

Forward, Backward, Inward, Outward and Omniward Chaining

Nil Geisweiller

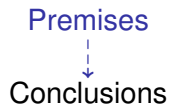
SingularityNET Foundation

Inference Tree

- Formal proof as *tree*
- Axioms as *leaves*
- Theorem as *root*

$$\frac{\frac{\frac{}{P} (P) \quad \frac{\frac{}{P \rightarrow Q} (PQ) \quad \frac{\frac{}{Q \rightarrow R} (QR)}{P \rightarrow R} \text{ (Deduction)}}{P \rightarrow R} \text{ (Modus Ponens)}}{R}}$$

Forward Chaining



Premises: $P, P \rightarrow Q, Q \rightarrow R$

Forward Chaining

Premises
↓
Conclusions

Premises: $P, P \rightarrow Q, Q \rightarrow R$

$$\frac{\frac{\frac{P}{P} (P) \quad \frac{P \rightarrow Q}{P \rightarrow Q} (PQ) \quad \frac{Q \rightarrow R}{Q \rightarrow R} (QR)}{P \rightarrow Q} \quad Q \rightarrow R}{P} (P)$$

Forward Chaining

Premises
↓
Conclusions

Premises: $P, P \rightarrow Q, Q \rightarrow R$

$$\frac{\frac{\frac{P}{P} \text{ (P)} \quad \frac{P \rightarrow Q}{P \rightarrow Q} \text{ (PQ)}}{P \rightarrow R} \quad \frac{Q \rightarrow R}{Q \rightarrow R} \text{ (QR)}}{P \rightarrow R} \text{ (Deduction)}$$

Forward Chaining

Premises
↓
Conclusions

Premises: $P, P \rightarrow Q, Q \rightarrow R$

$$\frac{\overline{P} \text{ (P)}}{\quad} \quad \frac{\overline{P \rightarrow Q} \text{ (PQ)}}{\quad} \quad \frac{\overline{Q \rightarrow R} \text{ (QR)}}{\quad} \quad \frac{\quad}{P \rightarrow R} \text{ (Deduction)} \\ \frac{\quad}{R} \text{ (Modus Ponens)}$$

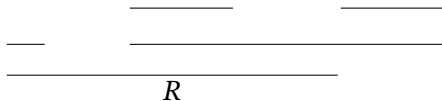
Backward Chaining



Backward Chaining

Premises
↑
Conclusions

Conclusion: R



Backward Chaining

Premises
↑
Conclusions

Conclusion: R

$$\frac{\frac{\overline{P}}{P} \text{ (P)} \quad \frac{\quad}{P \rightarrow R} \text{ (Modus Ponens)}}{R}$$

Backward Chaining

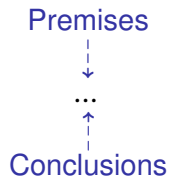
Premises
↑
Conclusions

Conclusion: R

$$\frac{\frac{\frac{\overline{P} \text{ (P)}}{\overline{P \rightarrow Q} \text{ (PQ)}}}{\overline{Q \rightarrow R} \text{ (QR)}}}{\overline{P \rightarrow R} \text{ (Deduction)}} \text{ (Modus Ponens)}$$

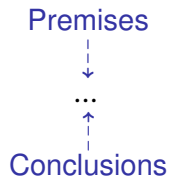
R

Inward Chaining



Premises: $P, P \rightarrow Q, Q \rightarrow R$, Conclusion: R

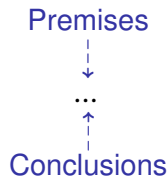
Inward Chaining



Premises: $P, P \rightarrow Q, Q \rightarrow R$, Conclusion: R

$$\frac{\frac{\overline{P} \text{ (P)}}{\quad} \quad \frac{\overline{P \rightarrow Q} \text{ (PQ)} \quad \overline{Q \rightarrow R} \text{ (QR)}}{\quad}}{R} \text{ (Modus Ponens)}$$

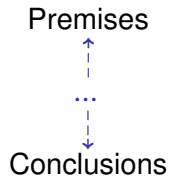
Inward Chaining



Premises: $P, P \rightarrow Q, Q \rightarrow R$, Conclusion: R

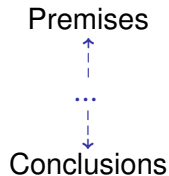
$$\frac{\frac{\frac{\overline{P} \text{ (P)}}{\quad} \quad \frac{\overline{P \rightarrow Q} \text{ (PQ)}}{\quad} \quad \frac{\overline{Q \rightarrow R} \text{ (QR)}}{\quad}}{\frac{P \rightarrow R \text{ (Modus Ponens)}}{R}} \text{ (Deduction)}$$

Outward Chaining



Premise: P , Lemma: $P \rightarrow R$

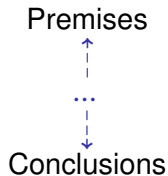
Outward Chaining



Premise: P , Lemma: $P \rightarrow R$

$$\frac{\frac{P}{(P)} \quad \frac{P \rightarrow R}{}}{}$$

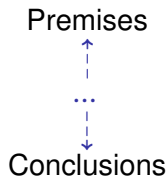
Outward Chaining



Premise: P , Lemma: $P \rightarrow R$

$$\frac{\frac{\frac{\overline{P} \text{ (P)}}{\quad} \quad \frac{\overline{P \rightarrow Q} \text{ (PQ)}}{\quad} \quad \frac{\overline{Q \rightarrow R} \text{ (QR)}}{\quad}}{\quad} \quad \frac{\quad}{P \rightarrow R} \text{ (Deduction)}$$

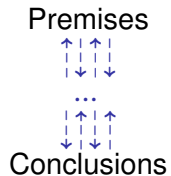
Outward Chaining



Premise: P , Lemma: $P \rightarrow R$

$$\frac{\frac{\overline{P} \text{ (P)}}{\quad} \quad \frac{\overline{P \rightarrow Q} \text{ (PQ)}}{\quad} \quad \frac{\overline{Q \rightarrow R} \text{ (QR)}}{\quad}}{\frac{P \rightarrow R \text{ (Deduction)}}{R} \text{ (Modus Ponens)}}$$

Omniward Chaining



Premise: $Q \rightarrow R$, Lemma: $P \rightarrow R$

Omniward Chaining

Premises

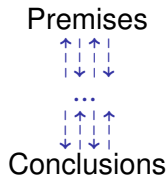
↑ ↑ ↓
↓ ↓ ↓
...
↓ ↑ ↑
↓ ↓ ↓

Conclusions

Premise: $Q \rightarrow R$, Lemma: $P \rightarrow R$

$$\frac{\frac{}{P \rightarrow R}}{Q \rightarrow R} (\text{QR})$$

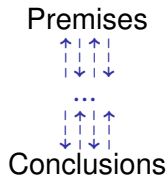
Omniward Chaining



Premise: $Q \rightarrow R$, Lemma: $P \rightarrow R$

$$\frac{\frac{}{P \rightarrow Q} \text{ (PQ)} \quad \frac{}{Q \rightarrow R} \text{ (QR)}}{P \rightarrow R} \text{ (Deduction)}$$

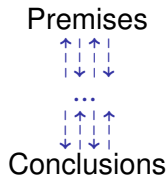
Omniward Chaining



Premise: $Q \rightarrow R$, Lemma: $P \rightarrow R$

$$\frac{\frac{\frac{}{P \rightarrow Q} \text{ (PQ)}}{P \rightarrow R} \quad \frac{\frac{}{Q \rightarrow R} \text{ (QR)}}{Q \rightarrow R} \text{ (Deduction)}}{R} \text{ (Modus Ponens)}$$

Omniward Chaining



Premise: $Q \rightarrow R$, Lemma: $P \rightarrow R$

$$\begin{array}{c}
 \frac{\overline{P} \text{ (P)}}{\quad} \quad \frac{\overline{P \rightarrow Q} \text{ (PQ)}}{\quad} \quad \frac{\overline{Q \rightarrow R} \text{ (QR)}}{\quad} \\
 \hline
 \frac{\quad}{R} \text{ (Modus Ponens)}
 \end{array}$$

(Deduction)