Temporal Probabilistic Logic Networks for Procedural Reasoning

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Abstract. TODO

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1 Introduction

The goal of this project is to make an agent as rational as possible, not necessarily as efficient as possible. This stems from the concern that in order to autonomously gain efficiency the agent must first be able to make the best possible decisions, starting first in the outer world, and then in the inner world.

The paper presents

The agent starts in a completely unknown environment

The idea is that reasoning is used at all levels, discovering patterns from raw observations, building plans and making decisions.

It is a work in progress.

Neural networks are excellent at interpolation, but are rather poor at extrapolation, what we need for true intelligence is a system that thinks critically.

Rarely do causes and effects take place over arbitrary temporal scales. For instance it is unlikely to find a cause that may produce the same effect, or an effect at all, after 1ms, 1 century or any time in between. For that reason we focus on a real time temporal logic.

2 Probabilistic Logic Networks Recall

PLN stands for *Probabilistic Logic Networks* [2]. It is a mixture of predicate and term logic that has been probabilitized to handle uncertainty. It has at least two types of rules for inferring relationships

1. from direct observations,
2. from existing relationships.

As such it is well suited for building a model of a given environment, and planning in that environment. All it needs is to be properly equipped with a vocabulary for representing temporal and procedural knowledge. The Chapter 14 of the PLN book [2] does just that. However we found that definitions are sometimes ambiguous or incorrect. This paper has been written to remedy that.

2.1 Elementary Notions

Graphically speaking, PLN statements are sub-hypergraphs[1] made of *Links* and *Nodes*, called *Atoms*, decorated with *Truth Values*. Syntactically speaking, PLN statements are not very different from statements expressed in another logic, except that they are usually formatted in prefixed-operator with a tree-style indentation to emphasize their graphical nature and to leave room for their truth values. For instance

$$\begin{array}{c} \mathit{Implication}\ \langle TV \rangle \\ P \\ Q \end{array}$$

represents an implication link between P and Q with truth value TV. For the sake of conciseness we also adopt the following representations. We adopt a flattened, as opposed to a tree-style, representation. For instance the implication link above is represented as

$$\mathit{Implication}(P, Q)\ \langle TV \rangle$$

We adopt a more mathematically looking symbolic representation. For instance that same implication can be represented as

$$P \rightarrow Q \stackrel{m}{=} TV$$

There is a large variety of constructs for PLN. Here, we will focus primarily on the ones for handling predicates. Let us recall that predicates are functions that output Boolean values. The domain of a predicate can be arbitrarily defined, but its range is always Boolean. In this paper, the letters a, b, c represent atoms of any type, x, y, z represent atoms that are variables, while the capital letter P, Q, R, ... represent atoms that are predicates, typed as follows:

$$P, Q, R, ... : \mathit{Domain} \mapsto \{\mathit{True}, \mathit{False}\}$$

Note that in PLN, predicates are not necessarily crisp because their outputs can be totally or partially unknown, thus potentially described by probabilities.

Truth values are, in essence, second order probability distributions, or probability distributions of probabilities. They are often described by two numbers: a strength, s, representing a probability, and a confidence, c, representing the confidence over that probability. Such truth values are called *Simple Truth Values* and are denoted as follows

$$< s, c >$$

Alternatively, the strength and the confidence of a simple truth value TV can be denoted $TV.s$ and $TV.c$ respectively. Underneath, a simple truth value is a beta distribution [1], similarly to an *opinion* in Subjective Logic [3]. The parameters of the corresponding beta distribution can be obtained as follows

$$\alpha(s, c) = \alpha_0 + \frac{s.c.k}{1 - c} \qquad \beta(s, c) = \beta_0 + \frac{(1 - s).c.k}{1 - c}$$

[1] because links can point to links, not just nodes

where k is a PLN parameter called the *Lookahead*, and α_0 and β_0 are usually set to 0.5 corresponding to Jeffreys prior. For truth values obtained from direct evidence, a simple truth value makes perfect theoretical sense. For truth values obtained from indirect evidence, not so much, even though they are often used in practice. When more precision is needed, to represent a multi-modal truth value for instance, a mixture of simple truth values can be used. Through out the paper, sometimes we may say *probability*, while what we really mean is *second order probability distribution*.

Below is a table of the constructs used in this paper with their flattened and symbolic representations, as well as precedence values to minimize parenthesis usage with the symbolic representation.

Flattened	Symbolic	Precedence
$Evaluation(P, a)$	$P(a)$	0
$Not(P)$	$\neg P$	1
$And(P, Q)$	$P \wedge Q$	2
$Or(P, Q)$	$P \vee Q$	2
$Implication(P, Q)$	$P \rightarrow Q$	4
$a\langle TV \rangle$	$a \overset{m}{=} TV$	5

For representing n-ary predicates one may use $P(a_1, \ldots, a_n)$ which may simply be understood as unary predicates applied to tuples. Let us now explain their semantics and how their truth values are interpreted.

- $\neg P$ is the predicate resulting from the pointwise negation of P.
- $P \wedge Q$ is the predicate resulting from the pointwise conjunction of P and Q.
- $P \vee Q$ is the predicate resulting from the pointwise disjunction of P and Q.
- $P(a) \overset{m}{=} TV$ states that $P(a)$ outputs *True* with a second order probability described by TV.
- $P \rightarrow Q \overset{m}{=} TV$ states that if $P(a)$ is *True* for some a in the domain of P, then $Q(a)$ is *True* with a second order probability described by TV. In simple probability terms, it represents $\mathcal{P}r(Q|P)$ [2], the conditional probability of Q knowing P. We may also say that such implication is a conditional predicate where Q is conditioned by P.
- $P \overset{m}{=} TV$ states that the prevalence of P being *True* is described by TV.

2.2 Inference Rules

Inferences rules are used to construct PLN statements and calculate their truth values. They fall into two groups, direct evidence based or otherwise. Rules

[2] To be precise, $\mathcal{P}r(Q|P)$ should be understood as $\mathcal{P}r(\mathcal{S}at(Q)|\mathcal{S}at(P))$, where $\mathcal{S}at(P)$ and $\mathcal{S}at(Q)$ are the satisfying sets of P and Q respectively.

from the former group infer abstract knowledge from direct evidence, while rules from the latter group infer knowledge by combining existing abstractions. There are dozens of inference rules, for now we only recall two, *Implication Direct Introduction* and *Deduction*.

The Implication Direct Introduction Rule (IDI) takes evaluations as premises and produces an implication as conclusion. It is formally depicted by the following proof tree

$$\frac{P(a_1) \stackrel{\text{m}}{=} TV_1^P \qquad Q(a_1) \stackrel{\text{m}}{=} TV_1^Q \qquad ... \qquad P(a_n) \stackrel{\text{m}}{=} TV_n^P \qquad Q(a_n) \stackrel{\text{m}}{=} TV_n^Q}{P \to Q \stackrel{\text{m}}{=} TV} \text{ (IDI)}$$

Assuming perfectly reliable direct evidence[3] then the resulting simple truth value can be calculated as follows

$$TV.s = \frac{\sum_{i=1}^{n} f_\wedge(TV_i^P.s, TV_i^Q.s)}{\sum_{i=1}^{n} TV_i^P.s} \qquad\qquad TV.c = \frac{n}{n+k}$$

where $f_\wedge$ is a function embodying a probabilistic assumption about the conjunction of the events. Such function typically ranges from the product (perfect independence) to the min (perfect overlap). Note that this inference rule takes an arbitrary number of premises. In practice it is not a problem as it is decomposed into two rules covering the base and the recursive cases, while storing evidence to avoid double counting.

The Deduction Rule (D) takes two implications as premises and produces a third one. Depending on the assumptions made there exists different variations of that rule. The simplest one is based on the Markov property

$$\mathcal{P}r(R|Q,P) = \mathcal{P}r(R|Q)$$

which gives rise to the rule depicted by the following proof tree

$$\frac{P \to Q \stackrel{\text{m}}{=} TV^{PQ} \qquad Q \to R \stackrel{\text{m}}{=} TV^{QR} \qquad P \stackrel{\text{m}}{=} TV^{P} \qquad Q \stackrel{\text{m}}{=} TV^{Q} \qquad R \stackrel{\text{m}}{=} TV^{R}}{P \to R \stackrel{\text{m}}{=} TV} \text{ (D)}$$

The reader may notice that three additional premises have been added, corresponding to the probabilities $\mathcal{P}r(P)$, $\mathcal{P}r(Q)$ and $\mathcal{P}r(R)$. This is a consequence of the Markov property. The exact formula for that variation is not recalled here but it merely derives from

$$\mathcal{P}r(R|P) = \mathcal{P}r(R|Q,P) \times \mathcal{P}r(Q|P) + \mathcal{P}r(R|\neg Q,P) \times \mathcal{P}r(\neg Q|P)$$

More information about this derivation can be found in Chapter 5, Section 5.3 of [2]. Finally, one may notice that the same conclusion may be inferred by different inference paths leading to different truth values. How to properly aggregate these truth values is not the subject of this paper and is discussed in Chapter 5, Section 5.10 of [2].

[3] A perfectly reliable piece of evidence has a confidence of 1. Dealing with unreliable evidence involves using convolution products and is outside of the scope of this paper.

3 Temporal Probabilistic Logic Networks

The temporal extension of PLN defined in [**?**] is somewhat partial and ambiguous. In that section we provide a possible completion. Let us begin by defining *Temporal Predicates*, also called *Fluents* as predicates tends to denote atemporal relationships, as regular predicates with a temporal dimension

$$P, Q, R, ... : Domain \times Time \mapsto \{True, False\}$$

Time TODO:list properties here is considered discrete, formally defined as a natural number.

3.1 Temporal Operators

Given temporal predicates we can now define a set of temporal operators.

Lag* and *Lead are temporal operators to shift the temporal dimension of a temporal predicate. They are similar to the metric variations of the *Past* and *Future* operators of Tense Logic, denoted P_n and F_n respectively [4], with the distinction that they are applied over temporal predicates, as opposed to Boolean modal expressions. The *Lag* operator is formally defined as follows

$$Lag(P, T) := \lambda x, t.P(x, t - T)$$

It allows to looks into the past, or one may say that it brings the past into the present. The *Lead* operator is the inverse of the *Lag* operator and is formally defined as follows

$$Lead(P, T) := \lambda x, t.P(x, t + T)$$

It allows to look into the future, or one may that it brings the future into the present. As such, one may observe the following equivalence

$$Lag(Lead(P, T), T) \equiv P$$

SequentialAnd is a temporal conjunction where one of the temporal predicate arguments have been temporally shifted. There are at least two variations one can define. A first where the past of the first predicate is brought into the present. A second where the future of the second predicate is brought into the present. In this paper we provide the second one, formally defined as

$$SequentialAnd(T, P, Q) := And(P, Lead(Q, T))$$

which results into a temporal predicate that is *True* at time t if and only if P is *True* at time t and Q is *True* at time $t + T$. Since we do not know at that point which one of the two variations is best, in practice we have implemented both, but in that paper we settle to one for the sake of simplicity.

SequentialOr is a temporal disjunction where one of the temporal predicate arguments have been temporally shifted. Like for *SequentialAnd* we settle to the variation where the future is brought into the present, defined as

$$SequentialOr(T, P, Q) := Or(P, Lead(Q, T))$$

which results into a temporal predicate that is *True* at time t if and only if P is *True* at time t or Q is *True* at time $t + T$.

PredictiveImplication likewise is an implication where the future of the second predicate has been brought into the present, defined as

$$PredictiveImplication(T, P, Q) := Implication(P, Lead(Q, T))$$

resulting into a conditional predicate, that in order to be defined at time t requires that P is *True* at time t, and if so, to be *True* at t requires that Q is *True* at $t + T$.

Let us introduce a symbolic representation for these temporal constructs with precedence values to minimize parenthesis usage.

Flattened	Symbolic	Precedence
$Lag(P, T)$	$\vec{P}^T$	1
$Lead(P, T)$	$\overleftarrow{P}^T$	1
$SequentialAnd(T, P, Q)$	$P \nearrow^T Q$	3
$SequentialOr(T, P, Q)$	$P \swarrow^T Q$	3
$PredictiveImplication(T, P, Q)$	$P \rightsquigarrow^T Q$	4

The *Lag* (resp. *Lead*) operator is symbolized by an overlining arrow going from the left to the right (resp. from the right to the left) because it brings the past (resp. the future) to the present.

3.2 Temporal Rules

Given these operators we can now introduce a number of temporal inference rules.

The Predictive Implication to Implication Rule (PI) takes a predictive implication as premise and produces an equivalent implication, as depicted by the following proof tree

$$\frac{P \rightsquigarrow^T Q}{P \rightarrow \overleftarrow{Q}^T} \text{ (PI)}$$

Note that because the conclusion is equivalent to the premise, the truth values do not need to be indicated in the rule definition.

The Implication to Predictive Implication Rule (IP) takes an implication as premise and produces an equivalent predictive implication, as depicted by the following proof tree

$$\frac{P \rightarrow \overleftarrow{Q}^{T}}{P \rightsquigarrow^{T} Q} \text{ (IP)}$$

The Temporal Shifting Rule (S) takes a temporal predicate and shits its temporal dimension, as depicted by the following proof tree

$$\frac{P \overset{m}{=} TV}{\overleftarrow{P}^{T} \overset{m}{=} TV} \text{ (S)}$$

Shifting does not change the truth value of the predicate. Indeed, the prevalence of being *True* remains the same, only the origin of the temporal dimension changes. Note however that the predicate itself changes, it has been shifted. Therefore, unlike for the IP and PI inference rules that produce equivalent predicates, the truth values must be included in the rule definition, otherwise the rule of replacement would incorrectly apply. Additionally, one may define a number of variations of that rule. For the sake of conciseness we will not enumerate them all here. Let us instead show one more variation of that rule over conditional predicates

$$\frac{P \rightarrow Q \overset{m}{=} TV}{\overleftarrow{P}^{T} \rightarrow \overleftarrow{Q}^{T} \overset{m}{=} TV} \text{ (S)}$$

The Predictive Implication Direct Introduction Rule (PIDI) is similar to the implication direct introduction rule of Section 2 but accounts for temporal differences between evaluations. It is formalized by the following proof tree

$$\frac{\left(P(a_i, t_i) \overset{m}{=} TV_i^P\right)_{i=1,\ldots,n} \qquad \left(Q(a_i, t_i + T) \overset{m}{=} TV_i^Q\right)_{i=1,\ldots,n}}{P \rightsquigarrow^{T} Q \overset{m}{=} TV} \text{ (PIDI)}$$

The truth value formula is identical to that of the implication direct introduction rule. In fact, such rule can be trivially derived from the implication direct introduction rule, the implication to predictive implication rule and the definition of the *Lead* operator.

The Temporal Deduction Rule (TD) is similar to the deduction rule of Section 2 but operates on predictive implications. It is formally depicted by the following proof tree

$$\frac{P \rightsquigarrow^{T_1} Q \overset{m}{=} TV^{PQ} \qquad Q \rightsquigarrow^{T_2} R \overset{m}{=} TV^{QR} \qquad P \overset{m}{=} TV^{P} \qquad Q \overset{m}{=} TV^{Q} \qquad R \overset{m}{=} TV^{R}}{P \rightsquigarrow^{T_1+T_2} R \overset{m}{=} TV} \text{ (TD)}$$

As it turns out, the truth value formula is also identical to that of the deduction rule, but the proof is not as trivial as in the case of the predictive implication direct introduction rule. In order to convince us that it is the case, let us construct an inference tree that can perform the same inference without requiring the temporal deduction rule. The resulting inference tree is depicted below

$$\dfrac{\dfrac{\dfrac{P \rightsquigarrow^{T_1} Q \;\equiv_{TV}^{PQ}}{P \to \bar{Q}^{T_1} \equiv_{TV}^{PQ}}\,(\text{PI}) \qquad \dfrac{\dfrac{Q \rightsquigarrow^{T_2} R \equiv_{TV}^{QR}}{Q \to \bar{R}^{T_2} \equiv_{TV}^{QR}}\,(\text{PI})}{\bar{Q}^{T_1} \to \bar{R}^{T_1+T_2} \equiv_{TV}^{QR}}\,(\text{S}) \qquad P \equiv_{TV}^{P} \qquad \dfrac{Q \equiv_{TV}^{Q}}{\bar{Q}^{T_1} \equiv_{TV}^{Q}}\,(\text{S}) \qquad \dfrac{R \equiv_{TV}^{R}}{\bar{R}^{T_1+T_2} \equiv_{TV}^{R}}\,(\text{S})}{\dfrac{P \to \bar{R}^{T_1+T_2} \equiv_{TV}}{P \rightsquigarrow^{T_1+T_2} R \equiv_{TV}}\,(\text{IP})}\,(\text{D})$$

As you may see, the premises and the conclusion of that inference tree match exactly the premises and the conclusion of the temporal deduction rule. Since none of the intermediary formulae, beside the deduction formula, alter the truth values, we may conclude that the formula of the temporal deduction rule is identical to that of the deduction rule..

3.3 Procedural Reasoning

Now we come to the reason of that work, which is to perform procedural reasoning.

Likewise, we can use the same temporal to regular deduction mapping to build inference rules for procedural reasoning. Given two cognitive schematics

$$P \wedge \widehat{A} \rightsquigarrow^{T_1} Q$$

expressing that executing A in context P likely leads to context Q after T_1 time units, and

$$Q \wedge \widehat{B} \rightsquigarrow^{T_2} R$$

expressing that executing B in context Q likely leads to context R after T_2 time units, the question becomes how to infer the likelihood that executing A and B in sequence starting from context P leads to context R after $T_1 + T_2$ time units, corresponding to the cognitive schematic

$$P \wedge \widehat{A} \nearrow^{T_1} \widehat{B} \rightsquigarrow^{T_1+T_2} R$$

Using the same rules to map *PredictiveImplication* to regular *Implication* and vice versa, as well as rules about the *Lead* operator, we can construct the following inference tree, in fact the PLN engine can construct it for us

$$\dfrac{\dfrac{\dfrac{\dfrac{P \wedge \widehat{A} \rightsquigarrow^{T_1} Q}{P \wedge \widehat{A} \to^{T_1} \bar{Q}^{T_1}}\,(\text{PI})}{P \wedge \widehat{A} \wedge \bar{\widehat{B}}^{T_1} \to \bar{Q}^{T_1} \wedge \bar{\widehat{B}}^{T_1}}\,(\text{C}) \qquad \dfrac{\dfrac{Q \wedge \widehat{B} \rightsquigarrow^{T_2} R}{Q \wedge \widehat{B} \to \bar{R}^{T_2}}\,(\text{PI})}{\bar{Q}^{T_1} \wedge \bar{\widehat{B}}^{T_1} \to \bar{R}^{T_1+T_2}}\,(\text{S}) \qquad P \wedge \widehat{A} \wedge \bar{\widehat{B}}^{T_1} \qquad \dfrac{Q \wedge \widehat{B}}{\bar{Q}^{T_1} \wedge \bar{\widehat{B}}^{T_1}}\,(\text{S}) \qquad \dfrac{R}{\bar{R}^{T_1+T_2}}\,(\text{S})}{\dfrac{P \wedge \widehat{A} \wedge \bar{\widehat{B}}^{T_1} \to \bar{R}^{T_1+T_2}}{P \wedge \widehat{A} \nearrow^{T_1} \widehat{B} \rightsquigarrow^{T_1+T_2} R}\,(\text{IP})}\,(\text{D})$$

which reduces to the following inference tree after retaining only the premises and the conclusion

$$\dfrac{P \wedge \widehat{A} \rightsquigarrow^{T_1} Q \qquad Q \wedge \widehat{B} \rightsquigarrow^{T_2} R \qquad P \wedge \widehat{A} \wedge \bar{\widehat{B}}^{T_1} \qquad Q \wedge \widehat{B} \qquad R}{P \wedge \widehat{A} \nearrow^{T_1} \widehat{B} \rightsquigarrow^{T_1+T_2} R}\,(\text{ASD})$$

providing an Action Sequence Deduction (ASD) rule ready to be used for procedural reasoning. Three additional premises have been added $P \wedge \widehat{A} \wedge \bar{\widehat{B}}^{T_1}$, $Q \wedge \widehat{B}$ and R.

4 Conclusion

TODO: implement more temporal and procedural rules, support temporal intervals, behavior trees, introduce Temporal Truth Value. Integrate Event Calculus.

Hint regarding intervals: define a notion of parameterized union, where for temporal interval the parameters would be temporal variables ranging over intervals.

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