Probabilistic Logical Networks

Intensional Inference

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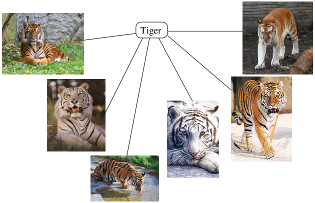
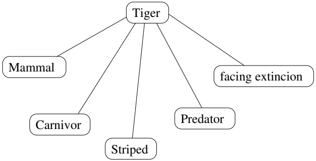
Outline

1. Extensionality vs Intensionality
2. Extensional Inheritance
3. Intensional Inheritance
4. Conclusion

Intensionality vs Extensionality



Extension of a term:
set of objects



Intension of a term:
set of properties

Intensionality vs Extensionality

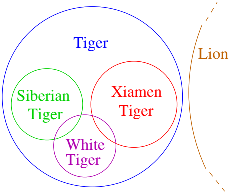
definition

- ExtInh = ExtensionalInheritanceLink
- IntInh = IntensionalInheritanceLink

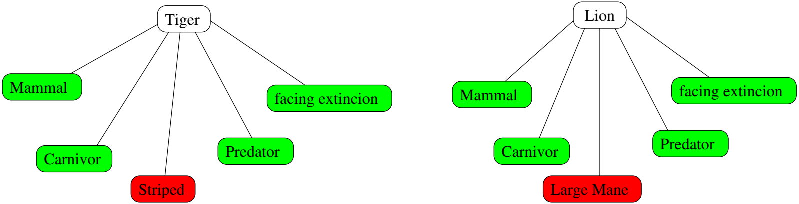
```
ExtInh<1>
      Siberian_Tiger
      Tiger
```

```
ExtInh<1>
      Xiamen_tiger
      Tiger
```

```
ExtInh<0>
      Lion
      Tiger
```



Intensionality vs Extensionality



Lion Inherits Tiger Intensionally

Tiger and Lion share many properties:

```
IntInh<0.78>
    Lion
    Tiger
```

Mixed Inheritance

Mixed Inheritance

Disjunction of Extensional and Intensional Inheritance.

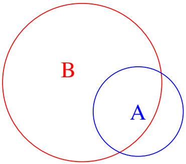
```
Inheritance A B <tv>

≡

OR<tv>
    ExtInh A B
    IntInh A B
```

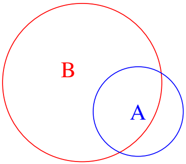
Outline

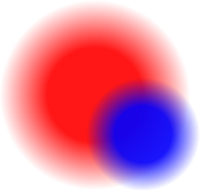
Extensional Inheritance $\equiv$ SubSet



`ExtInh A B <p>` $\equiv$ `SubSet A B <p>` $\equiv$ $P(B|A) = p$

Extensional Inheritance $\equiv$ SubSet





`ExtInh A B <p>` $\equiv$ `SubSet A B <p>` $\equiv$ $P(B|A) = p$

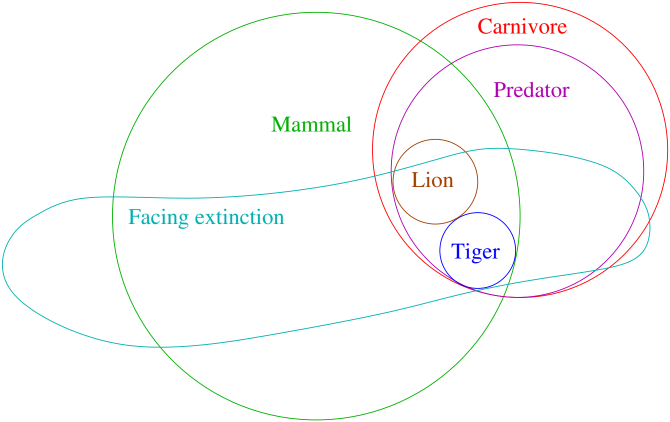
- Fuzzy set $\Rightarrow$ heuristic to compute $P(B|A)$

$$P(B|A) = \frac{\sum_x \min(A(x), B(x))}{\sum_x A(x)}$$

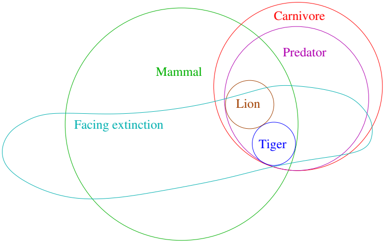
Outline

What is a property?

Property = Super-set

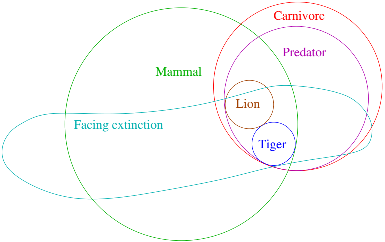


What is a property?



- `SubSet T Fe <1>`
- `SubSet T P <1>`
- `SubSet T C <1>`
- `SubSet T M <1>`
- `SubSet L Fe <0.8>`
- `SubSet L T <0>`

What is a property?



- SubSet T Fe <1>
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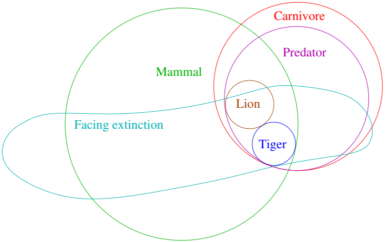
- SubSet L T <0>

- $P(Fe|T) = 1$
- $P(P|T) = 1$
- $P(C|T) = 1$
- $P(M|T) = 1$
- $P(Fe|L) = 0.8$
- $P(T|L) = 0$

What is a property?



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- $P(Fe|L) = 0.8$
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$\Rightarrow$

F is a property of G if

$P(F|G)$ is sufficiently high

What is an interesting property?

But potentially. . . infinity of super-sets (i.e. properties) of a term

Interesting property of a term

1. Help differentiate that term
2. Less complex than the term itself

Interesting property: help differentiate

being_part_of_the_universe

$P(being\_part\_of\_the\_universe|Tiger) = 1$
$P(being\_part\_of\_the\_universe|Lion) = 1$
$P(being\_part\_of\_the\_universe|Chair) = 1$

...

Interesting property: help differentiate

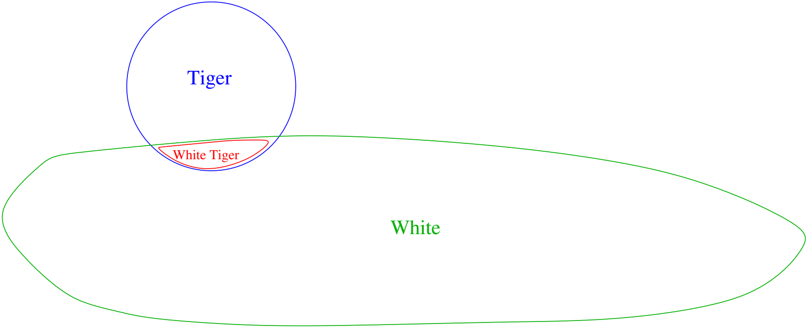
being_part_of_the_universe

$P(being\_part\_of\_the\_universe | Tiger) = 1$
$P(being\_part\_of\_the\_universe | Lion) = 1$
$P(being\_part\_of\_the\_universe | Chair) = 1$
...

being_part_of_the_universe not interesting property of Tiger

because it does not help to differentiate it in any manner

Interesting property: help differentiate



White is an interesting property of White_Tiger

- $P(White|White\_Tiger) = 1$
- $P(White|Golden\_Tiger) = 0$

Interesting property: help differentiate

F helps differentiate G, or F is associated with G iff

$P(F|G) > P(F|\neg G)$

$$ASSOC(F, G) = [P(F|G) - P(F|\neg G)]^+$$

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Examples

1. $ASSOC(being\_part\_of\_the\_universe, Tiger)$

$$\begin{aligned} &= [\ P(being\_part\_of\_the\_universe|Tiger) \\ &\quad -P(being\_part\_of\_the\_universe|\neg Tiger)]^+ \\ &= [1 - 1]^+ = 0 \end{aligned}$$

Interesting property: help differentiate

F helps differentiate G, or F is associated with G iff

$P(F|G) > P(F|\neg G)$

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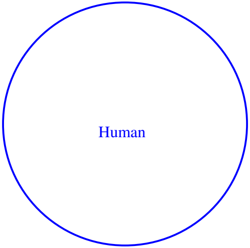
Examples

1. $ASSOC(being\_part\_of\_the\_universe, Tiger)$
$$\begin{aligned} = & \quad [\quad P(being\_part\_of\_the\_universe|Tiger) \\ & \qquad -P(being\_part\_of\_the\_universe|\neg Tiger)]^+ \\ = & \quad [1 - 1]^+ \quad = \quad 0 \end{aligned}$$
2. $ASSOC(White, White\_Tiger)$
$$\begin{aligned} = & \quad [P(White|White\_Tiger) - P(White|\neg White\_Tiger)]^+ \\ = & \quad [1 - 0.3]^+ = 0.7 \end{aligned}$$

Interesting property: Less complex than the term itself



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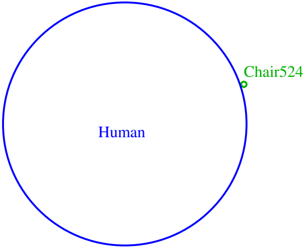

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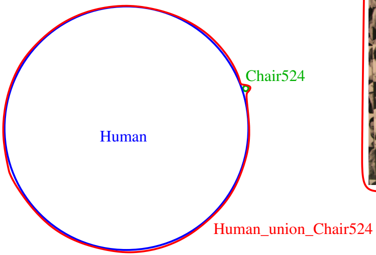

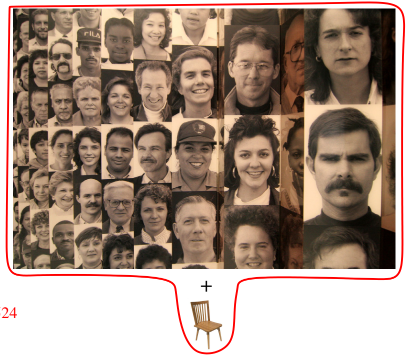

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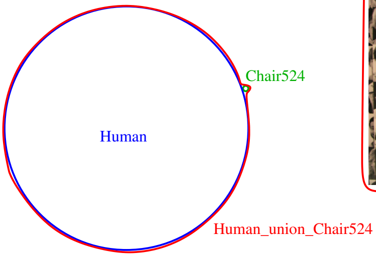
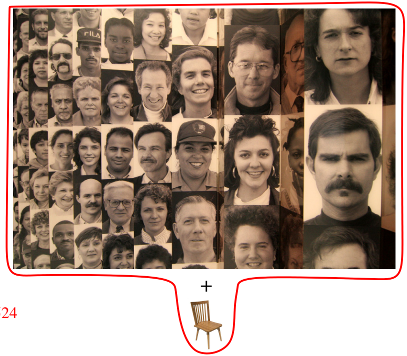
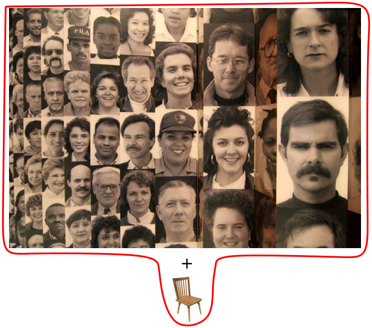




ASSOC(*Human_union_Chair*524, *Human*) is high, yet *Human_union_Chair*524 is not an interesting property of Human.

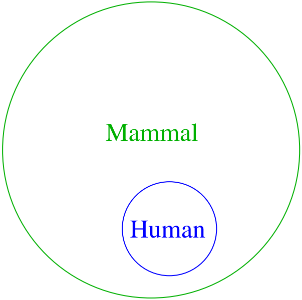
Interesting property: Less complex than the term itself



$Human\_union\_Chair524$ is more complex than Human

$c(Human\_union\_Chair524) \approx c(Human) + c(Chair524)$

Interesting property: Less complex than the term itself



Mammal is less complex than Human, c(Mammal) < c(Human)

A human is a mammal + additional characteristics

Interesting property: Less complex than the term itself

If F is a property of G we want

c(F)<c(G)

$$[c(G) - c(F)]^+$$

Complexity of F, $c(F)$ is for example the shortest description of F in some language L.

Interesting property: Less complex than the term itself

If F is a property of G we want

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$$[c(G) - c(F)]^+$$

Examples:

- $[c(Human) - c(Human\_union\_Chair524)]^+ = 0$

Complexity of F, $c(F)$ is for example the shortest description of F in some language L.

Interesting property: Less complex than the term itself

If F is a property of G we want

c(F)<c(G)

$$[c(G) - c(F)]^+$$

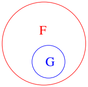
Examples:

- $[c(Human) - c(Human\_union\_Chair524)]^+ = 0$
- $[c(Human) - c(Mammal)]^+ > 0$

Complexity of F, $c(F)$ is for example the shortest description of F in some language L.

What is an interesting property? A Pattern

1. Help differentiate that term, $ASSOC(F, G)$[1]
2. Less complex than the term itself, $[c(G) - c(F)]^+$



[1] $ASSOC(F, G) = [P(F|G) - P(F|\neg G)]^+$

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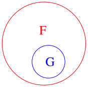

F is a pattern of G if

F is associated with G and F is less complexe than G

$$PAT(F, G) = [c(G) - c(F)]^+ \times ASSOC(F, G)$$

[1] $ASSOC(F, G) = [P(F|G) - P(F|\neg G)]^+$

Intensional Inheritance = Pattern Inheritance

A Intensionally Inherits *B*

How many patterns *A* inherits from *B*

Intensional Inheritance = Pattern Inheritance

A Intensionally Inherits B

How many patterns A inherits from B

More formally...

Definition: X_PAT is the fuzzy set of all patterns of X, that is:

$$X\_PAT(G) = PAT(G, X)$$

Intensional Inheritance = Pattern Inheritance

A Intensionally Inherits *B*

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More formally...

Definition: X_PAT is the fuzzy set of all patterns of X, that is:

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For example:

$$\boxed{\begin{array}{lcc} Tiger\_PAT(Predator) & = & 0.3 \\ Tiger\_PAT(Mammal) & = & 0.4 \\ Tiger\_PAT(Carnivore) & = & 0.2 \\ Tiger\_PAT(Facing\_extinction) & = & 0.15 \\ Tiger\_PAT(Striped) & = & 0.3 \end{array}}$$

Intensional Inheritance $\Rightarrow$ Extensional Inheritance

$$\texttt{IntInh A B} \equiv \texttt{ExtInh A\_PAT B\_PAT}$$
$$\equiv$$
$$\texttt{SubSet A\_PAT B\_PAT} = P(B\_PAT|A\_PAT)$$

Intensional Inheritance $\Rightarrow$ Extensional Inheritance

For example: `IntInh Tiger Lion <?>`

Intensional Inheritance $\Rightarrow$ Extensional Inheritance

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1. Determine Tiger_PAT and Lion_PAT

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Intensional Inheritance ⇒ Extensional Inheritance

For example: `IntInh Tiger Lion <?>`

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2.

$$P(Tiger\_PAT|Lion\_PAT) = \frac{\sum_x \min(Lion\_PAT(x), Tiger\_PAT(x))}{\sum_x Lion\_PAT(x)}$$

Intensional Inheritance ⇒ Extensional Inheritance

For example: `IntInh Tiger Lion <?>`

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Intensional Inheritance $\Rightarrow$ Extensional Inheritance

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2.

$$\begin{aligned} P(Tiger\_PAT \mid Lion\_PAT) &= \frac{\sum_x \min(Lion\_PAT(x), Tiger\_PAT(x))}{\sum_x Lion\_PAT(x)} \\ &= \frac{0.3 + 0.4 + 0.2 + \min(0.1, 0.15) + \min(0, 0.3) + \min(0.3, 0.01)}{0.3 + 0.4 + 0.2 + 0.1 + 0 + 0.3} = 0.78 \end{aligned}$$

3. `IntInh Lion Tiger <0.78>`

Outline

Conclusion

- `ExtensionalInheritance A B <p>` $\equiv P(B|A) = p$

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Conclusion

- `ExtensionalInheritance A B <p>` $\equiv$ $P(B|A) = p$
- A_PAT is the fuzzy set of interesting properties of A, called patterns
- `IntensionalInheritance A B <p>`

  $\equiv$

  `ExtensionalInheritance A_PAT B_PAT <p>`