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Scheduling a maintenance activity and due-window assignment on a single machine

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ABSTRACT

We study a single machine scheduling and due-window assignment problem. In addition to the traditional decisions regarding sequencing the jobs and scheduling the due-window, we allow an option for performing a maintenance activity. This activity requires a fixed time interval during which the machine is turned off and no production is performed. On the other hand, after the maintenance time, the machine becomes more efficient, as reflected in the new shortened job processing times. The objective is to schedule the jobs, the due-window and the maintenance activity, so as to minimize the total cost consisting of earliness, tardiness, and due-window starting time and size. We introduce an efficient (polynomial time) solution for this problem.

1. Introduction

Liman et al. [1] considered the following scheduling and due-window assignment problem: n jobs need to be processed on a single machine around a common due-window; jobs completed within the due-window are considered as on-time jobs, whereas early and tardy jobs are penalized. There are four cost components: for earliness, tardiness, due-window starting time and due-window size. The objective is to schedule all jobs and to determine the due-window size and location such that the total cost is minimized. Liman et al. introduced an $O(n \log n)$ solution for this problem.

In this note we extend the above model to include an additional (optional) maintenance activity. This activity requires a fixed time interval during which the machine is turned off and production is stopped. On the other hand, after the maintenance time, the machine becomes more efficient, as reflected in the new shortened job processing times. Thus, in addition to the traditional job scheduling and due-window assignment decisions, our objective includes a decision regarding the time for scheduling the maintenance activity.

Scheduling a maintenance activity became a popular topic among researchers in the last decade. Among the recent papers, Ji et al. [2] studied a model of scheduling a periodic maintenance on a single machine, and Yang et al. [3] focused on a separated maintenance activity in flow-shops. Lee and Leon [4] introduced the basic scheduling model with a maintenance activity that improves the machine's efficiency (a rate-modifying activity). They studied several classic measures such as makespan, flow-time, weighted flow-time and maximum tardiness. Mosheiov and Oron [5] studied the same setting with a due-date assignment problem. Most other references considered either (i) a given *fixed* period of maintenance (i.e. a setting of non-availability time constraints—see e.g. Lee [6]), or (ii) a situation where the time of the maintenance activity is determined by the scheduler, but must be completed within a time interval and does not affect the machine efficiency (see e.g. Lee and Chen [7] and Kubzin and Strusevich [8]). Our problem includes scheduling decisions regarding (i) the jobs, (ii) the due-window, and (iii) the maintenance activity. We introduce a polynomial time solution requiring $O(n^4)$ time (where n is the number of jobs).

In the second section we provide the notation and the formulation of the problem. The third section contains the important properties of an optimal schedule. The (polynomial time) solution method is presented in Section 4, followed by a detailed numerical example given in the fifth section. Conclusion and ideas for future research are discussed in the last section.

2. Formulation

We study a non-preemptive, n-job, single-machine setting. The scheduler has an option to schedule a maintenance activity which lasts t units of time. During the maintenance no production is performed. The processing time of job j is denoted by P_j if the job is processed prior to the maintenance activity, and $\lambda_j P_j (0 < \lambda_j \leqslant 1)$ if it is scheduled after it, $j = 1, \ldots, n$. λ_j is the *modifying rate* of job j. For a given job sequence, the processing time of the job in the j-th position, and the modifying rate of the job in the j-th position are denoted by $P_{(j)}$ and $\lambda_{(j)}$, respectively. The completion time

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of job j is denoted by C_j, $j=1,\dots,n$. We assume that all the jobs share a common due-window. Let d_1 and d_2 ($\geqslant d_1$) denote the starting time and the finishing time of the due-window, respectively. Let $D=d_2-d_1$ denote its size. The earliness of job j is given by $E_j=\max\{0, d_1-C_j\}$, $j=1,\dots,n$. Similarly, the tardiness of job j is given by $T_j=\max\{0, C_j-d_2\}$, $j=1,\dots,n$. There are four cost components: α and β represent the earliness and tardiness unit costs, respectively, γ denotes the unit cost of (delaying) the due-window starting time, and δ is the unit cost of (increasing) the due-window size. All four unit cost components are clearly non-negative. The objective is to determine (i) the job schedule, (ii) the time to schedule the maintenance activity, and (iii) the time and length of the due-window, such that the following objective function is minimized:

$$Z=\sum_{j=1}^{n}\{\alpha E_j+\beta T_j+\gamma d_1+\delta D\}.$$

3. Properties of an optimal schedule

We begin by introducing several properties of an optimal solution. One trivial property is that an optimal schedule starts at time zero and contains no idle time between consecutive jobs. Another trivial property is that an optimal schedule exists in which the maintenance activity is performed prior to the starting time of one of the jobs. (Otherwise, if the maintenance starts $\varDelta$ units of time after the starting time of, say, job k, performing the maintenance activity $\varDelta$ time units earlier does not affect the completion time of earlier jobs, while the completion time of job k and the following jobs is reduced.) The third property is presented in the following lemma:

Lemma 1. *(i) If $\gamma>\delta$, an optimal schedule exists in which the due-window starts at time zero. (ii) If $\beta<\min\{\gamma,\delta\}$, an optimal schedule exists in which the due-window is reduced to a due-date that starts (and is completed) at time zero.*

Proof. (i) Suppose that $\gamma>\delta$ and $d_1>0$. We shift d_1, X units of time to the left. The change in the total cost is given by: $\Delta Z=X\delta n-X\gamma n-X\alpha k$, where k denotes the number of early jobs. Clearly, $\Delta Z<0$ (since $\gamma>\delta$ and $\alpha\geqslant 0$). Therefore, a shift of d_1 (until $d_1=0$) can only decrease the total cost.

(ii) Consider a due-window which starts at time $d_1>0$. Since $\beta<\gamma$, a shift of the entire window to start at time zero (without changing its size) can only reduce the cost. Since $\beta<\delta$, a further shift of d_2 to coincide with d_1 (at time zero) can only reduce the cost. □

Lemma 2 provides an additional property with respect to the due-window starting and finishing times in the general case:

Lemma 2. *An optimal schedule exists in which the due-window starting time (d_1), and the due-window completion time (d_2) coincide with job completion times.*

Proof. Suppose that there exists a schedule starting at time zero, and containing jobs at the k-th and the k+m-th positions such that: $C_k<d_1<C_{k+1}$ and $C_{k+m}<d_2<C_{k+m+1}$.

First we show that a small shift of d_2 either to the right or to the left can only decrease (does not increase) the total cost. When we shift d_2, X units of time to the right, the total tardiness cost decreases, whereas the cost of the due-window size increases. The change in the total cost is given by

$$\begin{aligned}\Delta Z&=-\beta(n-(k+m))X+\delta nX\\&=X(\delta n-\beta(n-(k+m))).\end{aligned}$$

When we shift d_2, X units of time to the left, the total tardiness cost increases, whereas the cost of the due-window size decreases. The change in the total cost is clearly given by $-\Delta Z$. If ΔZ is positive, a shift of d_2 to the left reduces the total cost. Otherwise, a shift of d_2 to the right reduces the cost. (If $\Delta Z=0$, then a shift to either side does not increase the total cost.) Therefore, there exists an optimal schedule in which d_2 coincides with a job completion time.

Now we show that a small shift of d_1 either to the right or to the left decreases (does not increase) the total cost. When d_1 is shifted X units of time to the left, the total earliness cost and the cost of the due-window starting time decrease, whereas the cost of the due-window size increases. The change in the total cost is given by

$\Delta Z=-\alpha kX-\gamma nX+\delta nX=X(\delta n-\gamma n-\alpha k)$. When we shift d_1, X units of time to the right, the change in the total cost is easily shown to be $-\Delta Z$. Again, a shift of d_1 either to the right or to the left does not increase the total cost.

Therefore, an optimal schedule exists such that both d_1 and d_2 coincide with job completion times. □

Comment: Based on Lemmas 1 and 2, both d_1 and d_2 can either be equal to zero, or coincide with a job completion time.

Given Lemma 2, let k and k+m denote the indices of the jobs completed at times d_1 and d_2, respectively, i.e., $C_k=d_1$ and $C_{k+m}=d_2$. The following lemma indicates that k and k+m are functions of the cost parameters and *independent* of the location and size of the maintenance activity.

Lemma 3. *An optimal schedule exists in which the index of the job completed at the due-window starting time is $k=\lceil n(\delta-\gamma)/\alpha\rceil$, and the index of the job completed at the due-window finishing time is $k+m=\lceil n(\beta-\delta)/\beta\rceil$.*

Proof. By the standard technique of small perturbations (of d_1 and d_2 to both sides). □

We are now ready to discuss the optimal schedule of the maintenance activity. Lemma 4 restricts the number of possible options for its position. Let l denote the index of the first job scheduled after the maintenance activity:

Lemma 4. *An optimal solution exists in which one of the following three cases holds*:

Case 1: *The maintenance activity starts at time zero* (*i.e.*, $l=1$).

Case 2: *The maintenance activity is not performed at all.*

Case 3: *The maintenance activity is scheduled after the completion time of the due window* (*therefore, the relevant values of l are*: $l=k+m+1,\dots,n$).

Proof. Recall that an optimal schedule exists in which the maintenance activity is scheduled prior to the starting time of one of the jobs. Case 3 complies with this property. Case 1 clearly holds for sufficiently small t values ($t\to 0$). On the other hand, for sufficiently large λ_i values ($\lambda_i\to 1$ for all i) and a positive t, an optimal schedule exists without performing the maintenance activity altogether, i.e., Case 2 holds.

We now prove that scheduling the maintenance activity *prior* to the due-window (unless at time zero) is never optimal. Suppose that the maintenance activity is performed at the completion time of, say job j, scheduled prior to the window. A shift of the maintenance activity to start at time zero, reduces the earliness cost of jobs $1,2,\dots,j$: they are shorter (their new processing time is $\lambda_i P_i$), and the maintenance activity has been removed (thus, their earliness decreases by t units of time). An additional cost reduction is due to the decrease of the due-window starting time.

Next we prove that scheduling the maintenance activity at the completion time of one of the jobs *inside* the due-window, is never optimal. In such a case, a shift of the maintenance to start at the beginning of the window reduces the window size and does not affect its starting time and earliness and tardiness costs.

We conclude that the candidate positions for scheduling the maintenance activity are either (i) at time zero, or (ii) at the beginning of the due-window, or (iii) prior to one of the jobs scheduled after the window. We show, however, that case (ii) is never optimal, unless the window starts at time zero. When comparing cases (i) and (ii), the earliness cost and the cost of the due-window size are smaller in case (i), whereas the cost of the due-window starting time may be smaller in case (ii). Consider an extreme case where $\lambda_i = 1$ for all early jobs. In this situation, the cost in case (ii) is smaller than that of case (i) by $t(\gamma - \delta)$. Case (ii) is better if $\gamma > \delta$. Here, however, an optimal due-window starts at time zero (see Lemma 1). Thus, cases (i) and (ii) coincide: in both, the maintenance activity is scheduled to start at time zero. We conclude that an optimal schedule exists in which either Cases 1, 2, or 3 is optimal. $\square$

4. A polynomial time solution

In this section we analyze separately the solution procedures for each of the three cases specified in Lemma 4. We show that the running time for the procedures in Cases 1 and 2 is $O(n\log n)$, while the complexity of the solution procedure in Case 3 is $O(n^4)$.

Case 1 (*The maintenance activity starts at time zero*). Note that this case is a special case of Liman et al. [1], where the sequence starts at time t (at the completion time of the maintenance activity), and the job processing times are $\lambda_j P_j$. The total cost is given by

$$\begin{aligned}
Z &= a\sum_{j=1}^{n} E_j + \beta\sum_{j=1}^{n} T_j + \gamma n d_1 + \delta n D \\
&= \alpha\sum_{j=1}^{k}(d_1 - C_j) + \beta\sum_{j=k+m+1}^{n}(C_j - d_2) \\
&\quad + \gamma n\left(t + \sum_{j=1}^{k}\lambda_{(j)}P_{(j)}\right) + \delta n\left(\sum_{j=k+1}^{k+m}\lambda_{(j)}P_{(j)}\right) \\
&= \gamma n t + \alpha\sum_{j=1}^{k}(j-1)\lambda_{(j)}P_{(j)} + \beta\sum_{j=k+m+1}^{n}(n-j+1)\lambda_{(j)}P_{(j)} \\
&\quad + \gamma n\sum_{j=1}^{k}\lambda_{(j)}P_{(j)} + \delta n\left(\sum_{j=k+1}^{k+m}\lambda_{(j)}P_{(j)}\right) \\
&= \gamma n t + \sum_{j=1}^{k}[\alpha(j-1) + \gamma n]\lambda_{(j)}P_{(j)} \\
&\quad + \sum_{j=k+1}^{k+m}[\delta n]\lambda_{(j)}P_{(j)} + \sum_{j=k+m+1}^{n}[\beta(n-j+1)]\lambda_{(j)}P_{(j)} \\
&= \gamma n t + \sum_{j=1}^{n} W_j(\lambda_{(j)}P_{(j)}),
\end{aligned}$$

where

$$W_j = \begin{cases} \alpha(j-1) + \gamma n, & j = 1, \ldots, k \\ \delta n, & j = k+1, \ldots, k+m \\ \beta(n-j+1), & j = k+m+1, \ldots, n. \end{cases}$$

As in Liman et al., we calculate $\alpha(j-1) + \gamma n$, δn and $\beta(n-j+1)$ for each $j = 1, \ldots, n$. The relevant positional weight is the smallest of these three (for a given j).

We define: $W_j = \min\{\alpha(j-1) + \gamma n, \delta n, \beta(n-j+1)\}$ for $j = 1, \ldots, n$. Note that based on the above positional weights, k is easily obtained as the largest j value such that $\alpha(j-1) + \gamma n \leqslant \min\{\delta n, \beta(n-j+1)\}$; If no j value satisfies this inequality, then clearly $k = 0$, i.e., there are no early jobs (either the due-window starts at time zero if $\gamma > \delta$, or the due-window starts at time t if $\gamma < \delta$). Similarly, $k+m$ is the largest j value such that $\delta n \leqslant \min\{\alpha(j-1) + \gamma n, \beta(n-j+1)\}$; If no j value satisfies this inequality, then $m = 0$ (the due-window is reduced to a due-date). Recall that k and m can be calculated directly as indicated in Lemma 3.

The complexity of the solution procedure in this case is equivalent to a standard matching of processing times to positional weights, which is known to be $O(n\log n)$.

Case 2 (*The maintenance activity is not performed at all*). This case is identical to Liman et al. [1]. The running time was shown to be $O(n\log n)$.

Case 3 (*The maintenance activity is scheduled after the due-window*). Given l (the index of the job scheduled after the maintenance activity), the total cost is given by

$$\begin{aligned}
Z &= a\sum_{j=1}^{n} E_j + \beta\sum_{j=1}^{n} T_j + \gamma n d_1 + \delta n D \\
&= \alpha\sum_{j=1}^{k}(d_1 - C_j) + \beta\left[\sum_{j=k+m+1}^{l-1}(C_j - d_2) + \sum_{j=l}^{n}(C_j - d_2)\right] \\
&\quad + \gamma n\sum_{j=1}^{k} P_{(j)} + \delta n\left(\sum_{j=k+1}^{k+m} P_{(j)}\right) \\
&= \alpha\sum_{j=1}^{k}(j-1)P_{(j)} \\
&\quad + \beta\left[\sum_{j=k+m+1}^{l-1}(n-j+1)P_{(j)} + (n-l+1)t\right. \\
&\quad \left. + \sum_{j=l}^{n}(n-j+1)\lambda_{(j)}P_{(j)}\right] \\
&\quad + \gamma n\sum_{j=1}^{k} P_{(j)} + \delta n\sum_{j=k+1}^{k+m} P_{(j)} \\
&= \sum_{j=1}^{k}[\alpha(j-1) + \gamma n]P_{(j)} + \sum_{j=k+1}^{k+m}\delta n P_{(j)} \\
&\quad + \sum_{j=k+m+1}^{l-1}\beta(n-j+1)P_{(j)} \\
&\quad + \sum_{j=l}^{n}\beta(n-j+1)\lambda_{(j)}P_{(j)} + \beta(n-l+1)t.
\end{aligned}$$

Given k, m and l, the cost associated with job j if scheduled in position r, denoted by $Z(j,r)$, is given by

$$Z(j,r) = [\alpha(r-1) + \gamma n]P_j \quad r = 1, \ldots, k$$

$$Z(j,r) = \delta n P_j \quad r = k+1, \ldots, k+m$$

$$Z(j,r) = \beta(n-r+1)P_j \quad r = k+m+1, \ldots, l-1$$

$$Z(j,r) = \beta(n-r+1)\lambda_j P_j + \beta t \quad r = l, \ldots, n.$$

Comment: The last cost component in the above objective function, i.e., $\beta(n-l+1)t=\sum_{j=l}^{n}\beta t$ refers to the set of jobs scheduled after the maintenance activity. Therefore, we add βt to the cost associated with each of the $n-l+1$ jobs included in this set.

As mentioned before, we already know the number of early, tardy and on time (located inside the window) jobs. Therefore, for a given location of the maintenance activity, our problem becomes identical to the classical assignment problem and can be solved in $O(n^3)$ time (using the well-known Hungarian method). Since the maintenance activity can be scheduled prior to any one of the tardy jobs $(k+m+1,\dots,n)$, we should solve $n-k-m$ assignment problems. Thus, the complexity of this (dominant) case is $O(n^4)$.

We conclude that an optimum for our problem is found by solving each of the three cases. The best solution found is the global optimum. The overall complexity is $O(n^4)$ due to the solution of Case 3.

5. A numerical example

A solution of an 8-job problem is demonstrated in the following. The job processing times and the modifying rates are: $P_1=15$, $P_2=9$, $P_3=26$, $P_4=104$, $P_5=10$, $P_6=2$, $P_7=25$, $P_8=82$ and $\lambda_1=0.6$, $\lambda_2=0.42$, $\lambda_3=0.16$, $\lambda_4=0.39$, $\lambda_5=0.7$, $\lambda_6=0.36$, $\lambda_7=0.51$, $\lambda_8=0.11$. The cost parameters are: $\alpha=2$, $\beta=25$, $\gamma=15$ and $\delta=15.6$. The duration of the maintenance activity is $t=75$.

We solve each case separately:

Case 1 (*The maintenance activity starts at time zero*). $\alpha(j-1)+\gamma n=(120,122,124,126,128,130,132,134)$.

$\delta n=(124.8, 124.8, 124.8, 124.8, 124.8, 124.8, 124.8, 124.8)$.

$\beta(n-j+1)=(200, 175, 150, 125, 100, 75, 50, 25)$.

Thus,

$W_j=(120, 122, 124, 124.8, 100, 75, 50, 25)$.

$\lambda_j P_j=(7, 4.16, 3.78, 0.72, 9, 9.02, 12.75, 40.56)$.

The optimal solution for Case 1 consists of three early jobs, one job inside the due-window, and four tardy jobs. The starting and the finishing times of the due-window are $d_1=89.94$ and $d_2=90.66$. The total cost in Case 1 is

$$Z=\gamma nt+\sum_{j=1}^{n}W_j(\lambda_j P_j)=9000+5134.096=14,134.096.$$

Case 2 (*The maintenance activity is not performed al all*). $W_j=(120, 122, 124, 124.8, 100, 75, 50, 25)$.

$P_j=(15, 10, 9, 2, 25, 26, 82, 104)$.

As in Case 1, the optimal solution consists of a single job scheduled inside the window. $d_1=34$, $d_2=36$. The total cost is: $Z=\sum_{j=1}^{n}W_jP_j=15,535.6$.

Case 3 (*The maintenance activity is scheduled after the due-window*). For $l=k+m+1=5$ (the maintenance activity is scheduled before the fifth job), the cost associated with job j if scheduled in position r is given by

$$Z(j,r)=[\alpha(r-1)+\gamma n]P_j, \qquad r=1,2,3$$
$$Z(j,r)=\delta nP_j, \qquad r=4$$
$$Z(j,r)=\beta(n-r+1)\lambda_jP_j+\beta t, \quad r=5,\dots,8.$$

Thus,

Z(j,r)	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$	$r=8$
$j=1$	1800	1830	1860	1872	2775	2550	2325	2100
$j=2$	1080	1098	1116	1123.2	2253	2158.5	2064	1969.5
$j=3$	3120	3172	3224	3244.8	2291	2187	2083	1979
$j=4$	12,480	12,688	12,896	12,979.2	5931	4917	3903	2889
$j=5$	1200	1220	1240	1248	2575	2400	2225	2050
$j=6$	240	244	248	249.6	1947	1929	1911	1893
$j=7$	3000	3050	3100	3120	3150	2831.25	2512.5	2193.75
$j=8$	9840	10,004	10,168	10,233.6	2777	2551.5	2326	2100.5

The solution of the assignment problem leads to the following job sequence: (1, 5, 2, 6, 3, 8, 7, 4). $d_1=34$, $d_2=36$. The total cost is $Z=14,629.6$.

We repeat this procedure (a solution of an assignment problem) for $l=6$, 7, and 8.

For $l=6$: the job sequence is (1, 5, 2, 6, 7, 3, 8, 4). $d_1=34$, $d_2=36$. The total cost is $Z=14,287.6$.

For $l=7$: the job sequence is (1, 5, 2, 6, 7, 3, 8, 4). $d_1=34$, $d_2=36$. The total cost is $Z=14,050.6$.

For $l=8$: the job sequence is (1, 5, 2, 6, 7, 3, 8, 4). $d_1=34$, $d_2=36$. The total cost is $Z=15,824.6$.

When comparing the costs in Cases 1, 2, and 3 ($l=5$, 6, 7, 8), we conclude that the global optimum is obtained in Case 3 for $l=7$. As specified above, the job sequence is (1, 5, 2, 6, 7, 3, 8, 4), the due-window is of size 2, ($d_1=34$, $d_2=36$), three jobs are early, one job is scheduled inside the window, and four jobs are tardy. The maintenance activity is scheduled after the due-window between the sixth and the seventh jobs. The total cost is $Z=14,050.6$.

6. Conclusion

The option for performing a maintenance activity in scheduling problems has been studied by several researchers in recent years. We focus on a due-window assignment on a single machine, in which the objective is to minimize the costs of earliness, tardiness, due-window starting time, and due-window size. This classical problem is known to be solved in $O(n\log n)$ time. We extend this setting to allow a maintenance activity, after which the machine becomes more efficient. We introduce an $O(n^4)$ solution algorithm for this problem. Standard possible extensions for future research contain e.g. (i) job-dependent earliness and tardiness costs (a problem which is easily shown to be NP-hard), (ii) general non-decreasing earliness, tardiness and due-window costs, and (iii) a deteriorating maintenance activity (similar to the setting assumed in the recent paper of Kubzin and Strusevich, [8]).

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