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Theory and Methodology

Single-machine scheduling with maintenance and repair rate-modifying activities

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Abstract

Most papers in the scheduling field are based on the assumption that machines are always available at constant speed. However, in industry applications, it is very common for a machine to be in subnormal condition after running for a certain period of time. Motivated by a problem commonly found in the surface-mount technology of electronic assembly lines, this paper deals with scheduling problems involving repair and maintenance rate-modifying activities. When a machine is running at less than an efficient speed, a production planner can decide to stop the machine and maintain it or wait and maintain it later. If the choice is made to continue running the machine without fixing it, it is possible that the machine will break down and repair will be required immediately. Both maintenance and repair activities can change the machine speed from a sub-normal production rate to a normal one. Hence, we call them rate-modifying activities. Our purpose here is to simultaneously sequence jobs and schedule maintenance activity to optimize regular performance measures. In this paper, we assume that processing time is deterministic, while machine break down is a random process following certain distributions. We consider two types of processing cases: resumable and nonresumable. We study problems with objective functions such as expected makespan, total expected completion time, maximum expected lateness, and expected maximum lateness, respectively. Several interesting results are obtained, especially for the nonresumable case. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Machine scheduling; Maintenance; Breakdown; Rate-modifying activity

1. Introduction

Machine scheduling is concerned with the problem of optimally scheduling available resources to process jobs. This is a decision-making process that exists in most production systems, service systems, and

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information-processing environments. In the last four decades, many papers have been published in the scheduling area. Most of them deal with problems assuming that machines are always available at constant speed. However, in today's industry applications, it has become very common for a machine to be in subnormal condition after running for a certain period of time. For example, in the surface-mount technology lines of electronic assembly systems, some of the pick-and-place nozzles may not be working (Fig. 1) which results in the machine running at a less efficient speed (Lee and Leon, 2001). A production planner can decide to stop the machine and fix it, or can wait and fix it later. For notational convenience, we call the fixing activity a *maintenance*. On the other hand, if the production planner continues to run the machine without fixing it, it is possible that the machine will break down and will have to be repaired immediately. We call this activity a *repair*. Both maintenance and repair activity can change the machine speed from a sub-normal production rate to a normal one. Hence we call both activities *rate-modifying activities* since they can be expected to change the speed of the machine.

This paper deals with the problem of scheduling jobs and the rate-modifying activities of maintenance and repair. Our problem can be stated as follows: There are n jobs to be scheduled on a single machine. Associated with each job j, $j = 1, \dots, n$, there is a due date d_j, and there are two known processing durations p_j and q_j, representing the duration of job j if it is processed *before* and *after* a rate-modifying activity, respectively. Further, we assume $q_j = \alpha p_j$, where $0 < \alpha \leqslant 1$ is a given positive real number. A maintenance activity takes t units of time while a repair activity is a nonnegative random variable V with an expected value of $E[V] = v$ units of time. We assume $v > t$. The decisions under consideration are (i) when to schedule the maintenance activity, and (ii) how to sequence the jobs, to optimize some regular performance measure. Hence there are two features of this problem, (i) a machine may not be always available due to machine breakdown or maintenance, and (ii) a machine may have different speeds before and after maintenance or repair.

In this paper, we consider two types of processing cases: *resumable* and *nonresumable*. Suppose that a machine breakdown or maintenance interrupts the processing of a job. If the job will continue after the machine becomes available again, we call the problem resumable. On the other hand, the problem is called nonresumable if the job has to restart from the beginning when the machine becomes available (Lee, 1996). Note that resumable and nonresumable are also referred to as preempt-resume and preempt-repeat, respectively, in the literature (see Pinedo, 1995). In this paper, we study problems representing both cases with

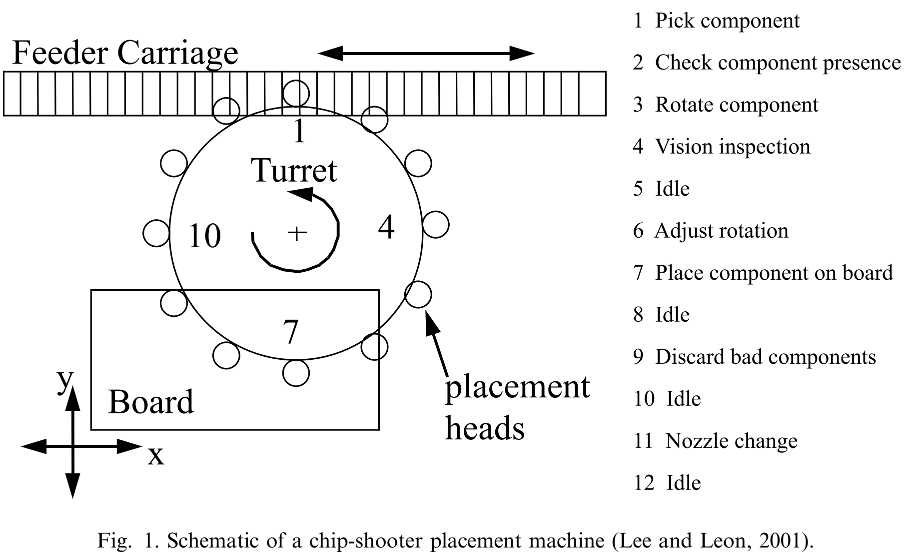

Fig. 1. Schematic of a chip-shooter placement machine (Lee and Leon, 2001).

such objective functions as expected makespan, total expected completion time, maximum expected lateness, and expected maximum lateness, respectively.

2. Literature review

This problem can be viewed as a scheduling problem within the category of machine scheduling with an availability constraint where the constraint can be a machine breakdown or maintenance or lack of machine availability because the machine is still processing jobs left from the previous planning horizon. This type of problem is important since it happens often in industry and has attracted much recent attention in the literature. For the literature that study the problem with the unavailability periods *fixed* in advance, please see for example Schmidt (1984, 1988), Adiri et al. (1989), Lee and Liman (1992, 1993), Lee (1991), Mosheiov (1994), Lee (1996, 1997, 1999), Leon and Wu (1992). Lee et al. (1997), Sanlaville and Schmidt (1998), and Schmidt (2000) provide surveys of the topic. For the problem where the unavailability constraints are due to maintenance and maintenance periods are also *decision variables*, please see Qi et al. (1997), Graves and Lee (1999), and Lee and Chen (2000). In particular, Qi et al. (1997) study the scheduling problem with multiple maintenances. They present an NP-completeness proof, and a branch and bound algorithm for solving the problem. Motivated by industry applications, Graves and Lee (1999) study problems where the machine maintenance can happen at most one or two times during the planning horizon. They assume that if a job is not processed to completion before the machine is stopped for maintenance, an additional setup is necessary when the processing is resumed. Lee and Chen (2000) study the problem of processing a set of jobs on parallel machines in which each machine must be maintained once in the planning horizon. Two cases are studied in their paper; the case where there is a sufficient amount of resource and hence different machines can be maintained simultaneously if necessary, and the case where only one machine can be maintained at any given time. They propose branch and bound algorithms based on the column generation approach for solving both cases of the problem. Their algorithms are capable of solving optimally medium sized problems within a reasonable computational time.

On the other hand, the unavailability can be due to machine breakdown. The single-machine scheduling problem subject to breakdowns was first considered by Glazebrook (1984). When a breakdown occurs, all processing is immediately interrupted and is resumed after the machine is repaired. A discounting Markov decision process was utilized to model a single-machine scheduling problem. Glazebrook (1987) obtains results for a similar problem where the machine uptimes are geometrically distributed and the preemption of jobs at any point in time is allowed by the decision-maker. Both papers consider the minimization of the expected discount cost. Adiri et al. (1989) investigate a single-machine scheduling problem where the machine is restricted to a single breakdown, and the job processing time is assumed to be deterministic. They proved that if the distribution function of the random variable time-to-breakdown is concave, then the shortest processing time (SPT) minimizes the flowtime (makespan) stochastically. When the time-to-breakdown is exponentially distributed, then SPT minimizes the expected flowtime for the case of multiple breakdowns. Pinedo and Rammouz (1988) use a different approach for a similar problem. In their study, the job processing time is assumed to be random and precedence constraints of jobs are allowed. They consider the following objective functions: the weighted sum of the completion times, the weighted sum of an exponential function of the completion times, and the weighted number of late jobs. Birge and Glazebrook (1988) provide some new results by extending their previous research to include the case of random processing times of jobs. Birge et al. (1990) provide analysis for a case with a more general breakdown process and different classes of objective functions. For the weighted sum of completion time model, strong bounds were derived for the difference between the optimal static policy and the weighted shortest processing time (WSPT) policy. Frostig (1991) deals with a problem where the uptimes are exponentially distributed. Optimal policies for the nonresumable models are obtained for the problems of minimizing the expected weighted sum of completion times and the number of tardy jobs. Glazebrook (1991) studies a modeling approach that enables one to consider both resumable and nonresumable models as special cases in a "damage" process. Subject only to fairly mild restriction on this process, the existence of an optimal policy that is nonpreemptive is demonstrated.

Most of the above papers assume that the processing time (or distribution of the processing time) is fixed regardless of whether the job is scheduled before or after the period when the machine is unavailable. Recently, Lee and Leon (2001) present a problem where the job processing times vary depending on whether a job is scheduled before or after the rate-modifying (maintenance) activity. They consider the start-time of the maintenance activity as a decision variable. However, their model excludes the possibility of machine breakdown.

In this paper, we integrate maintenance and breakdowns into the production scheduling decisions. We investigate a problem where the time for carrying out maintenance is a component of the scheduling decisions. In addition, the job processing time is reduced after maintenance or repair is performed. We use static list policy, which means that, before the machine starts to process the jobs, we order the jobs according to a certain priority list and decide when (or not) to insert the maintenance activity. The job order will not change during the processing. Our decision to insert the maintenance activity at a certain point in the process will not change during the processing unless the machine is broken before we reach the insertion point. In such case, we will not insert the maintenance activity. Furthermore, it is assumed that the repair or maintenance activity will bring the machine to its normal speed until the end of the planning horizon and that the machine will not breakdown during the remainder of the planning horizon. This is motivated by our collaboration with industry applications where we are dealing with shop floor scheduling and hence, during a planning horizon, it is very rare to have machine breakdown again after we implement repair or maintenance activity.

3. Notation

Let us summarize the notation that has been used and introduce some additional notation. Let n denote the number of jobs to be processed on a single machine. For each job j, $j = 1, \dots, n$, we define the following notation:

- p_j denotes the processing time if job j is processed before rate-modifying activity.
- q_j denotes the processing time if job j is processed after rate-modifying activity. We assume $q_j = \alpha p_j$, where $0 < \alpha \leqslant 1$ is a given positive real number.
- C_j denotes the completion time of job j.
- L_j denotes the lateness of job j. $L_j = C_j - d_j$.
- t denotes the duration of the maintenance activity. This is a fixed number.
- V denotes the duration of the repair activity. It is a random variable with an expected value equal to $E[V] = v$ units of time. We assume that $v > t$.
- X denotes the instant that machine breaks down if no maintenance activity was inserted. It is a random variable with $F(x)$ and $f(x)$ representing the cumulative distribution function and the probability density function of X, respectively. We assume that X and V are independent.
- $C = \sum_{i=1}^{n} p_i$.

We assume that no job processing can occur while a rate-modifying activity is taking place. In order to refer to the problem under study more precisely, we follow the standard notation used in the scheduling literature (Pinedo, 1995; Lee and Leon, 2001). We use $1 \mid rm, r-a \mid Z(C)$ to denote a single-machine scheduling problem with *rate-modifying* activities, *resumable* availability constraint and an objective function $Z(C)$. Similarly, we use $1 \mid rm, nr-a \mid Z(C)$ to denote the same problem except that we consider the *nonresumable* availability constraint. In this paper, we study $1 \mid rm, r-a \mid E[C_{\max}]$,

$1 \mid rm, r-a \mid \sum E[C_i]$, $1 \mid rm, r-a \mid \max E[L_i]$ and $1 \mid rm, r-a \mid E[\max L_i]$ for resumable cases. We also study $1 \mid rm, nr-a \mid E[C_{\max}]$, $1 \mid rm, nr-a \mid \sum E[C_i]$, $1 \mid rm, nr-a \mid \max E[L_i]$, and $1 \mid rm, nr-a \mid E[\max L_i]$ for nonresumable cases.

Given a sequence σ of jobs, let $[j]$ denote the jth job in the sequence and $P_{[j]} = \sum_{i=1}^{j} p_{[i]}$. Let $E[C_j(\sigma)]$ denote the expected completion time of job j in sequence σ.

4. The resumable case

In this section we deal with the resumable case. The first result concerns scheduling a maintenance activity under a regular performance measure. *Regular performance measure are functions that are nondecreasing in* $C_1, \ldots, C_n$ (Pinedo, 1995, p. 15).

Lemma 1. *If the objective function is an expectation of a regular performance measure, then there exists an optimal solution such that a maintenance activity will not be inserted during the processing of a job.*

Proof. Consider a sequence σ such that there exists a job in the kth position that starts before the maintenance activity and completes after the maintenance activity. Let S_r denote the starting time of the maintenance activity. Let $E[C_{[j]}]$ denote the expected completion time of the jth position job in σ. Let $E[C_{[j]}|V=y]$ denote the expected completion time of the jth position job in σ conditioning on the repair time V. We have

$$E[C[k]|V=y] = \int_0^{S_r} [x+y+\alpha(P_{[k]}-x)]f(x)\,\mathrm{d}x + \int_{S_r}^{\infty} [S_r+t+\alpha(P_{[k]}-S_r)]f(x)\,\mathrm{d}x$$
$$= (1-\alpha)\int_0^{S_r} xf(x)\,\mathrm{d}x + \alpha P_{[k]} + (y-t)F(S_r) + t + S_r(1-\alpha)(1-F(S_r)). \quad (1)$$

Now consider a sequence σ' which is the same as σ except that the maintenance activity is scheduled at the beginning of the kth position job, and denote the expected completion time of the jth position job of σ' as $E[C'_{[j]}|V=y]$. We have

$$E[C'_{[k]}|V=y] = \int_0^{P_{[k-1]}} [x+y+\alpha(P_{[k]}-x)]f(x)\,\mathrm{d}x + \int_{P_{[k-1]}}^{\infty} (P_{[k-1]}+t+p_{[k]})f(x)\,\mathrm{d}x$$
$$= (1-\alpha)\int_0^{P_{[k-1]}} xf(x)\,\mathrm{d}x + \alpha P_{[k]} + (y-t)F(P_{[k-1]}) + t + (1-\alpha)P_{[k-1]}(1-F(P_{[k-1]})). \quad (2)$$

Now, using the assumption that X and V are independent and the relationship $E[C_{[k]}] = E[E[C_{[k]}|V=y]]$, we obtain

$$E[C_{[k]}] = \int_0^{S_r} [x+v+\alpha(P_{[k]}-x)]f(x)\,\mathrm{d}x + \int_{S_r}^{\infty} [S_r+t+\alpha(P_{[k]}-S_r)]f(x)\,\mathrm{d}x$$
$$= (1-\alpha)\int_0^{S_r} xf(x)\,\mathrm{d}x + \alpha P_{[k]}] + (v-t)F(S_r) + t + S_r(1-\alpha)(1-F(S_r)), \quad (1')$$

$$E[C'_{[k]}] = \int_0^{P_{[k-1]}} [x+v+\alpha(P_{[k]}-x)]f(x)\,\mathrm{d}x + \int_{P_{[k-1]}}^{\infty} (P_{[k-1]}+t+\alpha P_{[k]})f(x)\,\mathrm{d}x$$
$$= (1-\alpha)\int_0^{P_{[k-1]}} xf(x)\,\mathrm{d}x + \alpha P_{[k]} + (v-t)F(P_{[k-1]}) + t + (1-\alpha)P_{[k-1]}(1-F(P_{[k-1]})). \quad (2')$$

Note this approach of obtaining an expected completion time will be employed in the remainder of the paper.

Now subtracting $E[C'_{[k]}]$ from $E[C_{[k]}]$, we obtain

$$E[C_{[k]}] - E[C'_{[k]}] = \int_{P_{[k-1]}}^{S_r} [(1-\alpha)(x - P_{[k-1]}) + (v - t)]f(x)\,\mathrm{d}x + (1-\alpha)(S_r - P_{[k-1]})(1 - F(P_{[k-1]})). \quad (3)$$

Since $v \geqslant t$ and $S_r \geqslant P_{[k-1]}$, we see that $E[C_{[k]}] - E[C'_{[k]}] \geqslant 0$. Similarly, it can be checked that $E[C_{[j]}] \geqslant E[C'_{[j]}]$ for all $j \neq k$. $\square$

Remark 1.
1. As a result of Lemma 1, we focus on problems in the remainder of this section where the maintenance activity is scheduled *at the completion of a job or before any job has started.*
2. The Eqs. (1′) and (2′) are exactly the same as (1) and (2), respectively, except that we replace y by v. Namely, instead of using a random variable V as the repair time, *our problem can be treated as the one with constant repair time* $v = E(V)$. Hence, in the remainder of the paper, except Theorems 5 and 10, we will just use v as the repair time.

Now we consider the problem $1 \mid rm, r-a \mid E[C_{\max}]$. For convenience, we say that the maintenance activity is in position k if it is scheduled right before the beginning of the job at the kth position and let $E[C_{\max}(k)]$ denote the corresponding expected makespan. We also use $E[C_{\max}(n+1)]$ to denote the expected makespan corresponding to a schedule in which the maintenance activity is not to be carried out.

The following lemma gives a condition when a maintenance activity is desirable. It also shows that if a maintenance activity is needed it must be carried out before the processing of any job.

Lemma 2. *For* $1 \mid rm, r-a \mid E[C_{\max}]$ *problem,*
(i) $E[C_{\max}(1)]$ *and* $E[C_{\max}(n+1)]$ *are independent of the sequence of jobs.*
(ii) *For any given sequence of jobs,* $E[C_{\max}(1)] \leqslant E[C_{\max}(k)]$ *for* $1 \leqslant k \leqslant n$.
(iii) $E[C_{\max}(n+1)] \leqslant E[C_{\max}(1)]$ *for every schedule when*

$$(1-\alpha)C - \int_0^C (C - x)f(x)\mathrm{d}x + vF(C) \leqslant t.$$

Proof.

$$\begin{aligned} E[C_{\max}(k)] &= \int_0^{P_{[k-1]}} [x + v + \alpha(C - x)]f(x)\,\mathrm{d}x + \int_{P_{[k-1]}}^{\infty} [P_{[k-1]} + t + \alpha(C - P_{[k-1]})]f(x)\,\mathrm{d}x \\ &= [t + (1-\alpha)P_{[k-1]}][1 - F(P_{[k-1]})] + (1-\alpha)\int_0^{P_{[k-1]}} xf(x)\,\mathrm{d}x + vF(P_{[k-1]}) + \alpha C, \end{aligned} \quad (4)$$

If $k = 1$, then $P_{[k-1]} = F(P_{[k-1]}) = 0$. Hence $E[C_{\max}(1)] = t + \alpha C$. (5)

If $k = n+1$, then we set $t = 0$, $P_{[k-1]} = C$, and $F(P_{[k-1]}) = F(C)$. It follows that

$$E[C_{\max}(n+1)] = C - (1-\alpha)\int_0^C (C - x)f(x)\,\mathrm{d}x + vF(C). \quad (6)$$

Based on the above equations, (i) and (iii) are clear. (ii) can be shown easily by using the fact that $v > t$. $\square$

Theorem 1. *For $1 \mid rm, r-a \mid E[\sum C_i]$ problem, if we decide not to do the maintenance activity, then the optimal schedule is to sequence jobs in the SPT order.*

Proof. This proof involves a pairwise job interchange argument. Consider a job sequence $\sigma_1 = (\rho, i, j, \omega)$ where i and j are two consecutive jobs, and ρ and ω are two subsequences of jobs which precede job i and follow job j, respectively. Consider another sequence $\sigma_2 = (\rho, j, i, \omega)$ which is exactly the same as σ_1 except that we interchange the positions of jobs i and j. Assume that $p_i \leqslant p_j$ and let B denote the sum of the processing times of all jobs, excluding the possible repair time, in subsequence ρ. Conditioning on the random variable X, the expected completion times of jobs i and j under sequence σ_1 can be found as follows:

$$\begin{aligned} E[C_i(\sigma_1)] &= \int_0^{B+p_i} [x + v + \alpha(B + p_i - x)] f(x)\,\mathrm{d}x + \int_{B+p_i}^{\infty} (B + p_i) f(x)\,\mathrm{d}x \\ &= B + p_i + (1-\alpha)\int_0^{B+p_i} x f(x)\,\mathrm{d}x - (1-\alpha)(B+p_i)F((B+p_i)) + vF(B+p_i), \end{aligned} \tag{7}$$

$$\begin{aligned} E[C_j(\sigma_1)] &= \int_0^{B+p_i+p_j} [x + v + \alpha(B + p_i + p_j - x)] f(x)\,\mathrm{d}x + \int_{B+p_i+p_j}^{\infty} (B + p_i + p_j) f(x)\,\mathrm{d}x \\ &= B + p_i + p_j + (1-\alpha)\int_0^{B+p_i+p_j} x f(x)\,\mathrm{d}x - (1-\alpha)(B+p_i+p_j)F(B+p_i+p_j) \\ &\quad + vF(B+p_i+p_j). \end{aligned} \tag{8}$$

Similarly, the expected completion times of jobs i and j in sequence σ_2 can be obtained as

$$E[C_j(\sigma_2)] = B + p_j + (1-\alpha)\int_0^{B+p_j} x f(x)\,\mathrm{d}x - (1-\alpha)(B+p_j)F(B+p_j) + vF(B+p_j), \tag{9}$$

$$\begin{aligned} E[C_i(\sigma_2)] &= B + p_j + p_i + (1-\alpha)\int_0^{B+p_j+p_i} x f(x)\,\mathrm{d}x - (1-\alpha)(B+p_j+p_i)F(B+p_j+p_i) \\ &\quad + vF(B+p_j+p_i). \end{aligned} \tag{10}$$

Note that $E[C_j(\sigma_1)] = E[C_i(\sigma_2)]$. It can also be shown that $E[C_k(\sigma_1)] = E[C_k(\sigma_2)]$ for each job k in ρ and ω. Hence it is sufficient to show that $E[C_i(\sigma_1)] - E[C_j(\sigma_2)] \leqslant 0$

$$\begin{aligned} E[C_i(\sigma_1)] - E[C_j(\sigma_2)] &= \alpha\int_0^{B+p_i} p_i f(x)\,\mathrm{d}x - \alpha\int_0^{B+p_j} p_j f(x)\,\mathrm{d}x - \int_{B+p_i}^{B+p_j} [(1-\alpha)x + \alpha B + v] f(x)\,\mathrm{d}x \\ &\quad + \int_{B+p_i}^{\infty} (B+p_i) f(x)\,\mathrm{d}x - \int_{B+p_j}^{\infty} (B+p_i) f(x)\,\mathrm{d}x \\ &= \int_{B+p_i}^{B+p_j} [(1-\alpha)B - (1-\alpha)x - v] f(x)\,\mathrm{d}x - \alpha\int_{B+p_i}^{B+p_j} p_i f(x)\,\mathrm{d}x + \int_{B+p_i}^{B+p_j} p_i f(x)\,\mathrm{d}x \\ &\quad + \int_{B+p_i}^{\infty} (p_i - p_j) f(x)\,\mathrm{d}x \\ &= \int_{B+p_i}^{B+p_j} [(1-\alpha)(B+p_i) - (1-\alpha)x - v] f(x)\,\mathrm{d}x + \int_{B+p_i}^{\infty} (p_i - p_j) f(x)\,\mathrm{d}x \leqslant 0. \qquad \square \end{aligned}$$

Remark 2. Adiri et al. (1989) has shown a similar result for the problem where the repair activity does not change the machine speed.

Theorem 2. *For the* $1|rm, r-a|\sum E[C_i]$ *problem, it is optimal to sequence jobs in the SPT order.*

Proof. It is clear that after the maintenance activity, jobs should be sequenced in the SPT order. Also, from Theorem 1, we see that jobs scheduled before the maintenance activity should also follow the SPT order. Hence, we only need to show that a job scheduled before the maintenance activity is not greater than any job scheduled after the maintenance activity. Consider a job sequence $\sigma_1 = (\rho, i, M, j, \omega)$ where ρ and ω are two subsequences of jobs which precede job i and follow job j, respectively. Also the maintenance activity is done right after the completion of job i and before the start of job j. Consider another sequence $\sigma_2 = (\rho, j, M, i, \omega)$ which is exactly the same as σ_1 except that we interchange the position of jobs i and j. Assume that $p_i \leqslant p_j$ and let B denote the sum of processing times of all jobs in the subsequence ρ. Conditioning on the random variable X, the expected completion times $E[C_i(\sigma_1)]$ and $E[C_j(\sigma_2)]$ are given in the proof of the previous theorem. While the expected completion time of jobs j and i in sequences σ_1 and σ_2, respectively, can be expressed as follows:

$$E[C_j(\sigma_1)] = \int_0^{B+p_i} [x + v + \alpha(B + p_i + p_j - x)]f(x)\,\mathrm{d}x + \int_{B+p_i}^{\infty} (B + t + p_i + \alpha p_j)f(x)\,\mathrm{d}x, \tag{11}$$

$$E[C_i(\sigma_2)] = \int_0^{B+p_j} [x + v + \alpha(B + p_j + p_i - x)]f(x)\,\mathrm{d}x + \int_{B+p_j}^{\infty} (B + t + p_j + \alpha p_i)f(x)\,\mathrm{d}x. \tag{12}$$

We first show that $E[C_j(\sigma_1)] \leqslant E[C_i(\sigma_2)]$.

$$\begin{aligned}
E[C_j(\sigma_1)] - E[C_i(\sigma_2)] &= -\int_{B+p_i}^{B+p_j} [(1-\alpha)x + v + \alpha B + \alpha p_i + \alpha p_j]f(x)\,\mathrm{d}x \\
&\quad + \int_{B+p_i}^{B+p_j} (B + t)f(x)\,\mathrm{d}x + \int_{B+p_i}^{\infty} (p_i + \alpha p_j)f(x)\,\mathrm{d}x - \int_{B+p_j}^{\infty} (p_j + \alpha p_i)f(x)\,\mathrm{d}x \\
&= \int_{B+p_i}^{B+p_j} [t + v + (1-\alpha)(B - x) - \alpha p_i - \alpha p_j]f(x)\,\mathrm{d}x + \int_{B+p_i}^{B+p_j} (p_i + \alpha p_j)f(x)\,\mathrm{d}x \\
&\quad + \int_{B+p_i}^{\infty} (p_i + \alpha p_j)f(x)\,\mathrm{d}x - \int_{B+p_j}^{\infty} (p_j + \alpha p_i)f(x)\,\mathrm{d}x \\
&= \int_{B+p_i}^{B+p_j} [t - v + (1-\alpha)(B + p_i - x)]f(x)\,\mathrm{d}x + \int_{B+p_i}^{\infty} (1-\alpha)(p_i - p_j)f(x)\,\mathrm{d}x \leqslant 0,
\end{aligned}$$

since both terms are nonpositive.

It can be shown that $E[C_k(\sigma_1)] \leqslant E[C_k(\sigma_2)]$ for each job k in ω. Furthermore, the expected completion time of each job in the subsequence ρ is not affected by the interchange of jobs i and j. Hence it is sufficient to show that $\Delta = (E[C_i(\sigma_1)] + E[C_j(\sigma_1)]) - (E[C_j(\sigma_2)] + E[C_i(\sigma_2)]) \leqslant 0$. Since in the previous theorem we have shown that $E[C_i(\sigma_1)] - E[C_j(\sigma_2)] \leqslant 0$, combining these results with the above discussion we have shown that $\Delta \leqslant 0$. □

Suppose that we reindex jobs in SPT order, i.e., $p_1 \leqslant p_2 \leqslant \cdots \leqslant p_n$ and let $P_i = \sum_{j=1}^{i} p_j$. Let $\mathrm{TC}(k)$ denote the total expected cost if we decide to insert the maintenance at the kth position, $k = 1, \ldots, n+1$. Then

$$\mathrm{TC}(k) = \sum_{i=1}^{k-1} \left(\int_0^{P_i} (x + v + \alpha(P_i - x)) f(x)\,\mathrm{d}x + \int_{P_i}^{\infty} P_i f(x)\,\mathrm{d}x \right) + \sum_{i=k}^{n} \left(\int_0^{P_{k-1}} (x + v + \alpha(P_i - x)) f(x)\,\mathrm{d}x + \int_{P_{k-1}}^{\infty} (P_{k-1} + t + \alpha(P_i - x)) f(x)\,\mathrm{d}x \right).$$

We can calculate $\mathrm{TC}(k)$ for $k = 1, \ldots, n+1$ and choose the index k that corresponds to the minimal cost as the optimal position for inserting the maintenance activity.

Theorem 3. *For $1 \mid rm, r-a \mid \max E[L_i]$ problem, if we decide not to do the maintenance activity, then the optimal schedule is to sequence jobs in the earliest due date (EDD) order.*

Proof. Consider a job sequence $\sigma_1 = (\rho, i, j, \omega)$ where i and j are two consecutive jobs, and ρ and ω are two subsequences of jobs which precede job i and follow job j, respectively. Consider another sequence $\sigma_2 = (\rho, j, i, \omega)$ which is exactly the same as (ρ, i, j, ω) except that we interchange the position of jobs i and j. Assume $d_i \leqslant d_j$. Let B denote the sum of processing times of all jobs in the subsequence ρ.

Let $Z(\sigma_1)$ and $Z(\sigma_2)$ denote the objective function obtained from the sequences σ_1 and σ_2, respectively. Then we can express these two objective functions as $Z(\sigma_1) = \max\{Z_1(\rho), E[L_i(\sigma_1)], E[L_j(\sigma_1)], Z_1(\omega)\}$ and $Z(\sigma_2) = \max\{Z_2(\rho), E[L_j(\sigma_2)], E[L_i(\sigma_2)], Z_2(\omega)\}$, where $Z_k(\rho)$ and $Z_k(\omega)$, $k = 1, 2$, denote the objective function value of jobs in the subsequences ρ and ω, respectively. Since $E[C_j(\sigma_1)] = E[C_i(\sigma_2)]$ and $d_i \leqslant d_j$, we have $E[L_j(\sigma_1)] = E[C_j(\sigma_1)] - d_j \leqslant E[C_i(\sigma_2)] - d_i = E[L_i(\sigma_2)]$. Also, it can be checked easily that $Z_1(\omega) = Z_2(\omega)$. Furthermore, the expected completion time of jobs in the subsequence ρ are not affected by the interchange of jobs i and j, it follows that $Z_1(\rho) = Z_2(\rho)$. Finally, $E[L_i(\sigma_1)] \leqslant E[L_i(\sigma_2)]$ since $E[C_i(\sigma_1)] \leqslant E[C_i(\sigma_2)]$. Hence, $Z(\sigma_1) \leqslant Z(\sigma_2)$. □

Theorem 4. *For $1 \mid rm, r-a \mid \max E[L_i]$ problem, it is optimal to sequence jobs in the EDD order.*

Proof. It is clear that after the maintenance activity, jobs should be sequenced in the EDD order. Also, from Theorem 3, we see that jobs scheduled before the maintenance activity should also follow the EDD order. Hence, we only need to show that the due date of a job scheduled before the maintenance activity is not greater than that of any job scheduled after the maintenance activity. Consider a job sequence $\sigma_1 = (\rho, i, M, j, \omega)$ where ρ and ω are two subsequences of jobs that precede job i and follow job j, respectively. Also the maintenance activity is performed right after the completion of job i and before the start of job j. Consider another sequence $\sigma_2 = (\rho, M, j, i, \omega)$ which is exactly the same as σ_1 except that we move job i to be processed immediately after job j. Assume that $d_i > d_j$ and let B denote the sum of processing times of all jobs in the subsequence ρ. We can calculate other expected completion times as follows:

$$E[C_j(\sigma_1)] = \int_0^{B+p_i} [x + v + \alpha(B + p_i + p_j - x)] f(x)\,\mathrm{d}x + \int_{B+p_i}^{\infty} (B + t + p_i + \alpha p_j) f(x)\,\mathrm{d}x, \tag{13}$$

$$E[C_j(\sigma_2)] = \int_0^{B} [x + v + \alpha(B + p_j - x)] f(x)\,\mathrm{d}x + \int_{B}^{\infty} (B + t + \alpha p_j) f(x)\,\mathrm{d}x, \tag{14}$$

$$E[C_i(\sigma_2)] = \int_0^B [x + v + \alpha(B + p_j + p_i - x)]f(x)\,\mathrm{d}x + \int_B^\infty (B + t + \alpha p_j + \alpha p_i)f(x)\,\mathrm{d}x. \tag{15}$$

Since $v > t$, it can be shown that $E[C_i(\sigma_2)] \leqslant E[C_j(\sigma_1)]$. Thus the assumption $d_i > d_j$ implies that $E[L_i(\sigma_2)] = E[L_i(\sigma_2)] - d_i < E[C_j(\sigma_1)] - d_j = E[L_j(\sigma_1)]$. It can also be shown easily that $E[C_j(\sigma_2)] \leqslant E[C_j(\sigma_1)]$. Hence we have $E[L_j(\sigma_2)] \leqslant E[L_j(\sigma_1)]$. Thus $\max\{E[L_i(\sigma_2)], E[L_j(\sigma_2)]\} \leqslant \max\{E[L_i(\sigma_1)], E[L_j(\sigma_1)]\}$. Furthermore, it can be shown that $E[L_k(\sigma_2)] \leqslant E[L_k(\sigma_1)]$ for each job k in ω. Clearly, $E[L_k(\sigma_2)] = E[L_k(\sigma_1)]$ for any job k in subsequence ρ. □

Theorem 5. *For $1 \mid rm, r-a \mid E[\max L_i]$ problem, it is optimal to sequence jobs in the EDD order.*

Proof. Similar to the proof in Pinedo (1995, pp. 189), we should proceed with the proof by conditioning on the random variable X (time-to-breakdown) and the repair time V. For each given instance of X and V, the problem becomes deterministic. Two cases should be considered depending on whether the maintenance activity is carried out before or after the occurrence of the breakdown. Consider a particular instance $X = x$ and $V = v$.

Case 1: The maintenance activity is carried out before the occurrence of the breakdown. In this case, after the rate-modification activity the breakdown will never occur. We need to show that the jobs should be scheduled based on the EDD order.

It is clear that after the maintenance activity, jobs should be sequenced in the EDD order. Also, jobs scheduled before the maintenance activity should also follow the EDD order. Hence, we only need to show that the due date of a job scheduled before the maintenance activity is not greater than that of any job scheduled after the maintenance activity. Consider a job sequence $\sigma_1 = (\rho, i, M, j, \omega)$, where ρ and ω are two subsequences of jobs that precede job i and follow job j, respectively. Also the maintenance activity is performed right after the completion of job i and before the start of job j. Consider another sequence $\sigma_2 = (\rho, M, j, i, \omega)$ which is exactly the same as σ_1 except that we move job i to be processed immediately after job j. Assume that $d_i > d_j$ and let B denote the sum of processing times of all jobs in the subsequence ρ. We can calculate the completion times as follows:

$$L_i(\sigma_1) = B + p_i - d_i,$$

$$L_j(\sigma_1) = B + p_i + t + \alpha p_j - d_j,$$

$$L_i(\sigma_2) = B + t + \alpha p_j + \alpha p_i - d_i,$$

$$L_j(\sigma_2) = B + t + \alpha p_j - d_j.$$

The assumption $d_i > d_j$ implies that $L_j(\sigma_1) - L_i(\sigma_2) = (1-\alpha)p_i + (d_i - d_i) \geqslant 0$. It can also be shown that $L_j(\sigma_2)] \leqslant L_j(\sigma_1)$. Thus, $\max\{L_i(\sigma_2), L_j(\sigma_2)\} \leqslant \max\{L_i(\sigma_1), L_j(\sigma_1)\}$. Furthermore, it can be shown that $L_k(\sigma_2) \leqslant L_k(\sigma_1)$ for each job k in ω. Clearly, $L_k(\sigma_2) = L_k(\sigma_1)$ for any job k in subsequence ρ.

Case 2: The maintenance activity is carried out after the occurrence of the breakdown. In this case, the repair activity is carried out right after the breakdown, which the maintenance activity is never executed. Again, we need to prove that jobs should be scheduled based on the EDD order. Assuming that the breakdown occurs while a job i is being processed.

We consider a job sequence $\sigma_1 = (\rho, i1, R, i2, j, \omega)$ where $i1$ and $i2$ represented the two parts of job i that are separated by the repair activity, and ρ and ω are two subsequences of jobs that precede job i and follow job j, respectively. Consider another sequence σ_2 that is exactly the same as σ_1 except we move job i to be processed immediately after job j. Now, depending on the length of the processing time of job j, this interchange of jobs i and j could result in two forms of the sequence σ_2. Namely, $\sigma_2 = (\rho, j1, R, j2, i, \omega)$, and $\sigma_2 = (\rho, j, i1', R, i2', \omega)$. We will proceed with the proof in these two subcases. In the subcase (1), job j is divided by the repair activity in σ_2. While in the subcase (2) job j is too small to be divided by the repair activity. Instead job i is divided into two parts $i1'$ and $i2'$ which may be different from those in the sequence σ_1. We show these two subcases in the remainder of this proof.

Subcase 1: Assume that $d_i > d_j$ and let B denote the sum of processing times of all jobs in the subsequence ρ. We can calculate the lateness of jobs i and j as follows:

$$L_i(\sigma_1) = B + p_{i1} + v + \alpha p_{i2} - d_i,$$

$$L_j(\sigma_1) = B + p_{i1} + v + \alpha p_{i2} + \alpha p_j - d_j,$$

$$L_i(\sigma_2) = B + p_{j1} + v + \alpha p_{j2} + \alpha p_i - d_i,$$

$$L_j(\sigma_2) = B + p_{j1} + v + \alpha p_{j2} - d_j.$$

We obtain $L_j(\sigma_1) - L_i(\sigma_2) = (1-\alpha)(p_{i1} - p_{j1}) + (d_i - d_j) \geqslant 0$ since $p_{i1} = p_{j1}$ and $d_i > d_j$. Also, $L_j(\sigma_1) - L_j(\sigma_2) = \alpha(p_{i2} + p_{j1}) \geqslant 0$. Thus $\max\{L_i(\sigma_2), L_j(\sigma_2)\} \leqslant \max\{L_i(\sigma_1), L_j(\sigma_1)\}$. Furthermore, it can be shown that $L_k(\sigma_2) = L_k(\sigma_1)$ for each job k in ω. Clearly, $L_k(\sigma_2) = L_k(\sigma_1)$ for any job k in subsequence ρ.

Subcase 2: We can calculate the lateness of jobs i and j as follows:

$$L_i(\sigma_1) = B + p_{i1} + v + \alpha p_{i2} - d_i,$$

$$L_j(\sigma_1) = B + p_{i1} + v + \alpha p_{i2} + \alpha p_j - d_j,$$

$$L_i(\sigma_2) = B + p_j + p_{i1'} + v + \alpha p_{i2'} - d_i,$$

$$L_j(\sigma_2) = B + p_j - d_j.$$

We can obtain $L_j(\sigma_1) - L_i(\sigma_2) = [p_{i1} - (p_j + p'_{i1})] + \alpha[(p_{i2} + p_j) - p'_{i2}] + (d_i - d_j) \geqslant 0$ since $p_{i1} = p_j + p_{i1'}$, $p_{i2} + p_j = p'_{i2}$, and $d_i > d_j$. Also, $L_j(\sigma_2) - L_j(\sigma_1) = (p_{i1} - p_j) + v + \alpha(p_{i2} + p_j) \geqslant 0$ since $p_{i1} \geqslant p_j$. Thus $\max\{L_i(\sigma_2), L_j(\sigma_2)\} \leqslant \max\{L_i(\sigma_1), L_j(\sigma_1)\}$. Furthermore, it can be shown that $L_k(\sigma_2) = L_k(\sigma_1)$ for each job k in ω. Clearly, $L_k(\sigma_2) = L_k(\sigma_1)$ for any job k in subsequence ρ. In summary, we have shown that EDD sequence minimizes the $\max L_i$ for *any instance of X and V*. Hence it also minimizes $E[\max L_i]$. □

5. The nonresumable case

In this section we consider the nonresumable case where a job interrupted by machine breakdown or maintenance activity has to restart when the machine becomes available again. In such a case, it is clear that if the objective function is an expected value of a regular performance measure, then it is optimal to schedule the maintenance activity at the beginning or the completion of a job. Namely, it is not optimal to insert a maintenance activity during the process of a job.

A distribution function F is said to be *concave* on [0, a] if the function F is continuous and satisfies $F(x+s) - F(x) \geqslant F(y+s) - F(y)$ for each x, y, s such that $0 \leqslant x < y$, $s > 0$, and $y + s \leqslant a$, $F(a) = 1$ and $F(x) = 0$ for $x < 0$. Next we say that a distribution function F is *convex* on [0, a] if the function is continuous, $F(a) = 1$, $F(x) < 1$ for $x < a$, $F(x) = 0$ for $x < 0$, and $F(x+s) - F(x) \leqslant F(y+s) - F(y)$ for each x, y, s such that $0 \leqslant x < y$, $s > 0$, and $y + s \leqslant a$. Note that F is concave on [0, ∞] if it is concave on [0, a]. However, the same is not true of convex function. Examples of concave distribution functions include all Decreasing Failure Rate (DFR) distributions, the exponential distribution, the gamma distribution with shape parameter less than or equal to 1, and the uniform distribution on interval [0, a] where $a > 0$. Note that a distribution function is called Decreasing (Increasing) Failure Rate distribution if $f(t)/(1-F(t))$ is decreasing (increasing) of t.

In the remainder of the paper if we say the distribution function $F(x)$ is convex we mean that it is convex up on [0, C] where $C = \sum p_j$ (see Adiri et al., 1989).

Theorem 6. *For the* $1 \mid rm, nr-a \mid E[C_{\max}]$ *problem, the following statements hold.*

(i) $E[C_{\max}(1)]$ *is independent of any schedule.*

(ii) $E[C_{\max}(1)] \leqslant E[C_{\max}(k)]$ *for every schedule, where* $1 \leqslant k \leqslant n$.

(iii) *If we decide not to do maintenance activity, then it is optimal to sequence jobs in the SPT order when the distribution function* $F(x)$ *is concave, and in the longest processing time (LPT) order when the distribution function* $F(x)$ *is convex up to* C.

Proof. (i) Since the objective function $E[C_{\max}(1)] = t + \alpha C$, it is easy to see that $E[C_{\max}(1)]$ is independent of any schedule.

(ii) The objective function $E[C_{\max}(k)]$ can be written as

$$\begin{aligned} E[C_{\max}(k)] &= \sum_{i=1}^{k-1}\left[\int_{P_{[i-1]}}^{P_{[i]}}\left(x+\sum_{j=i}^{n}\alpha p_{[j-1]}+v\right)f(x)\,\mathrm{d}x\right]+\int_{P_{[k-1]}}^{\infty}\left(P_{[k-1]}+\sum_{i=k}^{n}\alpha p_{[i]}+t\right)f(x)\,\mathrm{d}x \\ &= \int_0^{S_r} xf(x)\,\mathrm{d}x+\sum_{i=1}^{k-1}\left(\alpha p_{[i]}F(P_{[i]})\right)+(1-\alpha)\left(\alpha P_{[k-1]}\right)(1-F(P_{[k-1]}))+(v-t)F(P_{[k-1]}) \\ &\quad +t+\alpha C. \end{aligned} \tag{16}$$

It is easy to see that $E[C_{\max}(1)] \leqslant E[C_{\max}(k)]$ since $v > t$.

(iii) The objective function $E[C_{\max}(n+1)]$ can be written as

$$\begin{aligned} E[C_{\max}(n+1)] &= \sum_{i=1}^{n}\left[\int_{P_{[i-1]}}^{P_{[i]}}\left(x+\alpha(C-P_{[i-1]})+v\right)f(x)\,\mathrm{d}x\right]+\int_C^{\infty} Cf(x)\,\mathrm{d}x \\ &= \int_0^C xf(x)\,\mathrm{d}x+\sum_{i=1}^{n}\left(\alpha p_{[i]}F(P_{[i]})\right)+vF(C)+C(1-F(C)). \end{aligned} \tag{17}$$

Since the first, third and fourth terms in the above expression are independent of schedules, it follows that $E[C_{\max}(n+1)]$ is minimized when the second term is minimized. This proof involves a pairwise job interchange argument. Consider two arbitrary job sequences $\sigma_1 = (\rho, i, j, \omega)$ and $\sigma_2 = (\rho, j, i, \omega)$ where ρ and ω are two subsequences of jobs before jobs i and j, and after jobs i and j, respectively. Assume that there are $k-1$ jobs in ρ. Assume also $p_i \leqslant p_j$.

Let $Z(\sigma_1)$ and $Z(\sigma_2)$ denote the second term in the objective function (17) obtained for the sequences σ_1 and σ_2, respectively. Since only the positions of jobs i and j are altered in the interchange, we can obtain the expression of $\Delta = Z(\sigma_1) - Z(\sigma_2)$ as follows:

$$\begin{aligned} \Delta &= Z(\sigma_1)-Z(\sigma_2) \\ &= \alpha[p_iF(P_{[k-1]}+p_i)+\alpha p_jF(P_{[k-1]}+p_i+p_j)]-[\alpha_jF(P_{[k-1]}+p_j)+\alpha p_iF(P_{[k-1]}+p_j+p_i)] \\ &= \alpha[p_iF(P_{[k-1]}+p_i)-p_jF(P_{[k-1]}+p_j)]+\alpha(p_j-p_i)F(P_{[k-1]}+p_j+p_i) \\ &= \alpha\{(pp_j-p_i)F(P_{[k-1]}+p_j+p_i)-[p_jF(P_{[k-1]}+p_j)-p_iF(P_{[k-1]}+p_i)]\}. \end{aligned}$$

Suppose that F is concave, then

$$F(P_{[k-1]} + p_j) \geqslant \frac{p_i}{p_j} F(P_{[k-1]} + p_i) + \left(1 - \frac{p_i}{p_j}\right) F(P_{[k-1]} + p_i + p_j),$$

namely,

$$(p_j - p_i) F(P_{[k-1]} + p_i + p_j) \leqslant p_j F(P_{[k-1]} + p_j) - p_i F(P_{[k-1]} + p_i). \tag{18}$$

Equivalently, $\Delta \leqslant 0$. Hence, the optimal solution is to sequence jobs in the SPT order. On the other hand, if F is convex, then

$$F(P_{[k-1]} + p_j) \leqslant \frac{p_i}{p_j} F(P_{[k-1]} + p_i) + \left(1 - \frac{p_i}{p_j}\right) F(P_{[k-1]} + p_i + p_j),$$

namely,

$$(p_j - p_i) F(P_{[k-1]} + p_i + p_j) \geqslant p_j F(P_{[k-1]} + p_j) - p_i F(P_{[k-1]} + p_i).$$

Thus, $\Delta \geqslant 0$ and the optimal solution is to sequence jobs in the LPT order. □

Remark 3. We can compare $E[C_{\max}(1)]$ and $E[C_{\max}(n+1)]$ and choose the minimal one as the optimal solution value.

Theorem 7. *For the $1 \mid rm, nr - a \mid E[\sum C_i]$ problem, if we decide not to schedule the maintenance activity during the makespan of the entire job set, then it is optimal to sequence jobs in SPT order when the distribution function F is concave.*

Proof. We consider a job sequence $\sigma_1 = (\rho, i, j, \omega)$ where i and j are two consecutive jobs, and ρ and ω are two subsequences of jobs which precede job i and follow job j, respectively. Consider another sequence $\sigma_2 = (\rho, j, i, \omega)$ which is exactly the same as σ_1 except that we interchange the position of jobs i and j. Assume that there are $k-1$ jobs in ρ. Assume that $p_i \leqslant p_j$. Conditioning on the random variable X, the expected completion times of jobs i and j under sequence σ_1 can be found as follows:

$$\begin{aligned} E[C_i(\sigma_1)] = {} & \sum_{t=1}^{k-1} \left[\int_{P_{[t-1]}}^{P_{[t]}} \left(x + v + \sum_{r=t}^{k-1} \alpha p_{[r]} + \alpha p_i \right) f(x)\,\mathrm{d}x \right] + \int_{P_{[k-1]}}^{P_{[k-1]}+p_i} (x + v + \alpha p_i) f(x)\,\mathrm{d}x \\ & + \int_{P_{[k-1]}+p_i}^{\infty} \big(P_{[k-1]} + p_i\big) f(x)\,\mathrm{d}x, \end{aligned} \tag{19}$$

$$\begin{aligned} E[C_j(\sigma_1)] = {} & \sum_{t=1}^{k-1} \left[\int_{P_{[t-1]}}^{P_{[t]}} \left(x + v + \sum_{r=t}^{k-1} \alpha p_{[r]} + \alpha p_i + \alpha p_j \right) f(x)\,\mathrm{d}x \right] \\ & + \int_{P_{[k-1]}}^{P_{[k-1]}+p_i} \big(x + v + \alpha p_i + \alpha p_j\big) f(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_i+p_j} \big(x + \alpha p_j + v\big) f(x)\,\mathrm{d}x \\ & + \int_{P_{[k-1]}+p_i+p_j}^{\infty} \big(P_{[k-1]} + p_i + p_j\big) f(x)\,\mathrm{d}x. \end{aligned} \tag{20}$$

Similarly, we can obtain the following expected completion times of job j and job i in sequence σ_2,

$$E[C_j(\sigma_2)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\sum_{r=t}^{k-1}\alpha p_{[r]}+\alpha p_j\right)f(x)\,\mathrm{d}x\right]+\int_{P_{[k-1]}}^{P_{[k-1]}+p_j}\big(x+v+\alpha p_j\big)f(x)\,\mathrm{d}x$$
$$+\int_{P_{[k-1]}+p_j}^{\infty}\big(P_{[k-1]}+p_j\big)f(x)\,\mathrm{d}x, \tag{21}$$

$$E[C_i(\sigma_2)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\sum_{r=t}^{k-1}\alpha p_{[r]}+\alpha p_j+\alpha p_i\right)f(x)\,\mathrm{d}x\right]$$
$$+\int_{P_{[k-1]}}^{P_{[k-1]}+p_j}\big(x+v+\alpha p_j+\alpha p_i\big)f(x)\,\mathrm{d}x+\int_{P_{[k-1]}+p_j}^{P_{[k-1]}+p_j+p_i}(x+\alpha p_i+v)f(x)\,\mathrm{d}x$$
$$+\int_{P_{[k-1]}+p_j+p_i}^{\infty}\big(P_{[k-1]}+p_j+p_i\big)f(x)\,\mathrm{d}x. \tag{22}$$

Note that

$$E[C_j(\sigma_1)]-E[C_i(\sigma_2)] = \alpha[p_iF(P_{[k-1]}+p_i)-p_jF(P_{[k-1]}+p_j)+(p_j-p_i)F(P_{[k-1]}+p_i+p_j)]\leqslant 0. \tag{23}$$

The last inequality follows by an argument similar to the one in the proof of Theorem 3 and Eq. (18) since F is concave. Furthermore,

$$E[C_j(\sigma_2)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\sum_{r=t}^{k-1}\alpha p_{[r]}+\alpha p_j\right)f(x)\,\mathrm{d}x\right]+\int_{P_{[k-1]}}^{P_{[k-1]}+p_j}\big(x+v+\alpha p_j\big)f(x)\,\mathrm{d}x$$
$$+\int_{P_{[k-1]}+p_j}^{\infty}\big(P_{[k-1]}+p_j\big)f(x)\,\mathrm{d}x$$
$$= \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\sum_{r=t}^{k-1}\alpha p_{[r]}+\alpha p_j\right)f(x)\,\mathrm{d}x\right]+\int_{P_{[k-1]}}^{P_{[k-1]}+p_i}\big(x+v+p_j\big)f(x)\,\mathrm{d}x$$
$$+\int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}\big(x+v+\alpha p_j\big)f(x)\,\mathrm{d}x+\int_{P_{[k-1]}+p_j}^{\infty}\big(P_{[k-1]}+p_j\big)f(x)\,\mathrm{d}x. \tag{24}$$

Comparing these results with those from $E[C_i(\sigma_1)]$ in (19), we see that the first two terms of (24) are not smaller than those in (19). The third and fourth terms are again not smaller than the third term in (19). Hence $E[C_i(\sigma_1)]\leqslant E[C_j(\sigma_2)]$.

Since the expected completion time of jobs in the subsequence ρ are not affected by the interchange of jobs i and j, now it is sufficient to show that $E[C_{[m]}(\sigma_1)]-E[C_{[m]}(\sigma_2)]\leqslant 0$ for all $m>k+1$.

$$
\begin{aligned}
E[C_{[m]}(\sigma_1)] = {} & \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\alpha\sum_{r=t}^{k-1}p_{[r]}+\alpha p_i+\alpha p_j+\alpha\sum_{r=k+2}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x\right] \\
& +\int_{P_{[k-1]}}^{P_{[k-1]}+p_i}\left(x+v+\alpha p_i+\alpha p_j+\alpha\sum_{r=k+2}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x \\
& +\int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_i+p_j}\left(x+v+\alpha p_j+\alpha\sum_{r=k+2}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x \\
& +\sum_{t=k+2}^{m-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\alpha\sum_{r=t}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x\right] \\
& +\int_{P_{[m-1]}}^{P_{[m]}}\left(x+v+\alpha p_{[m]}\right)f(x)\,\mathrm{d}x+\int_{P_{[m]}}^{\infty}P_{[m]}f(x)\,\mathrm{d}x,
\end{aligned}
$$

$$
\begin{aligned}
E[C_{[m]}(\sigma_2)] = {} & \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\alpha\sum_{r=t}^{k-1}p_{[r]}+\alpha p_j+\alpha p_i+\alpha\sum_{r=k+2}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x\right] \\
& +\int_{P_{[k-1]}}^{P_{[k-1]}+p_j}\left(x+v+\alpha p_j+\alpha p_i+\alpha\sum_{r=k+2}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x \\
& +\int_{P_{[k-1]}+p_j}^{P_{[k-1]}+p_j+p_i}\left(x+v+\alpha p_i+\alpha\sum_{r=k+2}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x \\
& +\sum_{t=k+2}^{m-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+v+\alpha\sum_{r=t}^{m}p_{[r]}\right)f(x)\,\mathrm{d}x\right] \\
& +\int_{P_{[m-1]}}^{P_{[m]}}\left(x+v+\alpha p_{[m]}\right)f(x)\,\mathrm{d}x+\int_{P_{[m]}}^{\infty}P_{[m]}f(x)\,\mathrm{d}x.
\end{aligned}
$$

Following a similar argument we can see that $E[C_{[m]}(\sigma_1)]-E[C_{[m]}(\sigma_2)]\leqslant 0$ for all $m>k+1$. □

Theorem 8. *For the* $1\mid rm, nr-a\mid\sum E[C_i]$ *problem, it is optimal to sequence jobs in the SPT order when the distribution function* $F(x)$ *is concave.*

Proof. From the proof in Theorem 7, we see that jobs processed before the maintenance activity must follow the SPT order. It is also well known that jobs processed after the maintenance activity must follow the SPT order. However, it remains to be shown that jobs to be processed before the maintenance activity have a smaller processing time than those to be processed after the maintenance activity. Consider a sequence, $\sigma_1=(\rho,i,M,j,\omega)$ where ρ and ω are two of jobs which precede job i and follow job j, respectively. The maintenance activity is done right after the completion of job i and before job j in sequence σ_1. Consider another sequence $\sigma_2=(\rho,j,M,i,\omega)$ which is exactly the same as (ρ,i,M,j,ω) except that we interchange jobs i and j. We assume that there are $k-1$ jobs in the subsequences ρ. Assume $p_i\leqslant p_j$. Conditioning on the random variable X, $E[C_i(\sigma_1)]$ and $E[C_j(\sigma_2)]$ are given in the proof of the last theorem. The expected completion times of job j in sequence σ_1, and job i in sequence σ_2, respectively, can be found as follows:

$$E[C_j(\sigma_1)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+\sum_{r=t}^{k-1}\alpha p_{[r]}+\alpha p_i+\alpha p_j+v\right)f(x)\,\mathrm{d}x\right] + \int_{P_{[k-1]}}^{P_{[k-1]}+p_i}\big(x+\alpha p_i+\alpha p_j+v\big)f(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_i}^{\infty}\big(P_{[k-1]}+p_i+\alpha p_j+t\big)f(x)\,\mathrm{d}x, \tag{25}$$

$$E[C_i(\sigma_2)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x+\sum_{r=t}^{k-1}\alpha p_{[r]}+\alpha p_j+\alpha p_i+v\right)f(x)\,\mathrm{d}x\right] + \int_{P_{[k-1]}}^{P_{[k-1]}+p_j}\big(x+\alpha p_j+\alpha p_i+v\big)f(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_j}^{\infty}\big(P_{[k-1]}+p_j+\alpha p_i+t\big)f(x)\,\mathrm{d}x.$$

Hence,

$$E[C_j(\sigma_1)]-E[C_i(\sigma_2)] = \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}(P_{[k-1]}-x+t-v)f(x)\,\mathrm{d}x - \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}(\alpha p_i+\alpha p_j)f(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_i}^{\infty}(p_i+\alpha p_j)f(x)\,\mathrm{d}x - \int_{P_{[k-1]}+p_j}^{\infty}(p_j+\alpha p_i)f(x)\,\mathrm{d}x.$$

Note that

$$\int_{P_{[k-1]}+p_i}^{\infty}(p_i+\alpha p_j)f(x)\,\mathrm{d}x = \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}(p_i+\alpha p_j)f(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_j}^{\infty}(p_i+\alpha p_j)f(x)\,\mathrm{d}x \leqslant \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}(p_i+\alpha p_j)f(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_j}^{\infty}(p_j+\alpha p_i)f(x)\,\mathrm{d}x.$$

Hence, we have

$$E[C_j(\sigma_1)]-E[C_i(\sigma_2)] = \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}(P_{[k-1]}-x+t-v)f(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}(1-\alpha)p_if(x)\,\mathrm{d}x.$$

Since $t \leqslant v$, we have

$$E[C_j(\sigma_1)]-E[C_i(\sigma_2)] \leqslant -\int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}p_if(x)\,\mathrm{d}x + \int_{P_{[k-1]}+p_i}^{P_{[k-1]}+p_j}(1-\alpha)p_if(x)\,\mathrm{d}x \leqslant 0.$$

Since we have shown that $E[C_i(\sigma_1)]-E[C_j(\sigma_2)] \leqslant 0$ in the previous theorem, combining this result with the above discussion we have $\{E[C_i(\sigma_1)]+E[C_j(\sigma_1)]\}-\{E[C_j(\sigma_2)]+E[C_i(\sigma_2)]\} \leqslant 0$. It is obvious that the expected completion time of jobs in the subsequence ρ is not affected by the interchange of jobs i and j. Following a similar argument as used in the proof of Theorem 7, we can see that $E[C_{[m]}(\sigma_1)]-E[C_{[m]}(\sigma_2)] \leqslant 0$ for all $m > k+1$. $\square$

We call an instance of a problem *agreeable*, when the following condition satisfies: "If $d_i < d_j$, then $p_i \leqslant p_j$ for all $1 \leqslant i, j \leqslant n$". Similarly, we call an instance of a problem *reverse-agreeable*, when the following condition satisfies: "If $d_i < d_j$, then $p_i \geqslant p_j$ for all $1 \leqslant i, j \leqslant n$".

Theorem 9. *For the* $1 \mid rm, nr - a \mid \max E[L_i]$ *problem, if the instance satisfies with the agreeable (reverse-agreeable) assumption then it is optimal to sequence jobs in the EDD order when the distribution function* $F(x)$ *is concave (convex).*

Proof. It is clear that after the maintenance activity, jobs should be sequenced in the EDD order. Now we show that the jobs scheduled before the maintenance activity should also follow the EDD order when the instance satisfies with the agreeable (reverse-agreeable) assumption and the distribution function $F(x)$ is concave (convex). Consider a job sequence $\sigma_1 = (\rho, i, j, \omega)$ where i and j are two consecutive jobs, and ρ and ω are two subsequences of jobs which precede job i and follow job j, respectively. Consider another sequence $\sigma_2 = (\rho, j, i, \omega)$ which is exactly the same as σ_1 except that we interchange the position of jobs i and j. Assume $d_i < d_j$. Let B denote the total sum of processing time of jobs in the subsequence ρ.

From Eq. (23), we have

$$\begin{aligned} E[L_j(\sigma_1)] - E[L_i(\sigma_2)] &= E[C_j(\sigma_1)] - E[C_i(\sigma_2)] - d_j + d_i \\ &= \alpha[p_i F(P_{[k-1]} + p_i) - p_j F(P_{[k-1]} + p_j) + (p_j - p_i) F(P_{[k-1]} + p_i + p_j)] + (d_i - d_j). \end{aligned} \tag{26}$$

Suppose the instance is agreeable, then $p_i \leqslant p_j$. In such a case, if F is concave then by (18) we have

$$(p_j - p_i) F(P_{[k-1]} + p_i + p_j) \leqslant p_j F(P_{[k-1]} + p_j) - p_i F(P_{[k-1]} + p_i).$$

Hence, it follows that $E[L_j(\sigma_1)] \leqslant E[L_i(\sigma_2)]$. Following a similar proof to that of Theorem 7, we have $E[C_{[m]}(\sigma_1)] \leqslant E[C_{[m]}(\sigma_2)]$, hence $E[L_{[m]}(\sigma_1)] \leqslant E[L_{[m]}(\sigma_2)]$ for all $m > k + 1$. Clearly, $E[L_{[m]}(\sigma_1)] = E[L_{[m]}(\sigma_2)]$ for all $m < k$.

On the other hand, if the instance is reverse-agreeable, then $p_j \leqslant p_i$. In such a case, if F is convex then similar to (18), we have

$$F(P_{[k-1]} + p_i) \geqslant \frac{p_j}{p_i} F(P_{[k-1]} + p_j) + \left(1 - \frac{p_j}{p_i}\right) F(P_{[k-1]} + p_i + p_j).$$

Namely,

$$(p_i - p_j) F(P_{[k-1]} + p_i + p_j) \geqslant (p_i F(P_{[k-1]} + p_i) - p_j F(P_{[k-1]} + p_j),$$

or equivalently,

$$(p_j - p_i) F(P_{[k-1]} + p_i + p_j) \leqslant p_j F(P_{[k-1]} + p_j) - p_i F(P_{[k-1]} + p_i).$$

Hence $E[L_j(\sigma_1)] \leqslant E[L_i(\sigma_2)]$. Following a similar proof to that of Theorem 7, we have $E[C_{[m]}(\sigma_1)] \leqslant E[C_{[m]}(\sigma_2)]$, hence $E[L_{[m]}(\sigma_1)] \leqslant E[L_{[m]}(\sigma_2)]$ for all $m > k + 1$. Clearly, $E[L_{[m]}(\sigma_1)] = E[L_{[m]}(\sigma_2)]$ for all $m < k$.

Hence, we see that jobs scheduled before the maintenance activity should also follow the EDD order when the instance satisfies with an agreeable (reverse-agreeable) assumption and the distribution function $F(x)$ is concave (convex). Thus the remaining task is to show that in an optimal solution, the due date of a job scheduled before the maintenance activity is not greater than that of any job scheduled after the maintenance activity. Consider a job sequence $\sigma_1 = (\rho, i, M, j, \omega)$ where ρ and ω are two subsequences of jobs which precede job i and follow job j, respectively. Also the maintenance activity is done right after the completion of job i and before the start of job j. Consider another sequence $\sigma_2 = (\rho, M, j, i, \omega)$ which is exactly the same as (ρ, i, M, j, ω) except that we move the job i to be processed immediately behind job j. Assuming $d_i > d_j$, we first calculate several expected completion times as follows:

$$E[C_j(\sigma_1)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x + v + \sum_{r=t}^{k-1}\alpha p_{[r]} + \alpha p_i + \alpha p_j\right)f(x)\,\mathrm{d}x\right] + \int_{P_{[k-1]}}^{P_{[k-1]}+p_i}\left(x + v + \alpha p_i + \alpha p_j\right)f(x)\,\mathrm{d}x$$
$$+ \int_{P_{[k-1]}+p_i}^{\infty}\left(P_{[k-1]} + p_i + t + \alpha p_j\right)f(x)\,\mathrm{d}x,$$

$$E[C_j(\sigma_2)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x + v + \sum_{r=t}^{k-1}\alpha p_{[r]} + \alpha p_i + \alpha p_j\right)f(x)\,\mathrm{d}x\right] + \int_{P_{[k-1]}}^{\infty}\left(P_{[k-1]} + \alpha p_j + t\right)f(x)\,\mathrm{d}x,$$

$$E[C_i(\sigma_2)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x + v + \sum_{r=t}^{k-1}\alpha p_{[r]} + \alpha p_i + \alpha p_j\right)f(x)\,\mathrm{d}x\right] + \int_{P_{[k-1]}}^{\infty}\left(P_{[k-1]} + t + \alpha p_j + \alpha p_i\right)f(x)\,\mathrm{d}x.$$

Since $v > t$, it can be shown that $E[C_i(\sigma_2)] \leqslant E[C_j(\sigma_1)]$. Thus the assumption $d_i > d_j$ implies that $E[L_i(\sigma_2)] = E[L_i(\sigma_2)] - d_i \leqslant E[C_j(\sigma_1)] - d_j = E[L_j(\sigma_1)]$. It can also be shown easily that $E[C_j(\sigma_2)] \leqslant E[C_j(\sigma_1)]$. Hence we have $E[L_j(\sigma_2)] \leqslant E[L_j(\sigma_1)]$. Thus $\max\{E[L_i(\sigma)_2], E[L_j(\sigma_2)]\} \leqslant \max\{E[L_i(\sigma_1)], E[L_j(\sigma_1)]\}$. Furthermore,

$$E[C_{[m]}(\sigma_1)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x + v + \sum_{r=t}^{k-1}\alpha p_{[r]} + \alpha p_i + \alpha p_j + \sum_{r=k+2}^{m}\alpha p_{[r]}\right)f(x)\,\mathrm{d}x\right]$$
$$+ \int_{P_{[k-1]}}^{P_{[k-1]}+p_i}\left(x + v + \alpha p_i + \alpha p_j + \sum_{r=k+2}^{m}\alpha p_{[r]}\right)f(x)\,\mathrm{d}x$$
$$+ \int_{P_{[k-1]}+p_i}^{\infty}\left(P_{[k-1]} + p_i + t + \alpha p_j + \sum_{r=k+2}^{m}\alpha p_{[r]}\right)f(x)\,\mathrm{d}x,$$

$$E[C_{[m]}(\sigma_2)] = \sum_{t=1}^{k-1}\left[\int_{P_{[t-1]}}^{P_{[t]}}\left(x + v + \sum_{r=t}^{k-1}\alpha p_{[r]} + \alpha p_i + \alpha p_j + \sum_{r=k+2}^{m}\alpha p_{[r]}\right)f(x)\,\mathrm{d}x\right]$$
$$+ \int_{P_{[k-1]}}^{\infty}\left(P_{[k-1]} + t + \alpha p_j + \alpha p_i + \sum_{r=k+2}^{m}\alpha p_{[r]}\right)f(x)\,\mathrm{d}x.$$

It can be shown that $E[C_{[m]}(\sigma_2)] \leqslant E[C_{[m]}(\sigma_1)]$ and thus $E[L_{[m]}(\sigma_2)] \leqslant E[L_{[m]}(\sigma_1)]$ for $m > k+1$. Clearly, $E[L_{[m]}(\sigma_1)] = E[L_{[m]}(\sigma_2)]$ for all $m < k$. □

Theorem 10. *For* $1 \mid rm, nr - a \mid E[\max L_i]$ *problem, it is optimal to sequence jobs in the EDD order.*

Proof. Since this proof is similar to that in Theorem 5 of the resumable case, we only mention briefly the differences in this proof. Again, we proceed with the proof by conditioning on the random variable X (time-to-breakdown) and the repair time V. The same two cases are to be considered here.

Case 1: The maintenance activity is carried out before the occurrence of the breakdown. In this case, the proof is identical to that of Theorem 5. Hence it is omitted.

Case 2: The maintenance activity is carried out after the occurrence of the breakdown. In this case, we proceed with the proof in the same two subcases as discussed in the proof of Theorem 5.

Subcase 1: Assume that $d_i > d_j$ and let B denote the sum of processing times of all jobs in the subsequence ρ. We can calculate the lateness of jobs i and j as follows:

$$L_i(\sigma_1) = B + p_{i1} + v + \alpha p_i - d_i,$$

$$L_j(\sigma_1) = B + p_{i1} + v + \alpha p_i + \alpha p_j - d_j,$$

$$L_i(\sigma_2) = B + p_{j1} + v + \alpha p_j + \alpha p_i - d_i,$$

$$L_j(\sigma_2) = B + p_{j1} + v + \alpha p_j - d_j.$$

We obtain $L_j(\sigma_1) - L_i(\sigma_2) = (p_{i1} - p_{j1}) + (d_i - d_i) \geqslant 0$ since $p_{i1} = p_{j1}$ and $d_i > d_j$. Also, $L_j(\sigma_1) - L_j(\sigma_2) = \alpha p_i \geqslant 0$. Thus $\max\{L_i(\sigma_2), L_j(\sigma_2)\} \leqslant \max\{L_i(\sigma_1), L_j(\sigma_1)\}$. Furthermore, it can be shown that $L_k(\sigma_2) = L_k(\sigma_1)$ for each job k in ω. Clearly, $L_k(\sigma_2) = L_k(\sigma_1)$ for any job k in subsequence ρ.

Subcase 2: We can calculate the lateness of jobs i and j as follows:

$$L_i(\sigma_1) = B + p_{i1} + v + \alpha p_i - d_i,$$

$$L_j(\sigma_1) = B + p_{i1} + v + \alpha p_i + \alpha p_j - d_j,$$

$$L_i(\sigma_2) = B + p_j + p_{i1'} + v + \alpha p_i - d_i,$$

$$L_j(\sigma_2) = B + p_j - d_j.$$

We can obtain $L_j(\sigma_1) - L_i(\sigma_2) = \alpha p_j + (d_i - d_i) \geqslant 0$ since $p_{i1} = p_j + p'_{i1}$, and $d_i > d_j$. Also, $L_j(\sigma_1)L_j - (\sigma_2) = (p_{i1} - p_j) + v + \alpha(p_i + p_j) \geqslant 0$ since $p_{i1} \geqslant p_j$. Thus $\max\{L_i(\sigma_2), L_j(\sigma_2)\} \leqslant \max\{L_i(\sigma_1), L_j(\sigma_1)\}$. Furthermore, it can be shown that $L_k(\sigma_2) = L_k(\sigma_1)$ for each job k in ω. Clearly, $L_k(\sigma_2) = L_k(\sigma_1)$ for any job k in subsequence ρ.

In summary, we have shown that EDD sequence minimizes the $\max L_i$ for any instance of X and V, hence it also minimizes $E[\max L_i]$. □

6. Conclusions and future research

Motivated by a problem commonly found in the surface-mount technology of electronic assembly lines, this paper studies a scheduling problem involving repair and maintenance rate-modifying activities. Several interesting results are obtained, especially for the nonresumable case. For example, (i) for the problem of minimizing the expected makespan, if we decide not to do maintenance activity then it is optimal to sequence jobs in the SPT order when the distribution function $F(x)$ is concave and in the LPT order when the distribution function $F(x)$ is convex up to the total sum of job processing times; (ii) for the problem of

Table 1
Summary of results

Problem	Result
$1 \mid rm, r-a \mid E[C_{\max}]$	Lemma 2
$1 \mid rm, r-a \mid E[\sum C_i]$	Theorem 2 (SPT rule)
$1 \mid rm, r-a \mid \max E[L_i]$	Theorem 4 (EDD rule)
$1 \mid rm, r-a \mid E[\max L_i]$	Theorem 5 (EDD rule)
$1 \mid rm, nr-a \mid E[C_{\max}]$	Theorem 6
$1 \mid rm, nr-a \mid E[\sum C_i]$	Theorem 8
$1 \mid rm, nr-a \mid \max E[L_i]$	Theorem 9
$1 \mid rm, nr-a \mid E[\max L_i]$	Theorem 10

minimizing total expected completion times, it is optimal to sequence jobs in the SPT order when the distribution function $F(x)$ is concave; (iii) for the problem of minimizing maximal expected lateness when the instance satisfies with the agreeable (reverse-agreeable) assumption, it is optimal to sequence jobs in the EDD order when the distribution function $F(x)$ is concave (convex). The results are summarized in Table 1.

Future research will focus on the problem with $q_j = \alpha_j p_j$ where α_j can vary for different j. We are also working on parallel machine as well as flow shop problems, where a sub-normal production rate machine is one of the machines in the system.

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