distance from wire

Thumb: $\vec{B}$
Fingers: I

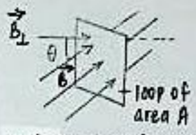
$$B = \frac{\mu_0 N I}{2R}$$
N: number of loop
R: radius of loop

Thumb: $\vec{B}$
Fingers: I

$$B = \mu_0 n I$$
n: number of turns per unit length

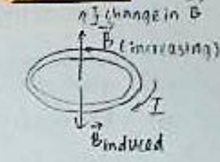
Electromagnetic Induction

Magnetic flux
$\Phi = BA\cos\theta$



Unit: tesla meter squared / Tm^2 Wb (Weber)

Induced quantities
$\boxed{\Delta\Phi} \rightarrow \boxed{\mathcal{E}_{Ind.}} \xrightarrow{\text{complete circuit}} \boxed{I_{induced}} \rightarrow \boxed{B_{ind}} \rightarrow \boxed{\Phi_{induced}}$

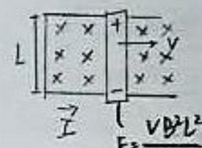
Lenz's law


Faraday's law
$$|\mathcal{E}| = N\frac{|\Delta\Phi|}{\Delta t} = N\frac{|\Phi_f - \Phi_i|}{\Delta t}$$
N = number of loops

For solenoid, $|\mathcal{E}| = \frac{\mu N(\pi r^2)}{L}\left|\frac{\Delta I_{sol}}{\Delta t}\right|$

$$I_{induced} = \frac{|\mathcal{E}|}{R}$$

Motional emf
$\mathcal{E} = vBL$



$F = \frac{vB^2L^2}{R}$

Electromagnetic Waves

Electromagnetic waves
$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$
$E = cB$, $c = \lambda f$

- energy:

$u_E = \frac{1}{2}\varepsilon_0 E^2$, $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$

$u_{EM} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$

- Intensity

$I = \frac{U}{A\Delta t} = cu_{EM} = c\varepsilon_0 E^2 = \frac{c}{\mu_0}B^2$

- Average intensity (W/m²)

$I_{av} = \frac{1}{2}c\varepsilon_0 E_{max}^2 = \frac{1}{2}\frac{c}{\mu_0}B_{max}^2$

Doppler effect
$$f_o = f_s\left(1 \pm \frac{u}{c}\right)$$
+: moving closer
−: moving further
f_o = observed frequency
f_s = frequency emitted by source

Polarization of light
$I_{transmitted} = I_{incident}\cos^2\theta$

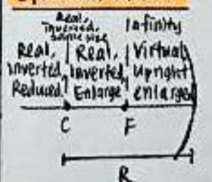
$= \frac{1}{2}I_{incident}$

θ = angle between electric field and polarizer axis

Reflection of light

Law of reflection
$\theta_i = \theta_r$

$f = \pm\frac{R}{2}$ +: concave −: convex

Spherical mirror


Mirror equation & magnification
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
d_o, d_i: + : real object (in front of mirror)
− : virtual (behind mirror)

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$
h_o: always +
h_i: +: upright
−: inverted

Refraction of light

Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

Apparent depth

$$\frac{d'}{d} = \frac{\tan\theta_2}{\tan\theta_1}$$

Total internal Reflection

- Critical angle ; θ_c

$n_1 \sin\theta_c = n_2 \sin 90°$

$$\sin\theta_c = \frac{n_2}{n_1}$$

Polarization by reflection

- can be completely polarized, partially polarized, unpolarized.
- Brewster's angle (beam is fully polarized)

$$\tan\theta_B = \frac{n_2}{n_1}$$

Thin lenses

Converging lens

- formed 5 possible images same as concave mirror
- $+f$

Diverging lens

- $-f$

Lens equation & magnification

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

d_o/d_i : + = real object/image, infront/behind lens

d_o/d_i : − : virtual object/image, behind/infront of lens

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Refractive power of lens

$P = \frac{1}{f}$ unit: Diopter / m^{-1}

Human eyes

$f_{lensed} = +22.7mm$

- Nearsightedness

$$\frac{1}{f_{lens}} = \frac{1}{+\infty} + \frac{1}{-d_{far}}$$

$\Rightarrow f_{lens} = -d_{far}$ (diverging lens)

- Farsightedness

$$\frac{1}{f_{lens}} = \frac{1}{25cm} + \frac{1}{-d_{near}}$$

$$f_{lens} = \frac{(25)(-d_{near})}{d_{near} - 25}$$

(converging lens)

Interference of light

Double slit interference

$$\left.\begin{aligned} \theta &\approx \frac{m\lambda}{nd} \\ y &= \frac{m\lambda L}{nd} \end{aligned}\right\} \text{constructive Interference}$$

$\delta = nd\sin\theta$

$$\left.\begin{aligned} \theta &= \left(m+\tfrac{1}{2}\right)\frac{\lambda}{nd} \\ y &= \left(m+\tfrac{1}{2}\right)\frac{\lambda L}{nd} \end{aligned}\right\} \text{destructive Interference}$$

d = distance between 2 slit.

Thin film interference

- Phase shift

$n_1 > n_2$, the reflected wave does not have a phase change.

$n_1 < n_2$, the reflected wave does have a phase change. $= \frac{\lambda}{2}$

Interference	0/2 phase change	1 phase change
constructive	$2nt = m\lambda$	$2nt = (m+1/2)\lambda$
destructive	$2nt = (m+1/2)\lambda$	$2nt = m\lambda$

Diffraction and grating

Single slit diffraction

$\frac{\lambda}{W}$, ↑W, ↓diffraction

- Width of central maxima = 2× of side maxima.
- locating minima

$nW\sin\theta = m\lambda$

$m = 1, 2, 3, \ldots$

W = width of slit

- small angle approximation ; $\sin\theta \approx \tan\theta$

$$y = \frac{m\lambda L}{nW}$$

$$\theta = \frac{m\lambda}{nW}$$

Diffraction grating

$nd\sin\theta = m\lambda$

$y = L\tan\theta$

Circular aperture

$$\theta_{obj} \geq \theta_{min} = \frac{1.22\lambda}{Dn}$$

$$\theta_{obj} \approx \frac{y}{L}$$

y = separation between objects

L = distance between objects and aperture

* ↑θ_{obj} , ↓θ_{min}, better resolution

Wave-Particle Duality

Blackbody radiation

- Photon

$E = hf = \frac{hc}{\lambda}$

Photoelectric effect

- Metal's work function : W_0
  - ↳ min energy needed to free an electron from its surface.
- Max kinetic energy

$K_{max} = hf - W_0$

- Cutoff / threshold f/λ

$f_0 = \frac{W_0}{h}$, $\lambda_0 = \frac{hc}{W_0}$

$f > f_0$, $\lambda < \lambda_0$ to release electrons.

de Broglie's wavelength

$$\lambda = \frac{h}{P}$$

P = momentum = mv

- stopping potential

$eV_0 = K_{max}$

$V_0 = \frac{h}{e}(f - f_0)$

Atomic physics

Line spectra

- Balmer series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \ n = 3, 4, 5$$

$R = 1.097 \times 10^7 \ m^{-1}$

general :

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

- Lyman series

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \ n = 2, 3, 4$$

- Paschen series

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

- Brackett series, 4
- Pfund series, 5

Bohr's model

- $\left.\begin{aligned} E_i - E_f &= hf \\ E_i &> E_f \end{aligned}\right\}$ electron jumps from higher level to lower level
- Bohr's angular momentum quantization

$mrv = \frac{nh}{2\pi}$

- coulomb attraction as centripetal force.

$$\frac{ke^2}{r_n^2} = \frac{mv_n^2}{r_n}$$

$$r_n = \epsilon_0 \frac{n^2h^2}{\pi m e^2}, \quad v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh}$$

- quantization of orbit

$a_0 = \epsilon_0 \frac{h^2}{\pi m e^2} = 0.0529 \ nm$

$r_n = n^2 a_0 = n^2 (0.0529 \ nm)$

Bohr's model (continued)

- energy of orbit

$$E = -\frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{n^2 a_0}$$

$$E_n = -\frac{13.6 \ eV}{n^2}$$

- hydrogen-like atoms

$$E_n = -\frac{Z^2(13.6 \ eV)}{n^2}$$

Z = atomic number

Nuclear physics

Atomic mass unit (u) :

- Electron : 5.485799×10^{-4}
- Proton : 1.007276
- Neutron : 1.008665
- Hydrogen : 1.007825

$u = 931.5 \ MeV/c^2$

Rest energy

$E_R = mc^2$

$= 1.4924 \times 10^{-10} J$

- Mass defect & binding energy

$\Delta M = Zm_p + Nm_n - M$

$E_B = (\Delta M)c^2$

Radioactivity & decay law

- Alpha decay

${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

- Beta decay

${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e^-$

$\rightarrow {}^{A}_{Z-1}Y + {}^{0}_{-1}e^+$

- Gamma decay

${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$

- Nuclear activity

$\frac{\Delta N}{\Delta t} = -\lambda N$

$A = \left|\frac{\Delta N}{\Delta t}\right|$

1 Bq = 1 decay/s

1 Ci = 3.7×10^{10} Bq

- Nuclear decay

$N = N_0 e^{-\lambda t}$

$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

$N = N_0 2^{-t/T_{1/2}}$