- Hamiltonian:

$$H = \frac{\Delta \sigma_x}{2} + g \sigma_z (a + a^\dagger) + \underbrace{\omega_{cav} a^\dagger a}_{\text{cavity mode}} + \underbrace{\sum_k \omega_k a^\dagger_k a_k}_{\text{bath modes}}$$

$$+ \; a^\dagger \sum_k \gamma_k a_k + a \sum_k \gamma_k^* a_k^\dagger$$

I wrote here the cavity/bath coupling as single photon loss, but more generic form can be also described, e.g. terms like $a^\dagger \sum_k (\gamma_k a_k + \beta_k a_k^\dagger)$

Let's combine the cavity + bath modes as a common field $a_p^\dagger \equiv \begin{cases} a^\dagger & \text{if } p = 0 \\ a^\dagger_k & \text{if } p = k \neq 0 \end{cases}$

$$\Rightarrow \quad H = \frac{\Delta \sigma_x}{2} + g \sigma_z (a_0^\dagger + a_0) + \sum_{pp'} h_{pp'} a_p^\dagger a_{p'}$$

with $\begin{cases} h_{00} = \omega_{cav} \\ h_{kk} = \omega_k \\ h_{0k} = h_{k0}^* = \gamma_k \\ h_{pp'} = 0 \quad \text{else} \end{cases}$

Diagonalizing the matrix $h_{pp'}$ provides normal modes b_p of the problem:

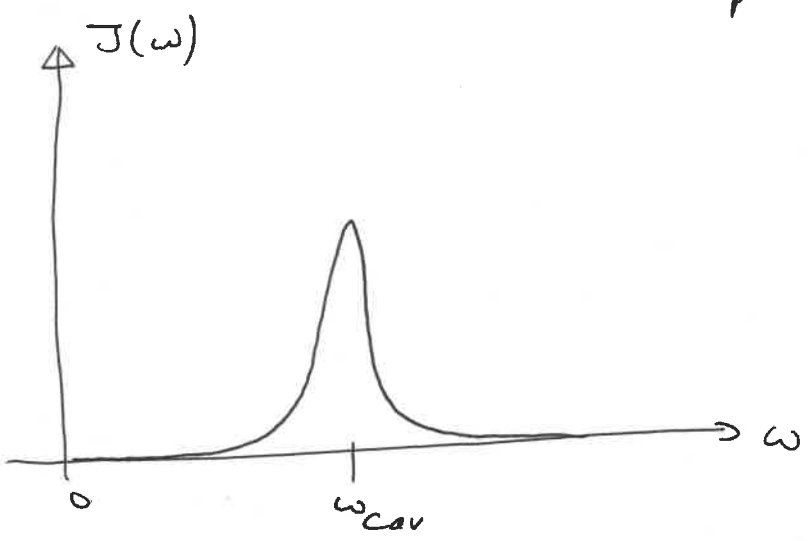
$$\sum_{pp'} h_{pp'} a_p^\dagger a_{p'} = \sum_p \Omega_p b_p^\dagger b_p$$

$$a^\dagger = \sum_p u_{0p} b_p^\dagger \quad \text{and} \quad a_k^\dagger = \sum_p u_{kp} b_p^\dagger$$

eigenvectors of $h_{pp'}$

$$\Rightarrow H = \frac{\Delta \sigma_x}{2} + g\,\sigma_z \sum_p u_{0p}\left(b_p^\dagger + b_p\right) + \sum_p \Omega_p\, b_p^\dagger b_p$$

Spectral density: $J(\omega) = \pi \sum_p (g\, u_{0p})^2\, \delta(\omega - \Omega_p)$



We end up with a spin boson model with a structured (Lorentzian) spectral density.