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ORIGINAL ARTICLE

Single-machine-based production scheduling model integrated preventive maintenance planning

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Abstract Manufacturing and production plants operate physical machine that will deteriorate with increased usage and time. Maintenance planning which can keep machines in good operation is thus required for smooth production. However, in previous research, production scheduling and maintenance planning are usually performed individually and not studied as an integrated model. In order to balance the trade-offs between them, this study proposes an integrated scheduling model by incorporating both production scheduling and preventive maintenance planning for a single-machine problem with the objective of minimizing the maximum weighted tardiness. In this model, a variable maintenance time subjected to machine degradation is considered. Finally, a numerical example using this improved production scheduling model is shown. The computational results prove its efficiency.

Keywords Production scheduling · Preventive maintenance · Tardiness · Weight · Effective age

1 Introduction

As production scheduling becomes one of the most critical issues in the planning and managing of manufacturing processes, a great amount of production scheduling models have been studied to be deterministic optimization models designed to maximize some measure of customer satisfaction. However, previous theoretical models usually do not take machine availability constraints into account and assume machines are always available for processing the tasks during the planning time horizon, yet this assumption is inappropriate in many real-world scheduling cases [1, 2].

In real situations, machine breakdown is common after long time operation, hence, machines will be unavailable while processing tasks due to failures. For machine breakdown will reduce production efficiency, maintenance as an important part in manufacturing systems is used to keep machines in good condition to decrease failures [3]. Thus, maintenance planning becomes more and more important in manufacturing processes [4]. Like machine breakdown, when performing maintenance activities, the task being processed is stopped. So the machine will be unavailable when an accidental breakdown or a maintenance activity occurs. Based on this scheme, a more realistic production scheduling model should well consider maintenance planning. Because machine breakdown is stochastic and maintenance is deterministic, maintenance planning could be treated as availability constraint in a production scheduling model.

As production scheduling models with the consideration of maintenance operations arouse great interest of researchers, the machine availability problem for production scheduling models has been widely discussed in the past decades [5]. However, those production scheduling models with maintenance planning usually do not consider machine degradation. Most of them just pre-design a fixed maintenance interval and arrange job sequence based on this maintenance interval, which cannot show maintenance operation's real influence on production scheduling model. The related literature review is given below in Section 2. Hence, in order to describe the interaction between

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production scheduling and maintenance planning, this study is devoted to propose an improved production scheduling model for single-machine problem by integrating production scheduling and preventive maintenance simultaneously.

This paper is structured as follows. Section 2 gives the literature review about previous research. Section 3 introduces the development of this improved production scheduling model. Problem description and notation are given in Section 4. Then, Section 5 presents the improved production scheduling model considering preventive maintenance planning subjected to machine degradation. A numerical example given in Section 6, and discussions are provided to compare the result obtained by this proposed scheduling model with those obtained by some previous production scheduling models. Finally, the conclusion and outlines for future research are in Section 7.

2 Literature review

Scheduling is one of the most widely researched areas of operational research, which is largely due to the rich variety of different problem types within the field. And in this paper, the types of scheduling problems with maintenance operations that arise in production industries are focused on.

In the past decades, some research has been done to study production scheduling models with maintenance operations based on different customer requirements (i.e., objectives). Adiri, Frostig, and Rinnooy studied a scheduling model with maintenance operations for a single machine. In their paper, a single-machine breakdown is assumed and the number of tardy jobs is minimized [6]. Based on single-machine scheduling problem, Lee and Liman considered a two parallel-machine scheduling problem with the aim to minimize the total completion time. In this scheduling model, one machine which has the availability constraint is discussed [7]. Then, Mosheiov studied on the same problem. In his scheduling model, he assumed each machine was unavailable in an interval [8]. Later, for the flow-shop scheduling research, Espinouse and Formanowicz worked on a flow-shop scheduling problem. They assumed a limited machine availability in their scheduling model to arrange maintenance operations with the aim of minimizing the makespan [9]. In these research, some basic concepts are given to study scheduling models with maintenance operations. However, most of them assumed there was only one interval in which machine was unavailable. It seems not practical. Based on this scheme, researchers began to consider several maintenance intervals. For example, Qi, Chen and Tu studied a scheduling problem with periodic maintenance planning. In their model, they assumed several intervals in which machine was unavailable and took them to be decision variables in order to minimize the total completion time [10]. With periodic maintenance constraints (i.e. fixed maintenance interval), some heuristic algorithms were developed. Lee considered a single-machine scheduling problem in which the job was nonresumable. The result showed that the longest processing time rule had a tight worst-case ratio of 4/3 [11]. Then, Liao and Chen used the branch-and-bound algorithm to derive the optimal sequence for a scheduling problem. And they also proposed a heuristic algorithm for large-sized problems [12]. Sadfi, Penz, Rapine, Błażewicz and Formanowicz developed a modified algorithm MSPT for a single-machine scheduling problem with the aim of minimizing the total completion time. Using their algorithm, the worst-case ratio was 20/17 [13]. Wodecki considered a single-machine job scheduling problem where the objective was to minimize the weighted sum of earliness and tardiness penalties of jobs. He developed a new fast local search algorithm based on a tabu search method [14]. For flow-shop scheduling problem, Allaoui and Artiba integrated simulation and optimization to schedule a hybrid flow-shop scheduling problem with maintenance constraints [15]. Further, based on their previous result, they researched on two-stage hybrid flow-shop scheduling problem [16]. Haq and Ramanan considered the sequencing of jobs that arrived in s flow-shop in different combinations over time. They focused on scheduling for a flow-shop with ‘m’ machines and ‘n’ jobs, and used artificial neural network with acquired scheduling knowledge in making the future sequencing decisions [17].

Although these previous research have shown their performance to solve production scheduling problem, most of them considered periodic maintenance which had fixed maintenance interval. As periodic maintenance predetermines a fixed time interval to perform maintenance activities (i.e. characteristics of the unavailability periods known in advance), machine idleness will appear due to the incoordination of the total processing time of prior batch and the fixed time interval. Hence, though these researches have considered periodic maintenance planning into production scheduling model, they seem not economic due to machine idleness. If maintenance activities could be well planned according to a jobs' processing time and machine's status, much more resources will be saved. Based on this scheme, preventive maintenance is studied for the optimization of those cases in which maintenance activity is controllable. Now, there are some researches considering preventive maintenance during production processes, however, most of them take production scheduling and preventive maintenance planning as two independent problems despite that there should be a close relationship between them [2, 18–20]. Maintenance activity occupies production time, and no jobs can be processed at that time, thus frequent maintenance activities will delay production. However, if maintenance activity is delayed to keep the production, machine failures will increase [21]. Thus, in order to balance the trade-offs between them, an improved production scheduling model is proposed in this study to solve a single-machine problem by integrating production scheduling and preventive maintenance simultaneously. More precisely, how to minimize the maximum tardiness of jobs is considered as the objective, which can help maximize customer satisfaction (e.g. shorten delivery time or lead time).

3 Development of this improved production scheduling model

Compare with those previous production scheduling models with maintenance planning, this study develops an improved scheduling model with production scheduling and preventive maintenance planning focusing on the following parts.

1. As machine idleness may occur while performing periodic maintenance activities, this study considers preventive maintenance to avoid this situation. Maintenance interval for preventive maintenance can be controllable and solved by the stochastic production scheduling model (either mathematical or simulative), the job sequence can be well designed to avoid machine idleness so as to make full use of the machine availability. Usually, preventive maintenance models are typically stochastic models accompanied by optimization techniques designed to maximize the machine availability or to minimize the total cost [21]. There are some papers published to show the great use of preventive maintenance planning [20, 22–27]. Although most of the relevant research considers preventive maintenance planning into the scheduling model, they usually take production scheduling and preventive maintenance planning as two independent problems, which makes the scheduling model not a real integrated model. So the result may be not optimal. Hence, this study explicitly integrates production scheduling and preventive maintenance planning into an improved scheduling model.
2. In reality, machines suffer increasing wear with increased age and usage due to degradation, which causes low reliability of the machine [28]. In most of previous scheduling models with maintenance planning, they usually do not consider machine's deteriorating process, and just pre-design a fixed maintenance interval and then arrange the job sequence based on this time interval. It is obvious that the inflexible maintenance interval will ignore machine breakdown occurring during the planning time horizon. Hence, in order to meet more practical situations, this study considers that machine breakdown be subjected to a deterioration process, and repair activity to restore the machine should be performed. Moreover, due to machine degradation, if each preventive maintenance activity could restore the machine to be "as good as new", much more resources should be needed when the machine locates at a worse condition. Thus, a concept of effective age is proposed in this study to describe machine's condition. Based on this scheme, maintenance time is proposed to be variable according to machine's condition subjected to machine degradation.
3. With the development of the single multi-functional machine for manufacturing requirements, the jobs to be processed on a single machine may have different values, but not always the same. In order to handle the real-world scheduling cases in which jobs have different weights, besides the effect of maintenance activity, this study adds weight into the scheduling model. Therefore, the aim of this proposed scheduling model is devoted to minimize the maximum weighted tardiness. In addition, this scheduling model will schedule multiple maintenance activities, but not schedule only one maintenance activity during the whole planning horizon [12, 29, 30].

With these above-mentioned considerations, this study tries to develop an improved production scheduling model by incorporating production scheduling and preventive maintenance planning to bridge the gaps between theory and practical situations.

4 Problem description

This paper studies a single-machine scheduling problem in which the machine is not continuously available due to machine breakdown; hence, preventive maintenance activities with flexible time intervals during the planning horizon are performed. Assume there are a set of n jobs to be processed on a single machine, and all the jobs are ready to be released at time zero. For simplicity, job's processing time, job's due date and job's weight are supposed to be an integer. While processing, only one job can be processed at one time on the machine. Suppose that one job preempts another is not permitted (i.e. the job is nonresumable and it cannot be interrupted after starting its service on the machine). The similar assumption can be found in the published papers by Merten and Muller [31] and Schrage [32]. When the machine fails due to degradation, minimal repair is performed to restore the machine to be "as bad as old" (i.e. the operating condition just before failures). Preventive maintenance activities which reduce the increasing risk of machine failures are supposed to be able to restore the machine to be "as good as new" (i.e. the machine is renewed and its effective age return to zero). The time to perform each preventive maintenance activity is variable according to machine's effective age. By considering job's different weight, the maximum weighted tardiness is to minimize for this single-machine scheduling problem (seen in Fig. 1).

Notations

n	Number of jobs to be processed at time zero
p_j	Processing time of job j, $j \in \{1, 2, ..., n\}$
d_j	Due date of job j
w_j	Weight of job j
C_j	Completion time of job j
L_j	Lateness of job j
T_j	Tardiness of job j
$J_{[i]}$	Ordinal of jobs in the sequence, $i \in \{1, 2, ..., n\}$
$p_{[i]}$	Processing time of ith job in the sequence
$w_{[i]}$	Weight of i^{th} job in the sequence
$C_{[i]}$	Completion time of ith job in the sequence
$L_{[i]}$	Lateness of ith job in the sequence
$T_{[i]}$	Tardiness of ith job in the sequence
$\lambda(t)$	System's failure rate
a	Effective age for the machine
a_0	The initial effective age for the machine
t_{mr}	time for performing minimal repair
$M_{[l]}$	Ordinal of maintenance periods in the sequence, $l \in N$
$t_{p,a}$	Time for performing preventive maintenance activity
T_{max}	Maximum tardiness

5 The improved production scheduling model

In this section, the mathematical framework for the improved production scheduling model with the aim to minimize the maximum weighted tardiness on a single machine is established to prove structural characteristics of the optimum. Some assumptions are presented as follows.

Assumptions:

1. a single machine is studied;
2. the machine is repairable and subjected to a deterioration process;
3. the machine begins a new degradation process after preventive maintenance;
4. the machine failure is stochastic, but can be described by system failure functions;
5. the unexpected failure can be inspected at once when it happens.

In order to obtain the tardy time of each job to minimize the maximum weighted tardiness, the completion time of each job should be obtained at the beginning. In this scheduling model, the set of n jobs includes J_1, J_2, J_3,…, J_n, and there are multiple maintenance periods $M_{[l]}$ (i.e. no less than one maintenance period). As C_j denotes the completion time of job j, the lateness of job j can be defined as

$$L_j = w_j \cdot (C_j - d_j) \quad \text{for } j = 1, 2, ..., n \tag{1}$$

If one job is processed ahead of its due date, it has no tardy time. So $L_j < 0$ makes T_j=0. Thus, the tardiness of job j should be inferred as

$$T_j = \max\{0, L_j\} \tag{2}$$

With the aim to minimize the maximum weighted tardiness of all the jobs, the corresponding $T_{max} = \max\{0, T_j\}$ should be minimized. Suppose $x_{i,j}$ as one decision variable and let

$$x_{i,j} = \begin{cases} 1, \text{if the } i\text{th job performed is job } j \\ 0, \text{otherwise} \end{cases} \tag{3}$$

for i=1, 2, ..., n, j=1, 2, ..., n

Based on this decision variable $x_{i,j}$, the processing time and the weight of the ith job in the sequence can be deduced as

$$p_{[i]} = \sum_{j=1}^{n} (p_j \cdot x_{i,j}) \tag{4}$$

$$w_{[i]} = \sum_{j=1}^{n} (w_j \cdot x_{i,j}) \tag{5}$$

Then, another decision variable y_i for integrating preventive maintenance activity is supposed as

$$y_i = \begin{cases} 1, \text{if preventive maintenance activity is performed} \\ \quad \text{prior to the } i\text{th job} \\ 0, \text{otherwise} \end{cases} \tag{6}$$

for i=1, 2, ..., n

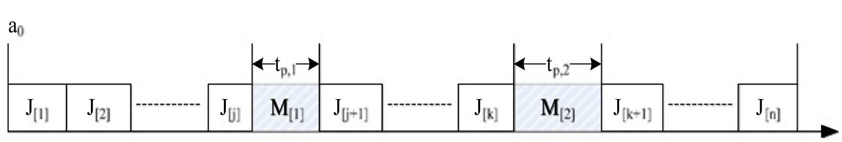

Fig. 1 Illustration of a schedule considering preventive maintenance planning

Considering the development of this improved production scheduling model discussed in Section 3, the completion time for each job as a random variable should be influenced by

1. the completion time of the previous jobs;
2. the processing time of this job;
3. the possibility of machine failures while processing this job;
4. time for minimal repair;
5. the effective age of the machine prior to this job;
6. decision variable for preventive maintenance activity;
7. time for performing preventive maintenance activity.

Hence, for the first job in the sequence, its expected value of the completion time $C_{[1]}$ can be constructed as

$$E(C_{[1]}) = \sum_{j=1}^{n} (p_j \cdot x_{1,j}) + y_1 \cdot t_{p,a_0} + y_1 \cdot t_{\mathrm{mr}} \cdot \int_0^{p_j} \lambda(t)\mathrm{d}t + (1-y_1) \cdot t_{\mathrm{mr}} \cdot \int_{a_0}^{a_0+p_j} \lambda(t)\mathrm{d}t \quad (7)$$

which includes three parts: the processing time of the first job, the probable time for performing preventive maintenance activity prior to the first job, the probable time for performing minimal repair. In Eq. 7, $\sum_{j=1}^{n} (p_j \cdot x_{1,j})$ represents the processing time of $J_{[1]}$, and $y_1 \cdot t_{p,a_0}$ represents the probable time for performing preventive maintenance activity prior to $J_{[1]}$. Y_1 is the decision variable for the selection of preventive maintenance activity prior to $J_{[1]}$. Because preventive maintenance activity is assumed to restore the machine to be "as good as new" (i.e. the machine is renewed), if one preventive maintenance activity is performed prior to $J_{[1]}$, machine's initial effective age before production becomes zero. Hence, the probable time for performing minimal repair should be $t_{\mathrm{mr}} \cdot \int_0^{p_j} \lambda(t)\mathrm{d}t$. But if no preventive maintenance activity is performed, machine's initial effective age is still a_0, thus, the probable time for performing minimal repair should be $t_{\mathrm{mr}} \cdot \int_{a_0}^{a_0+p_j} \lambda(t)\mathrm{d}t$. Based on Eq. 7, the expected value of the lateness of $J_{[1]}$ can be inferred as

$$E(L_{[1]}) = w_{[1]} \cdot [E(C_{[1]}) - d_j] = \sum_{j=1}^{n} (w_j \cdot x_{1,j}) \cdot [E(C_{[1]}) - d_j] \quad (8)$$

So the expected value of the tardiness of $J_{[1]}$ should be

$$E(T_{[1]}) = \max\{0, E(L_{[1]})\} \quad (9)$$

In this proposed scheduling model, suppose Q_i (i ∈ {1, 2, ..., n}) to be a set of jobs having been selected (e.g. if job $J_{[1]}$ has been selected to be processed, before processing the second job in the sequence, there is Q_1= {$J_{[1]}$}). Thus, for the second job in the sequence, the expected value of the completion time $C_{[2]}$ should be

$$\begin{aligned} E(C_{[2]}) = & \sum_{j\in I} (p_j \cdot x_{2,j}) + y_2 \cdot \left[y_1 \cdot t_{p,E(C_{[1]})} + (1-y_1) \cdot t_{p,a_0+E(C_{[1]})} \right] \\ & + y_2 \cdot t_{\mathrm{mr}} \cdot \int_0^{p_j} \lambda(t)\mathrm{d}t \\ & + (1-y_2) \cdot t_{\mathrm{mr}} \cdot \left[y_1 \cdot \int_{E(C_{[1]})}^{E(C_{[1]})+p_j} \lambda(t)\mathrm{d}t + (1-y_1) \cdot \int_{a_0+E(C_{[1]})}^{a_0+E(C_{[1]})+p_j} \lambda(t)\mathrm{d}t \right] \end{aligned} \quad (10)$$

for $I = \{1, 2, ..., n\} \cap \overline{Q}_1$

The expected value of the lateness of $J_{[2]}$ is

$$E(L_{[2]}) = w_{[2]} \cdot [E(C_{[2]}) - d_j] \quad (11)$$

where the weight of $J_{[2]}$ in Eq. 11 is

$$w_{[2]} = \sum_{j\in I} (w_j \cdot x_{2,j}) \quad (12)$$

for $I = \{1, 2, ..., n\} \cap \overline{Q}_1$

Thus, the expected value of the tardiness of $J_{[2]}$ is

$$E(T_{[2]}) = \max\{0, E(L_{[2]})\} \quad (13)$$

Based on the expected value of the completion time of $J_{[1]}$ and $J_{[2]}$ shown in Eqs. 7 and 10, the expected value of the completion time $C_{[i]}$ of $J_{[i]}$ could be deduced as

$$E(C_{[i]}) = \sum_{j\in I} (p_j \cdot x_{i,j}) + T_{\mathrm{pm}} + T_{\mathrm{mr}} \quad (14)$$

Subject to $I = \{1, 2, ..., n\} \cap \overline{Q}_{i-1}$

$$\begin{aligned} T_{\text{pm}} = & y_i \cdot y_{i-1} \cdot t_{p,E\left(C_{[i-1]}\right)} \\ & + y_i \cdot (1 - y_{i-1}) \cdot y_{i-2} \cdot t_{p,E\left(C_{[i-2]}\right)+E\left(C_{[i-1]}\right)} \\ & + \cdots\cdots \\ & + y_i \cdot (1 - y_{i-1}) \cdot (1 - y_{i-2}) \cdot \cdots \cdot y_1 \cdot t_{p,E\left(C_{[1]}\right)+\cdots+E\left(C_{[i-2]}\right)+E\left(C_{[i-1]}\right)} \\ & + y_i \cdot (1 - y_{i-1}) \cdot (1 - y_{i-2}) \cdot \cdots \cdot (1 - y_1) \cdot t_{p,a_0+E\left(C_{[1]}\right)+\cdots+E\left(C_{[i-2]}\right)+E\left(C_{[i-1]}\right)} \end{aligned} \tag{15}$$

$$\begin{aligned} T_{\text{mr}} = & y_i \cdot t_{\text{mr}} \cdot \int_0^{p_j} \lambda(t) \mathrm{d}t \\ & + (1 - y_i) \cdot y_{i-1} \cdot t_{\text{mr}} \cdot \int_{E\left(C_{[i-1]}\right)}^{E\left(C_{[i-1]}\right)+p_j} \lambda(t) \mathrm{d}t \\ & + \cdots\cdots \\ & + (1 - y_i) \cdot \cdots \cdot (1 - y_2) \cdot y_1 \cdot t_{\text{mr}} \cdot \int_{E\left(C_{[1]}\right)+\cdots+E\left(C_{[i-1]}\right)}^{E\left(C_{[1]}\right)+\cdots+E\left(C_{[i-1]}\right)+p_j} \lambda(t) \mathrm{d}t \\ & + (1 - y_i) \cdot \cdots \cdot (1 - y_2) \cdot (1 - y_1) \cdot t_{\text{mr}} \cdot \int_{a_0+E\left(C_{[1]}\right)+\cdots+E\left(C_{[i-1]}\right)}^{a_0+E\left(C_{[1]}\right)+\cdots+E\left(C_{[i-1]}\right)+p_j} \lambda(t) \mathrm{d}t \end{aligned} \tag{16}$$

$$\sum_{j \in I} x_{i,j} = 1,\ I = \{1, 2, ..., n\} \cap \overline{Q}_{i-1} \tag{17}$$

$$\sum_{i=1}^{n} x_{i,j} = 1, i = 1, 2, ..., n \tag{18}$$

$$x_{i,j} \text{ binary}, i = 1, 2, ..., n, j \in \{1, 2, ..., n\} \cap \overline{Q}_{i-1} \tag{19}$$

$$y_i \text{ binary}, i = 1, 2, ..., n \tag{20}$$

where T_{pm} denotes the probable time for performing preventive maintenance activities prior to the ith job and T_{mr} denotes the probable time for performing minimal repair. The expected value of the lateness of $J_{[i]}$ should be

$$E\left(L_{[i]}\right) = w_{[i]} \cdot \left[E\left(C_{[i]}\right) - d_j\right] \tag{21}$$

In Eq. 21, the corresponding weight of $J_{[i]}$ is

$$w_{[i]} = \sum_{j \in I} \left(w_j \cdot x_{i,j}\right) \tag{22}$$

for $I = \{1, 2, ..., n\} \cap \overline{Q}_{i-1}$

Finally, the expected value of the tardiness of $J_{[i]}$ is obtained as

$$E\left(T_{[i]}\right) = \max\left\{0, E\left(L_{[i]}\right)\right\} \tag{23}$$

With the aim of minimizing the maximum weighted tardiness for the single-machine scheduling problem, the resulting mathematical programming formulation can be given by

$$\text{Minimize } T_{\max} = \max\left\{E\left(T_{[i]}\right)\right\} \tag{24}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{i,j} = 1,\ j = 1, 2, ..., n \tag{25}$$

$$\sum_{i=1}^{n} x_{i,j} = 1, i = 1, 2, ..., n \tag{26}$$

$$x_{i,j} \text{ binary}, i = 1, 2, ..., n,\ j = 1, 2, ..., n \tag{27}$$

$$y_i \text{ binary}, i = 1, 2, ..., n \tag{28}$$

In Eq. 24, $E(T_{[i]})$ denotes the expected value of the tardiness of $J_{[i]}$ (i.e. the ith job in the sequence). As each job sequence has the corresponding maximum weighted tardiness, after comparing the maximum weighted tardiness for different job sequences, the optimal job sequence will be solved with selected minimum weighted tardiness value. The whole computation process can be viewed as a recursive process.

6 Case study

6.1 A numerical example

As Weibull distribution has been widely used to describe system's failure rate in mechanical and electronical engineering, it has been usually adopted to study maintenance problems. For example, Weibull distribution was used for a replacement policy proposed by Chen and Feldman [33]; a preventive maintenance model was researched by Reineke, Murdock, Pohl and Rehmert in which wear-out failure mode was modelled by a 2-parameter Weibull distribution [34]; Kumar, Crocker and Knezevic considered Weibull distribution in a maintenance model for an aircraft engine [35]; Nakagawa and Nakamura studied an entoropy model with the application of a maintenance policy in which machine's failure time satisfied Weibull distribution [36]. Thus, in this study, Weibull distribution is used to describe system's failure rate function that is shown as

$$\lambda(t) = \frac{\beta}{\theta} \cdot \left(\frac{t}{\theta}\right)^{\beta-1} \tag{29}$$

Generally, the parameters of Weibull distribution for system's failure rate function are deduced from the history maintenance data. Suppose a machine is subjected to failures, and the time to failure for this machine is governed by a Weibull distribution. The shape parameter of this Weibull distribution should be greater than 1, which is practical to perform preventive maintenance on the machine to reduce the increasing risk of machine failures [37]. Suppose shape parameter β=2 and scale parameter θ=100. In real manufacturing processes, maintenance engineers usually should be responsible for the pre-design of related maintenance parameters. Suppose t_{mr}=15 and $t_{p,a} = 5 \times ((a+500)/500)^3$. In Table 1, the parameters of each job is presented, including job's processing time, job's due date and job's weight. For this single machine, its initial effective age is supposed to be 88 (i.e. a_0=88).

Table 1 The original data for example (in hours)

J_j	J_1	J_2	J_3	J_4	J_5	J_6
p_j	25	7	42	36	18	29
d_j	61	102	86	44	150	134
w_j	3	7	1	3	1	4

By using the mathematical model given in Section 5 to solve this single-machine scheduling problem, the job sequence including preventive maintenance decisions with the overall minimum objective function value can be identified as the global optimal solution. First, based on Eq. 14, the expected value of completion time of these jobs can be calculated. The probable time for performing preventive maintenance activities prior to the ith job (i.e. T_{pm}) and the probable time for performing minimal repair (i.e. T_{mr}) can then be estimated by the current job sequence. Then, by computing the expected value of the lateness of $J[i]$ that is $E(L_{[i]})$ in Eq. 21, the expected value of the tardiness of $J_{[i]}$ can be obtained by using Eq. 23. Finally, with the objective function in Eq. 24, the maximum weighted tardiness for different job sequences is compared, and the optimal job sequence is solved with selected minimum weighted tardiness value through the whole recursive process. The computational result with the aim to minimize the maximum weighted tardiness for this example is presented in Table 2. The optimal schedule is

$$\mathrm{job4} - \mathrm{PM} - \mathrm{job1} - \mathrm{job2} - \mathrm{job6} - \mathrm{PM} - \mathrm{job3} - \mathrm{job5}$$

During the whole process, there will be two preventive maintenance activities as the optimal maintenance decision. The first one should be performed after job4, and the corresponding time for this preventive maintenance activity is $t_{p,a} = 10(a = 124)$. The second preventive maintenance activity should be performed after job6, and its corresponding maintenance time is $t_{p,a} = 7(a = 61)$. From the computational result, it can be found that there are three tardy jobs: job1, job3 and job5. The corresponding expected weighted tardiness for these three jobs is: 30, 70 and 24, respectively. Obviously, job3 (the fifth job in the sequence) has the maximum weighted tardiness which could be identified as the global optimal solution.

6.2 Results discussion

As this study explicitly tries to integrate production scheduling and preventive maintenance into one optimized scheduling model, in order to show its better performance

Table 2 The optimal job sequence obtained by this proposed scheduling model

J_j	J_4	PM	J_1	J_2	J_6	PM	J_3	J_5
$E(C_{[i]})$	36	46 (10)	71	78	107	114 (7)	156	174
$E(T_{[i]})$	0		30	0	0		70	24

than those of previous production scheduling models, three cases are discussed:

1. production scheduling model without maintenance planning;
2. individual production scheduling and preventive maintenance planning;
3. individual integrated production scheduling model and job's weight.

6.2.1 Case 1: Individual production scheduling without maintenance planning

In this section, job sequence is obtained by traditional production scheduling model. For the same example, as machine availability constraint is not taken into account (i.e. assume the machine is always available for processing jobs during the planning time horizon), there are no machine breakdowns and maintenance time, thus the solution is solved mainly based on job's processing time, without considering machine's degradation. The job sequence is obtained as

job4 − job1 − job2 − job3 − job6 − job5

The third row of Table 3 presents the tardiness of each job in this job sequence, the maximum tardiness is 24 of job3 without considering machine's failure. For failure rate in the continuous sense, continuous failure rate depends on a failure distribution which in this study is Weibull distribution. Then, as known in reliability engineering, the probability of failure prior to time t can be described by the cumulative failure distribution function, that is

$$P(T \le t) = F(t) = 1 - R(t) \tag{30}$$

And there is

$$F(t) = \int_0^t \lambda(t)\mathrm{d}t \tag{31}$$

As the total processing time of these jobs is over 157 and machine's initial effective age is 88, based on Eqs. 30 and 31 above, there should be at least one machine failure occurring because the probability of failure exceeds 1. For the probability of failure prior to time t can be calculated after each repair operation, then, considering the processing time of each job, the probability of failure exceeds 1 before processing job 1, 6 and 5 according to the obtained job sequence job4 − job1 − job2 − job3 − job6 − job5. Hence, three machine failures are supposed to occur after job 4, 3 and 6. Considering the time for performing minimal repair is 15, four tardy jobs will appear and the maximum tardiness is over 140. Hence, it is obvious that this job sequence performs much worse than the one obtained by the proposed improved scheduling model. If much more jobs are required to be processed, the machine provided scheduled maintenance should be much more effective than the ones without maintenance planning. Maintenance planning should be considered into production scheduling models, as maintenance is a much more comprehensive activity than repair, perhaps corresponding to the replacement of several key components in the machine.

Table 3 The job sequence obtained by traditional production scheduling (no maintenance planning)

J_j	J_4	J_1	J_2	J_3	J_6	J_5
$E(C_{[i]})$ (no failure)	36	61	68	110	139	157
$E(T_{[i]})$ (no failure)	0	0	0	24	20	7
$E(C_{[i]})$ (with failure)	36	76	83	125	169	202
$E(T_{[i]})$ (with failure)	0	45	0	39	140	52

6.2.2 Case 2: Individual production scheduling and preventive maintenance planning

This section discusses the example solved by considering individual production scheduling and preventive maintenance planning. First, the job sequence is obtained by production scheduling model. Then, preventive maintenance activities are planned. As shown in Section 6.2.1, if machine availability constraint is not taken into account in production scheduling model, the obtained job sequence is job4 − job1 − job2 − job3 − job6 − job5.

Then, preventive maintenance planning should be scheduled (i.e. pre-determine the optimal time intervals for maintenance activities). Because the objective of maintenance planning is to maximize machine availability, with the popular applications of Poisson distribution which is widely used to describe the number of machine failures during the operation horizon, the expected value of the number of machine failures during one operating cycle can be obtained as

$$E[\mathrm{Num}(t)] = \int_0^t \lambda(t)\mathrm{d}t = \int_0^t \frac{\beta}{\theta} \cdot \left(\frac{t}{\theta}\right)^{\beta-1} \mathrm{d}t \tag{32}$$

where Num (t) denotes the number of machine failures in t time units of machine's operation. Note that preventive maintenance activity is supposed to be able to restore machines to be "as good as new", hence, the operation of maintenance activities can be modelled as a renewal process, and the renewal points are the initiation of machine's operation and the end of the corresponding preventive maintenance activity. As each cycle between the

Table 4 The job sequence rescheduled by adding individual maintenance planning

J_j	PM	J_4	PM	J_1	J_2	PM	J_3	PM	J_6	J_5
$E(C_{[i]})$	8 (8)	44	50 (6)	75	82	88 (6)	130	136 (6)	154	183
$E(T_{[i]})$		0		42	0		44		80	33
Idleness		22		26		16				

renewal points consists of operation time, time for performing minimal repair and time for performing preventive maintenance activity, the resulting steady-state availability of the machine is constructed as

$$A(t) = \frac{t}{t + E[\mathrm{Num}(t)] \cdot t_{\mathrm{mr}} + t_{\mathrm{pm}}} \tag{33}$$

where $A(t)$ denotes the steady-state machine availability. Based on Eq. 33, an optimal time interval for preventive maintenance activities by differentiation can be yielded as

$$t^* = \theta \cdot \left[\frac{t_{\mathrm{pm}}}{t_{\mathrm{mr}} \cdot (\beta - 1)} \right]^{(1/\beta)} \tag{34}$$

Thus, for this example, the time interval for preventive maintenance activity solved by Eq. 34 is t^*=57.7 (i.e. one preventive maintenance activity should be performed every 57.7 time units of machine's operation, which is considered as periodic preventive maintenance process). Based on this maintenance time interval, the job sequence is rescheduled by adding maintenance activities (shown in Table 4), that is

$$\mathrm{PM} - \mathrm{job4} - \mathrm{PM} - \mathrm{job1} - \mathrm{job2} - \mathrm{PM} - \mathrm{job3} - \mathrm{PM}$$
$$- \mathrm{job6} - \mathrm{job5}$$

Compare with the results in Case 1, adding maintenance planning can reduce the tardiness and enhance the productivity, and save breakdown cost as well. However, there appears one issue. The reschedule job sequence brings the idleness due to the fixed maintenance time interval. Because machine's initial effective age is $a_0 = 8857.7$, one maintenance activity should be performed before the first job (job4). As the total processing time of job4 and job1 is 61, another maintenance activity should be performed before job1. The idleness is thus 57.7–36≈22. In this case, there are four maintenance activities and the total idleness reaches 64. Compare with the result obtained by the proposed scheduling model, the job sequence considering individual production scheduling and preventive maintenance planning has longer tardiness and even brings idleness. Hence, the improved production scheduling model with controllable time interval for maintenance activity can make full use of machine availability and avoid machine idleness, which will optimize real production processes.

6.2.3 Case 3: Individual integrated production scheduling model and weight of jobs

This section discusses the influences of job's weight in the construction of improved production scheduling model. First, the job sequence is obtained by using this integrated scheduling model without the consideration of job's weight. Then, each job's weighted tardiness is the product of its obtained tardiness and its corresponding weight. In real systems, job's weight can describe their different value. It usually appears when multiple products should be processed on a single product line or machine.

When simplifying this improved scheduling model to meet no weight constraint, suppose that each job has the same value ratio while being processed (i.e. each job's weight is pre-defined to be 1 in Eq. 21). Hence, the expected value of the lateness should be

$$E\left(L_{[j]}\right) = E\left(C_{[j]}\right) - d_j \tag{35}$$

The expected value of the corresponding tardiness is

$$E\left(T_{[j]}\right) = \max\left\{0, E\left(L_{[j]}\right)\right\} = \max\left\{0, E\left(C_{[j]}\right) - d_j\right\} \tag{36}$$

Based on Eq. 36, the job sequence is obtained as

$$\mathrm{job4} - \mathrm{PM} - \mathrm{job1} - \mathrm{job3} - \mathrm{job2} - \mathrm{PM} - \mathrm{job6} - \mathrm{job5}$$

In Table 5, the maximum tardiness of the job sequence without weight constraint is 27 (seen in the third row). Then, by adding job's weight, the maximum tardiness of the job sequence is 126 (seen in the fourth row). The numerical result is much larger than that obtained by our proposed scheduling model (i.e. 70 shown in Table 2). Thus, the proposed improved production scheduling model can

Table 5 The job sequence obtained by considering individual job's weight

J_j	J_4	PM	J_1	J_3	J_2	PM	J_6	J_5
$E(C_{[i]})$	36	46 (10)	71	113	120	127 (7)	156	174
$E(T_{[i]})$ (no weight)	0		10	27	18		22	24
$E(T_{[i]})$ (with weight)	0		30	27	126		88	24

exactly reduce the tardiness. For the due-date-based function of scheduling problems with the consideration of weighted tardiness, it does not necessarily emphasize processing jobs with shorter due date earlier in the sequence. The decision maker should integrate job's weight into scheduling models to obtain the optimal job sequence, which can greatly help optimize real-production processes and reduce time to market.

7 Conclusion and future work

As the importance of maintenance management in manufacturing processes has been gradually recognized by the decision maker, it becomes essential to schedule maintenance in manufacturing systems. However, a lot of previous research just considers performing production scheduling and maintenance planning individually, but does not consider them as an integrated model. In order to balance the trade-offs between them, this study is devoted to improve a single-machine scheduling model by incorporating both production scheduling and preventive maintenance planning with the aim to minimize the maximum tardiness. In order to be more practical, this improved scheduling model considers machine's deterioration process. In this model, maintenance time is variable subjected to machine degradation, which seems more reasonable in real-world scheduling cases. Moreover, job's weight is also considered in this proposed scheduling model so as to meet more complex manufacturing processes. If the jobs to be processed are the same in simple production scheduling cases, this improved production scheduling model could be also simplified to solve them. Through cases studies, the computational results show this improved production scheduling model performs better than those previous production scheduling models. Thus, this proposed scheduling model proves its efficiency of helping reduce tardiness as well as keep machines in good operation condition.

In spite of many meaningful extensions that this proposed scheduling model has lend itself to, there is still further research needed. As this study discusses a single-machine scheduling problem, extensions of multiple machines and/or flow-shop scheduling problems can be further researched. We intend to explore these extensions as well as improve more accurate and heuristic solution procedures in future research.

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