# Partial Operator Induction with Beta Distribution


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AGI-18

SingularityNET
OpenCog Foundations

Problem:

Combining Models from Different Contexts

Theory:

Solomonoff Operator Induction and Beta Distribution

Practice:

Inference Control Meta-Learning

# Outline

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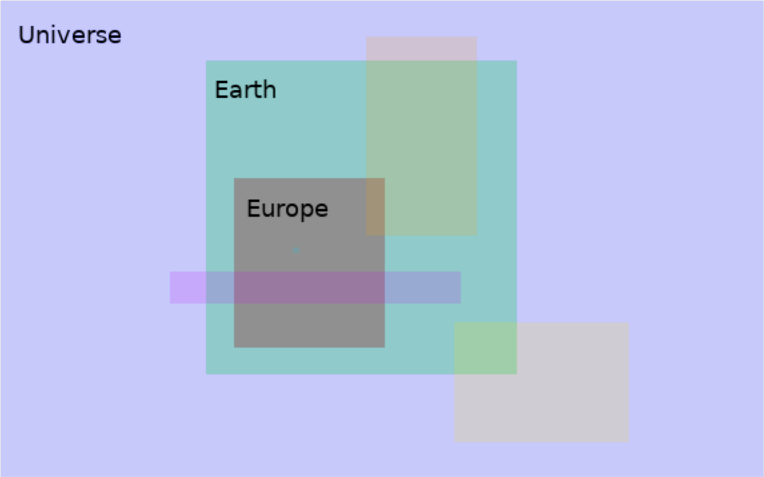
# Problem

How to combine models obtained from different contexts?



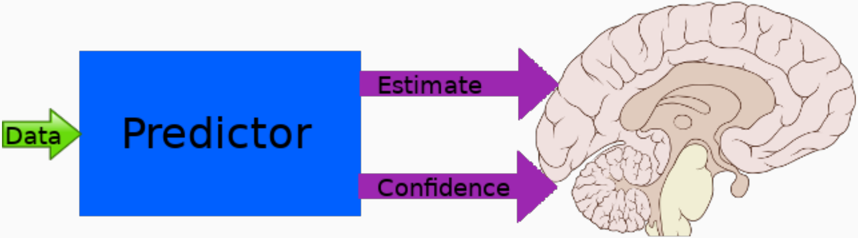
# Solution

Bayesian Model Averaging (esp. Solomonoff Operator Induction)

+ partial models (obtained from different data sets)

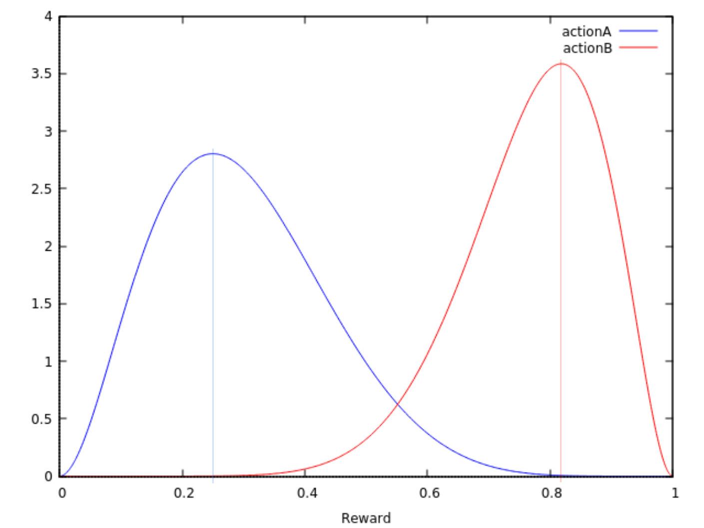
TODO: draw a line with overlapping lines of data

# Preserve Uncertainty



# Preserve Uncertainty

Exploration vs Exploitation (Thompson Sampling)



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## Solomonoff Operator Induction

Probability Estimate:

$$\hat{P}(A_{n+1}|Q_{n+1}) = \sum_j a_0^j \prod_{i=1}^{n+1} O^j(A_i|Q_i)$$

where:

- $Q_i$ = $i^{th}$ question
- $A_i$ = $i^{th}$ answer
- $O^j$ = $j^{th}$ operator
- $a_0^j$ = prior of $j^{th}$ operator

# Second Order Solomonoff Operator Induction

Probability Estimate:

$$\hat{P}(A_{n+1}|Q_{n+1}) = \sum_j a_0^j \prod_{i=1}^{n+1} O^j(A_i|Q_i)$$

$$\Downarrow$$

Probability Distribution Estimate:

$$\hat{cdf}(A_{n+1}|Q_{n+1})(x) = \sum_{O^j(A_{n+1}|Q_{n+1}) \leq x} a_0^j \prod_{i=1}^{n} O^j(A_i|Q_i)$$

TODO: add estimate and cdf graphs

# Beta Distribution

Probability Density Function:

$$pdf_{\alpha,\beta}(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

Beta Function:

$$B_x(\alpha,\beta) = \int_0^x p^{\alpha-1}(1-p)^{\beta-1}dp$$

$$B(\alpha,\beta) = B_1(\alpha,\beta)$$

TODO: add graphs about beta distributions

Meta-model of reality: capture uncertainty

# Beta Distribution Operator

OpenCog implication link

```
ImplicationLink <TV>
  R
  S
```

$$\equiv$$

Class of parameterized operators

$$O_p^j(A_i|Q_i) = \text{if } R^j(Q_i) \text{ then } \begin{cases} p, & \text{if } A_i = A_{n+1} \\ 1-p, & \text{otherwise} \end{cases}$$

# Program Completion

$$O^j_{p,C}(A_i|Q_i) = \begin{array}{l} \text{if } R^j(Q_i) \text{ then } \begin{cases} p, & \text{if } A_i = A_{n+1} \\ 1-p, & \text{otherwise} \end{cases} \\ \text{else } C(A_i|Q_i) \end{array}$$

A *completion* $C$ of $O^j_p$ is a program that completes $O^j_p$ for the unaccounted data, when $R^j(Q_i)$ is false, such that the operator once completed is as follows

# Combing Solomonoff Operator Induction and Beta Distributions

$$\hat{cdf}(A_{n+1}|Q_{n+1})(x) \propto \sum_j a_0^j r^j B_x(m^j + \alpha, h^j + \beta) B(m^j + \alpha, h^j + \beta)$$

where

- $m^j$ = true positives explained by $j^{th}$ model
- $h^j$ = false positives explained by $j^{th}$ model
- $r^j$ = likelihood of the unexplained data

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$r^j =$???

# Combing Solomonoff Operator Induction and Beta Distributions

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where

- $m^j$ = true positives explained by $j^{th}$ model
- $h^j$ = false positives explained by $j^{th}$ model
- $r^j$ = likelihood of the unexplained data

$r^j = ??? \approx 2^{-v_j^{(1-c)}}$

- $v_j$ = size of unexplained data
- $c$ = compressability parameter
    - $c = 1 \rightarrow$ explains remaining data (down to one bit)
    - $c = 0 \rightarrow$ can't explain remaining data

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# Inference Control Meta-learning

Learn how to reason efficiently

Methodology:

1. Solve sequence of problems (via reasoning)

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Methodology:

1. Solve sequence of problems (via reasoning)
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4. Build control rules

```
Implication <TV>
  And
    <inference-pattern>
    <rule>
  <good-inference>
```

# Inference Control Meta-learning

Learn how to reason efficiently

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1. Solve sequence of problems (via reasoning)
2. Store inference traces
3. Mine traces to discover patterns
4. Build control rules

```
Implication <TV>
  And
    <inference-pattern>
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```
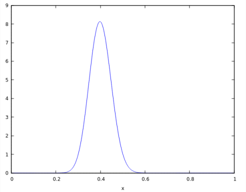

5. Combine control rules to guide future reasoning

# Combine Control Rules

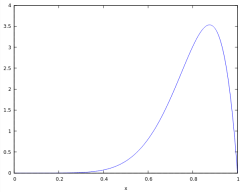
```
Implication <TV1>
  And
    <inference-pattern-1>
    deduction-rule
  <good-inference>
```

```
Implication <TV2>
  And
    <inference-pattern-2>
    deduction-rule
  <good-inference>
```




$\oplus$

$c = 0.01$

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