The second order differential equation to solve is the following

$$\ddot{x} = \frac{1}{\rho r_0 R_{max}} \left[(P_i(t) - P_o(t)) \cdot r_0 - M(x) - D\frac{h_0}{r_0}\dot{x} \right] ,$$

where

$$M(x) = E\frac{h_0}{r_0}x \ , x > x^*$$

$$M(x) = 4\frac{E}{\pi^2} \cdot \frac{h_0^2}{r_0^2} \cdot \frac{h_0}{h_0 + r_0 + x}x \ , x \leq x^*$$

Let's say

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix} \tag{1}$$

Then the derivative of y would be

$$\dot{y} = \begin{pmatrix} \dot{y_1} \\ \dot{y_2} \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} \tag{2}$$

So we can conclude the equation below.

$$\dot{y} = \begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} y_2 \\ \frac{1}{\rho r_0 R_{max}} \left[(P_i(t) - P_o(t)) \cdot r_0 - M(y_1) \cdot y_1 - D\frac{h_0}{r_0}y_2 \right] \end{pmatrix} \tag{3}$$

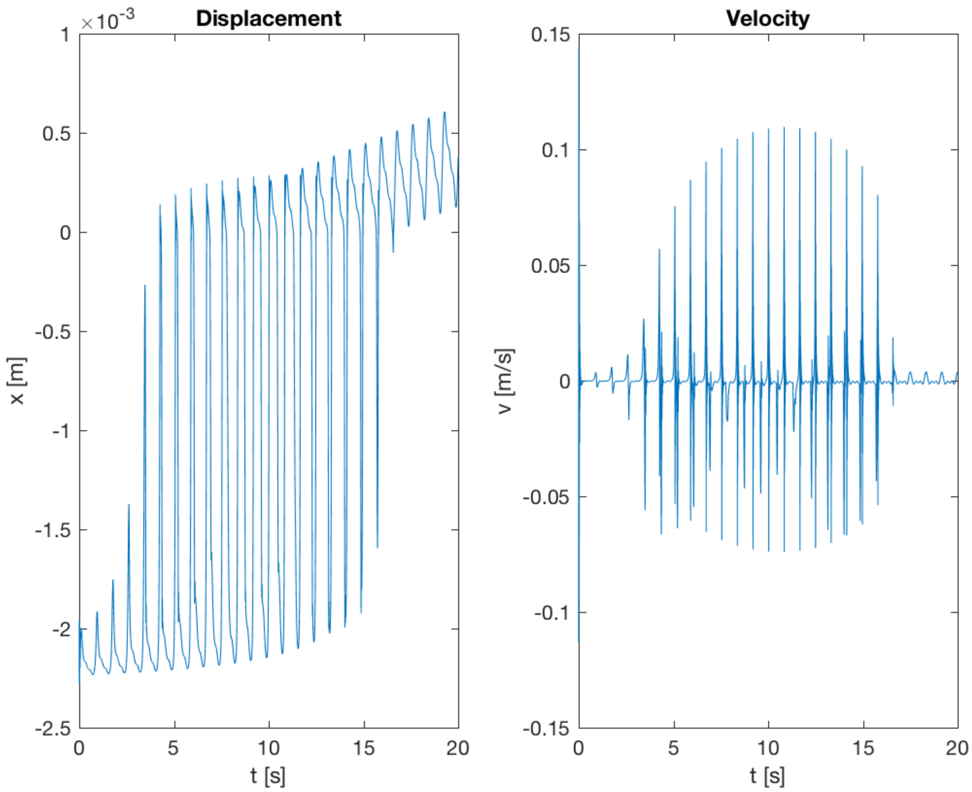

Figure 1: Displacement and velocity for the first 20 seconds with Explicit Euler ($h = 0.0001$)

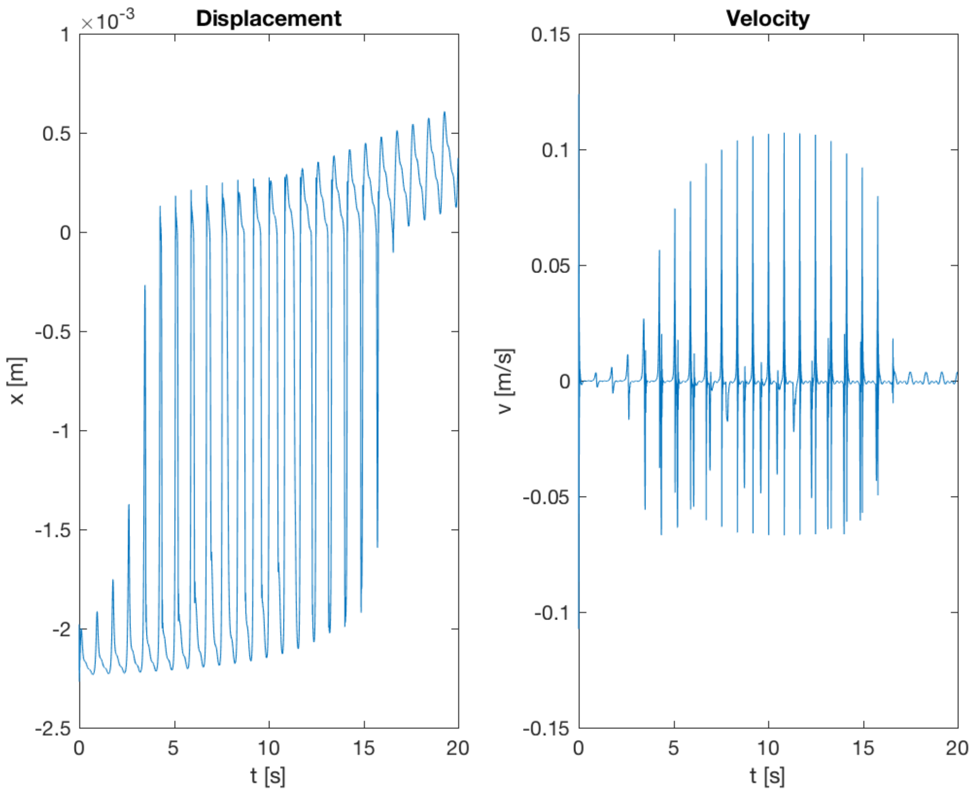

Figure 2: Displacement and velocity for the first 20 seconds with Runge Kutta 4 ($h = 0.001$)