Probabilistic Logic Networks for Temporal and Procedural Reasoning

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- Deduction (D)

$$\frac{P \rightarrow Q \stackrel{m}{=} TV^{PQ} \qquad Q \rightarrow R \stackrel{m}{=} TV^{QR} \qquad P \stackrel{m}{=} TV^P \qquad Q \stackrel{m}{=} TV^Q \qquad R \stackrel{m}{=} TV^R}{P \rightarrow R \stackrel{m}{=} \phi_D\left(TV^{PQ}, \ldots, TV^R\right)} \text{(D)}$$

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Temporal Inference Rules:

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- $P \leadsto^T Q \stackrel{\text{def}}{=} P \rightarrow \overleftarrow{Q}^T$

Temporal Inference Rules:

- Temporal Deduction (TD):

$$\frac{P \leadsto^{T_1} Q \stackrel{\text{m}}{=} TV^{PQ} \qquad Q \leadsto^{T_2} R \stackrel{\text{m}}{=} TV^{QR} \qquad P \stackrel{\text{m}}{=} TV^{P} \qquad Q \stackrel{\text{m}}{=} TV^{Q} \qquad R \stackrel{\text{m}}{=} TV^{R}}{P \leadsto^{T_1+T_2} R \stackrel{\text{m}}{=} \phi_{TD}\left(TV^{PQ}, \ldots, TV^{R}\right)} \text{(TD)}$$

$$\dfrac{P \rightsquigarrow^{T_1} Q \stackrel{\text{m}}{=} TV^{PQ} \qquad Q \rightsquigarrow^{T_2} R \stackrel{\text{m}}{=} TV^{QR} \qquad P \stackrel{\text{m}}{=} TV^{P} \qquad Q \stackrel{\text{m}}{=} TV^{Q} \qquad R \stackrel{\text{m}}{=} TV^{R}}{P \rightsquigarrow^{T_1+T_2} R \stackrel{\text{m}}{=} \phi_{TD}\left(TV^{PQ}, \ldots, TV^{R}\right)}\ \text{(TD)}$$

$$\equiv$$

$$\dfrac{\dfrac{\dfrac{P \rightsquigarrow^{T_1} Q \stackrel{\text{m}}{=} TV^{PQ}}{P \to \overleftarrow{Q}^{T_1} \stackrel{\text{m}}{=} TV^{PQ}}\ \text{(PI)} \qquad \dfrac{\dfrac{Q \rightsquigarrow^{T_2} R \stackrel{\text{m}}{=} TV^{QR}}{Q \to \overleftarrow{R}^{T_2} \stackrel{\text{m}}{=} TV^{QR}}\ \text{(PI)}}{\overleftarrow{Q}^{T_1} \to \overleftarrow{R}^{T_1+T_2} \stackrel{\text{m}}{=} TV^{QR}}\ \text{(S)} \qquad P \stackrel{\text{m}}{=} TV^{P} \qquad \dfrac{Q \stackrel{\text{m}}{=} TV^{Q}}{\overleftarrow{Q}^{T_1} \stackrel{\text{m}}{=} TV^{Q}}\ \text{(S)} \qquad \dfrac{R \stackrel{\text{m}}{=} TV^{R}}{\overleftarrow{R}^{T_1+T_2} \stackrel{\text{m}}{=} TV^{R}}\ \text{(S)}}{\dfrac{P \to \overleftarrow{R}^{T_1+T_2} \stackrel{\text{m}}{=} \phi_{D}\left(TV^{PQ}, \ldots, TV^{R}\right)}{P \rightsquigarrow^{T_1+T_2} R \stackrel{\text{m}}{=} \phi_{D}\left(TV^{PQ}, \ldots, TV^{R}\right)}\ \text{(IP)}}\ \text{(D)}$$

$$\Downarrow$$

$$\phi_{TD} = \phi_{D}$$

Procedural Reasoning

Procedural Construct:

- Cognitive Schematic

$$\mathit{Context} \wedge \mathit{Action} \leadsto^{T} \mathit{Goal} \stackrel{\mathrm{m}}{=} \mathit{TV}$$

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Procedural Inference Rule:

- Procedural Deduction (PD)

$$\frac{C_1 \wedge A_1 \rightsquigarrow^{T_1} C_2 \stackrel{\mathrm{m}}{=} \mathit{TV}^{12} \qquad C_2 \wedge A_2 \rightsquigarrow^{T_2} C_3 \stackrel{\mathrm{m}}{=} \mathit{TV}^{23} \qquad \ldots}{C_1 \wedge A_1 \nearrow^{T_1} A_2 \rightsquigarrow^{T_1+T_2} C_3 \stackrel{\mathrm{m}}{=} \phi_{PD}\left(\mathit{TV}^{12}, \mathit{TV}^{23}, \ldots\right)}$$

$$\dfrac{C_1 \wedge A_1 \rightsquigarrow^{T_1} C_2 \stackrel{m}{=} TV^{12} \quad C_2 \wedge A_2 \rightsquigarrow^{T_2} C_3 \stackrel{m}{=} TV^{23} \quad C_1 \wedge A_1 \wedge \overleftarrow{A_2}^{T_1} \stackrel{m}{=} TV^1 \quad C_2 \wedge A_2 \stackrel{m}{=} TV^2 \quad C_3 \stackrel{m}{=} TV^3}{C_1 \wedge A_1 \measuredangle^{T_1} A_2 \rightsquigarrow^{T_1+T_2} C_3 \stackrel{m}{=} \phi_{PD}(TV^{12}, TV^{23}, TV^1, TV^2, TV^3)}\text{(PD)}$$

$$\equiv$$

$$\dfrac{\dfrac{\dfrac{\dfrac{C_1 \wedge A_1 \rightsquigarrow^{T_1} C_2 \stackrel{m}{=} TV^{12}}{C_1 \wedge A_1 \rightarrow^{T_1} \overleftarrow{C_2}^{T_1}}\text{(PI)}}{C_1 \wedge A_1 \wedge \overleftarrow{A_2}^{T_1} \rightarrow \overleftarrow{C_2}^{T_1} \wedge \overleftarrow{A_2}^{T_1}}\text{(I)} \quad \dfrac{\dfrac{C_2 \wedge A_2 \rightsquigarrow^{T_2} C_3 \stackrel{m}{=} TV^{23}}{C_2 \wedge A_2 \rightarrow \overleftarrow{C_3}^{T_2}}\text{(PI)}}{\overleftarrow{C_2}^{T_1} \wedge \overleftarrow{A_2}^{T_1} \rightarrow \overleftarrow{C_3}^{T_1+T_2}}\text{(S)} \quad C_1 \wedge A_1 \wedge \overleftarrow{A_2}^{T_1} \stackrel{m}{=} TV^1 \quad \dfrac{C_2 \wedge A_2 \stackrel{m}{=} TV^2}{\overleftarrow{C_2}^{T_1} \wedge \overleftarrow{A_2}^{T_1}}\text{(S)} \quad \dfrac{C_3 \stackrel{m}{=} TV^3}{\overleftarrow{C_3}^{T_1+T_2}}\text{(S)}}{C_1 \wedge A_1 \wedge \overleftarrow{A_2}^{T_1} \rightarrow \overleftarrow{C_3}^{T_1+T_2}}\text{(D)}}{C_1 \wedge A_1 \measuredangle^{T_1} A_2 \rightsquigarrow^{T_1+T_2} C_3 \stackrel{m}{=} \phi_{PD}\left(TV^2, TV^{23}, TV^1, TV^2, TV^3\right)}\text{(IP)}$$

$$\Downarrow$$

$$\phi_{PD} = \phi_D$$