Math463: Bonus Assignment 1

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Part 1: Understanding Conditional Probability

Problem Statement A six-sided die is tossed. Let A be the event that the number rolled is even and let B be the event that the number rolled is greater than 3. Determine $P(A),\ P(B)$ and $P(A\cap B)$. Use R to compute $P(A|B)$ using the formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$.

The set of outcomes is $S = \{1, 2, 3, 4, 5, 6\}$. The set constituting event A is defined by the outcomes $A = \{2, 4, 6\}$ while the set constituting event B is defined by the outcomes $B = \{4, 5, 6\}$. The intersection between events A and B are defined by $A \cap B = \{4, 6\}$. The probabilities for each of these events are provided below.

$$P(A) = \frac{|A|}{|S|} = \frac{1}{2} \qquad P(B) = \frac{|B|}{|S|} = \frac{1}{2} \qquad P(A\cap B) = \frac{|A\cap B|}{|S|} = \frac{1}{3}$$

Based on the program in R, $P(A|B) = \frac{2}{3}$.

Part 2: Simulation of Traditional Dice Tossing

Problem Statement Write R Code to simulate 10,000 dice rolls of a 6 sided die. Experimentally determine $P(A), P(B),$ and $P(A|B)$ and compare with the theoretical values.

Experimentally $P(A) = 0.4982,\ P(B) = 0.5087\ P(A|B) = 0.6605$. Theoretically, $P(A) = 0.5,\ P(B) = 0.5\ P(A|B) = 0.6666$. Comparing the two results, the simulation appears to be reasonably accurate, and the discrepancies between the two are in the range of experimental error.

Part 3: Non-Traditional Dice Tossing

Problem Statement Assume a six sided die where the probabilities of rolling each face are non-uniform: $P(1) = 0.1, P(2) = 0.1, P(3) = 0.2, P(4) = 0.2, P(5) = 0.2, P(6) = 0.2$. Simulate 10,000 rolls of this weighted die in R. Let C be the event that then number rolled is a multiple of 3 and let D be the event that the number rolled is greater than or equal to 4. Determine $P(C), P(D), P(C\cap D), P(C|D)$ based on the simulation.

The results of the simulation indicate the following probabilities:

$$P(C) = 0.4031 \qquad P(D) = 0.5913 \qquad P(C\cap D) = 0.2012 \qquad P(C|D) = 0.3403$$

Part 4 Interpretation and Visualization

Expected Results To begin, the event C is defined by the set $\{3, 6\}$. Thus, the expected probability is determined as follows: $P(C) = P(3) + P(6) = 0.2 + 0.2 = 0.4$. The event D is defined by the set $\{4, 5, 6\}$. Thus, the expected probability is determined as follows: $P(D) = P(4) + P(5) + P(6) = 0.2 + 0.2 + 0.2 = 0.6$. The event $C \cap D$ is defined by the set $\{6\}$. Thus, the expected probability is determined as follows: $P(C \cap D) = P(6) = 0.2$.

Simulation Results Analysis The bargraphs in **figures 1** and **2** below provided the expected results and the simulation results for the weighted dice rolls across 10,000 simulated rolls. Looking closely at these results, the total error in the simulation does not deviate significantly from the expected result for $P(1), P(2), P(3), P(4)$, and $P(6)$. However, the error for $P(5)$ is beyond 1% which suggests that the random seed value used for the simulation resulted in a lower amount of 5's selected than what was expected. Overall $P(5)$ is off by roughly 1% from the expected value.

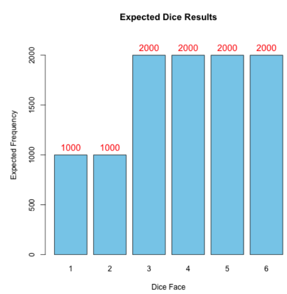

Figure 1: Expected Results

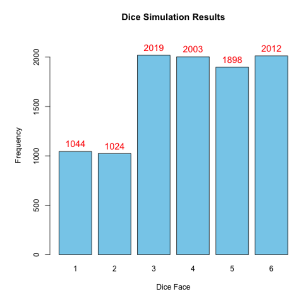

Figure 2: Simulation Results

Table 1 below summarizes the event probabilities for C and D. For each of the probabilities observed, the error is less than 1%, thus the discrepancies can be attributed to random experimental error from the sampling.

	Expected	Simulation	Error
$P(C)$	0.4	0.4031	0.0031
$P(D)$	0.6	0.5931	0.0069
$P(C \cap D)$	0.2	0.2012	0.0012
$P(C\|D)$	0.3333	0.3403	0.0070

Table 1: Comparison of Expected and Simulation Results