Towards porting PLN to MeTTa

Nil Geisweiller, Hedra Yusuf

AGI-22 Workshop

PLN Recall

$P, Q, \ldots : \mathit{Atom}^n \rightarrow \{\top, \bot\}$ (possibly fuzzy)

Atomese	MeTTa	Math
$(P\ \mathit{tv})$	$(\stackrel{\mathrm{m}}{=}\ P\ \mathit{tv})$	$\mathcal{P}r\,(\mathcal{S}(P)) \approx \mathit{tv.mean}$
$(\mathit{Not}\ \mathit{tv}\ P)$	$(\stackrel{\mathrm{m}}{=}\ (\neg\ P)\ \mathit{tv})$	$\mathcal{P}r\left(\overline{\mathcal{S}(P)}\right) \approx \mathit{tv.mean}$
$(\mathit{Or}\ \mathit{tv}\ P\ Q)$	$(\stackrel{\mathrm{m}}{=}\ (\vee\ P\ Q)\ \mathit{tv})$	$\mathcal{P}r\,(\mathcal{S}(P) \cup \mathcal{S}(Q)) \approx \mathit{tv.mean}$
$(\mathit{And}\ \mathit{tv}\ P\ Q)$	$(\stackrel{\mathrm{m}}{=}\ (\wedge\ P\ Q)\ \mathit{tv})$	$\mathcal{P}r\,(\mathcal{S}(P) \cap \mathcal{S}(Q)) \approx \mathit{tv.mean}$
$(\mathit{Implication}\ \mathit{tv}\ P\ Q)$	$(\stackrel{\mathrm{m}}{=}\ (P \rightarrow Q)\ \mathit{tv})$	$\mathcal{P}r\,(\mathcal{S}(Q) \vert \mathcal{S}(P)) \approx \mathit{tv.mean}$
$(\mathit{Evaluation}\ \mathit{tv}\ P\ (\mathit{List}\ X_1\ \ldots\ X_n))$	$(\stackrel{\mathrm{m}}{=}\ (P\ X_1\ \ldots\ X_n)\ \mathit{tv})$	$\mathcal{P}r\,(P(X_1, \ldots, X_n) = \top) \approx \mathit{tv.mean}$

PLN rules: Deduction

$$\frac{P \rightarrow Q \stackrel{\text{m}}{=} tv_{PQ} \qquad Q \rightarrow R \stackrel{\text{m}}{=} tv_{QR} \qquad P \stackrel{\text{m}}{=} tv_P \qquad Q \stackrel{\text{m}}{=} tv_Q \qquad R \stackrel{\text{m}}{=} tv_R}{P \rightarrow R \stackrel{\text{m}}{=} tv} \text{ (DED)}$$

$$tv.mean = tv_{PQ}.mean \times tv_{QR}.mean + \frac{(1 - tv_{PQ}.mean) \times (tv_R.mean - tv_Q.mean \times tv_{QR}.mean)}{1 - tv_Q.mean}$$

PLN rules: Implication Direct Introduction

$$\frac{(P\ a_1) \stackrel{\mathrm{m}}{=} tv_{Pa_1} \qquad (Q\ a_1) \stackrel{\mathrm{m}}{=} tv_{Qa_1} \qquad \ldots \qquad (P\ a_n) \stackrel{\mathrm{m}}{=} tv_{Pa_n} \qquad (Q\ a_n) \stackrel{\mathrm{m}}{=} tv_{Qa_n}}{P \rightarrow Q \stackrel{\mathrm{m}}{=} tv} \text{(IDI)}$$

$$tv.mean = \frac{\sum_x tv_{Px}.mean \times tv_{Qx}.mean}{\sum_x tv_{Px}.mean}$$

Evidential Truth Value

```
;; TruthValue type and constructors
(: TruthValue Type)
(: Bl (-> Bool TruthValue))
(: Pr (-> Number TruthValue))
(: PrCnt (-> Number Number TruthValue))

;; TruthValue methods
(: mode (-> TruthValue Number))
(: mean (-> TruthValue Number))
(: pos_count (-> TruthValue Number))
(: neg_count (-> TruthValue Number))

;; EvidentialTruthValue type and constructors
(: EvidentialTruthValue Type)
(: ETV (-> (Set $a) TruthValue EvidentialTruthValue)
```

Atomese	MeTTa	Math
(stv *s* (count->confidence *c*))	(PrCnt *s* *c*)	$<s\ c>$
-	(ETV *E* *tv*)	(E, tv)

PLN rules: Implication Direct Introduction (Recursive Decomposition)

- Base

$$\frac{}{P \rightarrow Q \stackrel{\mathrm{m}}{=} (\varnothing, <1\ 0>)} \text{ (IDI Axiom)}$$

- Recursion

$$\frac{(P\ a) \stackrel{\mathrm{m}}{=} tv_{Pa} \qquad (Q\ a) \stackrel{\mathrm{m}}{=} tv_{Qa} \qquad P \rightarrow Q \stackrel{\mathrm{m}}{=} (E, tv_{PQ}) \qquad a \notin E}{P \rightarrow Q \stackrel{\mathrm{m}}{=} (\{a\} \cup E, tv)} \text{ (IDI Induction)}$$

$$tv.count = tv_{PQ}.count + tv_{Pa}.mean$$

$$tv.mean = \frac{tv_{PQ}.pos\_count + tv_{Pa}.mean \times tv_{Qa}.mean}{tv.count}$$

PLN rules: Implication Direct Introduction Example

$$
\frac{(P\ b) \stackrel{m}{=} \top \qquad (Q\ b) \stackrel{m}{=} \bot \qquad \dfrac{(P\ a) \stackrel{m}{=} \top \qquad (Q\ a) \stackrel{m}{=} \top \qquad \dfrac{}{P \rightarrow Q \stackrel{m}{=} (\varnothing, <1\ 0>)}\,(\mathrm{IA}) \qquad \dfrac{}{a \notin \varnothing}\,(\mathrm{NE})}{P \rightarrow Q \stackrel{m}{=} (\{a\}, <1\ 1>)}\,(\mathrm{II}) \qquad \dfrac{}{b \notin \{a\}}\,(\mathrm{NS})}{P \rightarrow Q \stackrel{m}{=} (\{a, b\}, <0.5\ 2>)}\,(\mathrm{II})
$$

II : *Implication direct introduction Induction*

IA : *Implication direct introduction Axiom*

NE : *Nothing in Empty set*

NS : *element Not in differing Singleton*

Temporal Deduction $\mapsto$ Deduction

$$
\dfrac{
\dfrac{\dfrac{P \rightsquigarrow^{T_1} Q}{P \rightarrow \overleftarrow{Q}^{T_1}}\,\text{(PI2I)}
\qquad
\dfrac{\dfrac{Q \rightsquigarrow^{T_2} R}{Q \rightarrow \overleftarrow{R}^{T_2}}\,\text{(PI2I)}}{\overleftarrow{Q}^{T_1} \rightarrow \overleftarrow{R}^{T_1+T_2}}\,\text{(TS)}
\qquad P \qquad
\dfrac{Q}{\overleftarrow{Q}^{T_1}}\,\text{(TS)}
\qquad
\dfrac{R}{\overleftarrow{R}^{T_1+T_2}}\,\text{(TS)}}
{P \rightarrow \overleftarrow{R}^{T_1+T_2}}\,\text{(DED)}
}{P \rightsquigarrow^{T_1+T_2} R}\,\text{(I2PI)}
$$

DED : *Deduction*

TS : *Temporal Shift*

PI2I : *PredictiveImplication to Implication*

I2PI : *Implication to PredictiveImplication*

Temporal Procedural Deduction $\mapsto$ Deduction

$$
\dfrac{
\dfrac{
\dfrac{P \wedge \widehat{A} \rightsquigarrow^{T_1} Q}{P \wedge \widehat{A} \rightarrow \overleftarrow{Q}^{T_1}}\,\text{(PI2I)}
\qquad
\dfrac{\widehat{B}}{\overleftarrow{\widehat{B}}^{T_1}}\,\text{(TS)}
}{P \wedge \widehat{A} \wedge \overleftarrow{\widehat{B}}^{T_1} \rightarrow \overleftarrow{Q}^{T_1} \wedge \overleftarrow{\widehat{B}}^{T_1}}\,\text{(CI)}
\qquad
\dfrac{\dfrac{Q \wedge \widehat{B} \rightsquigarrow^{T_2} R}{Q \wedge \widehat{B} \rightarrow \overleftarrow{R}^{T_2}}\,\text{(PI2I)}}{\overleftarrow{Q}^{T_1} \wedge \overleftarrow{\widehat{B}}^{T_1} \rightarrow \overleftarrow{R}^{T_1+T_2}}\,\text{(TS)}
\qquad
P \wedge \widehat{A} \wedge \overleftarrow{\widehat{B}}^{T_1}
\qquad
\dfrac{Q \wedge \widehat{B}}{\overleftarrow{Q}^{T_1} \wedge \overleftarrow{\widehat{B}}^{T_1}}\,\text{(TS)}
\qquad
\dfrac{R}{\overleftarrow{R}^{T_1+T_2}}\,\text{(TS)}
}{
\dfrac{P \wedge \widehat{A} \wedge \overleftarrow{\widehat{B}}^{T_1} \rightarrow \overleftarrow{R}^{T_1+T_2}}{\left((P \wedge \widehat{A}) \bigslopedwedge^{T_1} \widehat{B}\right) \rightsquigarrow^{T_1+T_2} R}\,\text{(I2PI)}
}\,\text{(DED)}
$$

CI : *Conjunction Introduction*

TS : *Temporal Shift*

DED : *Deduction*

PI2I : *PredictiveImplication to Implication*

I2PI : *Implication to PredictiveImplication*

Conclusion

Demo Time