Task #1

Let's say $f(t) = \sum_{i=1}^{3} y_i(t)$ if $t \geq 0$! Then

$$f(t) = y_1(t) + y_2(t) + y_3(t)\,, \tag{1}$$

and

$$\dot{f}(t) = \dot{y_1}(t) + \dot{y_2}(t) + \dot{y_3}(t)\,. \tag{2}$$

The time derivatives are know from the given system. Let's substitute them back!

$$\dot{f}(t) = -\alpha y_1 + \beta y_2 y_3 + \alpha y_1 - \beta y_2 y_3 \gamma - y_2^2 + y_2^2 \tag{3}$$

From equation 3 it is clear that $\dot{f}(t) \equiv 0$, consequently $f(t)$ is a constant function. We know from the inital conditions that

$$f(0) = y_1(0) + y_2(0) + y_3(0) = 1 + 0 + 0 = 1 \tag{4}$$

Since f is a constant function, $f(t) = f(0) = 1$.

We can find the steady state if we solve

$$\dot{\boldsymbol{y}} = \boldsymbol{0}\,, \tag{5}$$

which means these three equations

$$\begin{aligned} -\alpha y_1 + \beta y_2 y_3 &= 0 \\ \alpha y_1 - \beta y_2 y_3 - \gamma y_2^2 &= 0\,. \\ \gamma y_2^2 &= 0 \end{aligned} \tag{6}$$

From the third equation we obtain $y_2 = 0$. If $y_2 = 0$ then $y_1 = 0$ according to the first equation. y_3 could be anything, however we know that $\sum_{i=1}^{3} y_i(t) = 1$. Since $y_1 = y_2 = 0$, y_3 must equal to 1. So the steady state is

$$\boldsymbol{y_s} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Task #2

Task #3