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1/ $f(x,y) = \frac{x}{\sin(x)} + y\ln(y)$

a/

$f\left(\frac{\pi}{2};1\right) = \frac{\frac{\pi}{2}}{\sin\left(\frac{\pi}{2}\right)} + \ln 1$

$= \frac{\pi}{2}$

b/ As $x \to 0$ we have $\lim_{x\to 0} \frac{x}{\sin(x)} = 1$

As $y \to 0$ we have $\lim_{y \to 0} y\ln(y) = -\infty$

So as x and y both go 0, $\lim_{x\to 0, y\to 0} f(x,y) \approx -\infty$

c/ $\frac{\partial f(x,y)}{\partial x} = \left(\frac{x}{\sin(x)}\right)' + 0$

$= \frac{1\sin x - x\cos x}{\sin^2(x)}$

$\Rightarrow \frac{\partial f(0,0)}{\partial x} =$ impossible because $\sin^2(x) = 0$ will make an equation of $\frac{0}{0}$

$\frac{\partial f\left(\frac{\pi}{2},1\right)}{\partial 1} = 1$

d/ $\frac{\partial f(x,y)}{\partial y} = (y\ln(y))'$

$= 1\ln(y) + \frac{y}{y}$

$= \ln(y) + 1. = *$

At $x=0$ and $y=0$

$\Rightarrow \frac{\partial f(x,y)}{\partial y} = -\infty$ as $\ln(y)$ will go to $-\infty$ as y go to 0 (question b)

At $x = \frac{\pi}{2}$, $y=1$

$\Rightarrow \frac{\partial f(x,y)}{\partial y} = 1$

2/ $f(t) = (t+t^2)i + e^{t/2}j + \cos(2\pi t)k.$

a/ $f'(t) = (1+2t)i + \left(\frac{1}{2}e^{t/2}\right)j + \left(-2\pi\sin(2\pi t)\right)k.$

$f''(t) = 2i + \left(\frac{1}{4}e^{t/2}\right)j + \left(-4\pi^2\cos(2\pi t)\right)k$

b/ $\int_0^1 f(t)\,dt.$

$\int f(t)\,dt = \left(\frac{t^2}{2} + \frac{t^3}{3}\right)i + \left(2e^{t/2}\right)j + \left(\frac{\sin(2\pi t)}{2\pi}\right)k$

$\Rightarrow \int_0^1 f(t)\,dt = \left(\frac{1}{2}+\frac{1}{3}\right)i + \left(2e^{1/2}\right)j + 0.k.$

$-[(0)i + 2j + 0.]$

$= \frac{5}{6} i + (2e^{1/2} - 2) j + 0k.$

c|

$f'(0) = i + \frac{1}{2} j + 0k.$

$\times \int_0^1 f(t) = \frac{5}{6} i + (2e^{1/2} - 2) j$

$\cancel{= (i + \frac{1}{2} j) \times [\frac{5}{6} i + (2e^{1/2} - 2) j]}$

$\cancel{= \frac{5}{6} i^2 + \frac{5}{12} ij + (2e^{1/2} - 2) ji + \frac{1}{2} j^2 (2e^{1/2} - 2)]}$

~~d|~~ $\cancel{f(t) \cdot f'(t) = (1+2t) i + (\frac{1}{2} e^{t/2}) j + }$

$= i(0) + (0) j + [(2e^{1/2} - 2) - \frac{1}{3}] \cdot k$

d| $f(t) \cdot f'(t)$,

$= i (t+t^2)(1+2t) + j e^{t/2} (\frac{1}{2} e^{t/2}) + k(-\pi \sin(4\pi t$

$= i(t + t^2 + 2t^2 + t^3) + j(\frac{1}{2} (e^{t/2})^2) + k(-\pi \sin(4\pi t$

$= i(t + 3t^2 + t^3) + j(\frac{1}{2} (e^{t/2})^2) + k(-\pi \sin(4\pi$

e| $\frac{1}{2} [(t+t^2)^2 + (e^{t/2})^2 + \cos(2\pi t)^2]$

3/ $r(t) = e^{at} i - e^{at} j + e^{-a.t} k$

$r''(t) = (a^2 e^{at}) i - (a^2 e^{at}) + (a^2 e^{-at}) k$

$= a^2 r(t)$ (because a is positive)

So $r''(t)$ is parallel to $r(t)$ for any time t, for all positive values of a.

4/ $f(x,y,z) = \frac{3xyz}{x+y+z} - y^2 \cos(\pi x z) + e^{xyz}$

a/ $\frac{\partial f(x,y,z)}{\partial x} = \frac{3yz(y+z)}{(x+y+z)^2} \; y^2 \pi z \sin(\pi xz) + yze^{xyz}$

at $y = 1 ; z = 1 ; x = 1$

$\Rightarrow \frac{\partial f(x,y,z)}{\partial x} = \cancel{3 + \pi . 0 + e = 3 + e} \; \frac{2}{3} + e$

b/ $h(xy) = \cancel{\frac{3xz}{1} \; \; 2y}$

$= \frac{3xz(x+z)}{(x+y+z)^2} - 2y\cos(\pi xz) + e^{xyz} xz$

at $y = -1, z = 1$

$h(y) = \frac{3x(x+1)}{x^2} + 2\cos(\pi x) + xe^{-x}$

c) $k(z) = \frac{3xy(x+y)}{(x+y+z)^2} + \pi xy^2 \sin(\pi x z) + e^{xyz} xy$

At $x=0$

$k(z) = \frac{0}{(y+z)^2} + 0 + 0$

$= 0$