# Calculus Workshop

Applications of Derivatives Problem Set

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*Answer the following questions to the best of your ability. Feel free to work with anyone in the cohort, though I would encourage attempting on your own first to make sure you fully understand the concepts.*

## Derivative Rules

1) Explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (constant, multiple, sum, product, etc.) you would use. [For extra practice, have a try at solving them!]

   a) $f(x) = \sqrt(x + 3)$

   b) $h(a) = \frac{\exp^{-a}}{a^2 - 5}$

   c) $g(t) = \left(\frac{5t^3 + 2}{2(t^2 - 2)}\right)^6$

2) On the graph, label all regions where the following statements are true:

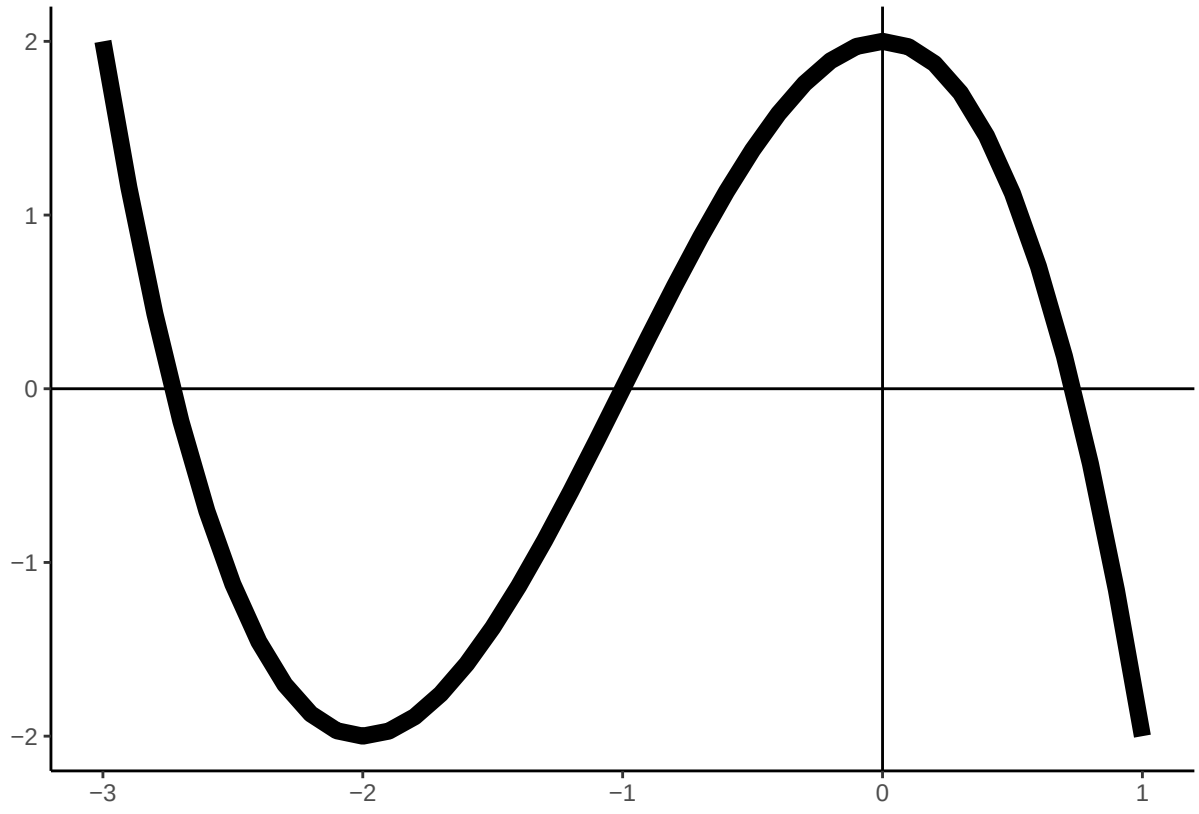
   a) $f'(x) = 0$

   b) $f'(x) > 0$

   c) $f'(x) < 0$

   d) [Bonus] $f''(x) > 0$

   e) [Bonus] $f''(x) < 0$




3) Find the derivative of these equations

   a) $f(x) = \exp^{x^2+3}$
   b) $f(x) = \sqrt{(}2x)((x^3 - 4x + 15))$
   c) $f(x) = \frac{x^3+2x}{x-1}$

4) Find the critical points of the function and determine whether they are maximum or minimum.

$$2x^3 - 5x^2 + 3x - 12$$

5) Let's solve the canonical Gordon-Schaefer fisheries production model. This model formed the basis of fisheries management for half a century and is still a widely used model. It starts by defining the change in the abundance or biomass of fish by the logistic growth equation:

$$\frac{dB}{dt} = rB(t)(1 - \frac{B(t)}{K}) \tag{1}$$

where $B$ is biomass, $t$ is time, $r$ is the intrinsic growth rate, and $K$ is the carrying capacity. Next, fishers catch fish based on their effort $E$, technological ability $q$ (also called the catchability coefficient), and the amount of fish $B$. For simplicity, we use an easy multiplicative form:

$$Y = qEB$$

This catch or harvest rate takes away from the growth of the fish stock, thus we can subtract it from the logistic growth equation 1.

$$\frac{dB}{dt} = rB(t)(1 - \frac{B(t)}{K}) - Y \tag{2}$$

**a)** Fishery managers are interested in finding the equilibrium level of biomass and harvest. We can set $\frac{dB}{dt} = 0$. Solve for the equilibrium biomass in terms of K,q,r,E.

**b)** Sub the equilibrium biomass you found in A into the harvest equation.

**c)** What effort leads to a maximum sustainable yield? Maximize the equation from b) with respect to effort $E$.

**d)** Interpret what you found in c). What are the two variables that drive fisher harvest levels?

**e)** Calculate what the maximum sustainable yield by substituting the optimal harvest level into the harvest equation.