

# EEG Signal Processing for Feature Extraction in Mental Arithmetic Tasks

Micol Malosso (10617863)

Dep. Biomedical Engineering  
Politecnico di Milano

Pascal Hansen (10986916)

Dep. Biomedical Engineering  
Politecnico di Milano

Nastaran Ghaffari Elkhechi (10900702)

Dep. Biomedical Engineering  
Politecnico di Milano

**Abstract-** This study investigates electroencephalogram (EEG) recordings from 6 healthy subjects, aiming to analyze differences between each subject's resting state and calculating state within  $\theta_1$ ,  $\theta_2$ ,  $\alpha$ ,  $\beta_1$ , and  $\beta_2$  frequency bands. We compare them using Power Spectral Density (PSD), which shows the signal power distribution across frequencies, and by Coherence, measuring brain region connectivity [1].

## I. INTRODUCTION

The analysis of EEG during mental arithmetic tasks is a common theme in literature, providing insights into the complexities of human cognitive processes. Our experiments specifically focus on studying EEG activity during stress evoking cognitive tasks, i.e., mentally subtracting two orally presented numbers. The analysis of Power Spectral Density (PSD) in EEG studies plays a crucial role as it offers a detailed view of power distribution in brain oscillations across different frequency bands. Variations in PSD between the resting state and mental calculation state can reveal cognitive load and other neural dynamics during such tasks, enhancing our understanding of brain activity. Additionally, we explore Coherence, a measure of functional connectivity, to interpret patterns of information exchange between brain regions during the mental arithmetic tasks.

## II. MATERIALS AND METHODS

### A. Data Collection and Preprocessing

EEG recordings from 6 participants were obtained using a Neurocom EEG 19-channel system [1,2], following the International 10-20 standard. Electrodes were placed on defined scalp regions, and the signals were referenced to interconnected ear electrodes. Impedance levels were maintained below 5 k $\Omega$ . The recordings had a sampling rate of 500 Hz. For preprocessing, a 45 Hz low-pass filter, a 0.5 Hz high-pass filter, and a 50 Hz notch filter for power line noise were applied. The experiment involved a 180-second rest condition and a 60-second mental arithmetic task (serial subtraction) [1,2].

### B. EEG Frequency Bands

To accurately capture alterations in EEG patterns associated with cognitive activation, the analysis focused on specific frequency sub-bands, namely:

$\theta_1$  (4.1 to 5.8) Hz,  $\theta_2$  (5.9 to 7.4) Hz,  $\alpha$  (7.5 to 12.9) Hz,  $\beta_1$  (13 to 19.9) Hz,  $\beta_2$  (20 to 25) Hz.

### C. Feature Extraction

#### Power Spectral Density

PSD is defined as the frequency spectrum (DFT) of the autocorrelation function of our signal  $x(n)$  and can be directly computed by the magnitude squared from our signal:

$$S_{xx}(f) = \frac{1}{N} \left| \sum_{n=0}^{N-1} x(n) e^{-j2\pi f n} \right|^2 = \frac{1}{N} |X(f)|^2$$

We estimated the PSD using the Welch periodogram method, employing 2-second windows with a 50% overlap. While a wider window could have provided higher frequency resolution, the narrower interval has been chosen to reduce the estimation variance. Additionally, the Hann window was employed to reduce spurious components. We computed the mean for all channel and frequency band combinations over all frequencies  $\overline{S_{xx}}$  as feature for comparing both states.

#### Coherence

(Quadratic) Coherence is defined as the ratio between the cross-PSD of the signals of two channels and the product of the two channels' PSDs:

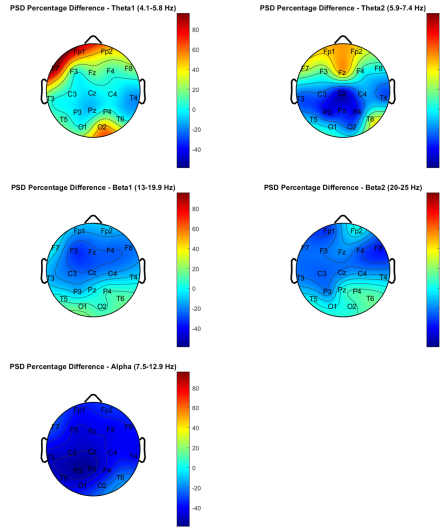
$$C_{xy}(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)}$$

The result is a number in the range of 0 and 1 for each frequency, where 0 means no synchronization and 1 complete synchronization between the two channels. The signals are divided into sub-signals and then averaged as otherwise each frequency would falsely be indicated as completely synchronized. Here, also the mean over all frequencies  $\overline{C_{xy}}$  is taken as feature, for all channel combinations and all frequency bands.

## III. RESULTS

To check for variations in PSD in resting and calculating state, we visualized the 'difference percentage' in form of topoplots (Figure 1), averaged over all 6 subjects. An increase in PSD within the  $\theta$  band is associated with heightened cognitive resource utilization, task complexity, working memory, and focused attention. Within the  $\theta_1$  and  $\theta_2$  bands, during mental arithmetic, a rise in PSD is observed at electrodes linked to the frontal, occipital, and temporal regions (F7, F8, FP2, FP1, O2, O1, and T6) due to their roles in encoding, concentration, and visual processing. Furthermore,

the  $\theta_1$  plot indicates a higher PSD in the left hemisphere, consistent with literature [2,3]. The alpha band is typically associated with the state of closed eyes, particularly when an individual is relaxed but remains alert. Here, there is a significant reduction in the PSD when switching to the calculating task, which is aligned with the observation that during task computation, subjects exhibit less relaxation and increased focus on tasks or visual stimuli [3]. The  $\beta$  band is associated with mental activation and attention. In  $\beta_1$  analysis, a higher PSD is seen in occipital regions (O1, O2), suggesting increased cognitive engagement in visual processing. The parietal area near electrode P4, involved in numerical cognition, exhibits an increase in PSD within the  $\beta_2$  band [3, 4]. Similar PSD values are observed in the T6 region, which is associated with working memory. This may be attributed to activities such as retaining a digit during mental computation. Meanwhile, PSD decreases in the  $\beta$  band in other electrodes may reflect neuronal selection to channel cognitive resources to crucial areas for the task, such as O1, O2, P4, and T6 [4].



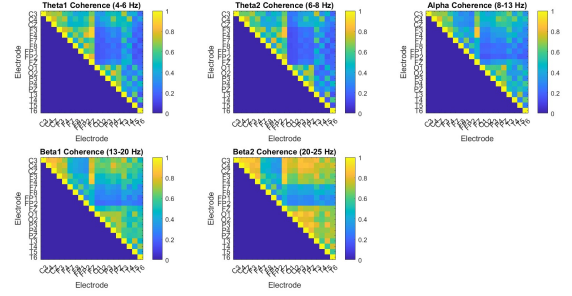
**Figure 1.** Percentage Difference between Calculation and Rest States through 5 Frequency Bands

In coherence analysis, coherence plots were created for all electrode combinations during rest and calculation states for various frequency bands, representing the degree of synchronized electrical activity between two electrodes.

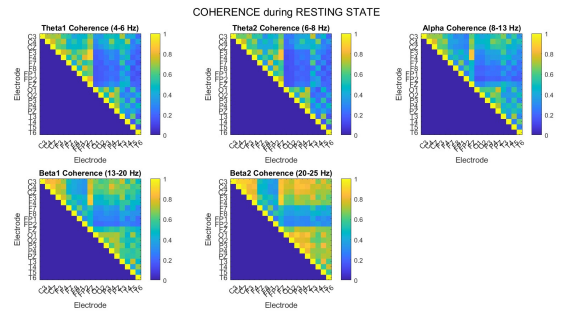
1. *Resting State:* The resting state exhibits strong synchronization between electrodes. In Figure 2, we can see that there is a strong synchronization between electrodes, particularly in the  $\beta_2$  band (with 0.8 coherence), demonstrating robust interregional communication during resting state.

2. *Calculating State:* In mental arithmetic tasks (Figure 3), regional specialization is evident. Different brain areas seem to focus more on their specific functions, resulting in reduced synchronization between them. For instance,

occipital and parietal areas might be primarily involved in visual and numerical processing, reducing coherence with other regions engaged in different functions, potentially improving efficiency in mental arithmetic calculations.



**Figure 2.** Coherence Plots during Rest State through 5 Frequency Bands



**Figure 3.** Coherence Plots during Calculation State through 5 Frequency Bands

#### IV. DISCUSSION

The study of changes in the Power Spectral Density (PSD) during mental arithmetic tasks has revealed how the brain's neural dynamics work. The observations highlighted a significant synergy in brain functions: specific regions show an increase in their activity in response to the cognitive load presented by the arithmetic task, while other regions demonstrate a reallocation of neural resources. This shows how complex and interconnected the brain's processes are during mental arithmetic. In the context of coherence analysis, it has been observed that reduced synchronization between electrodes during mental arithmetic may be indicative of a specialized distribution of the cognitive load among various brain regions. This specific load distribution appears essential to ensure efficient execution of complex neural activities such as arithmetic calculations. These insights can guide future research on brain functions and how different areas connect during thinking tasks.

#### V. REFERENCES

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