Partial Operator Induction with Beta Distribution

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AGI-18
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Problem:

Combining Models from Different Contexts

Theory:

Solomonoff Operator Induction and Beta Distribution

Practice:

Inference Control Meta-Learning

Outline

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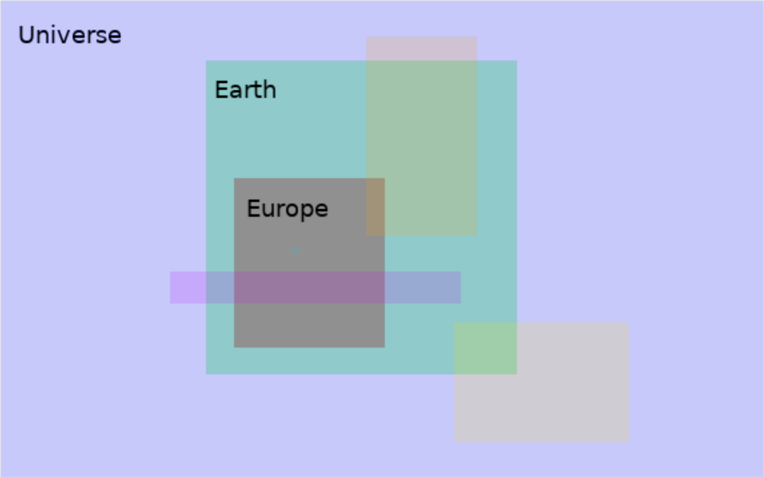
Problem

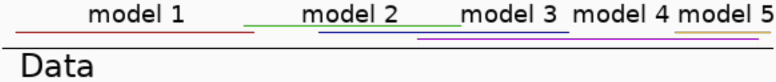
How to combine models obtained from different contexts?



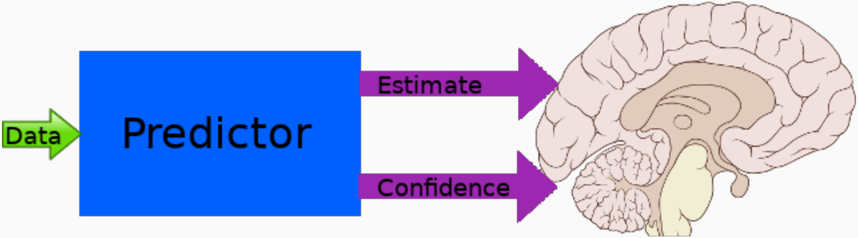
Solution

Bayesian Model Averaging (esp. Solomonoff Operator Induction)

+ partial models (obtained from different data sets)

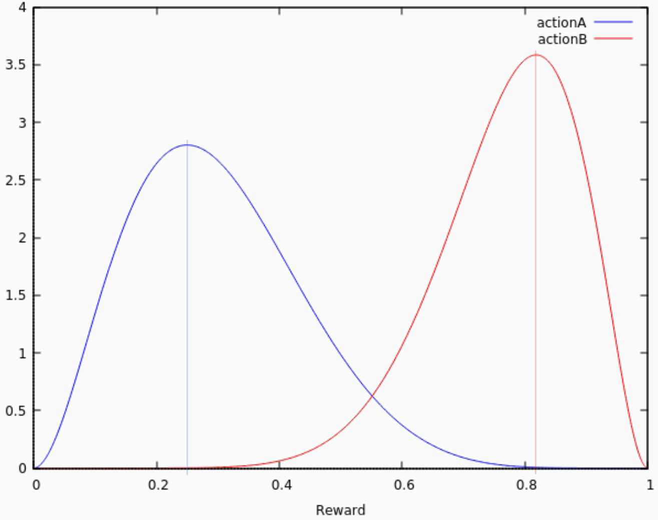


Preserve Uncertainty



Preserve Uncertainty

Exploration vs Exploitation (Thompson Sampling)



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Solomonoff Operator Induction

Probability Estimate:

$$\hat{P}(A_{n+1}|Q_{n+1}) = \sum_j a_0^j \prod_{i=1}^{n+1} O^j(A_i|Q_i)$$

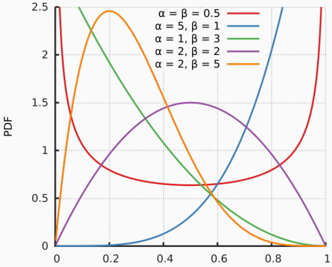
where:

- Q_i = i^{th} question
- A_i = i^{th} answer
- O^j = j^{th} operator
- a_0^j = prior of j^{th} operator

Beta Distribution

Probability Density Function:

$$pdf_{\alpha,\beta}(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$



Beta Function:

$$B_x(\alpha,\beta) = \int_0^x p^{\alpha-1}(1-p)^{\beta-1}dp$$

$$B(\alpha,\beta) = B_1(\alpha,\beta)$$

Conjugate Prior:

$$pdf_{m+\alpha,n-m+\beta}(x) \propto x^m(1-x)^{n-m} pdf_{\alpha,\beta}(x)$$

Beta Distribution Operator

OpenCog implication link

```
ImplicationLink <TV>
  R
  S
```

$$\equiv$$

Class of parameterized operators

$$O_p^j(A_i|Q_i) = \text{if } R^j(Q_i) \text{ then } \begin{cases} p, & \text{if } A_i = A_{n+1} \\ 1-p, & \text{otherwise} \end{cases}$$

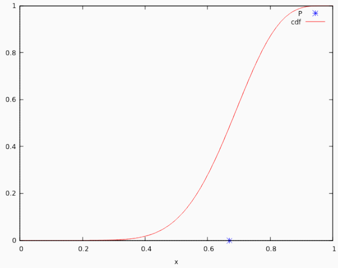
Second Order Solomonoff Operator Induction

Probability Estimate:

$$\hat{P}(A_{n+1}|Q_{n+1}) = \sum_j a_0^j \prod_{i=1}^{n+1} O^j(A_i|Q_i)$$

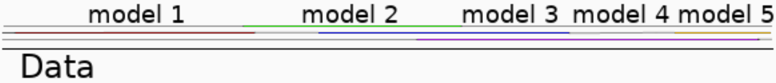
Probability Distribution Estimate:

$$\hat{cdf}_{(A_{n+1}|Q_{n+1})}(x) = \sum_{O^j(A_{n+1}|Q_{n+1}) \leq x} a_0^j \prod_{i=1}^{n} O^j(A_i|Q_i)$$



Program Completion

$$
\begin{aligned}
O^{j}_{p,C}(A_i|Q_i) = \quad & \text{if } R^{j}(Q_i) \text{ then } \begin{cases} p, & \text{if } A_i = A_{n+1} \\ 1-p, & \text{otherwise} \end{cases} \\
& \text{else } C(A_i|Q_i)
\end{aligned}
$$



Combing Solomonoff Operator Induction and Beta Distributions

$$\hat{cdf}_{(A_{n+1}|Q_{n+1})}(x) \propto \sum_j a_0^j r^j B_x(m^j+\alpha, n^j-m^j+\beta)B(m^j+\alpha, n^j-m^j+\beta)$$

where

- n^j = number of observations explained by j^{th} model
- m^j = number of true observations explained by j^{th} model
- r^j = likelihood of the unexplained data

$r^j =$???

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$r^j =??? \approx 2^{-v^{(1-c)}}$

- $v = n - n^j$ = number of unexplained observations
- c = compressability parameter
    - $c = 1 \rightarrow$ explains remaining data
    - $c = 0 \rightarrow$ can't explain remaining data

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Learn how to reason efficiently

Methodology:

1. Solve sequence of problems (via reasoning)

Inference Control Meta-learning

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4. Build control rules

```
Implication <TV>
  And
    <inference-pattern>
    <rule>
  <good-inference>
```

Inference Control Meta-learning

Learn how to reason efficiently

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1. Solve sequence of problems (via reasoning)
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```
Implication <TV>
  And
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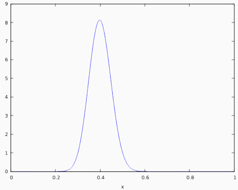
5. Combine control rules to guide future reasoning

Combine Control Rules

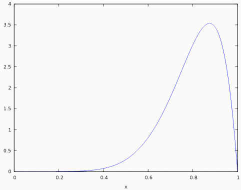
```
Implication <TV1>
  And
    <inference-pattern-1>
    deduction-rule
  <good-inference>
```

```
Implication <TV2>
  And
    <inference-pattern-2>
    deduction-rule
  <good-inference>
```



$\oplus$

$c = 0.01$



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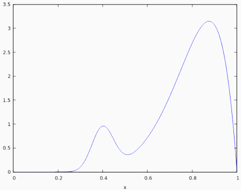

Conclusion

Contribution:

- Second Order Solomonoff Operator Induction
- Specialized for Beta Distribution
- Attempt to Deal with Partial Models

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Future Work:

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- More Experiments (Inference Control Meta-learning)

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Thank you!