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Scheduling n jobs and preventive maintenance in a single machine subject to breakdowns to minimize the expected total earliness and tardiness costs

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Abstract: This paper focuses on a stochastic scheduling problem in which n immediately available jobs are to be scheduled jointly with the preventive maintenance in a single machine subject to breakdowns. The objective is to minimize expected total earliness and tardiness costs with a common due-date. The problem of scheduling only n jobs to minimize total earliness and tardiness costs in a single machine subject to breakdowns is well known to be NP-hard. We first give a relevant literature review dealing with the single machine to minimize the ET-cost. After we introduce the process of breakdowns considered in this paper. Last we formulate a dynamic programming to solve the problem optimally.

1. INTRODUCTION

Production and maintenance are two important functions in any industrial process and are interrelated problems. In the past, production and maintenance have been treated as two separate functions. Nowadays because of the interdependence between them, there is an increasing interest to develop optimization models that take into consideration the integration of the two functions (Coudert *et al*. 2002, Raouf *et al*. 1995, Swanson 1999).

Most of the literature on scheduling assumes that machines are available at all times. However, due to maintenance activities machines cannot operate continuously without some kind of unavailability periods. In general, maintenance activities can be classified into two categories: preventive and corrective. For the corrective maintenance, the machine is repaired following any breakdown. On the other hand preventive maintenance PM is performed on the machine in order to reduce the increasing risk of machine breakdown. Thus for the preventive maintenance the machine is checked, repaired and re-calibrated before failure to be kept in an appropriate condition.

In this paper we consider the problem of jointly scheduling n available at time zero jobs and the preventive maintenance in a single machine subject to breakdowns to minimize expected total earliness and tardiness cost with a common due-date D. The processing times τ_j of the jobs j=1,...,n are deterministic and known. Suppose that pre-empting one job for another job is not permitted. On one hand, completion of a job j after the arrival of transporters will incur a much higher delivery cost. In this case the completion time of the job j is greater than or equal to the common due-date D, which is assumed as given. We denote the tardiness of the job j by $T_j=\max\{C_j-D,0\}$. On the other hand, the completion of a job before the arrival of transporters will incur an inventory cost. In this case the completion time of the job j is smaller than or equal to the common due-date D. The earliness of the job j will be denoted as $E_j=\max\{D-C_j,0\}$. The well known total earliness-tardiness cost which we refer to as the ET-cost is:

$$c_{ET}=\sum_{j=1}^{n}\alpha_j E_j+\sum_{j=1}^{n}\beta_j T_j \qquad (1)$$

Where α_j and β_j are, respectively, the per time unit penalties of the job j for being early or tardy. There are two categories of common due-date D; (i) D as a decision variable (unrestricted due date); (ii) and D as a given parameter (restricted due date). In this paper we deal only with the second case.

This type of performance measure is known to be a non-regular as opposed to a regular measure of performance which is an increasing function of the jobs' completion times (Lee *et al*. 1991). It corresponds to the just-in-time (JIT) concept where tardy or early jobs would generate penalties. We assume that jobs can not be pre-empted for PM and if a job is pre-empted for a breakdown it can be resumed after repair with or without penalty. In addition to choosing a job scheduling, one must also decide whether to perform or not PM prior to each job. The integrated problem is further complicated because the completions times are stochastic regarding to breakdowns. Although extensive research has been carried out on a single machine with common due date for minimizing ET-cost, to the best of our knowledge, the problem with the preventive maintenance and breakdowns has not been studied before. We first give a relevant literature review dealing with the single machine to minimize the ET-cost. After we introduce the process of breakdowns considered in this paper. Last we formulate a dynamic programming to solve the problem optimally.

2. LITERATURE REVIEW

The common due date single machine problems have been widely studied. For excellent surveys readers are referred to Raghavachari (Raghavachari 1988) and Baker and Scudder (Baker *et al.* 1990). The problem with restrictive common due-date is NP-hard even if $\alpha_j=\beta_j=1$. It has been proven for the first time by Hall *et al.* (Hall *et al.* 1991). That is problems with ($\alpha_j=\alpha$, $\beta_j=\beta$), ($\alpha_j=\beta_j$) and ($\alpha_j\neq\beta_j$) are also NP-hard (De *et al.* 1993, Hall *et al.* 1991, James *et al.* 1997). When ($\alpha_j=\beta_j=1$), Bagchi *et al.* (Bagchi *et al.* 1986) propose an algorithm enumerative in nature to solve the problem optimally. De *et al* (De *et al.* 1993) propose a solution methodology, based on dynamic programming that is pseudo-polynomial in its complexity. Sundararaghavan *et al.* (Sundararaghavan *et al.* 1984) propose a heuristic procedure. Hall *et al.* (Hall *et al.* 1991) provide a pseudo polynomial dynamic programming algorithm with O($n\sum_{i=1}^{n}\tau_i$) time running. This algorithm allows dealing with the problems up to 1000 jobs. Hoogeveen *et al.* (Hoogeveen *et al.* 1991) present a branch-and-bound algorithm based on Lagrangian lower and upper bounds and an O(nlogn) (4/3)-approximation algorithm. When ($\alpha_j=\alpha$ and $\beta_j=\beta$), Bagchi *et al.* (Bagchi *et al.* 1987) propose branching procedure which is suitable for the problems with a small size. For the problem with ($\alpha_j=\beta_j$), Hoogeveen *et al.* (Hoogeveen *et al. 1991*) give an O($n^2\sum_{i=1}^{n}\tau_i$) algorithm. Finally, concerning the problem with ($\alpha_j\neq\beta_j$), James (James. 1997) and other researchers use meta-heuristics to give approximation solutions.

The problem of scheduling a single machine subject to breakdowns to minimize the ET-cost has been studied by few researchers. Pinedo *et al.* (Pinedo *et al.* 1980) investigate a single machine problem in which the machine is subject to external shocks according to a non-homogeneous Poisson process. Glazebrook (Glazebrook 1984) focuses on a single machine problem and formulates it as a cost-discounted Markov decision process. Birge *et al.* (Birge *et al.* 1990) consider more general breakdown process. There are few researchers that explicitly try to integrate preventive maintenance and scheduling decisions on a single machine. All of them don't deal with the ET-cost and to the best of our knowledge the problem with the preventive maintenance to minimize the ET-cost has not been studied before. For instance Grave *et al.* (Grave *et al.* 1999) consider the problem to optimize weighted completion time and they take into consideration only one preventive maintenance period. Ji *et al.* (Ji *et al.* 2005) consider the same problem to minimize the makespan. Since the jointly scheduling jobs and preventive maintenance to minimize the ET-cost problem is never studied before, we develop a dynamic programming model to solve it optimally. The particularity of our model is that it could take into consideration more than one preventive maintenance period as well as the ET-cost.

3. PROBLEM STATEMENT

The problem of minimizing the earliness-tardiness cost when ($\alpha_j=\beta_j=1$), ($\alpha_j=\alpha$, $\beta_j=\beta$) and ($\alpha_j=\beta_j$) has been solved by pseudo polynomial dynamic programming algorithms. The new element we introduce is that of the preventive maintenance planning and a possible breakdown by the machine, which is a random event. The problem studied in this paper may be formulated to read as follows: What is the "best" time to perform overhaul on the machine in order to forestall the waste if it breaks down during the processing of a job? We assume a failure rate that increases with "age". The longer the machine is in use, the higher is the probability that it will fail. A known example of a distribution with increasing failure rate (IFR) is the Erlang distribution with c.d.f.

$$F(t) = 1 - \sum_{r=0}^{k-1} \frac{(\lambda t)^r e^{-\lambda t}}{r!} \qquad (2)$$

Now the problem decomposes into two parts: (i) How to schedule the n jobs so as to minimize the earliness-tardiness cost, and (ii) where would we place the maintenance period so as to minimize the expected loss. Observe that in Part (i) there is no randomness because we assume continuous machine availability. Therefore optimization is over a deterministic problem. However, Part (ii) relies on the probability distribution of machine failure and therefore we must take randomness into account. The major part of our model is the evaluation of cost. In Part (i) the costs are more-or-less straightforward, given the cost of carrying an item in inventory (earliness) and the cost of having the transporters waiting for the item (tardiness).

It is the determination of the cost in Part (ii) that requires careful analysis. Here is the reason why. If the machine were continuously available (no possibility of breakdown) then the answer given in Part (i) is optimal because we cannot do better, by the very definition of optimality of the ET-cost. However, Part (ii) implies that we shall intentionally insert the maintenance time immediately after the completion of job j, thus delaying all the jobs that are scheduled beyond that point in time; jobs j+1,...,n.

Consider time t at the completion of the job j. We have to choose between two decisions:

1. Do not perform maintenance on the machine. Here, there are two possible outcomes,

a. the machine does not fail during the processing of job j+1. This occurs with probability

$$\overline{F}(t + \tau_{j+1})$$

Where $\overline{F}(.)$ is the complementary distribution function (c.d.f.) of the life of the machine, and τ_{j+1} is the processing time of job j+1. In this eventuality the original schedule is kept intact, which we know is optimal, and the decision process moves on to time $t+\tau_{j+1}$ where we are faced with the same problem again but relative to job j+2.

b. the machine fails during the execution of job j+1, which occurs with probability

$$p = \int_{t}^{t+\tau_{j+1}} \frac{f(t)}{\overline{F}(t)} dt \qquad (3)$$

That means that a breakdown occurs in the machine. Now we are forced to perform the maintenance which is a repair activity on the machine and occupies a fixed duration t_R. Job j+1 and all the jobs that follow it (i.e., jobs j+1,...,n) are now delayed (at least) by an amount of time given by $Y+t_R$, in which Y is the time between the start of job j+1 and machine failure; Y is a random variable. The expected time to failure during that interval is given by

$$\overline{y} = \int_{t}^{t+\tau_{j+1}} \frac{f(t)\times t}{\overline{F}(t)} dt \qquad (4)$$

Therefore the expected delay of jobs j+1,...,n due to this machine failure, assuming no other machine failure will occur in the interval

$$\left[t+\overline{y}, t+\overline{y}+\sum_{i=j+1}^{n} \tau_i \right]$$

is

$$\overline{y}+t_R \qquad (5)$$

The cost of this delay (away from the original schedule) can be evaluated according to the equation 10. However, this cost will happen if the machine does not fail after time $t+\overline{y}$. This happens with probability

$$\overline{F}\left(\sum_{i=j+1}^{n} \tau_i\right)$$

With probability

$$1-\overline{F}\left(\sum_{i=j+1}^{n} \tau_i\right)$$

the machine will fail during this span of time, and additional delay will be experienced.

2. Perform maintenance on the machine. Now we know with certainty that jobs j+1,...,n will be delayed by t_R, and the cost of such delay can be evaluated according to the equation 9, again assuming the the machine does not fail after time $t+t_R$. We consider in this case that the repair activity and the preventive activity take the same duration of time which is t_R.

4. THE AGE OF MACHINE

Let us denote the number of failures in θ time units of machine operation by $N(\theta)$. $\Phi(\theta)$ will denote the expected value of $N(\theta)$. The age of the machine after processing the j[th] job in a given sequence is denoted by a_j. $\overline{a}_j$ will denote the age of the machine immediately prior to the j[th] job in the sequence (Fig.1.). We assume that the preventive maintenance renews the machine, and repair is minimal. Hence we have:

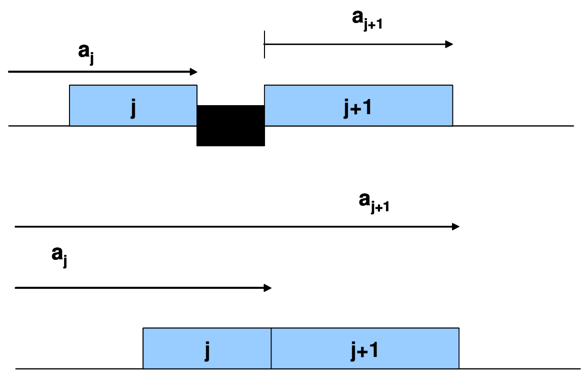
$$\overline{a}_{j+1} = a_j(1-d_j) \qquad (6)$$
$$a_{j+1} = \overline{a}_{j+1} + \tau_{j+1}$$

where:

$$d_j = \left\{ \begin{array}{l} 1 \text{ if PM is performed prior to the } j^{th} \text{ job} \\ 0 \text{ otherwise} \end{array} \right\} \qquad (7)$$

Then the expected value of the completion time of the j[th] job in the sequence can be formulated as follows:

$$\varepsilon(C_j) = \sum_{k=1}^{j-1} t_R d_k + t_R(\phi(a_{k+1}) - \phi(\overline{a}_k)) + \tau_{k+1} \qquad (8)$$



a_j : the machine age after processing the j[th] job in the sequence

Fig. 1. The age of machine

5. ILLUSTRATIVE EXAMPLE

To well illustrate the studied problem, we consider the following example with five jobs to treat under the single machine. We assume that pre-emption for preventive maintenance is not allowed and for a breakdown is with a penalty (the job has to be restarted).

	p_i	D	α_i	β_i
J_1	5	16	1	3
J_2	2	16	2	1
J_3	4	16	1	2
J_4	3	16	2	1
J_5	6	16	1	3

We suppose that the mean time between failure under the single machine is equal to 10 (MTBF=10). We consider that the maintenance activity renews the age of machine. In order to simplify the problem by not taking into consideration the expected value of the cost, we suppose that every 10 units of time we have a breakdown. We consider that the preventive duration (t_P) and the repairing duration after a breakdown (t_R) are equal (t_P=t_R=10). For instance, we evaluate the cost of the schedule ($J_1 \Rightarrow J_2 \Rightarrow J_3 \Rightarrow J_4 \Rightarrow J_5$). First, we consider the case where we don't perform the preventive maintenance during the time horizon (Fig.2.):

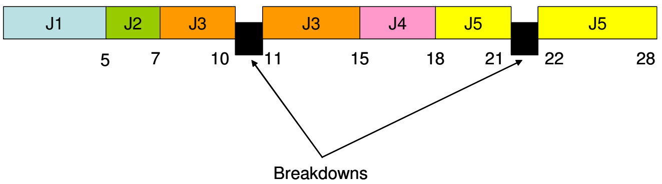


Fig.2. Without preventive maintenance

The objective function of this solution is

ET-cost = (11×1) + (9×2) + (1×1) + (2×1) + (12×2) = 56.

Now suppose that we perform once the preventive maintenance and we assume only one breakdown (Fig.3.). For the same job scheduling solution:

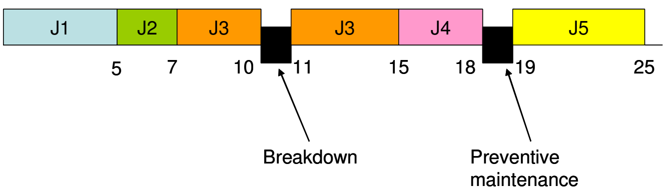


Fig.3. With one preventive maintenance period

The objective function of this solution is

ET-cost = (11×1) + (9×2) + (1×1) + (2×1) + (9×2) = 50.

Now suppose that we perform two times preventive maintenance which eliminates the risk of breakdown occurrence (Fig.4.). For the same job scheduling solution:

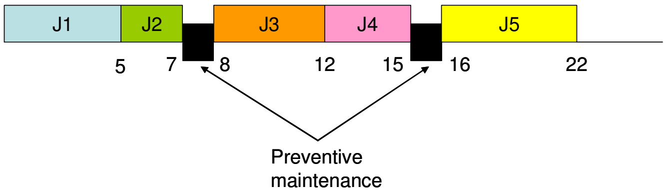


Fig.4. With two preventive maintenance periods

The objective function of this solution is

ET-cost= (11×1) + (9×2) + (4×1) + (6×2) = 45.

We have tested three solutions with different values of the objective function. In this case the fact to perform only preventive maintenance is better, but it's not always true. It depends on the scenario of breakdowns and the relationship between preventive maintenance and the age of machine. As we can see from this example, the studied problem is complex. We have to decide (1) when to start processing jobs? (2) in which sequence? (3) if we perform preventive maintenance after a given job or not?

6. DYNAMIC PROGRAMMING

In this section we formulate a dynamic programming when (α_j=β_j=1), (α_j=α, β_j=β) and (α_j=β_j). We can see from the above narrative the "recursive" nature of the problem. Indeed follow the procedure solving the problem without maintenance activities to end up with the optimal ET-schedule. It is given in a pseudopolinomial time running (see the literature review). Number the jobs in the order given by that schedule from 1 to n. Now iterate backwards. The stage of the DP iterations is the job number. The state of the DP model is the (absolute) time t and the "age" of the machine, which we shall denote by a. The decision at stage j shall be generically denoted by d_j . Clearly, d_j can take on two values: perform maintenance and do not perform maintenance, which we shall denote by 1 and 0; respectively. Let $v_j(t)$ denote the expected stage reward when job j is considered at time t; it is given by (in which $c_{ET}(\cdot)$ is the ET-cost),

$$d_j = 1 : v_j(t) = c_{ET}(t + t_R + \tau_j) \tag{9}$$

$$d_j = 0 : v_j(t) = \left\{ \begin{array}{c} c_{ET}(t + \overline{y}(a) + t_R + \tau_j), \text{ w.p } p \\ c_{ET}(t + \tau_j) \text{ w.p } 1 - p \end{array} \right\} \tag{10}$$

The rationale for (8) is that when maintenance is undertaken it will consume time t_R which is followed by the processing time of job j. The rationale for (9) is that when no maintenance is undertaken the machine will fail after time $\overline{y}(a)$, on the average, with probability p, and will not fail with probability 1-p. The value of p is given in (4) above. Therefore, the expected stage cost under decision $d_j=0$ is

$$d_j = 0 : \varepsilon\left[v_j(t)\right] = c_{ET}(t + \overline{y}(a) + t_R + \tau_j) \times p + c_{ET}(t + \tau_j) \times (1 - p) \qquad (11)$$

Finally, let $f_j(t,a)$ denote the minimal expected cost of stages j, j+1,...,n (the "tail" stages) when considered at time t with a machine of age a. We have,

$$f_n(t, a) = \min_{d_n=0,1} \varepsilon_Y \left[v_n(t_{(d_n)})\right] \qquad (12)$$

in which we indicated, for emphasis, that the completion time of job n depends on the decision made. Observe that the expectation is relative to the random variable. Y which signifies the behaviour of the machine during the period of processing job n, which is equal to τ_n. This initiates the iterations. For any stage j, $1 \leq j \leq n-1$, the extremal equation takes the form of

$$f_j(t, a) = \min_{d_j=0,1} \left\{ \begin{array}{c} \varepsilon_Y \left[v_j(t_{(d_j)})\right] \\ +\varepsilon\left[f_{j+1}(t', a')\right] \end{array} \right\}, \; j = n-1, ..., 1 \qquad (13)$$

in which both t′ and a′ are functions of the decision made:

- $t' = t + t_R + \tau_j$, if $d_j=1$;
- $t' = t + \tau_j$ if $d_j=0$ and the machine did not fail (which happens with probability 1-p);
- $t' = t + t_R + \tau_j + \overline{y}(a)$ if $d_j=0$ and the machine failed (which happens with probability p);
- $a' = a + t_R + \tau_j$, if $d_j=1$;
- $a' = a + \tau_j$ if $d_j=0$ and the machine did not fail (which happens with probability 1-p);
- $a' = a + \overline{y}(a) + t_R + \tau_j$ if $d_j=0$ and the machine failed (which happens with probability p).

Observe that $\overline{y}(.)$ in the above expressions is a function of the age of the machine, a. The problem is solved when we secure $f_1(0,0)$.

The running time of this algorithm is still manageable for problems with small sizes. For instance if the planning horizon is one day and the life of the machine is a random variable over a span of two weeks. One (rather crude) way to ameliorate the computational burden is to (i) assume the jobs to be "uniform", meaning they all take the same processing time $\overline{\tau}$, which is the average processing time of the population of jobs, and (ii) assume the horizon to be infinite (very large number of "uniform" jobs).

7. CONCLUSION

In this paper we have studied the problem of scheduling production jobs and planning preventive maintenance in a single machine subject to breakdowns. The objective is to minimize the expected total earliness-tardiness cost with a common due date. We have given a relevant literature considering on one hand the common due date in a single machine to minimize the ET-cost and on the other hand the integration of scheduling jobs and preventive maintenance planning. We have introduced the stochastic process of breakdowns considered for the problem studied in this paper. The major contribution of this paper is the formulation of a dynamic programming algorithm to solve the problem optimally. Our future works could be directed to focus on some properties of the optimal solution in order to reduce the running time of our algorithm and propose heuristics to give approximation solutions.

REFERENCES

Bagchi, U., Sullivan, R.S., Chang, Y.L., (1986). Minimizing mean absolute deviation of completion times about a common due date. Naval Research Logistics 33, 227--240.

Bagchi, U., Chang, Y.L., Sullivan, R.S., (1987). Minimizing absolute and squared deviations of completion times with different earliness and tardiness penalties and a common due date. Naval Research Logistics 34, 739--751.

Baker, K.R., Scudder, G.D., (1990). Sequencing with earliness and tardiness penalties: A review. Operations Research 38, 22--36.

Birge, J.R, Frenk, J.B.G., Mittenhall, J., and Rinnooy Kan, A.H.G., (1990), Single-machine scheduling subject to stochastic breakdowns, Naval Research Logistics. 37, 661-677.

Coudert, T., Grabot, B., and Archimède, B., (2002). Production/maintenance cooperative scheduling using multi-agents and fuzzy logic. International Journal of Production Research, 40-18, 4611 -- 4632.

De, P., Ghosh, J.B., Wells, C.E., (1993). On the general solution for a class of early/tardy problems. Computers and Operations Research 20, 141--149.

Glazebrook, K.D., (1984). Scheduling stochastic jobs on a single machine subject to breakdowns. Naval Research Logistics, 31, 251-264.

Hall, N.G., Kubiak, W., Sethi, S.P., (1991). Earliness--tardiness scheduling problems, II: Deviation of completion times about a restrictive common due date. Operations Research 39, 847--856.

Graves, G. H., and Lee, C.-Y., (1999). Scheduling Maintenance and Semiresumable Jobs on a Single Machine. Naval Research Logistics, 46, 845-863.

Hoogeveen, J.A., van de Velde, S.L., (1991). Scheduling around a small common due date. European Journal of Operational Research 55, 237--242.

James, R.J.W., (1997). Using tabu search to solve the common due date early/tardy machine scheduling problem. Computers and Operations Research 24, 199--208.

Ji, M., He, Y. and Cheng, T.C.E, (2005). Single-machine scheduling with periodic maintenance to minimize makespan. Computers & Operations Research, In P.C.P.

Lee, C.Y., Danusaputro, S.L. and Lin, C.S., (1991). Minimizing weighted number of tardy jobs and weighted earliness-tardiness penalties about a common due date. Computers & Operations Research, 18-4, 379-389.

Pinedo, M. and Ross, S.H, (1980). Scheduling jobs subject to nonhomogeneous Poisson shocks. Management Science, 26, 1250-1257.

Raghavachari, M., (1988). Scheduling problems with non-regular penalty functions -- a review. Operations Research 25, 144--164.

Raouf A., and Ben-Daya M., (1995). Total maintenance management: a systematic approach. Journal of Quality in Maintenance Engineering, 1-1, 6-14(9).

Sundararaghavan, P.S., Ahmed, M.U., (1984). Minimizing the sum of absolute lateness in single-machine and multimachine scheduling. Naval Research Logistics Quarterly 31, 325--333.

Szwarc, W., (1989). Single-machine scheduling to minimize absolute deviation of completion times from a common due date. Naval Research Logistics 36, 663--673.

Swanson, L., (1999). The impact of new production technologies on the maintenance function: an empirical study. International Journal of Production Research, 37-4, 849 -- 869.