UC SANTA BARBARA

Bren Calculus Workshop

Differential Equations and Numerical Calculus

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Team Review

How did everyone feel about the problem set?

Anything remaining confusing?

Discuss with Team

Exponents and natural log rules in Integration

$$\int e^x dx = e^x + C$$

If the x term gets more complex, we need integration by parts or substiution. Neither we will cover

$$\int \frac{1}{x} = \ln x + C$$

$$\int \ln x = x \ln x - x + C$$

This is solved by integration by parts

How do we solve?

$$\frac{dy}{dx} = 4y$$

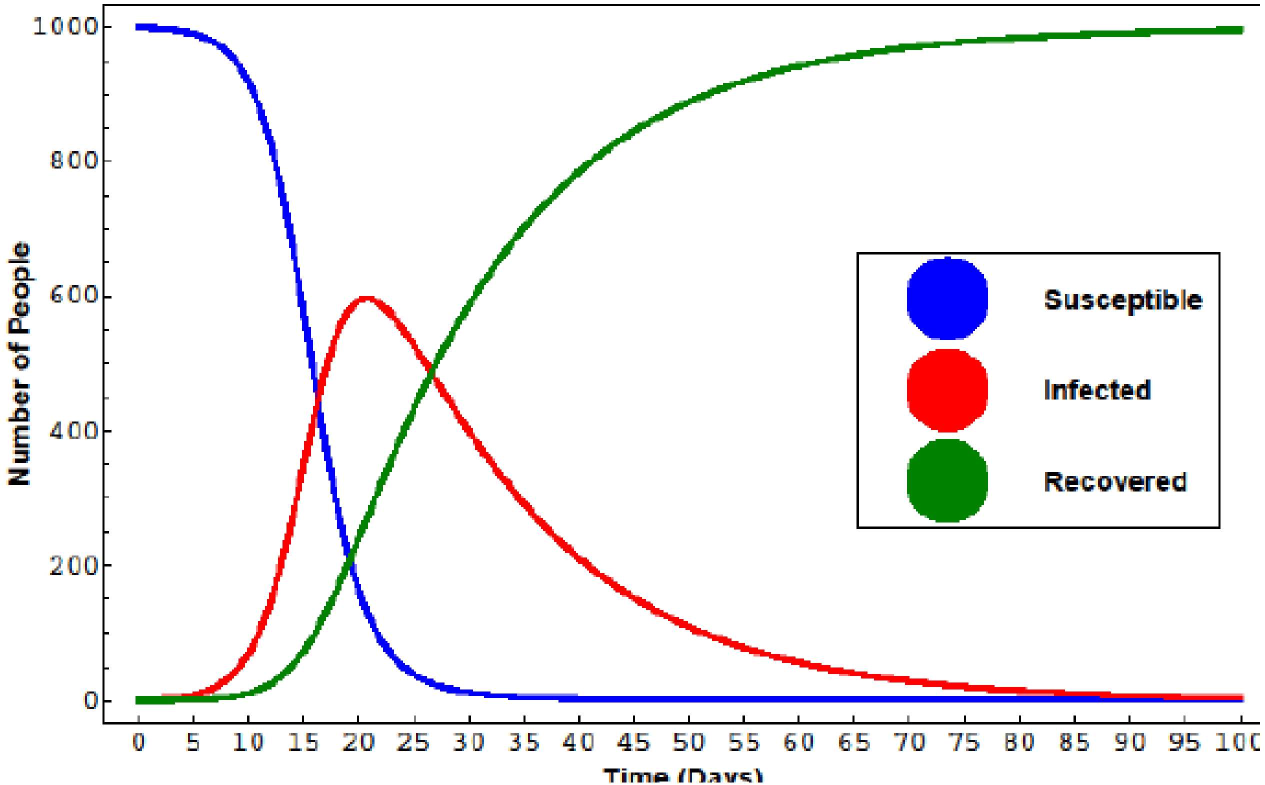
What are taking the intergral with respect too?

What do we do with the 4y?

What about the dy?

Differential Equations help us understand changing environments

Variables often change with each other and effects over time are extremely important



We'll focus on separable first order, ordinary differential equations

Separable - Break apart the function into an independent and dependent side

$$N(y)dy = M(x)dx$$

First Order - Only looking at first derivative rate of changes

$$\text{Only } \frac{dy}{dx} \text{ , no } \frac{d^2y}{dx^2}$$

Ordinary - We only have one independent variable

Steps for solving an Ordinary Differential Equation (ODE)

1. Move like terms to the same side, including differentials (dx,dy)
2. Apply the integral to both sides
3. Rearrange equations to isolate in terms of dependent variable
4. Use initial conditions (if given) to find C values
5. Evaluate the bounds if definite intervals are given

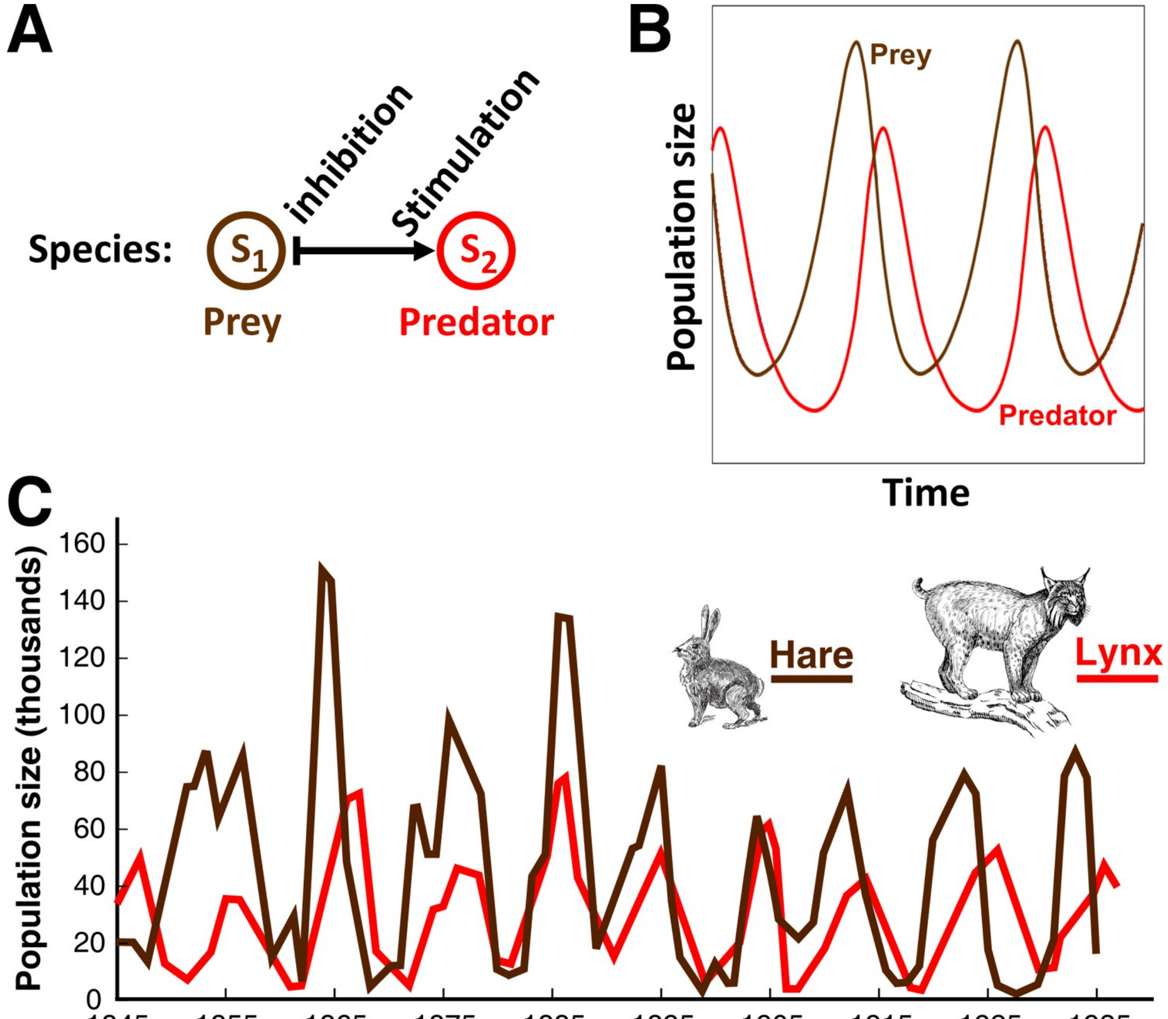
Start small

$$\frac{dy}{dx} = 4y$$

$$\frac{dy}{4y} = dx$$

$$\begin{aligned}
\int \frac{dy}{4y} &= \int dx \\
\frac{1}{4}\ln y &= x + C_1 \\
\ln y &= 4x + C_2 \\
y &= e^{4x+C_2} \\
y &= e^{C_2}e^{4x} \\
y &= Ce^{4x}
\end{aligned}$$

Go big with Lotka-Voltera



One predator and one prey species

Prey Species

$$\frac{dx}{dt} = \alpha x - \beta xy$$

x is the prey population

t is time

α is the prey growth rate

β is the effect of predators on the prey's growth rate

Predator Species

$$\frac{dy}{dt} = \delta xy - \gamma y$$

Where y is the population of the predator

δ is the effect of prey on predators growth rate

γ is the predator per capita death rate

Combine both equations by eliminating time

$$
\begin{aligned}
\frac{dx}{\alpha x - \beta xy} &= dt && \text{Solve for dt} \\
\frac{dy(\alpha x - \beta xy)}{dx} &= \delta xy - \gamma y && \text{Sub in dt} \\
\frac{dy}{dx} &= \frac{\delta xy - \gamma y}{\alpha x - \beta xy} \\
\frac{dy}{dx} &= -\frac{y}{x}\frac{\delta x - \gamma}{\beta y - \alpha} && \text{Pull out x and y}
\end{aligned}
$$

What does this model tells us?

Set up Diff eq steps

1) Separate equations

$$\begin{aligned}\frac{dy}{dx} &= -\frac{y}{x}\frac{\delta x-\gamma}{\beta y-\alpha} \\ \frac{dy(\beta y-\alpha)}{y} &= -\frac{(\delta x-\gamma)dx}{x}\end{aligned}$$

Diff eq steps

2) Apply integral

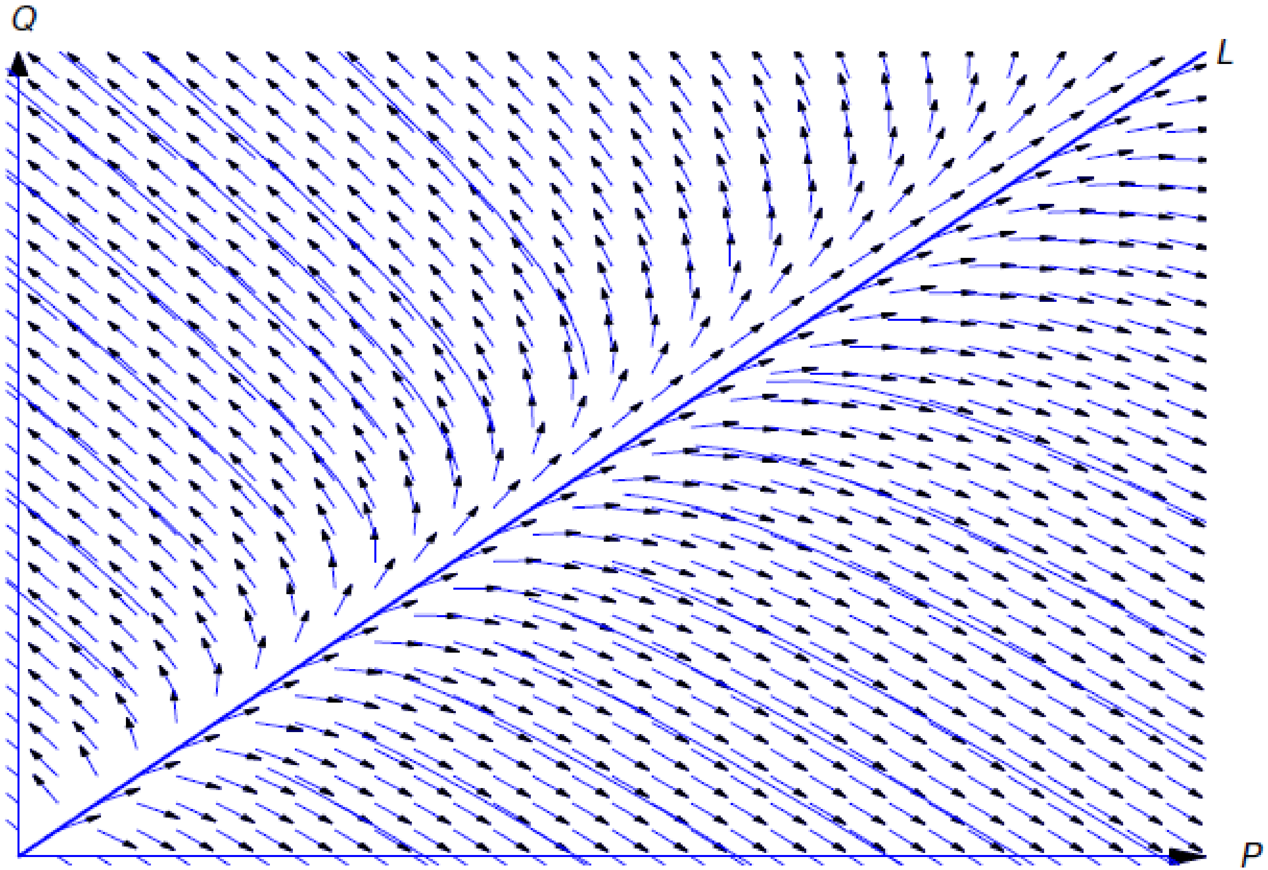
$$\int \frac{dy(\beta y - \alpha)}{y} = -\int \frac{(\delta x - \gamma)dx}{x}$$

$$\int \beta dy - \int \frac{\alpha}{y} dy = \int \frac{\gamma}{x} dx - \int \gamma dx \qquad \text{Distribute terms}$$

$$\beta y - \alpha \ln y = \delta \ln x - \gamma x + V$$

From here we could find intital conditions that lead to a changing population

Just because we solved it does not make it any easier to understand



We need computers to solve and visualize many Diff Eqs

- Lotka-Volterra and other dynamic population differential equation models will make appearances in ESM 201
- Rate of pollutant concentrations in ESM 202
- Solar forcing equations in ESM 203
- and many more in your time at Bren!

Team Assessment

An oil spill of the coast of Santa Barbara is spreading rapidly. Previous spills and an analysis of the current indicate that the oil is spreading at a daily rate of:

$$\frac{dA}{dt} = -0.001A + 60$$

If after the first day the oil has spread to 25 km^2, find an equation to show the spread of oil in total area.

When will the oil spill cover all of the Santa Barbara Channel (~5850 km^2)?

Hint:(The integral of $\frac{1}{ax+b} = \frac{ln(ax+b)}{a}$ Think reverse chain rule)

Numerical Calculus

Functions in R

Like functions in math, R functions are like baking recipes

```
#cake        #recipe name (Ingredients)
output    <-   mean          (x,...)
```

```
x<-mean(1:5,na.rm=TRUE)
x
```

```
## [1] 3
```

But where are the steps?

All functions have documentation

Any built in function in R describes the function and how to use it

```
?seq
```

Can always google too for more intuitive descriptions

Actual code is posted if you really want to break down a function

We can make our own functions

```r
#Name i will give    Tell R we want to      Ingredients
#my function         make a function       List
my_first_fun       <-   function           (x,a)   {

  # Steps
  y=x+2*a

  return(y)  #The cake output

} # End the function steps
```

```r
my_first_fun(x=1,a=2)
```

```
## [1] 5
```

Key pieces of R functions

1. *Name* - What are we going to call our function?
2. *Ingredients* - What goes into our function
3. *Steps* - All the instructions we apply within our function contained within {}
   - Order of steps matter (i.e. can't evaluate z=w+x if x or w have not been created)
4. *Output* - What do we want the function to put out? (By default it is the last object, explicit with `return()`)

Can you arrange this function this function in order to make it work?

```
b=(a+x)*y
```

```
}
```

```
a=x+y
```

```
add_multiply<-function(x,y){
```

```
  return(b)
```