Calculus Workshop

Integration Problem Set

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Answer the following questions to the best of your ability. Feel free to work with anyone in the cohort, though I would encourage attempting on your own first to make sure you fully understand the concepts.

Logarithms

1) Solve for x

$ln(x+2) = 12$

Solution

$$x + 2 = e^{12} \tag{1}$$
$$x = e^{12} - 2 \tag{2}$$
$$x = 162,752.79 \tag{3}$$

$e^{3x+1} = 16$

Solution

$$3x + 1 = \ln(16) \tag{4}$$
$$x = \frac{\ln(16) - 1}{3} \tag{5}$$
$$x = 1.26 \tag{6}$$

$6e^{4x} = 41e^{2x}$

Solution

$$\frac{e^{4x}}{e^{2x} = \frac{41}{6}} \tag{7}$$
$$e^{4x-2x} = \frac{41}{6} \tag{8}$$
$$e^{2x} = \frac{41}{6} \tag{9}$$
$$2x = \ln(41) - \ln(6) \tag{10}$$
$$x = \frac{\ln(41) - \ln(6)}{2} \tag{11}$$

2) Find the derivative:

$$G(a) = \frac{2\ln(4a-2)}{e^{3a}}$$

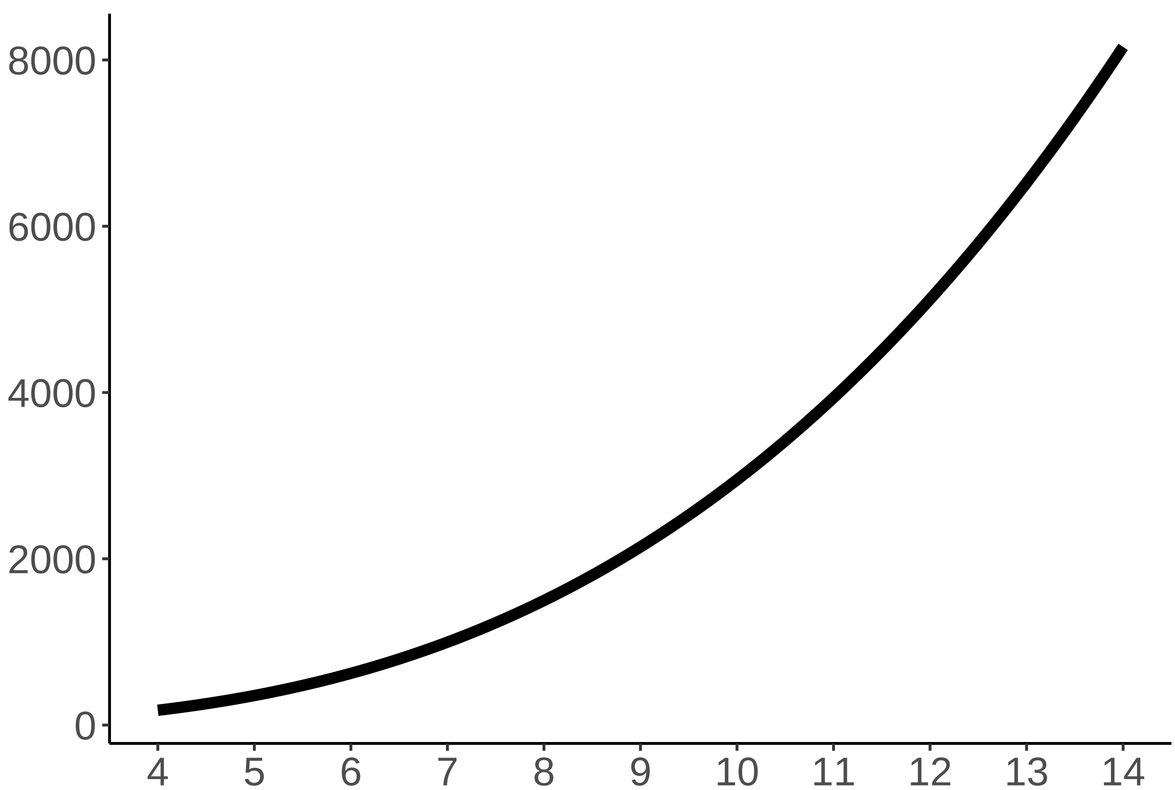
Solution: Break it down into parts. First we need the quotient rule. Then use chain rules for the f(x) and g(x) that you identify. Put it all together

$$\frac{dG}{da} = \frac{\frac{8e^{3a}}{4a-2} - 3e^{3a}\ln(4a-2)}{[e^{3a}]^2} \tag{12}$$

Reimann Sum

1) Approximate the area under the curve $f(x) = 3x^2 - 6x + 10$ on the interval $[6, 12]$ with 6 uniform rectangles.

Does the position of the rectangle make a difference? Describe in words,show mathematically, or draw on the graph how evaluating the rectangle in different ways might lead to slightly different approximations.



The position of the rectangles will slightly affect the approximation, but not much. We could have the squares evaluate at the midpoint of δx instead of at the end. Or we could evaluate to the right.

Evaluate the function at all values in between the interval. Then add them up.

```r
x=seq(6,12)

f=3*x^3-6*x+10

sum(f)
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## [1] 17269
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Intergrals

1)

The marginal benefit of abatement (e.g. reducing) for carbon is given by:

$$MB = 31 - 2Q$$

However there is also a marginal cost for carbon abatement given by:

$$MC = 6 + 3Q$$

Find the total net benefit of carbon abatement to society at equilibrium. (Hint: To get equilibrium and the bounds of the integral, first set marginal benefit equal to marginal cost and solve for Q^*. Then your integral bounds should be from 0 to Q^*)

Solution: First find the equilibrium by setting the MB and MC equations equal to each other and solve for Q

$$31 - 2Q = 6 + 3Q \tag{13}$$
$$25 = 5Q \tag{14}$$
$$5 = Q^* \tag{15}$$

Now we can find the total benefit and cost of abatement by taking the definite integral of each curve from 0 to 5 Marginal Benefit

$$\int_0^5 30 - 2Qdq \tag{16}$$
$$30Q - Q^2|_0^5 \tag{17}$$
$$30(5) - (5)^2 = 125 \tag{18}$$

Marginal Cost

$$\int_0^5 6 + 3Qdq \tag{19}$$
$$6Q + \frac{3x^2}{2}|_0^5 \tag{20}$$
$$6(5) + \frac{3(5)^2}{2} = 67.5 \tag{21}$$

Now we subtract the difference to get the total net benefits to society (in ESM 204 this will be called net welfare). $125 - 67.5 = 57.5$

2)

Take the Integrals

$$\text{A) } y = \frac{3}{x^2}, y(1) = 5 \qquad \text{B) } g(t) = 3t^5 - 2t^3 + 16t - 7 \qquad \text{C) } \int_2^4 \frac{1}{2}x \tag{22}$$

Solution:

A)

$$\int \frac{3}{x^2}dx \tag{23}$$
$$-3x^{-3} + C \tag{24}$$
$$-3(1)^{-3} + C = 5 \tag{25}$$
$$C = 8 \tag{26}$$
$$x^{-3} + 8 \tag{27}$$

B)

$$\int \; g(t) = 3t^5 - 2t^3 + 16t - 7dt \tag{28}$$
$$\frac{t^6}{2} - \frac{t^4}{2} + 8t^2 - 7t + C \tag{29}$$
$$\tag{30}$$

C)

$$\int_2^4 \frac{1}{2}xdx \tag{31}$$
$$\frac{1}{4}x^2|_2^4 \tag{32}$$
$$\frac{1}{4}(4)^2 - \frac{1}{4}(2)^2 = 3 \tag{33}$$

3)

A model for the rate of change in ozone concentrations over time between 1962-1984 is given by $\frac{dC}{dt} = 2t + 20$. Where C is the ozone concentration (ppm) and t is the elapsed time in years. Given that in 1964 the ozone concentration was 30 ppm, what was the ozone concentration in 1982?

$$\frac{dC}{dt} = 2t + 20 \tag{34}$$
$$dC = 2t + 20dt \tag{35}$$
$$\int dC = \int 2t + 20dt \tag{36}$$
$$C = t^2 + 20t + D \tag{37}$$
$$30 = 2^2 + 20(2) + D \tag{38}$$
$$D = -14 \tag{39}$$

Now that we know D we can solve for the concentration in 1982

$$20^2 + 20 * (20) - 14 = \text{ 786ppm}$$