Forward, Backward, Inward, Outward and Omniward Chaining

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SingularityNET Foundation

Inference Tree

- Formal proof as tree
- Axioms as leaves
- Theorem as root

$$\frac{P}{P}(P) = \frac{P \to Q}{P \to R} \frac{(PQ)}{P \to R} \frac{Q \to R}{(PQ)} \frac{(QR)}{(PQ)} \frac{(QR)}{(PQ$$

 $\underline{\mathsf{Premises}} \colon P,\, P \to Q,\, Q \to R$



$$\frac{\underline{\mathsf{Premises}} \colon P,\, P \to Q,\, Q \to R}{\underline{P \to Q} \; (\mathsf{PQ}) \quad \underline{\qquad \qquad } \mathsf{Q} \to R} \; (\mathsf{QR})$$



Premises:
$$P, P \to Q, Q \to R$$

$$\frac{\overline{P \to Q} \text{ (PQ)} \quad \overline{Q \to R}}{P \to R} \text{ (QR)}$$
(Deduction)

Premises: $P, P \rightarrow Q, Q \rightarrow R$

$$\frac{P}{P} \text{ (P)} \quad \frac{P \rightarrow Q}{P \rightarrow R} \text{ (PQ)} \quad \frac{Q \rightarrow R}{Q \rightarrow R} \text{ (Deduction)}$$

$$\frac{R}{\vdots} \quad \text{(Modus Ponens)}$$

Conclusion: R

Premises

Conclusions

Premises

Conclusions

Conclu	sion: <i>R</i>	
	R	

Premises

Conclusions

$$\frac{P}{P} (P) \qquad \frac{P \to R}{R} (Modus Ponens)$$

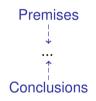
Premises

Conclusions

Conclusion: R

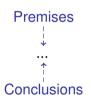
$$\frac{}{P} \text{ (P)} \quad \frac{\overline{P \to Q} \text{ (PQ)} \quad \overline{Q \to R}}{R} \text{ (QR)} \\ \frac{P \to R}{R} \text{ (Modus Ponens)}$$

Inward Chaining



<u>Premises</u>: $P, P \rightarrow Q, Q \rightarrow R, \underline{\text{Conclusion}}$: R

Inward Chaining

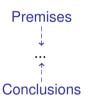


<u>Premises</u>: $P, P \rightarrow Q, Q \rightarrow R$, <u>Conclusion</u>: R

$$\frac{P}{P} (P) \qquad \frac{\overline{P \to Q} (PQ) \qquad \overline{Q \to R}}{R} (QR)$$

$$\frac{\overline{P} (PQ)}{R} \qquad (Modus Ponens)$$

Inward Chaining

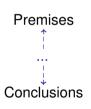


<u>Premises</u>: $P, P \rightarrow Q, Q \rightarrow R,$ <u>Conclusion</u>: R

$$\frac{}{P}\left(\mathsf{P}\right) \quad \frac{\overline{P \to Q} \, \left(\mathsf{PQ}\right) \quad \overline{Q \to R} \, \left(\mathsf{QR}\right)}{P \to R} \, \left(\mathsf{Modus} \, \mathsf{Ponens}\right)}$$

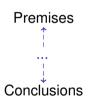


Premise: P, Lemma: $P \rightarrow R$



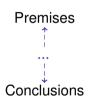
$$\underline{\mathsf{Premise}} \colon P, \, \underline{\mathsf{Lemma}} \colon P \to R$$

$$P \to R$$



Premise: P, Lemma: $P \rightarrow R$

$$\underline{\frac{P}(P)} \quad \underline{\frac{P \to Q}{P \to R}} (PQ) \quad \underline{\frac{Q \to R}{Q \to R}} (QR)$$



Premise:
$$P$$
, Lemma: $P \rightarrow R$

$$\frac{P}{P} (P) \qquad \frac{P \to Q}{P \to R} (PQ) \qquad \frac{Q \to R}{Q \to R} (QR)$$

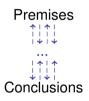
$$\frac{P \to R}{R} (Modus Ponens)$$

<u>Premise</u>: $Q \rightarrow R$, <u>Lemma</u>: $P \rightarrow R$

Premise:
$$Q \to R$$
, Lemma: $P \to R$

$$\frac{}{Q \to R} (QR)$$

$$\frac{}{P \to R}$$



Premise:
$$Q \to R$$
, Lemma: $P \to R$

$$\frac{P \to Q}{P \to R} (PQ) \frac{Q \to R}{Q \to R} (QR)$$
(Deduction)

$$\underline{\mathsf{Premise}} \colon Q \to R, \, \underline{\mathsf{Lemma}} \colon P \to R$$

$$\frac{P \to Q \text{ (PQ)} \quad \overline{Q \to R} \text{ (QR)}}{P \to R} \text{ (Deduction)}$$

$$\frac{P \to R}{R} \text{ (Modus Ponens)}$$

Premise:
$$Q \rightarrow R$$
, Lemma: $P \rightarrow R$

$$\frac{P}{P} (P) \qquad \frac{P \to Q}{P \to R} (PQ) \qquad \frac{Q \to R}{Q \to R} (QR)$$

$$\frac{P \to R}{R} (Modus Ponens)$$