Probabilistic Logic Networks for Temporal and Procedural Reasoning

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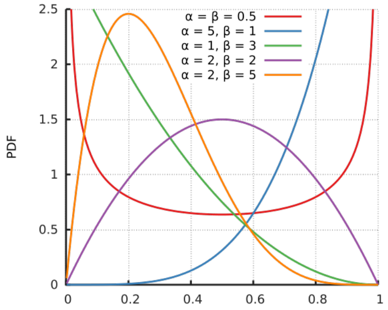

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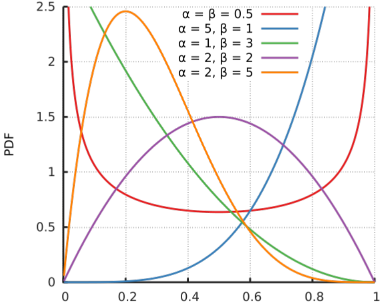

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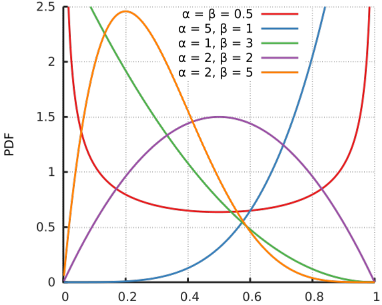

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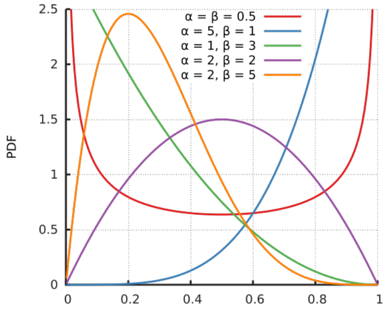

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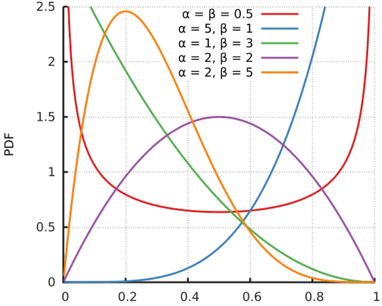

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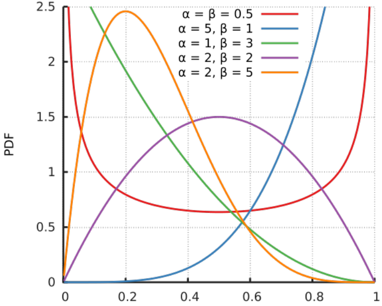

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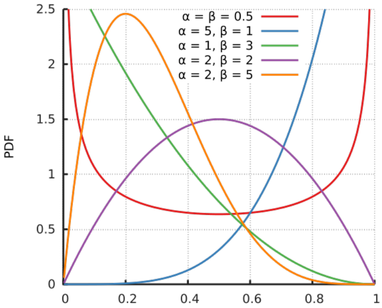

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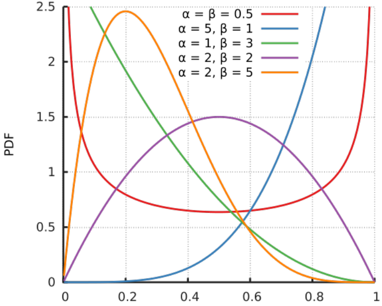

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- Implication Direct Introduction (IDI)

$$\frac{P(a_1) \stackrel{\mathrm{m}}{=} TV_1^P \qquad Q(a_1) \stackrel{\mathrm{m}}{=} TV_1^Q \qquad \ldots \qquad P(a_n) \stackrel{\mathrm{m}}{=} TV_n^P \qquad Q(a_n) \stackrel{\mathrm{m}}{=} TV_n^Q}{P \rightarrow Q \stackrel{\mathrm{m}}{=} \phi_{IDI}\left(TV_1^P, \ldots, TV_n^Q\right)} \text{(IDI)}$$

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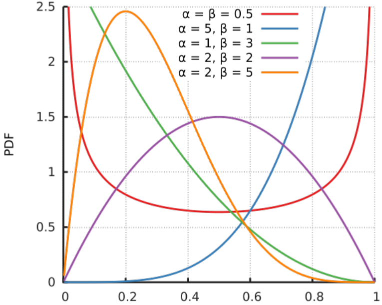

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- Deduction (D)

$$\frac{P \rightarrow Q \stackrel{m}{=} TV^{PQ} \qquad Q \rightarrow R \stackrel{m}{=} TV^{QR} \qquad P \stackrel{m}{=} TV^P \qquad Q \stackrel{m}{=} TV^Q \qquad R \stackrel{m}{=} TV^R}{P \rightarrow R \stackrel{m}{=} \phi_D\left(TV^{PQ}, \ldots, TV^R\right)} \text{(D)}$$

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- Temporal Deduction (TD):

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$$\equiv$$

$$\dfrac{\dfrac{\dfrac{P \rightsquigarrow^{T_1} Q \stackrel{m}{=} TV^{PQ}}{P \rightarrow \overleftarrow{Q}^{T_1} \stackrel{m}{=} TV^{PQ}} \text{ (PI)} \qquad \dfrac{\dfrac{Q \rightsquigarrow^{T_2} R \stackrel{m}{=} TV^{QR}}{Q \rightarrow \overleftarrow{R}^{T_2} \stackrel{m}{=} TV^{QR}} \text{ (PI)}}{\overleftarrow{Q}^{T_1} \rightarrow \overleftarrow{R}^{T_1+T_2} \stackrel{m}{=} TV^{QR}} \text{ (S)} \qquad P \stackrel{m}{=} TV^{P} \qquad \dfrac{Q \stackrel{m}{=} TV^{Q}}{\overleftarrow{Q}^{T_1} \stackrel{m}{=} TV^{Q}} \text{ (S)} \qquad \dfrac{R \stackrel{m}{=} TV^{R}}{\overleftarrow{R}^{T_1+T_2} \stackrel{m}{=} TV^{R}} \text{ (S)}}{\dfrac{P \rightarrow \overleftarrow{R}^{T_1+T_2} \stackrel{m}{=} \phi_{D}\left(TV^{PQ}, \ldots, TV^{R}\right)}{P \rightsquigarrow^{T_1+T_2} R \stackrel{m}{=} \phi_{D}\left(TV^{PQ}, \ldots, TV^{R}\right)} \text{ (IP)}} \text{ (D)}$$

$$\Downarrow$$

$$\phi_D = \phi_{TD}$$