

## Presentation of 5A project

Forecast of aircraft parts failures and  
optimization of spare parts stock management

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January 17, 2021

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# Introduction

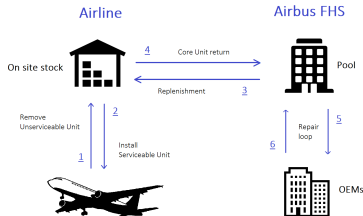


Figure: Airbus FHS principal activities

Due to the pandemic, the airlines have less aircraft flying and more contracts ending. Consequently, FHS need to fine-tune strategy “repair or stock unserviceable” to optimise the stock management.

→ define component lifetime forecast models and then by using these models.

→ determine if the current stock of spare parts is enough to full fill all the needs until the end of the contracts. If not, determine when the stock will be empty and will thus require to be replenished with new or repaired parts.

# Data set

## Data sheets:

- "Removals": records from 2012 until today of unscheduled removals.
- "SN list": status as of today of parts.
- "Airlines": normalised name of airlines, number of aircraft, flight hour per aircraft per month and end of contract of each airline.

## Glossary:

- PN (Part number): Identifier of a particular part design of an aircraft component.
- SN (Serial number): Unique identifier of a particular physical instantiating of a Part Number across its production.
- TSI (Time Since Installation): cumulative flight hours by a part between the date of its installation on aircraft and a later date.
- TSN (Time Since New): cumulative flight hours by a part between the date of its production and a later date.

## Basic notions in lifetime data analysis

### Reliability or survival function

#### Definition

Denote:

$$\forall t \geq 0, R(t) = \mathbb{P}(T > t)$$

where  $T$  is a random variable considered as a lifetime which is positive  $T > 0$ .

### Right censoring

#### Definition

The lifetime  $X$  is said to be right censored by a random variable  $C$  if, instead of observing  $X$ , one observe  $\min(X, C)$ .

## Parametric models

Exponential:

$$R(t) = \exp\left(\frac{-t}{\lambda}\right)$$

Weibull:

$$R(t) = \exp\left(-\left(\frac{t}{\lambda}\right)^{\rho}\right), \lambda > 0, \rho > 0$$

Log-Logistic:

$$R(t) = \left(1 + \left(\frac{t}{\alpha}\right)^{\beta}\right)^{-1}$$

Log-normal:

$$R(t) = 1 - \Phi\left(\frac{\log(t) - \mu}{\sigma}\right)$$

Piecewise Exponential model:  $R(t)$  is defined as the best model in multiple interval of  $t$ . Natural cubic splines:

$$R(t) = \exp\left(\phi_0 + \phi_1 \log t + \sum_{j=2}^N \phi_j v_j(\log t)\right).$$

Advantages and disadvantages:

**Not capable** of estimating the reliability function with **high precision**

Calculate the survival probability for any individual **at any point of time**.

# Kaplan-Meier estimator

## Kaplan-Meier Estimator

$$\hat{R}_n(x) = \prod_{i: T_{(i)} \leq x} \left(1 - \frac{\delta_{(i)}}{n - i + 1}\right).$$

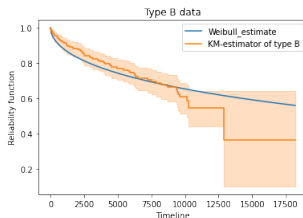


Figure: Comparing best parametric model and Kaplan-Meier estimator of PN "A".

### Advantages and disadvantages:

Estimation of the reliability function with **high precision**, especially, for a large population

Not capable of giving survival probability of any individual beyond the population.

## Current issues

For a given PN and a given number of spare parts of that PN available in stocks at time  $t$ :

- This stock will be sufficient to meet all future demands?
- Otherwise, a estimated date from which the stock will become insufficient, and then when and how many parts must then be sent for reparation? (taking in to account the TAT).

Knowing that Airbus FHS are aiming for a service level of 90% (out of 100 requests, they can respond favourably to 90), if they have  $x$  spare parts, they want to estimate a date that they are no longer able to maintain this service level.



## Forecast of failure number

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### Algorithm 1: Forecast of failure number (without difference distribution)

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**Input:**

$s_1, \dots, s_k$  is a list of TSIs of aircraft parts which are in service;

$m$  is an integer number;

$FH_{perMonth}$  is average flight hour per month of considering company;

**Output:**  $N$  is the number of failure we want to forecast in next  $m$  months;

**Algorithm:**

$FH := m \times FH_{perMonth}$ ;

**if**  $FH \leq t_n$  **then**

    Considering  $R(\cdot)$  as Kaplan-Meier model;

**else**

    Considering  $R(\cdot)$  as best parametric model;

**end**

$N := 0, i := 0$ ;

**while** individual  $i < k$  **do**

$i := i + 1$ ;

    The lifetime  $S^{(i)}$  is simulated by the conditional reliability  $R(\cdot | s_i)$  (or denoted  $R_{s_i}(\cdot)$ ):  $S^{(i)} := R_{s_i}^{-1}(U)$ ,  
    where  $U \sim U([0, 1])$ ;

$CumTime := S^{(i)}$ ;

**while**  $CumTime < FH$  **do**

$N := N + 1$ ;

$CumTime := CumTime + R_0^{-1}(U) (= CumTime + R^{-1}(U))$ , where  $U \sim U([0, 1])$ .

**end**

**end**

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## Estimated out-of-stock date

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### Algorithm 2: Forecast of out-of-stock date

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**Input:**

$comp_1, \dots, comp_m$  is a list of company;  
 $end_1, \dots, end_m$  is a corresponding end of contract (by month);  
 $x$  is the number of spare parts;  
 $los$  is the level of service;

**Output:**  $oos$  is estimated out-of-stock date;

**Algorithm:**

$P = \text{int}(x/los)$  ( $P$  is an integer part of  $x/los$ );

$j = 0$ ;

**while**  $j < m$  **do**

$j = j + 1$ ;

    We simulate the moments that the failures of  $comp_j$  occur before  $end_j$ :  $T_j = (T_{1j}, \dots, T_{k_{jj}})$ ;

**end**

$T = (T_1, \dots, T_m)$  (note that each  $T_j$  is a vector with the size of  $k_j$ );

We sort  $T$  in ascending order:  $T = (T^{(1)}, \dots, T^{(k)})$ , where  $k = \sum_{i=1}^m k_i$ ;

**if**  $P \leq k$  **then**

$oos = T^{(P)}$ : the  $P^{th}$  element of  $T$ ;

**else**

$oos = T^{(k)}$ : the last element of  $T$ .

**end**

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## Confidence interval of $\bar{X}$

Let  $X$  be a random sample (here is the number of failure or the out-of-stock date).  
Let  $X_1, X_2, \dots, X_n$  be random samples each of size  $n$  taken from a population with overall mean  $\mu$  and finite variance  $\sigma^2$ .

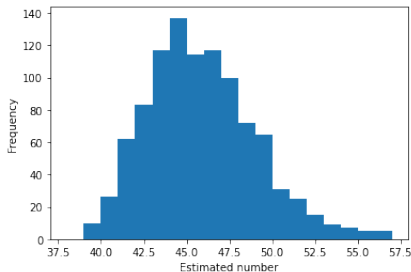


Figure: Histogram of estimated failure number of PN "B" in 1000 simulations.

$$CI_{1-\alpha}(\bar{X}) = \mu \pm q_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}.$$

( $q_{1-\alpha/2}$  is  $1 - \alpha/2$  quantile of standard normal distribution).

## Confidence interval of $X$

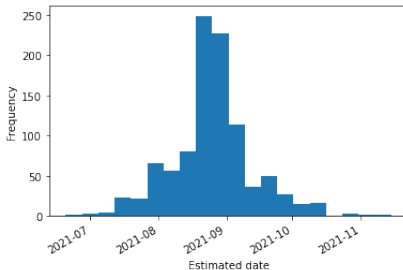


Figure: Histogram of estimated out-of-stock date of PN "A" in case of number-of-spare-parts=80.

**Without censor:**

$$Cl_{1-\alpha}(X) = [q_{\alpha/2}, q_{1-\alpha/2}]$$

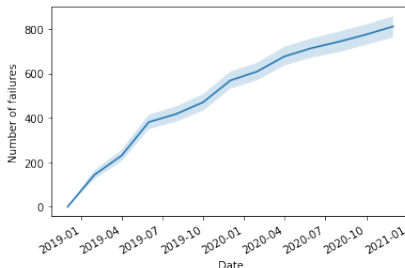
where  $q_{\alpha/2}$  and  $q_{1-\alpha/2}$  are  $\alpha/2$  and  $1 - \alpha/2$  quantile of  $X$ .

**With censor:**

$$Cl_{1-\alpha}(X) = [q_{\alpha}, +\infty)$$

where  $q_{\alpha}$  is  $\alpha$  quantile of  $X$ .

## Results and validation



**Figure:** Evolution of failure number of PN "C" from 2018 to 2020 by simulation.

### Explanation:

- The airline has less aircraft flying in the period 2019-2020 due to the pandemic. Consequently, there was less failure part in that time.
- The validation period observed a co-existence of 2 technologies ("C" and "C-new") for the same part. Some of aircraft parts of type "C" got replaced by "C-new" which is substantially more reliable. This also reduced the failure number.

## Conclusion and Future works

### Conclusion

The estimation of the survival probability of each PN was done by applying parametric models as well as non-parametric models in combination with homogeneity tests.

By using these models, we could write the algorithms which allow to forecast the number of failure in the future, the out-of-stock date and evaluate the failure number as a function of time, which could be a significant underlying factor in the proposals of stock optimisation strategy of Airbus FHS.

### Future works

- Make more use of the "TSN" by deploying a regression model.
- Better describe the transition between two technologies.
- Implement a reliability model on that simulated population  $X$  from the last algorithm.

**Thanks for your attention!**