CSE 151 Machine Learning

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Ensemble Learning

How to combine multiple classifiers into a single one

Works well if the classifiers are complementary

This class: two types of ensemble methods:

- Bagging
- Boosting

Bagging (Bootstrap AGgregatING)

Input: n labelled training examples (x_i, y_i), $i = 1,..,n$

Algorithm:

Repeat k times:

Select m samples out of n **with replacement** to get training set S_i

Train classifier (decision tree, k-NN, perceptron, etc) h_i on S_i

Output: Classifiers $h_1, .., h_k$

Classification: On test example x, output majority$(h_1, .., h_k)$

Example

Input: n labelled training examples (x_i, y_i), $i = 1,..,n$

Algorithm:

Repeat k times:

Select m samples out of n **with replacement** to get training set S_i

Train classifier (decision tree, k-NN, perceptron, etc) h_i on S_i

How to pick m?

Popular choice: $m = n$

Still different from working with entire training set. Why?

Bagging

Input: n labelled training examples $S = \{(x_i, y_i)\}$, $i = 1,..,n$

Suppose we select n samples out of n **with replacement** to get training set S_i

Still different from working with entire training set. Why?

$$\Pr(S_i = S) = \frac{n!}{n^n} \quad \text{(tiny number, exponentially small in n)}$$

$$\Pr((x_i, y_i) \text{ not in } S_i) = \left(1 - \frac{1}{n}\right)^n \approxeq e^{-1}$$

For large data sets, about 37% of the data set is left out!

Bias and Variance

Classification error = Bias + Variance

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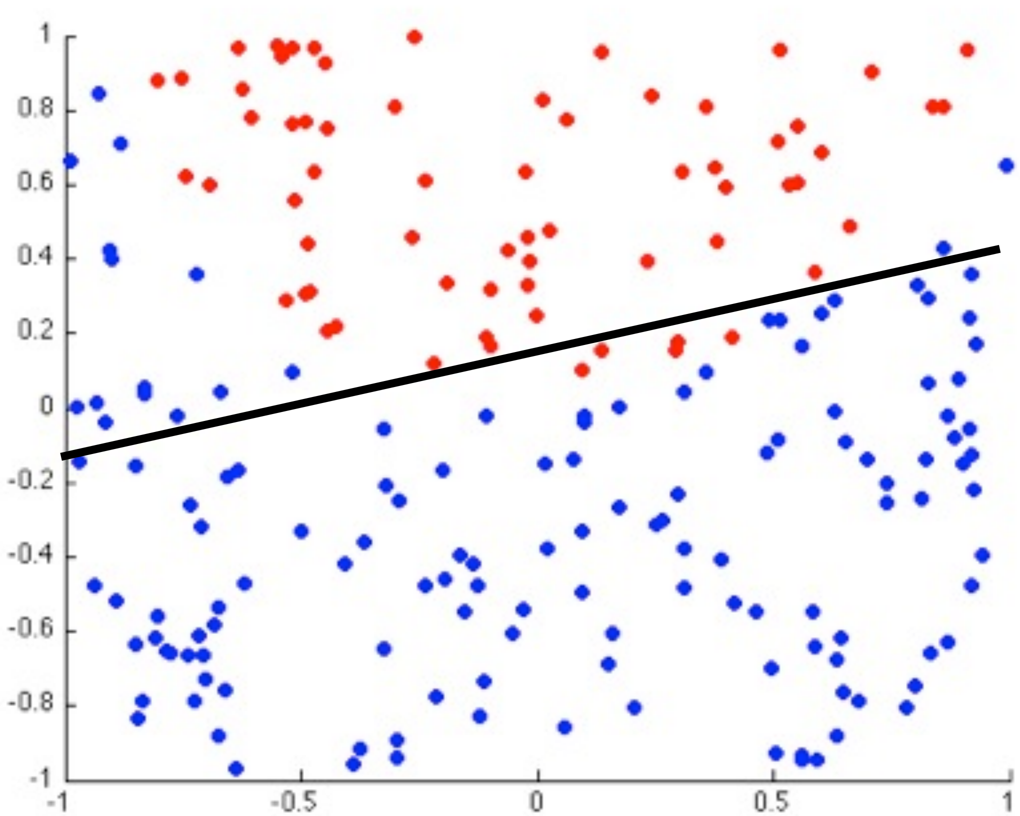
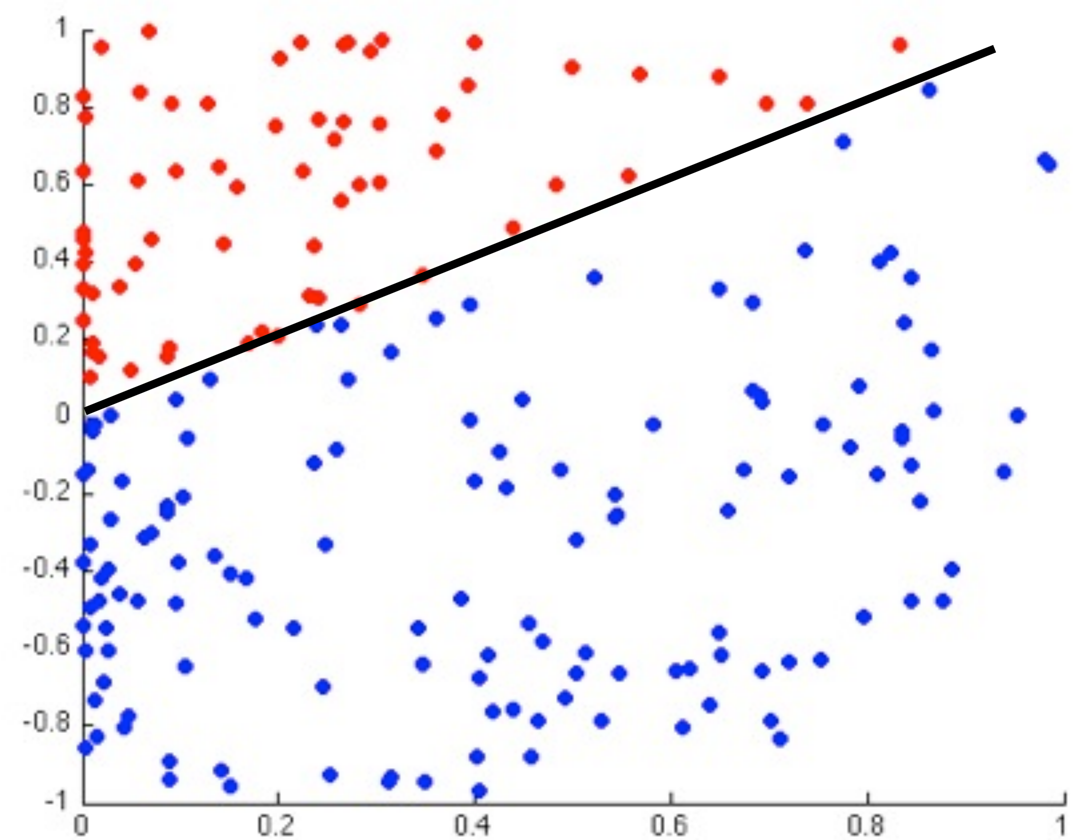
Bias is the true error of the best classifier in the concept class (e.g, best linear separator, best decision tree on a fixed number of nodes).

Bias is high if the concept class cannot model the true data distribution well, and does not depend on training set size.

Bias





High Bias **Low Bias**

Underfitting: when you have high bias

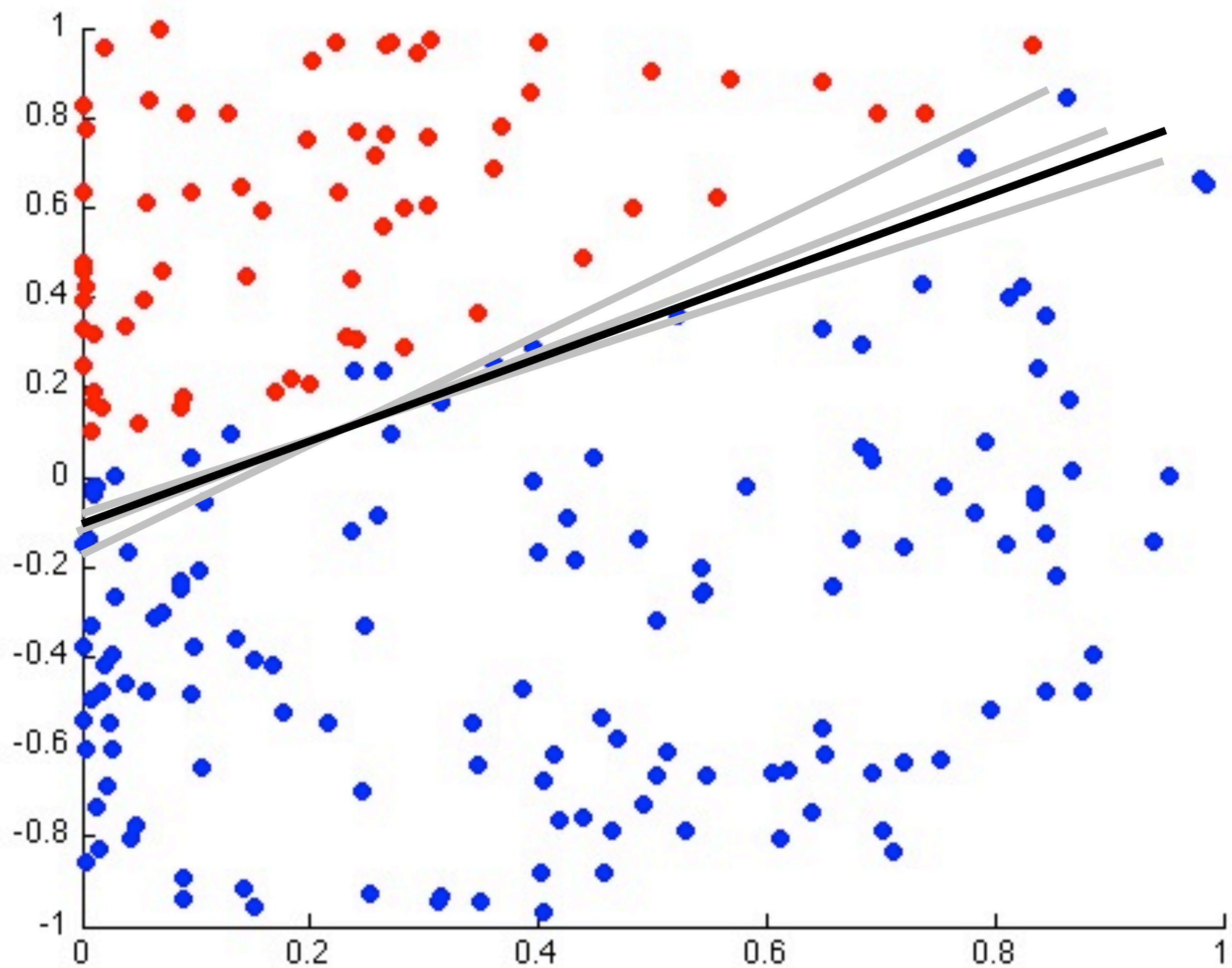
Bias and Variance

Classification error = Bias + Variance

Variance is the error of the trained classifier with respect to the best classifier in the concept class.

Variance depends on the training set size. It decreases with more training data, and increases with more complicated classifiers.

Variance



Overfitting: when you have extra high variance

Bias and Variance

Classification error = Bias + Variance

If you have high bias, both training and test error will be high

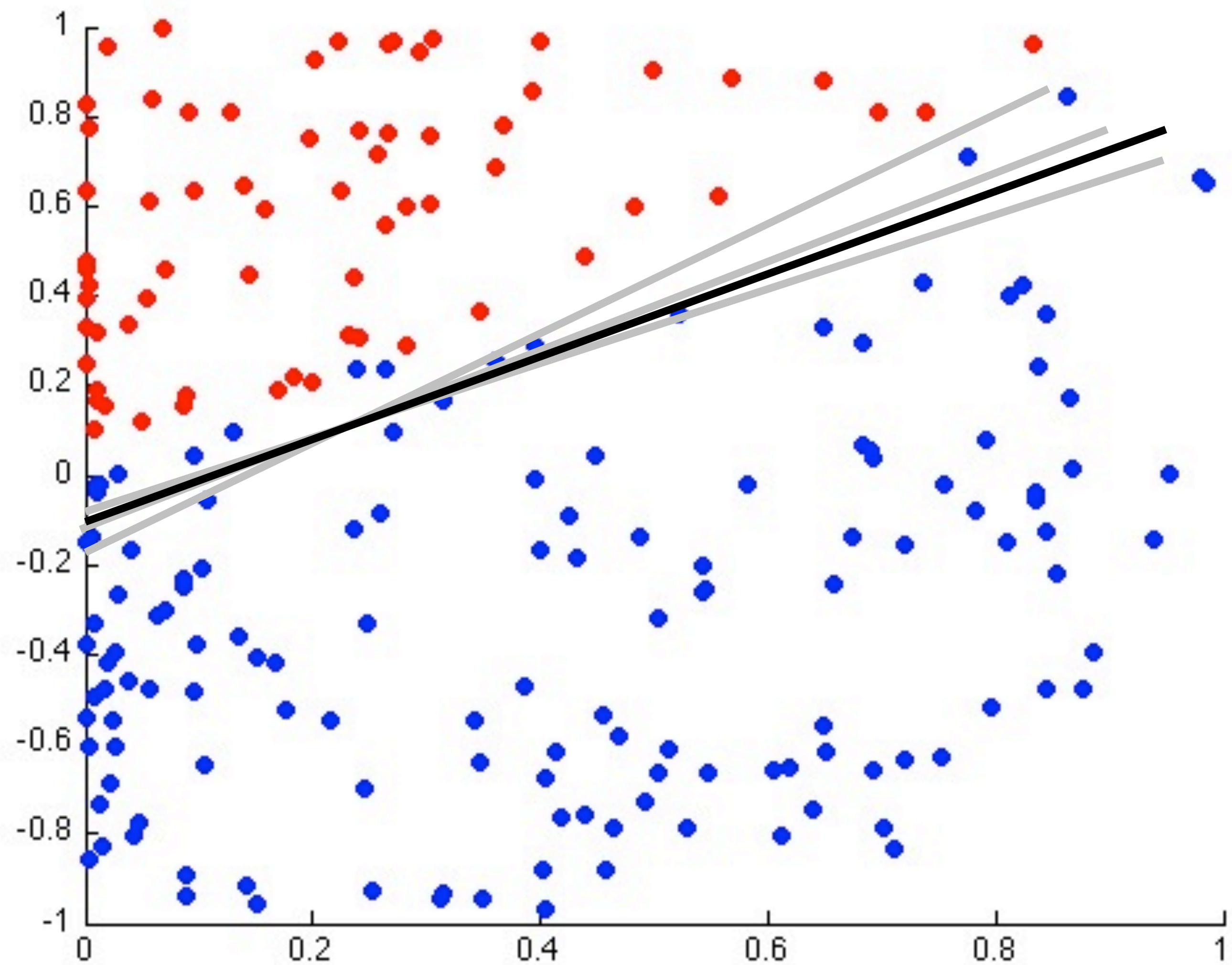
If you have high variance, training error will be low, and test error will be high

Bias Variance Tradeoff

If we make the concept class more complicated (e.g, linear classification to quadratic classification, or increase number of nodes in the decision tree), then bias decreases but variance increases.

Thus there is a bias-variance tradeoff

Why is Bagging useful?



Bagging reduces the variance of the classifier, doesn't help much with bias

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Boosting

Goal: Determine if an email is spam or not based on text in it

From: Yuncong Chen
Text: 151 homeworks are all graded...

Not Spam

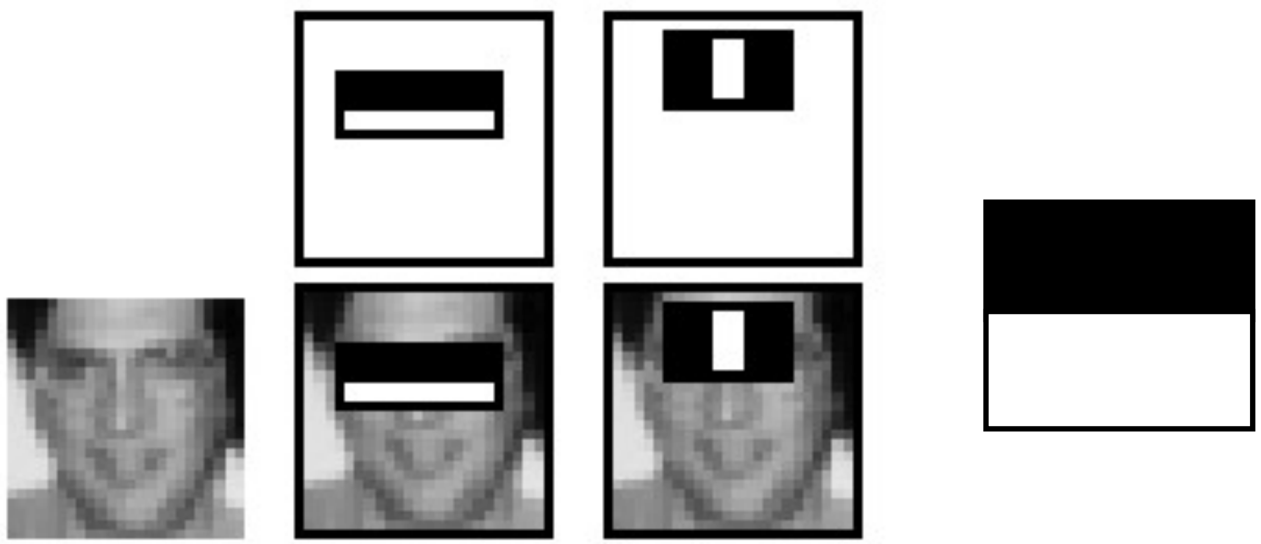
From: Work from home solutions
Text: Earn money without working!

Spam

Sometimes it is:
* Easy to come up with simple rules-of-thumb classifiers,
* Hard to come up with a single high accuracy rule

Boosting

Goal: Detect if an image contains a face in it



= Is the black region darker on average than the white?

Sometimes it is:
* Easy to come up with simple rules-of-thumb classifiers,
* Hard to come up with a single high accuracy rule

Boosting

Weak Learner: A simple rule-of-the-thumb classifier that doesn’t necessarily work very well

Strong Learner: A good classifier

Boosting: How to combine many weak learners into a strong learner?

Boosting

Procedure:

1. Design a method for finding a good rule-of-thumb
2. Apply method to training data to get a good rule-of-thumb
3. Modify the training data to get a 2nd data set
4. Apply method to 2nd data set to get a good rule-of-thumb
5. Repeat T times...

Boosting

1. How to get a good rule-of-thumb?

    Depends on application

    e.g, single node decision trees

2. How to choose examples on each round?

    Focus on the **hardest examples** so far -- namely, examples misclassified most often by previous rules of thumb

3. How to combine the rules-of-thumb to a prediction rule?

    Take a weighted majority of the rules

Some Notation

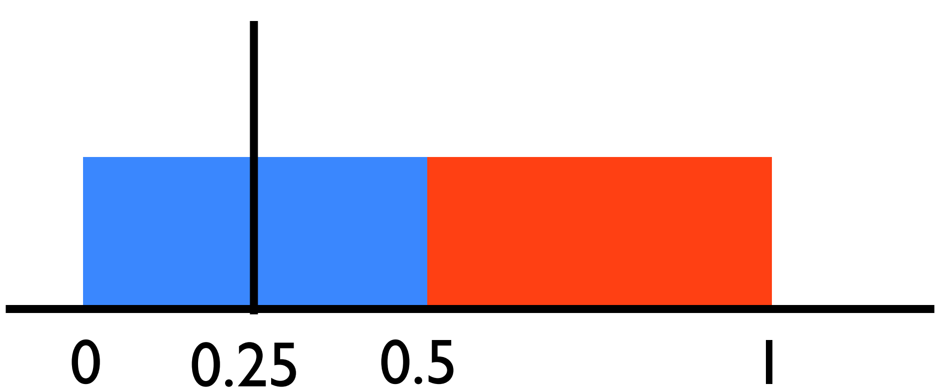
Let D be a distribution over examples, and h be a classifier

Error of h with respect to D is:

$$err_D(h) = Pr_{(X,Y)\sim D}(h(X) \neq Y)$$

Example:

Below X is uniform over [0, 1], and Y = 1 if X > 0.5, 0 ow



$err_D(h) = 0.25$

Some Notation

Let D be a distribution over examples, and h be a classifier

Error of h with respect to D is:

$$err_D(h) = Pr_{(X,Y)\sim D}(h(X) \neq Y)$$

h is called a **weak learner** if $err_D(h) < 0.5$

If you guess completely randomly, then the error is 0.5

Some Notation

Given training examples $\{(x_i, y_i)\}$, i=1,..,n, we can assign weights w_i to the examples. If the w_is sum to 1, then we can think of them as a distribution W over the examples.

The error of a classifier h wrt W is:

$$err_W(h) = \sum_{i=1}^{n} w_i 1(h(x_i) \neq y_i)$$

Note: 1 here is the indicator function

Boosting

Given training set $S = \{(x_1, y_1), ..., (x_n, y_n)\}$, y in $\{-1, 1\}$

For $t = 1, ..., T$

Construct distribution D_t on the examples

Find weak learner h_t which has small error $err_{D_t}(h_t)$ wrt D_t

Output final classifier

Initially, $D_1(i) = 1/n$, for all i (uniform)

Given D_t and h_t:

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} exp(-\alpha_t y_i h_t(x_i))$$

where:

$$\alpha_t = \frac{1}{2} \ln \left(\frac{1 - err_{D_t}(h_t)}{err_{D_t}(h_t)} \right)$$

Z_t = normalization constant

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$D_{t+1}(i)$ goes down if x_i is classified correctly by h_t, up otherwise

High $D_{t+1}(i)$ means hard example

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Higher if h_t has low error wrt D_t, lower otherwise. >0 if $err_{D_t}(h_t) < 0.5$

Z_t = normalization constant

Boosting

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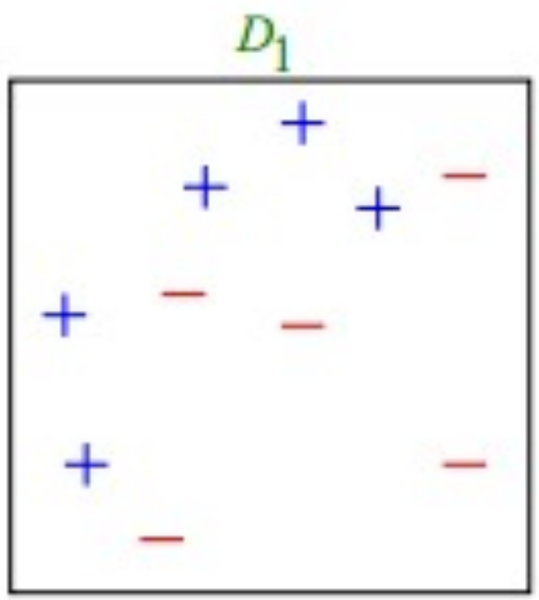
Z_t = normalization constant

Final classifier: $sign(\sum_{t=1}^{T} \alpha_t h_t(x))$

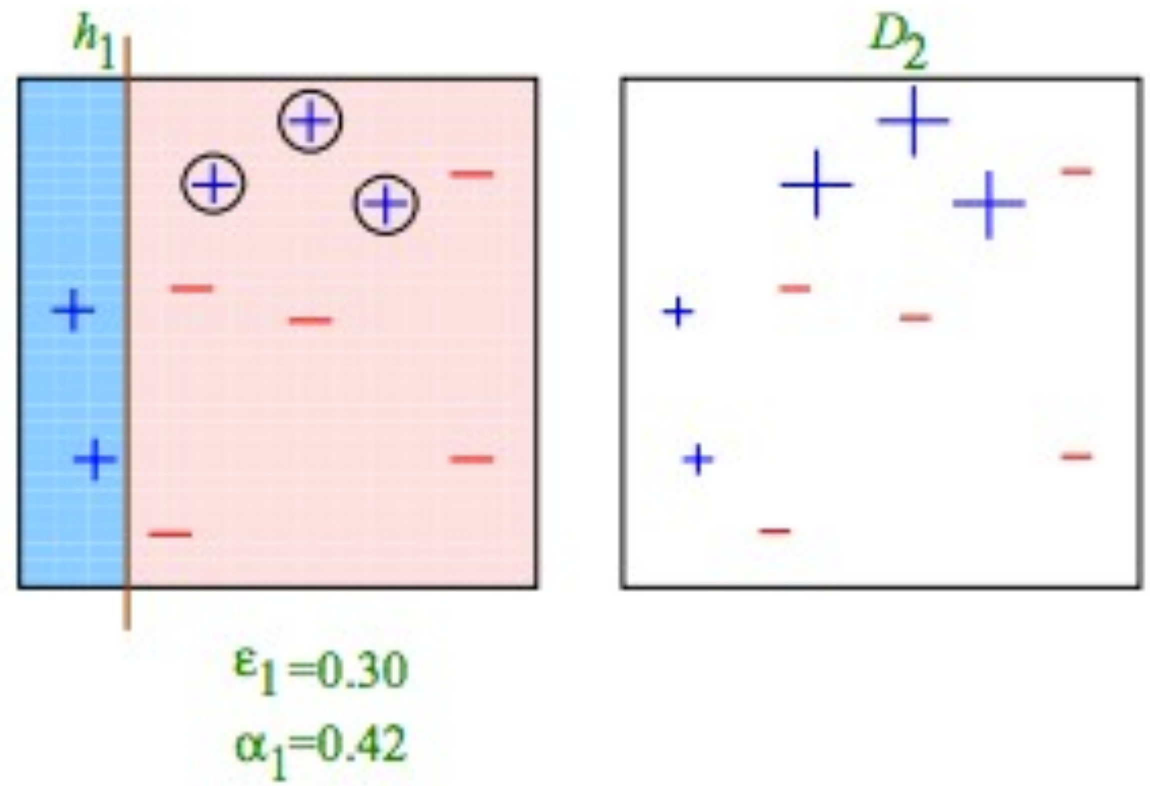
Boosting: Example



Schapire, 2011

weak classifiers: horizontal or vertical half-planes

Boosting: Example



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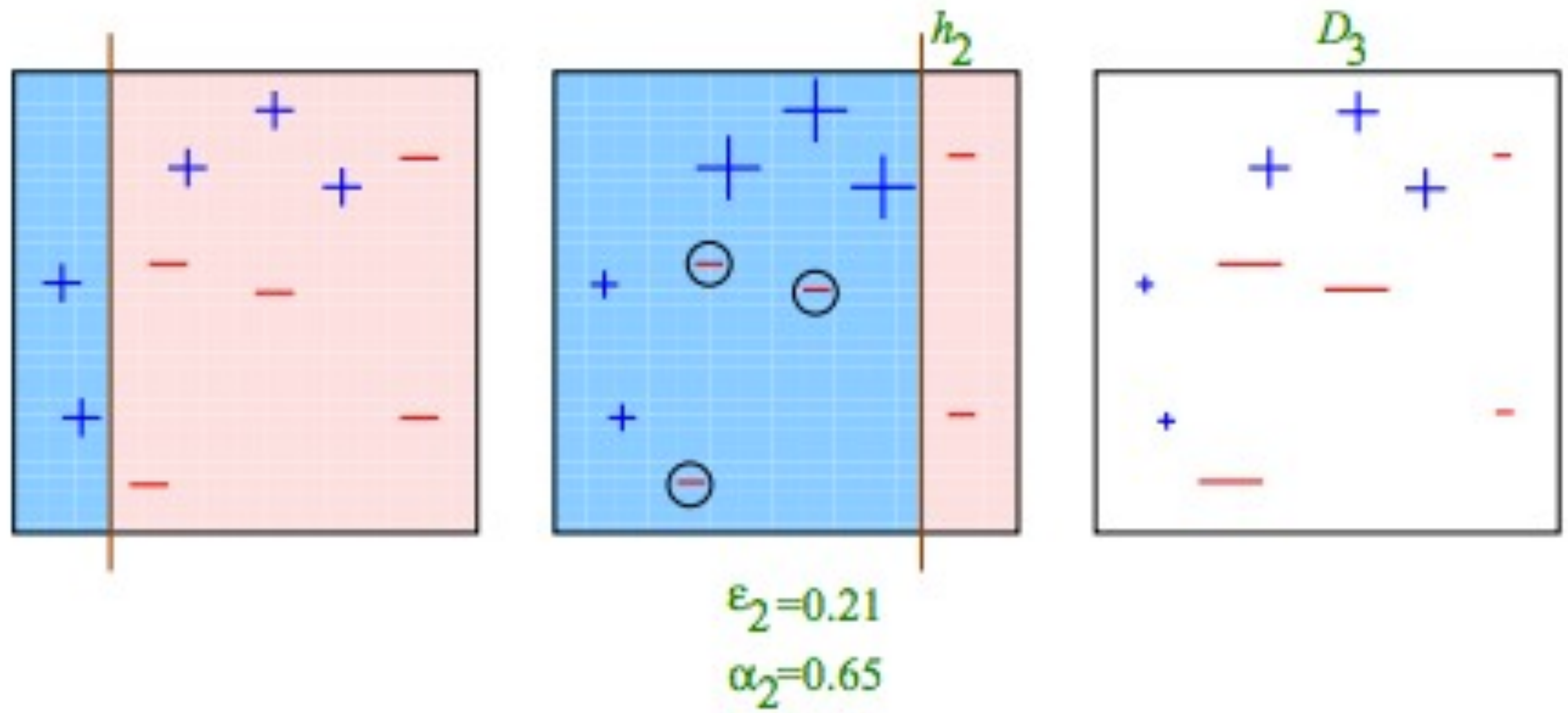

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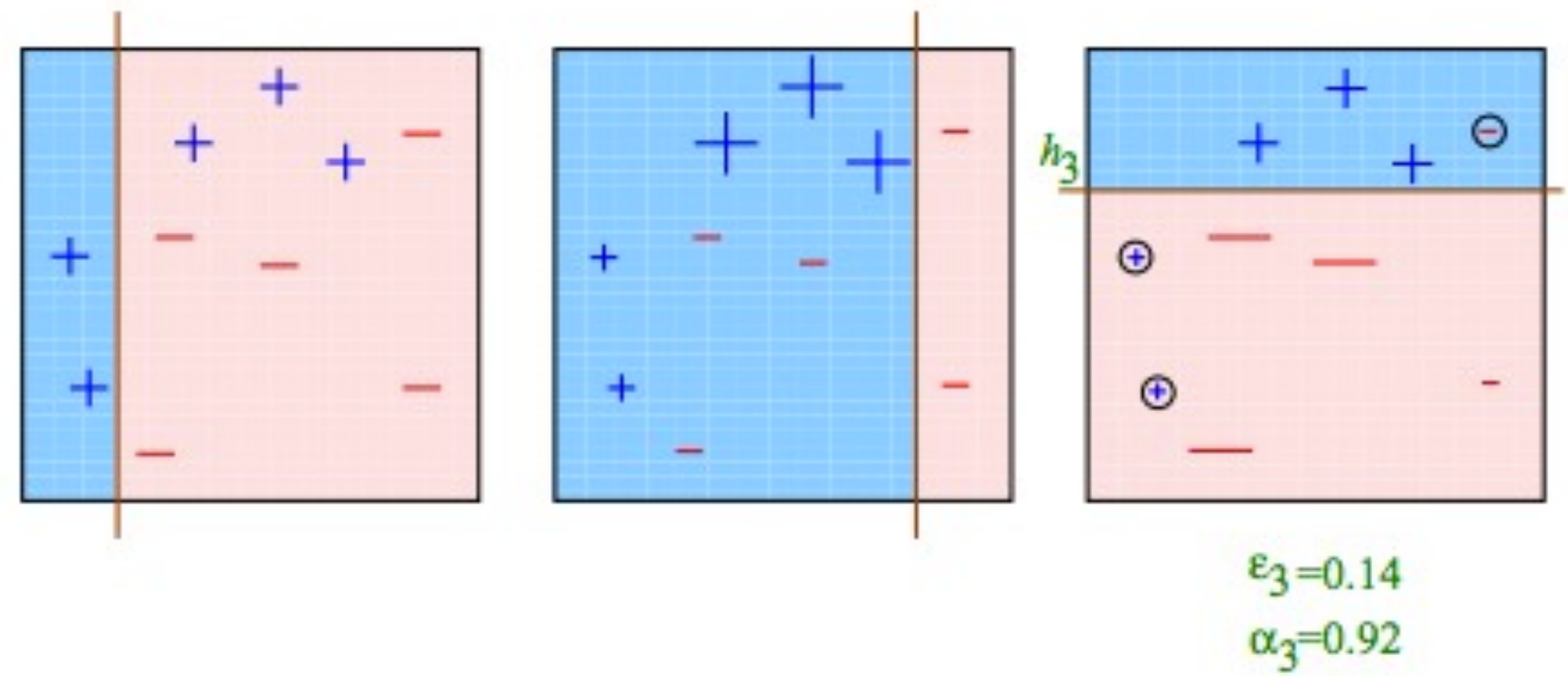

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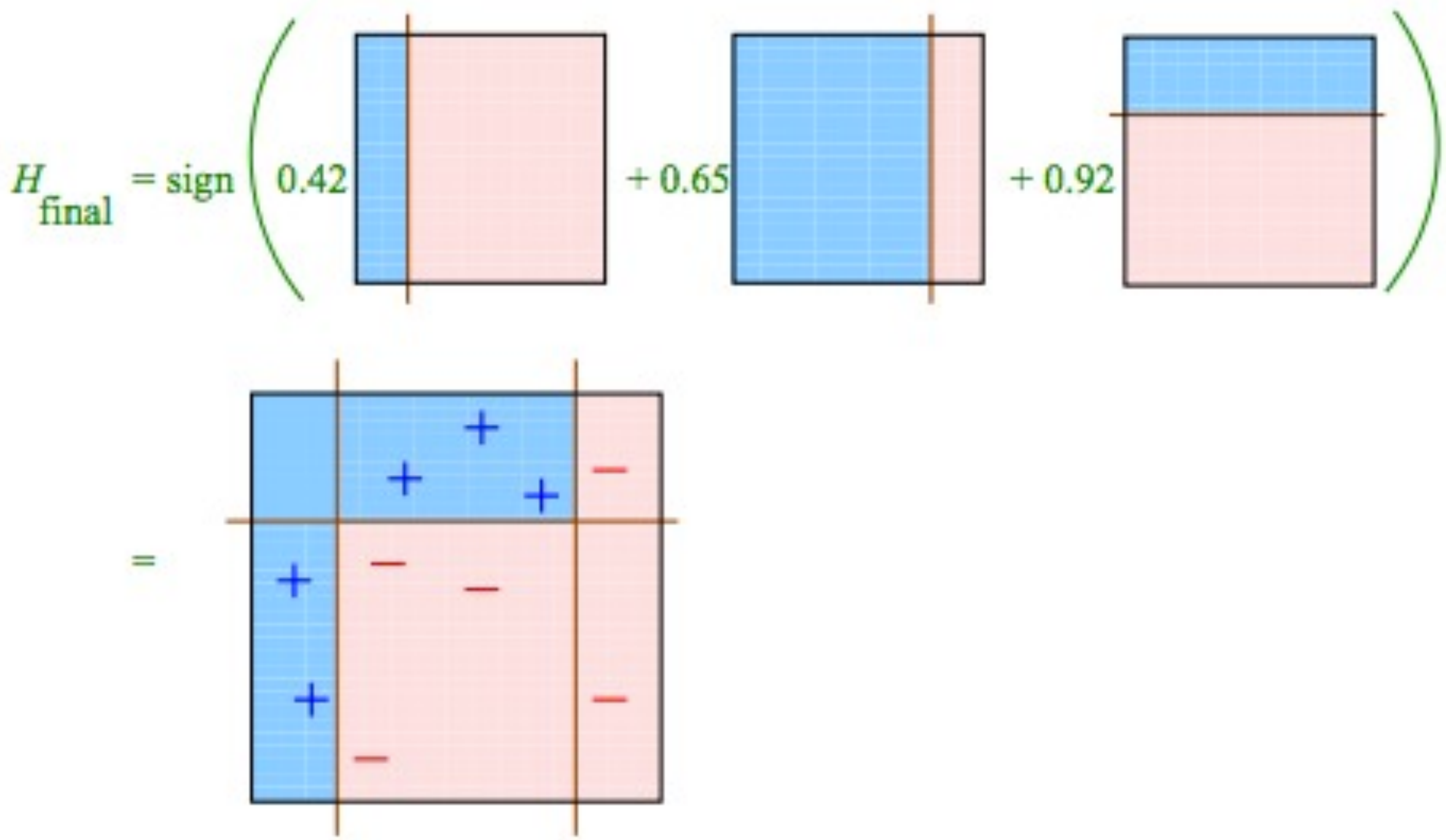
The Final Classifier



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weak classifiers: horizontal or vertical half-planes

How to Find Stopping Time

Given training set $S = \{(x_1, y_1), ..., (x_n, y_n)\}$, y in $\{-1, 1\}$

For $t = 1, ..., T$

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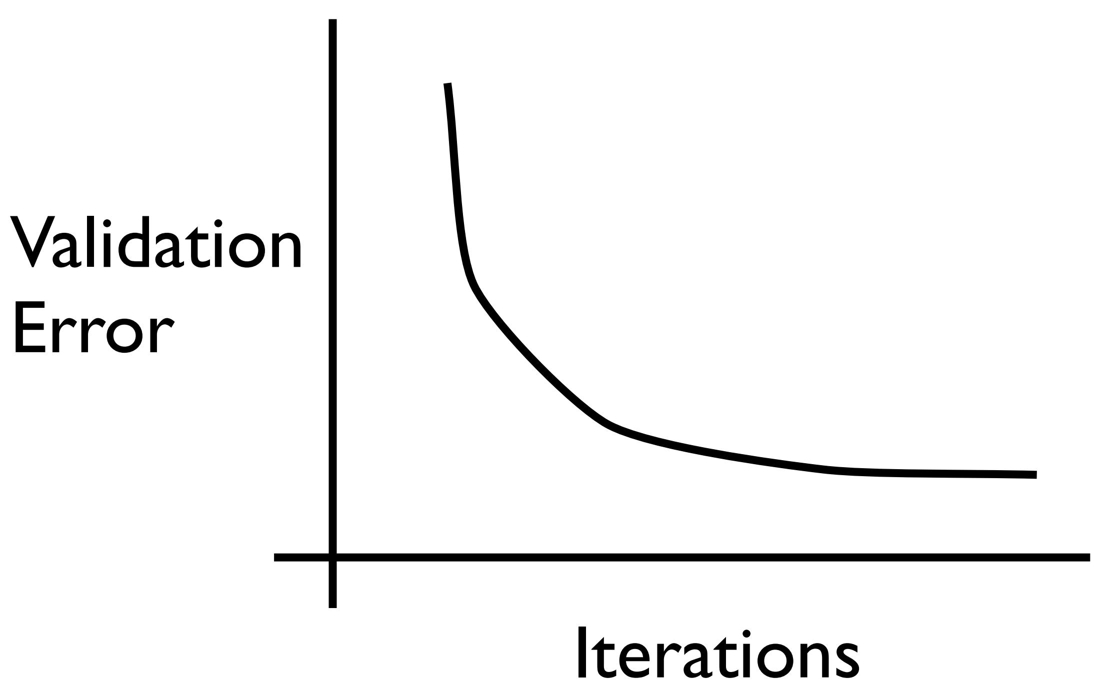
Output final classifier



To find stopping time, use a **validation dataset**.
Stop when the error on the validation dataset stops getting better, or when you can't find a good rule of thumb.