MACHINE LEARNING

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1 Multivariate Gaussian Distribution

Ex1. Proof: Σ is symmetric then Σ^{-1} is symmetric.

SOLVE

$$\Sigma\Sigma^{-1} = (\Sigma\Sigma^{-1})^T \tag{1}$$
$$\Sigma\Sigma^{-1} = (\Sigma^{-1}\Sigma)^T \tag{2}$$

We multiply Σ^{-1} :

$$\Sigma^{-1}(\Sigma\Sigma^{-1}) = \Sigma^{-1}((\Sigma^{-1}\Sigma)^T)$$

Plus we have:

$$I = \Sigma\Sigma^{-1}$$

$$I = I^{-1}$$

$$\text{So:}(2) \Rightarrow \Sigma = \Sigma^{-1}$$

Ex2. Conditional Gaussian Distribution
We have:

$$-\frac{1}{2}(x-\mu)\Sigma^{-1}(x-\mu)$$

$$= -\frac{1}{2}(x-\mu)A(x-\mu)$$

$$= -\frac{1}{2}(x_a-\mu_a)^T A_{aa}(x_a-\mu_a) - \frac{1}{2}(x_a-\mu_a)^T A_{ab}(x_b-\mu_b)$$

$$-\frac{1}{2}(x_b-\mu_b)^T A_{ba}(x_a-\mu_a) - \frac{1}{2}(x_b-\mu-b)^T A_{bb}(x_b-\mu_b)$$

$$= -\frac{1}{2}(x_a^T A_{aa}^{-1} x_a + x_a^T (A_{aa}\mu_a - A_{ab}(x_b - \mu_b)) + const$$

Set:

$$\Delta^2 = -\frac{1}{2} x^T \Sigma^{-1} x + x^T \Sigma^{-1} \mu + const$$

As a result:

$$\mu_{a|b} = \mu_a + \Sigma_{ab} + \Sigma_{bb}^{-1}(x_b - \mu_b)$$

$$\Sigma_{a|b} = \Sigma_{aa} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba}$$

$$\Rightarrow \text{p(xa—xb)} = \text{N}(\text{x}_{a|b}|\mu_{a|b}, \Sigma_{a|b})$$