Introduction to Robotics (CS223A) Homework #1

(Winter 2019/2020) Due: **Thursday, January 16, 11:59pm**

Name __________________________ **SUNet ID** __________________________

Instructions:

- **The assignments should be submitted via Gradescope.**
- Setup instructions for programming portions are detailed in the README file included with the code.
- Use abbreviations for trigonometric functions (e.g. $c\theta$ for $\cos(\theta)$, s_1 or $s\theta_1$ for $\sin(\theta_1)$) in situations where it would be tedious to repeatedly write sin,cos, etc.
- Unless instructed otherwise, leave square roots in symbolic form rather than writing out their decimal values.
- If you give a vector as an answer, make sure that you specify what frame it is given in (if it is not clear from context). The same rule applies to rotation and transformation matrices.

1. The vector ${}^{A}\mathbf{P}$ is first rotated about $\hat{Y}_A$ by θ degrees and then subsequently rotated about $\hat{Z}_A$ by ϕ degrees.

   (a) Give the 3×3 rotation matrix that accomplishes these rotations in the given order. You may leave your answer as a product of matrices.

   (b) What is the rotated vector if ${}^{A}\mathbf{P} = [2, 3, 1]^T$, $\theta = 120°$, and $\phi = 45°$? Be sure to specify the frame of your vector representation.

      i. Implement the appropriate functions in *rotations.py* (where it says *HW1 Q1b: Rotation operators*). This part of the homework will be graded with Gradescope Autograder.

      ii. Use the above functions to compute the answer and give your answer on your solution paper.

2. A frame {B} is initially coincident with a frame {A}. First, we rotate {B} about $\hat{X}_B$ by θ degrees. Next, we rotate the resulting frame {B} about $\hat{Y}_B$ by ϕ. Finally, we rotate the resulting frame {B} about $\hat{Z}_B$ by θ again.

   (a) Determine the 3×3 rotation matrix, ${}^{A}_{B}R$, that will change the description of a vector P in frame {B}, ${}^{B}\mathbf{P}$, to frame {A}, ${}^{A}\mathbf{P}$. You may leave your answer as a product of matrices.

   (b) What is the value of ${}^{A}_{B}R$ if $\theta = 30°$, $\phi = 45°$? Implement the appropriate functions in *rotations.py* (where it says *HW1 Q2b: Euler and Fixed angles*) and use them to compute the answer. Handle representation singularities explicitly in your code. Similar to Q1, please give your answer on your solution paper.

   (c) Compute the rotation matrix if $\phi = 90°$? Can you achieve the same final {B} using a different set of Euler Angles? Explain why.

3. Given the following transformation matrices:

$$T1 = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & -1 \\ 0 & 1 & 0 & 2 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, T2 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 1 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T3 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Are T1, T2, and T3 valid transformation matrices? Explain why or why not, and if there are multiple reasons why a matrix is invalid, include each. (We define a transformation matrix as a rotation and a translation, i.e. a "rigid body" transformation)

(b) Implement function *mat_to_quat* in *rotations.py* and find the Euler parameters that represent the rotations for the correct matrix (or matrices).

(c) Also find the unit vector that defines the axis of rotation, and the angle of rotation for the correct matrix (or matrices).

4. (a) Prove that 2D rotations commute. (You can show this either analytically or graphically)

   (b) Prove that 3D rotations do not necessarily commute. (You can show this either analytically or graphically)

(c) **[Extra Credit]** Prove that 3D infinitesimal rotations commute (clearly state the assumptions you make and their consequences).