# Partial Operator Induction with Beta Distribution

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NOVAMENTE

Problem:

Combining Models from Different Contexts

Theory:

Solomonoff Operator Induction and Beta Distribution

Practice:

Inference Control Meta-Learning

# Outline

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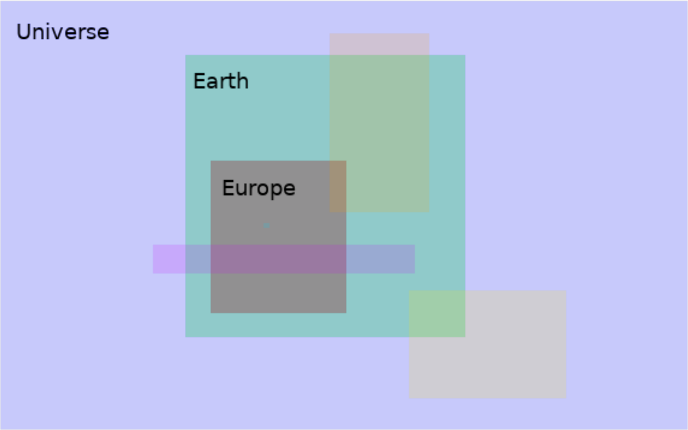
# Problem: Models from different contexts

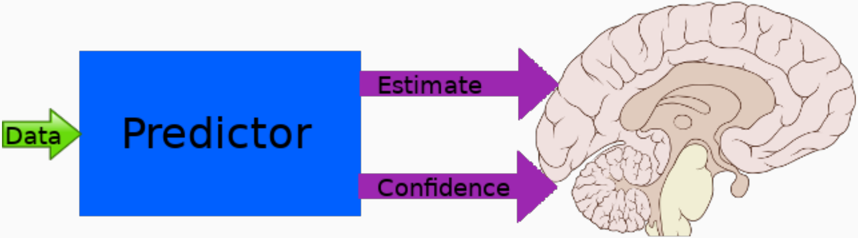
How to combine models obtained from different contexts?




Large Contexts $\rightarrow$ Underfit

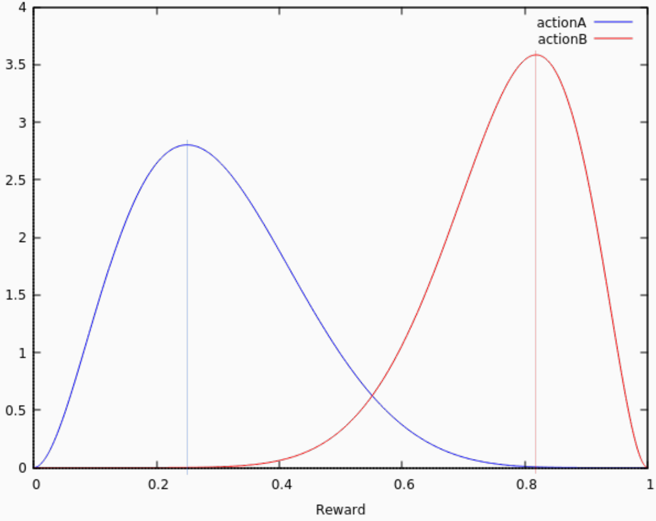
Small Contexts $\rightarrow$ Overfit

# Problem: Preserve Uncertainty



# Problem: Preserve Uncertainty

Exploration vs Exploitation (Thompson Sampling)



# Problem: ImplicationLink

```
ImplicationLink <TV>
  R
  S
```

$\equiv$

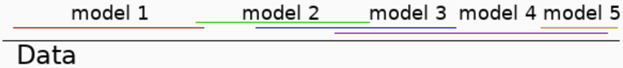
Second Order
$P(S|R)$

Beta Distribution in disguise

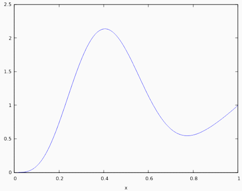
# Solution

Bayesian Model Averaging / Solomonoff Operator Induction, modified to:

1. Support partial models




2. Produce a probability distribution estimate, rather than probability estimate.




3. Specialize for Beta distributions

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# Solomonoff Operator Induction

Bayesian Model Averaging + Universal Distribution

Probability Estimate:

$$\hat{P}(A_{n+1}|Q_{n+1}) = \sum_j a_0^j \prod_{i=1}^{n+1} O^j(A_i|Q_i)$$

where:

- $Q_i$ = $i^{th}$ question
- $A_i$ = $i^{th}$ answer
- $O^j$ = $j^{th}$ operator
- $a_0^j$ = prior of $j^{th}$ operator

# Beta Distribution Operator

Specialization of Solomonoff Operator Induction

OpenCog implication link

```
ImplicationLink <TV>
  R
  S
```

$$\equiv$$

Class of parameterized operators

$$O_p^j(A_i|Q_i) = \text{if } R^j(Q_i) \text{ then } \begin{cases} p, & \text{if } A_i = A_{n+1} \\ 1-p, & \text{otherwise} \end{cases}$$

# Beta Distribution

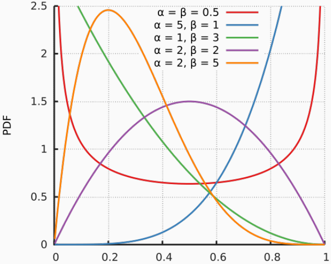
Probability Density Function:

$$pdf_{\alpha,\beta}(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

Beta Function:

$$B_x(\alpha,\beta) = \int_0^x p^{\alpha-1}(1-p)^{\beta-1}dp$$

$$B(\alpha,\beta) = B_1(\alpha,\beta)$$




Conjugate Prior:

$$pdf_{m+\alpha,n-m+\beta}(x) \propto x^m(1-x)^{n-m}pdf_{\alpha,\beta}(x)$$

# Artificial Completion

$$O_p^j \quad (A_i|Q_i) = \quad \text{if } R^j(Q_i) \text{ then } \begin{cases} p, & \text{if } A_i = A_{n+1} \\ 1-p, & \text{otherwise} \end{cases}$$

Data

# Artificial Completion

$$O^{j}_{p,C}(A_i|Q_i) = \begin{array}{l} \text{if } R^{j}(Q_i) \text{ then } \begin{cases} p, & \text{if } A_i = A_{n+1} \\ 1-p, & \text{otherwise} \end{cases} \\ \text{else } C(A_i|Q_i) \end{array}$$

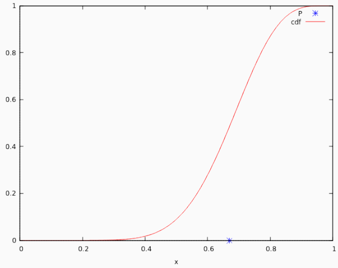
# Second Order Solomonoff Operator Induction

Probability Estimate:

$$\hat{P}(A_{n+1}|Q_{n+1}) = \sum_j a_0^j \prod_{i=1}^{n+1} O^j(A_i|Q_i)$$

Probability Distribution Estimate:

$$\hat{cdf}_{(A_{n+1}|Q_{n+1})}(x) = \sum_{O^j(A_{n+1}|Q_{n+1}) \leq x} a_0^j \prod_{i=1}^{n} O^j(A_i|Q_i)$$



# Combing Solomonoff Operator Induction and Beta Distributions

$$\hat{cdf}_{(A_{n+1}|Q_{n+1})}(x) \propto \sum_j a_0^j r^j B_x(m^j+\alpha, n^j-m^j+\beta)B(m^j+\alpha, n^j-m^j+\beta)$$

where

- $n^j$ = number of observations explained by $j^{th}$ model
- $m^j$ = number of true observations explained by $j^{th}$ model
- $r^j$ = likelihood of the unexplained data

$r^j =$???

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$r^j = ??? \approx 2^{-v^{(1-c)}}$

- $v = n - n^j$ = number of unexplained observations
- $c$ = compressability parameter
  - $c = 1 \rightarrow$ explains remaining data
  - $c = 0 \rightarrow$ can't explain remaining data

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Learn how to reason efficiently

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Learn how to reason efficiently

Methodology:

1. Solve sequence of problems (via reasoning)

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```
Implication <TV>
  And
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5. Combine control rules to guide future reasoning

# Combine Control Rules

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Implication <TV1>
  And
    <inference-pattern-1>
    deduction-rule
  <good-inference>
```

```
Implication <TV2>
  And
    <inference-pattern-2>
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```
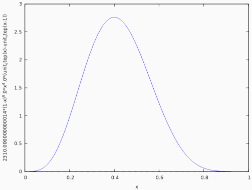
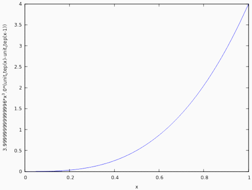



$$\oplus$$

$$c = 1$$



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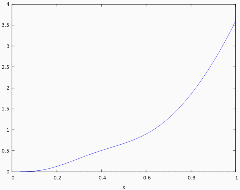

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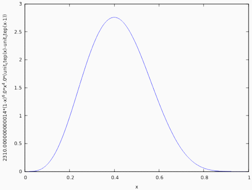
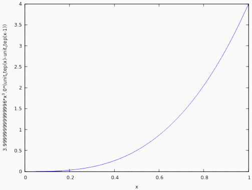


$$\oplus$$

$$c = 0.5$$



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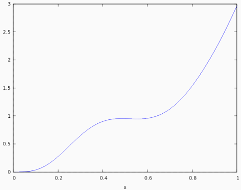

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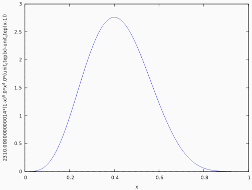
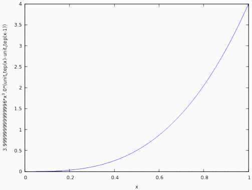
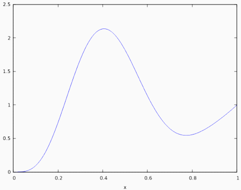


$$\oplus$$

$c = 0.1$



$$=$$



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Thank you!