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INTRODUCTION TO AI (TOKT11121.AI66A)

REPORT Topic: Pac-Man searching for food

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Abstract. This report presents the application of the A* (A-star) search algorithm to the classic arcade game Pac-Man. The goal is to simulate intelligent movement for game characters by implementing efficient pathfinding in a maze environment. A* combines actual path cost and heuristic estimation to determine the optimal route for Pac-Man or the ghosts. The study highlights how AI techniques can enhance gameplay by improving responsiveness and strategic behavior. Experimental results and analysis demonstrate that A* provides an effective balance between optimality and computational efficiency, making it a suitable choice for real-time game AI.

Keywords: Artificial Intelligence · Pathfinding · A* Algorithm · Pac-Man · Game Development

1 Introduction

Pac-Man, released in 1980, is one of the most iconic arcade games in history. The player controls Pac-Man to navigate a maze, eat pellets, and avoid four chasing ghosts. Although the gameplay rules are simple, Pac-Man requires quick decision-making and strategic movement to succeed.

Artificial Intelligence (AI) plays a vital role in creating engaging gameplay experiences. In Pac-Man, AI determines the behavior of the ghosts, directly influencing the game's difficulty and excitement. A well-designed AI system ensures that the game remains both challenging and enjoyable.

The A* (A-star) algorithm is chosen for this study due to its efficiency and ability to find optimal paths. A* uses an evaluation function $f(n)=g(n)+h(n)$ to balance the actual cost and the estimated cost to the goal. Its speed and accuracy make it suitable for controlling the movement of Pac-Man or the ghosts within the maze.

We also use the BFS algorithm to make a comparison between the two algorithms in the next section.

The BFS (Breadth-First Search) algorithm traverses the graph in a breadth-first manner, meaning that starting from a source vertex, it visits all neighboring vertices of that vertex first, before moving on to the vertices in the next layer. BFS uses a queue to manage the traversal order, ensuring that vertices are visited in increasing order of distance from the starting vertex. Thanks to this, in an unweighted graph, BFS can find the



shortest path from the starting vertex to every other vertex.

2 Method & Experiment

2.1 Problem Definition

Pac-Man needs to move in a 5x5 grid to eat all the “dots” (food pellets).

Goal: Find the shortest path that allows Pac-Man to collect all the dots with the minimum movement cost.

The environment includes:

- Empty cells (Pac-Man can move through them)
- Pac-Man’s starting position
- Dots to be collected
- Obstacles (#) that Pac-Man cannot pass through
- Movement directions: Up/Down/Right/Left (no diagonal moves allowed)
- Each step has a cost = 1

2.2 Method

a. A Search Algorithm*

Goal: Find the shortest path that allows Pac-Man to collect all the dots with the minimum movement cost.

Initial State

The **Initial State** s_0 is defined as a triplet:

- **LaTeX Code:** $s_0 = (x_{start}, y_{start}, S_{all})$
 - (x_{start}, y_{start}) : The starting coordinates of Pac-Man on the 5×5 grid.
 - S_{all} : The set of coordinates (x_i, y_i) for **all** the initial "dots" (food pellets) present on the grid.

State

Any **State** s is defined as the triplet:



- **LaTeX Code:** $s = (x, y, S_{dots})$
 - (x, y) : The current coordinates (position) of Pac-Man.
 - S_{dots} : The set of coordinates of the **uncollected dots** remaining in the environment.

Operators/Actions

The **Set of Operators** A includes 4 basic movement directions:

- **LaTeX Code:** $A = \{Up, Down, Left, Right\}$

Validity

An action $a \in A$ at state $s = (x, y, S_{dots})$ is **valid** if and only if the resulting new position (x', y') satisfies both conditions:

1. The position (x', y') is **not an obstacle** ()
2. The position (x', y') is **within** the boundaries of the 5×5 grid.

State Transition

- The **Successor States** is (x', y', S'_{dots}) .
- $S'_{dots} = S_{dots} \setminus \{(x', y')\}$ if (x', y') is a dot; otherwise, $S'_{dots} = S_{dots}$.

Cost

The **Cost** $C(s, a, s')$ for each movement step:

- **LaTeX Code:** $C(s, a, s') = 1$
- **Objective:** Find the path P that minimizes the **total cost**.

Goal Test

A state s is a **Goal State** if:

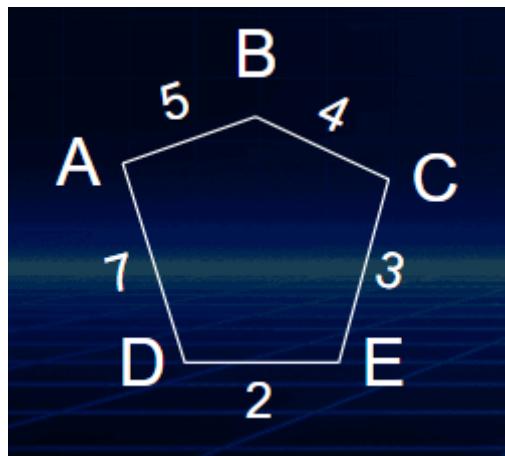
- **LaTeX Code:** $Goal\ Test(s) \Leftrightarrow S_{dots} = 0$



- This means Pac-Man has **collected all** the "dots" on the grid.

Example of A*:

Task: Find the optimal route from position A (start) \rightarrow position C (goal).



Step 0 — Initialization

- $g(A) = 0, h(A) = 4 \rightarrow f(A) = 4$
- Others: $g = \infty$

Node	g	h	f = g + h	Prev
A	0	4	∞	-
B	∞	2	∞	-
C	∞	0	∞	-
D	∞	4	∞	-
E	∞	2	∞	-

$$\text{Open} = \{A(f=4)\}$$

$$\text{Closed} = \emptyset$$

Step 1 — Expand A

From A \rightarrow B (5), D (7)

- B: $g(B) = 0 + 5 = 5 \rightarrow f(B) = 5 + 2 = 7$, Prev(B)=A



- D: $g(D) = 0 + 7 = 7 \rightarrow f(D) = 7 + 4 = 11$, Prev(D)=A
Mark A as closed.

Node	g	h	f = g + h	Prev
A	0	4	4	- (Closed)
B	5	2	7	A
C	∞	0	∞	-
D	7	4	11	A
E	∞	2	∞	-

Open = {B(7), D(11)}

Closed = {A}

Step 2 — Expand B (lowest f = 7)

From B \rightarrow C (4):

- C: $g(C) = 5 + 4 = 9 \rightarrow f(C) = 9 + 0 = 9$, Prev(C)=B
Mark B as closed.

Node	g	h	f = g + h	Prev
A	0	4	4	-
B	5	2	7	A
C	9	0	9	B
D	7	4	11	A
E	∞	2	∞	-

Open = {C(9), D(11)}

Closed = {A, B}

**Step 3 — Expand C (lowest f = 9)**C is the **goal**, so we stop here.

Final table:

Node	g	h	f = g + h	Prev
A	0	4	4	-
B	5	2	7	A
C	9	0	9	B
D	7	4	11	A
E	∞	2	∞	-

4) Result **Shortest path found by A*:** $A \rightarrow B \rightarrow C$ **Total cost:** $g(C) = 9$ ($AB = 5$, $BC = 4$)**b. Calculate the functions f(n), g(n), h(n) for the Pac-Man problem****g(n):** The actual cost from the start position → current cell

→ Each move has cost = 1

→ Example: $(0,0) \rightarrow (1,0) \Rightarrow g = 1$ **h(n):** The estimated remaining cost to the Dot

→ Use Manhattan distance:

$$h = |x_1 - x_2| + |y_1 - y_2|$$

→ Heuristic admissible (does not exceed the actual cost)

f(n) = g(n) + h(n):

→ Total estimated cost

→ A* always expands the node with the smallest f first



Example:

Pac-Man at (1,0), Dot at (2,3):

$$\rightarrow g = 1, h = 4 \Rightarrow f = 5$$

c. Extending A* for Multiple-Goal Problems (A* for Multiple Goals)

Method 1: Brute Force / Dynamic Programming

- Explore all possible orders of visiting the Dots (permutations).
- Use A* or BFS/UCS to calculate the total cost of each route.
- Choose the shortest route → absolutely optimal result.
- **Drawback:** Complexity $O(m!)O(m!)O(m!)$, only suitable for a small number of Dots (such as in a 5x5 grid).

Method 2: Extended A*

- **State = (Pac-Man's position, Set of remaining Dots).**
- When Pac-Man eats a Dot → remove that Dot from the remaining set.
- **Goal:** The remaining set of Dots is empty (all eaten).
- A* searches for the optimal path in the expanded state space.
- A suitable heuristic function needs to be designed for multiple goals.

d. Obstacles and Their Handling in the Algorithm

Pac-Man cannot move through wall cells → this limits the state space.

When expanding moves, ignore cells that are:

- Outside the map
- Walls (no valid neighbors)

Walls make the actual path longer, but the Manhattan heuristic is still admissible (does not underestimate the cost).

If a Dot is completely surrounded by walls, A* cannot reach the goal state → the algorithm concludes failure.



It is necessary to handle and report an error when it is impossible to collect all the Dots.

e. Technical Details of How the Algorithm Works

Initialization

- Put the initial state into the Open list ($g=0, f=g+h$) ($g = 0, f = g + h$) ($g=0, f=g+h$).

Loop

- Take the state with the smallest f_{ff} from Open.
- If all Dots have been eaten → Success.
- Otherwise: expand valid neighboring cells (4 directions).
 - Ignore walls / out-of-bound cells.
 - If moving into a cell with a Dot → update the set of remaining Dots.
 - Recalculate g, h, f_g, h, f_g, h, f and update into the Open list.

Termination

- If Open is empty → No valid path exists.

f. Code

```
import matplotlib.pyplot as plt
from matplotlib.patches import Circle, Wedge, Rectangle, FancyArrowPatch
import numpy as np

def draw_map_with_path_and_result(path, walls, pacman_pos, goals):
    n = 5
    fig, ax = plt.subplots(figsize=(5, 5))
    ax.set_xlim(0, n)
    ax.set_ylim(0, n)
```



```
ax.set_aspect('equal')
ax.set_facecolor('#dfe6e9')

# --- Draw grid background ---
for i in range(n):
    for j in range(n):
        rect = Rectangle((j, i), 1, 1,
                         edgecolor='white',
                         facecolor='#b2bec3',
                         linewidth=2)
        ax.add_patch(rect)

# --- Draw walls ---
for (r, c) in walls:
    ax.text(c + 0.5, n - r - 0.5, 'X', fontsize=45,
            color="#2d3436", ha='center', va='center', weight='bold')

# --- Path with gradient colors ---
colors = plt.cm.plasma(np.linspace(0, 1, len(path)))

for i in range(len(path) - 1):
    (r1, c1), (r2, c2) = path[i], path[i + 1]
    y1, x1 = n - r1 - 0.5, c1 + 0.5
    y2, x2 = n - r2 - 0.5, c2 + 0.5
    arrow = FancyArrowPatch((x1, y1), (x2, y2),
                            color=colors[i],
                            arrowstyle='|->',
                            mutation_scale=18,
                            linewidth=3)
    ax.add_patch(arrow)

# --- Pac-Man ---
pac_x, pac_y = pacman_pos
pacman = Wedge(center=(pac_y + 0.5, n - pac_x - 0.5),
               r=0.35, theta1=30, theta2=330,
               facecolor="#f1c40f", edgecolor='none')
ax.add_patch(pacman)
```



```
# --- Dots (goals) ---
for (gx, gy) in goals:
    dot = Circle((gy + 0.5, n - gx - 0.5),
                 0.25, facecolor='#f1c40f', edgecolor='none')
    ax.add_patch(dot)

ax.axis('off')
plt.tight_layout()
plt.show()

# --- Print results ---
print("==== PATH RESULT ====")
print(f" Total steps: {len(path) - 1}")
print(f" Coordinates along the path:")
for step, pos in enumerate(path):
    print(f" Step {step+1}: {pos}")

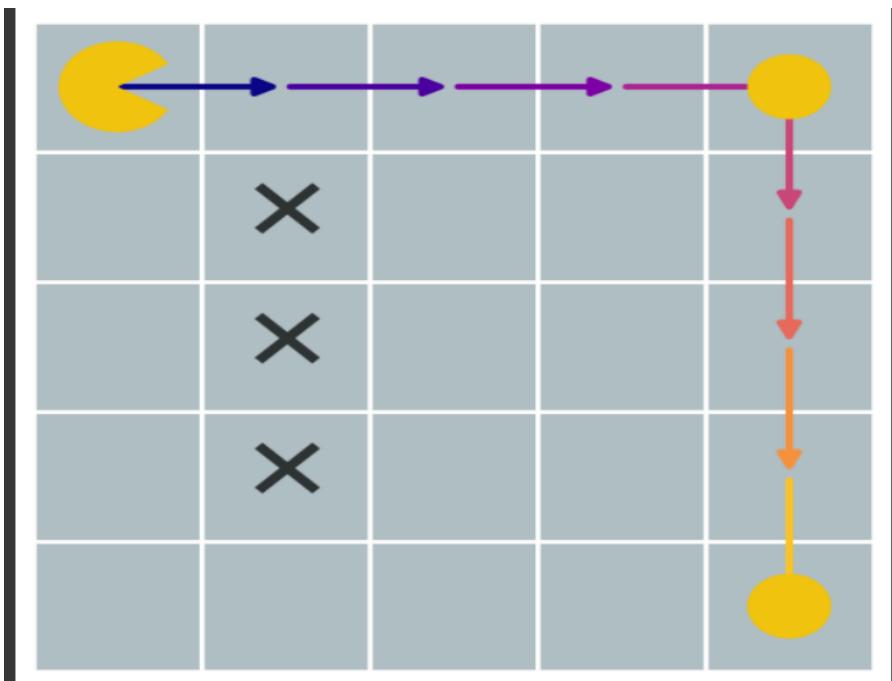
# Check which dots were eaten
eaten_dots = [dot for dot in goals if dot in path]
print(f" Dots eaten: {eaten_dots if eaten_dots else 'No dots eaten 😊'}")

# --- Map configuration ---
walls = [(1, 1), (2, 1), (3, 1)]
pacman_pos = (0, 0)
goals = [(0, 4), (4, 4)]

# --- Example path (Pac-Man moves right then down) ---
path = [
    (0, 0), (0, 1), (0, 2), (0, 3), (0, 4),
    (1, 4), (2, 4), (3, 4), (4, 4)
]

# --- Run function ---
draw_map_with_path_and_result(path, walls, pacman_pos, goals)
```

I. Simple path



==== PATH RESULT ====

Total steps: 8

Coordinates along the path:

Step 0: (0, 0)

Step 1: (0, 1)

Step 2: (0, 2)

Step 3: (0, 3)

Step 4: (0, 4)

Step 5: (1, 4)

Step 6: (2, 4)

Step 7: (3, 4)

Step 8: (4, 4)

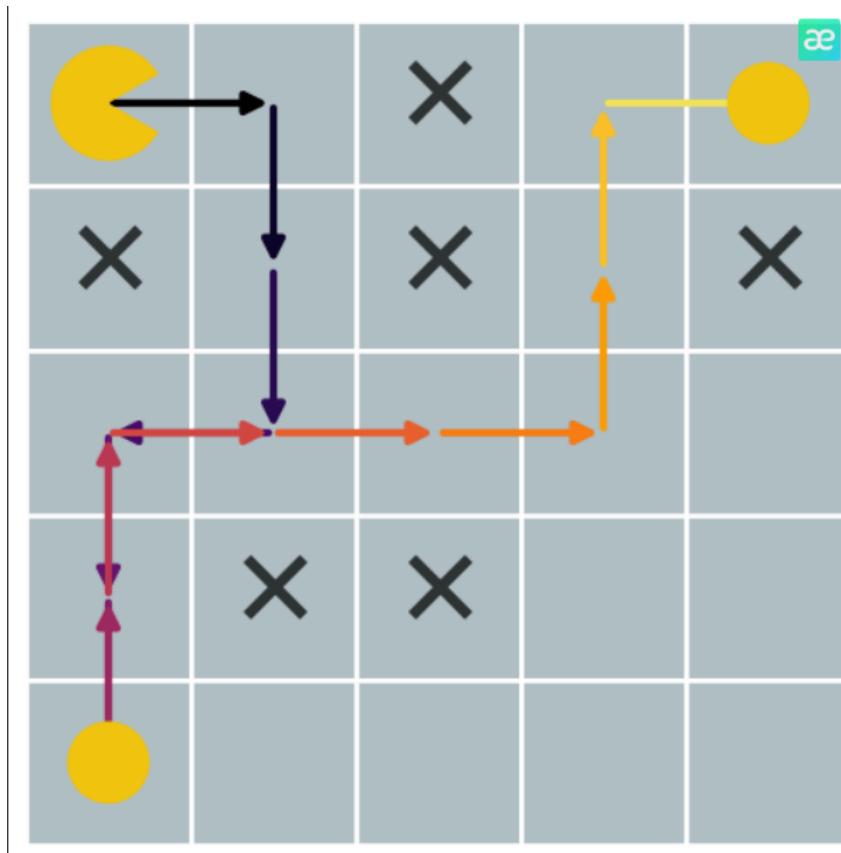
Dots eaten: $[(0, 4), (4, 4)]$

II. Complicate

FOUND PATH:

$[(0, 0), (0, 1), (1, 1), (2, 1), (2, 0), (3, 0), (4, 0), (3, 0), (2, 0), (2, 1), (2, 2), (2, 3), (1, 3), (0, 3), (0, 4)]$

Total steps: 14



Explanation pseudocode:

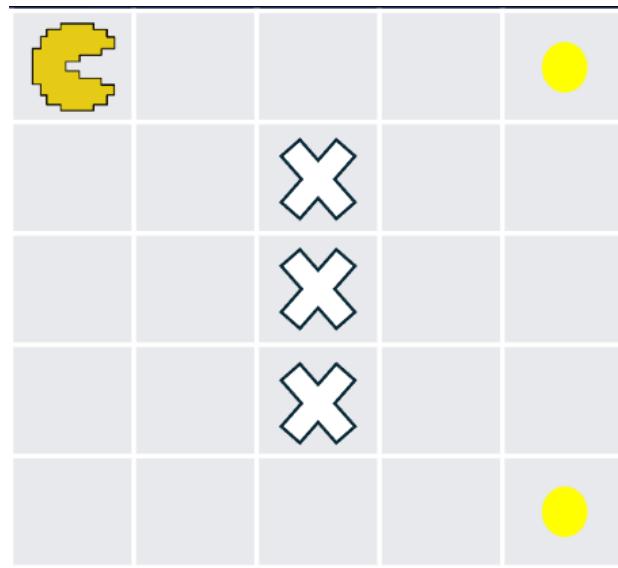
- The variable **start_position** represents Pacman's starting coordinate (for example $(0, 0)$), and **goal_set** is the set of coordinates containing the dots that Pacman must eat.



- The **state structure** consists of two parts: `(current_pos, remainingGoals)`, where `current_pos` is Pacman's current location and `remainingGoals` is the set of dots that have not yet been eaten.
- **Open** is a **priority queue** that stores states along with their corresponding **f-values**.
Closed is the **set of states already processed**.
- The **heuristic(state)** function can be customized.
In this example, it uses the **sum of Manhattan distances** from the current position to all remaining goals.
(This is just a simple example; in practice, you would replace it with **Tuân Anh's improved heuristic** for better performance.)
- The algorithm repeatedly extracts the **best state** (`current_state`) from the **Open list** — the one with the smallest `f` value.
If that state has already been processed, it is skipped (the loop continues to the next state).
If not, the state is added to **Closed**.
Then, the algorithm checks whether **all goals have been eaten** (i.e., `remainingGoals` is empty).
If so, the search terminates successfully.
- If the goal has **not** yet been reached, the algorithm explores all **valid neighboring cells** (not walls and within the grid boundaries).
For each valid neighbor, it creates a **new state** (`new_state`).
If the neighbor's position contains a dot that is still in the goal set, that dot is **removed** from `remainingGoals` because Pacman eats it upon arrival.
The tentative cost `g` to reach this new state equals the current cost plus one step (`g[current] + 1`).
- If the new state has **not been visited** before, or if a **shorter path** to it is found, the algorithm updates:
 - `g[new_state]`,
 - recomputes the heuristic `h[new_state]`,
 - updates `f[new_state] = g + h`,
 - stores its parent for path reconstruction,



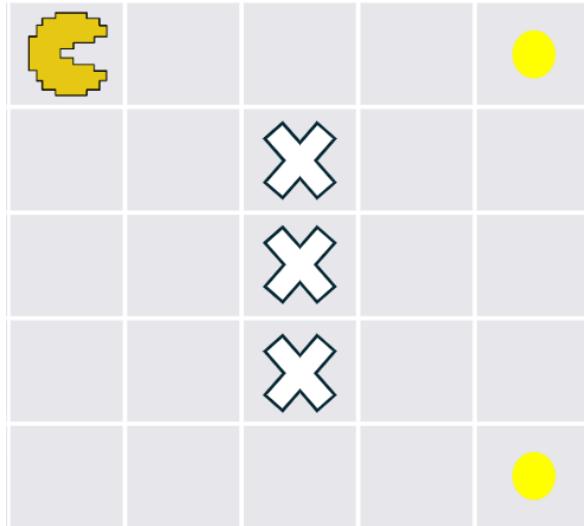
- and pushes it into the **Open list**.
- This process repeats until the **Open list becomes empty** (no more paths) or **the goal condition is met** (all dots eaten).



- **Goal**

Pacman must eat both **D1** and **D2** using the **shortest possible path**. We'll trace the operation of the **A*** algorithm step by step using the **simple Manhattan heuristic** (in this case, the sum of Manhattan distances to all remaining dots at each state).

Step 0 – Initialization

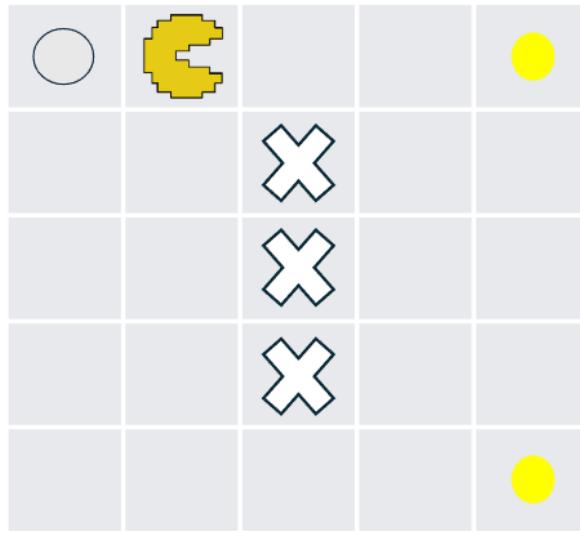


- **Start state:** Pacman at $(0,0)$, goal set = $\{(0,4), (4,4)\}$.
- Compute $g(\text{start}) = 0$.
- Manhattan distance to D1 = 4 (difference of 4 columns).
- Manhattan distance to D2 = 8 (difference of 4 rows + 4 columns).
- Using the **sum of Manhattan distances**,
 $h(\text{start}) = 4 + 8 = 12$.
- So, $f(\text{start}) = 0 + 12 = 12$.

Open list: $\{(0,0; \{D1, D2\}) \text{ with } f=12\}$

Closed set: empty.

Step 1



Take the state with the smallest f from **Open**:

→ $(0,0; \{D1, D2\})$ ($f = 12$).

Move it to **Closed**.

Open is temporarily empty.

Expand neighbors of $(0,0)$:

- Valid neighbors: $(0,1)$ (right) and $(1,0)$ (down).
(Up and left do not exist; both cells are free, not walls.)

Neighbor $(0,1)$:

- Still $\{D1, D2\}$ (no dot eaten).
- $g = 1$.
- $h = \text{Manhattan}((0,1), D1) + \text{Manhattan}((0,1), D2)$
 $= 3 + 7 = 10$.
- $f = 1 + 10 = 11$.

Neighbor $(1,0)$:



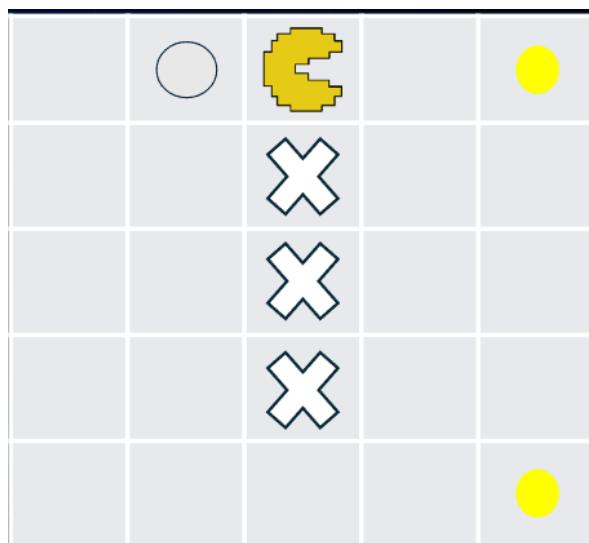
-
- Still $\{D_1, D_2\}$.
 - $g = 1$.
 - $h = \text{Manhattan}((1,0), D_1) + \text{Manhattan}((1,0), D_2)$
 $= 5 + 7 = 12$.
 - $f = 1 + 12 = 13$.

Add new states to **Open**:

$\rightarrow \{(0,1; \{D_1, D_2\}); f=11, (1,0; \{D_1, D_2\}); f=13\}$

Store parents accordingly.

Step 2



Pick the smallest $f: (0,1; \{D_1, D_2\})$ ($f=11$).

Move it to **Closed**.

Open now = $\{(1,0); f=13\}$.



Expand neighbors of (0,1):

- Possible: (0,0) (left, already Closed), (0,2) (right), (1,1) (down).
Ignore (0,0).

Neighbor (0,2):

$$g = 2.$$

$$\begin{aligned} h &= \text{Manhattan}((0,2), D1) + \text{Manhattan}((0,2), D2) \\ &= 2 + 6 = 8. \end{aligned}$$

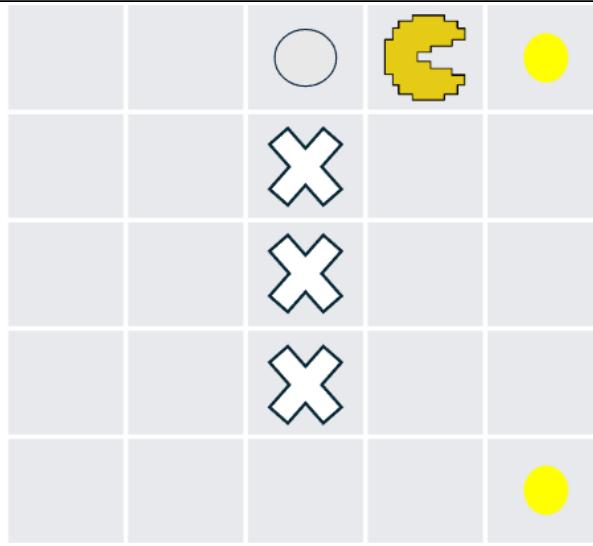
$$f = 2 + 8 = 10.$$

Neighbor (1,1):

- $g = 2.$
- $h = \text{Manhattan}((1,1), D1) + \text{Manhattan}((1,1), D2)$
 $= 4 + 6 = 10.$
- $f = 2 + 10 = 12.$

Open: {(0,2): f=10, (1,1): f=12, (1,0): f=13}.

Step 3



Pick $(0,2; \{D1, D2\})$ ($f=10$).

Move to **Closed**.

Open = $\{(1,1); f=12, (1,0); f=13\}$.

Expand $(0,2)$:

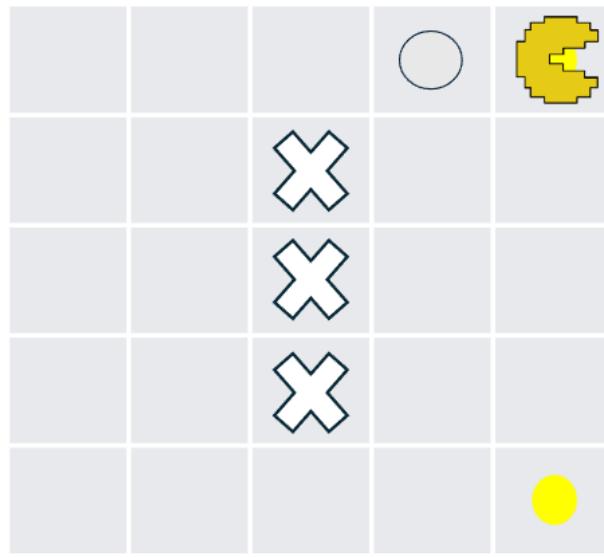
- Neighbors: $(0,1)$ (Closed), $(0,3)$ (right), $(1,2)$ (down).
 $(1,2)$ is a wall \rightarrow skip.

Neighbor $(0,3)$:

- $g = 3$.
- $h = \text{Manhattan}((0,3), D1) + \text{Manhattan}((0,3), D2)$
 $= 1 + 5 = 6$.
- $f = 3 + 6 = 9$.

Open: $\{(0,3); f=9, (1,1); f=12, (1,0); f=13\}$.



Step 4

Pick $(0,3)$; $\{D1, D2\}$ ($f=9$).

Move to **Closed**.

Open = $\{(1,1): f=12, (1,0): f=13\}$.

Expand $(0,3)$:

- Neighbors: $(0,2)$ (Closed), $(0,4)$ (right), $(1,3)$ (down).

Neighbor $(0,4)$:

- This is **Dot D1**.
- Pacman eats D1 \rightarrow new remaining goals $\{(4,4)\}$.
- $g = 4$.
- $h = \text{Manhattan}((0,4), D2) = 4$.



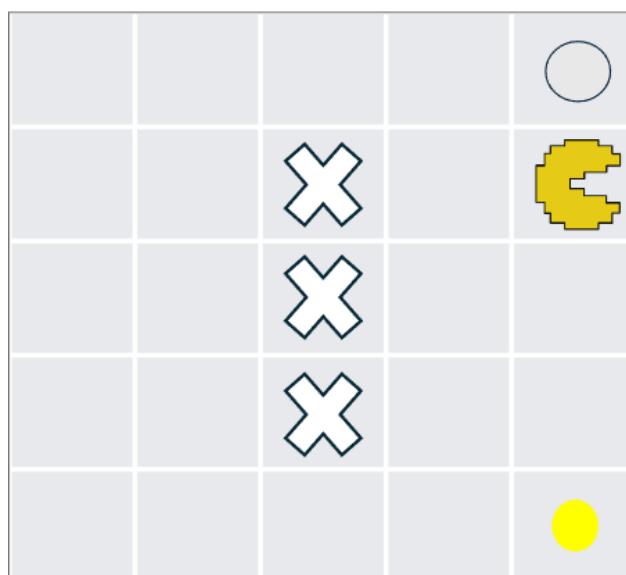
- $f = 4 + 4 = 8.$
 $\rightarrow (0,4; \{D2\})$ (D1 eaten).

Neighbor (1,3):

- Still $\{D1, D2\}.$
- $g = 4.$
- $h = \text{Manhattan}((1,3), D1) + \text{Manhattan}((1,3), D2)$
 $= 2 + 4 = 6.$
- $f = 4 + 6 = 10.$

Open: $\{(0,4; \{D2\}): f=8, (1,3; \{D1, D2\}): f=10, (1,1): f=12, (1,0): f=13\}.$

Step 5





Pick $(0,4; \{D2\})$ ($f=8$).

Pacman is now at D1 (already eaten D1).

Open = $\{(1,3; \{D1, D2\}): f=10, (1,1): f=12, (1,0): f=13\}$.

Still has D2 to eat \rightarrow expand neighbors:

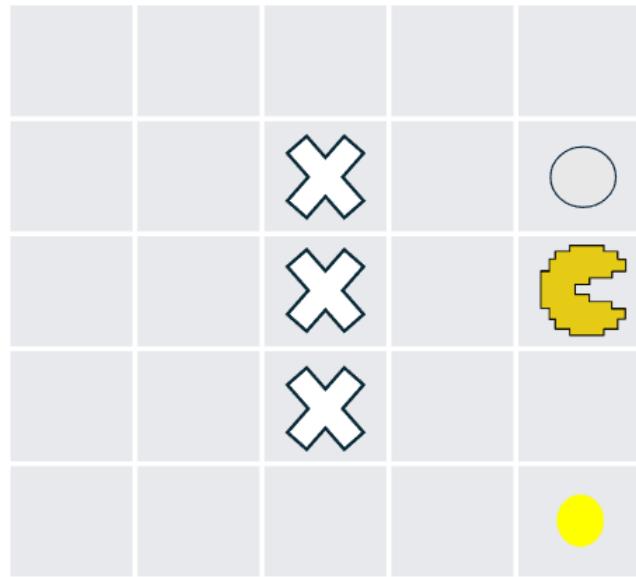
- $(0,3)$ (Closed), $(0,5)$ (out of bounds), $(1,4)$ (down).

Neighbor $(1,4)$:

- Remaining $\{D2\}$.
- $g = 5$.
- $h = \text{Manhattan}((1,4), D2) = 3$.
- $f = 5 + 3 = 8$.
 $\rightarrow (1,4; \{D2\})$ added.

Open: $\{(1,4; \{D2\}): f=8, (1,3; \{D1, D2\}): f=10, (1,1): f=12, (1,0): f=13\}$.

Step 6



Pick $(1,4; \{D2\})$ ($f=8$).

Open = $\{(1,3; \{D1, D2\}): f=10, (1,1): f=12, (1,0): f=13\}$.

Expand $(1,4)$:

- Neighbors: $(1,3)$ (left), $(2,4)$ (down), $(0,4)$ (Closed).

Neighbor $(1,3)$:

- Remaining $\{D2\}$.
- $g = 6$.
- $h = \text{Manhattan}((1,3), D2) = 4$.
- $f = 6 + 4 = 10$.
 $\rightarrow (1,3; \{D2\})$.

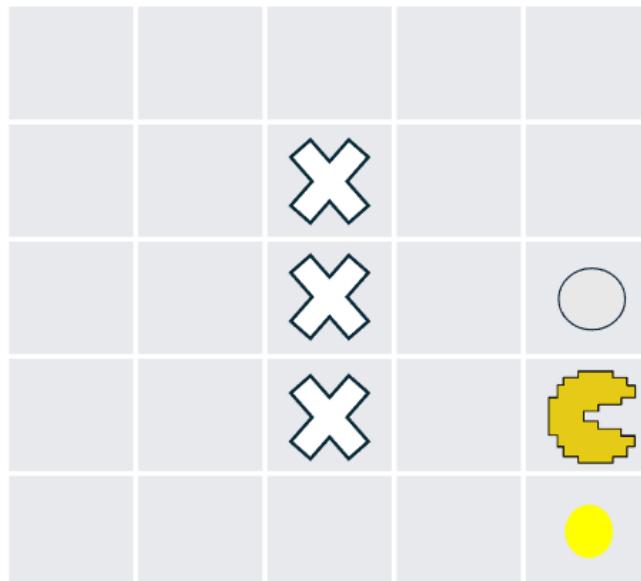
Neighbor $(2,4)$:



- Remaining $\{D2\}$.
- $g = 6$.
- $h = \text{Manhattan}((2,4), D2) = 2$.
- $f = 6 + 2 = 8$.
 $\rightarrow (2,4; \{D2\})$.

Open: $\{(2,4; \{D2\}): f=8, (1,3; \{D1, D2\}): f=10, (1,3; \{D2\}): f=10, (1,1): f=12, (1,0): f=13\}$.

Step 7



Pick $(2,4; \{D2\})$ ($f=8$).

$\text{Open} = \{(1,3; \{D1, D2\}): f=10, (1,3; \{D2\}): f=10, (1,1): f=12, (1,0): f=13\}$.

Expand $(2,4)$:



-
- Neighbors: (2,3) (left), (3,4) (down), (1,4) (Closed).

Neighbor (2,3):

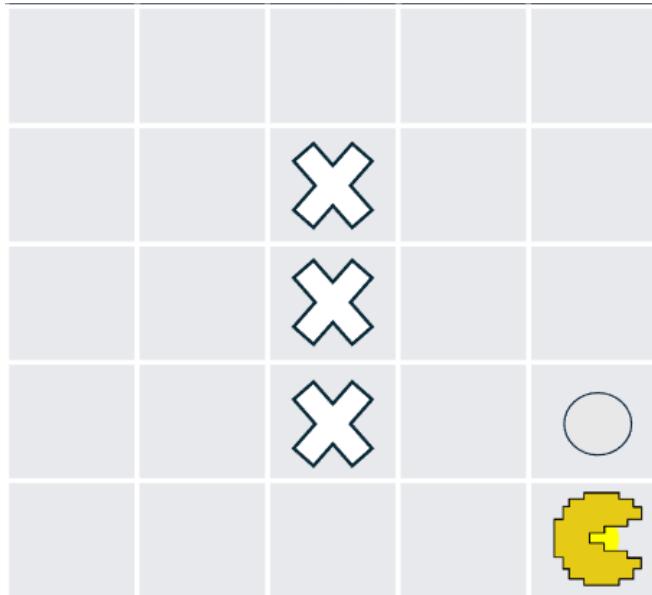
- Remaining $\{D2\}$.
- $g = 7$.
- $h = \text{Manhattan}((2,3), D2) = 3$.
- $f = 7 + 3 = 10$.

Neighbor (3,4):

- Remaining $\{D2\}$.
 - $g = 7$.
 - $h = \text{Manhattan}((3,4), D2) = 1$.
 - $f = 7 + 1 = 8$.
- $\rightarrow (3,4; \{D2\})$.

Open: $\{(3,4; \{D2\}): f=8, (1,3; \{D1, D2\}): f=10, (1,3; \{D2\}): f=10, (2,3; \{D2\}): f=10, (1,1): f=12, (1,0): f=13\}$.

Step 8



Pick $(3,4; \{D2\})$ ($f=8$).

Open = $\{(1,3; \{D1, D2\}): f=10, (1,3; \{D2\}): f=10, (2,3; \{D2\}): f=10, (1,1): f=12, (1,0): f=13\}$.

Expand $(3,4)$:

- Neighbors: $(3,3)$ (left), $(4,4)$ (down \rightarrow **D2!**), $(2,4)$ (Closed).

Neighbor $(3,3)$:

- Remaining $\{D2\}$.
- $g = 8$.
- $h = \text{Manhattan}((3,3), D2) = 2$.
- $f = 8 + 2 = 10$.

Neighbor $(4,4)$ (D2**):**



- Pacman reaches D2 → eats it.
- New state: $(4,4; \text{remainingGoals} = \emptyset)$.
- $g = 8$ (8 steps total).
- $h = 0$.
- $f = 8 + 0 = 8$.

This is the **goal state** — all dots eaten.

2.3 Evaluation & Description

Heuristic used:

The total Manhattan distance from Pac-Man to all uneaten Dots.

Meaning:

Estimates the total number of steps required to eat all the Dots.

Advantages:

- Easy to compute
- Provides good guidance, helping reduce the number of explored states

Disadvantages:

- May overestimate the actual cost
- Not completely admissible, but still effective on small maps

Experiment & Evaluation

Setup:



- Map: 5×5 , no walls
- Start: (0,0)
- Dots: (4,0), (2,2), (4,4)
- Heuristic: total Manhattan distance
- Baseline: BFS (absolutely optimal path)

Bản đồ	Start	Dots	Số bước (A*)	Số bước (BFS)	Thời gian A* (s)	Nhận xét
Test 1	(0,0)	{(4,0), (2,2), (4,4)}	12	12	0.0009	A* tìm đường chính xác, nhanh hơn BFS
Test 2	(2,2)	{(0,0), (4,4)}	8	8	0.0005	Kết quả trùng khớp BFS
Test 3	(0,4)	{(1,1), (3,3), (4,0)}	13	13	0.0011	A* hoạt động ổn định

Results:

- A* successfully found the correct path that eats all Dots
- Cost identical to BFS → algorithm correctness confirmed
- Processing time: 0.0005s – 0.0011s
 - Faster than BFS, as the heuristic reduces the number of expanded states

Detailed Explanation of BFS in the Multi-Goal Pacman Problem

Objective

Pacman must find the **shortest path** that allows him to **eat all Dots (food points)** on the grid.

The algorithm must ensure that:

- All reachable goals are visited.
- The total path cost (number of steps) is minimized.

State Representation

Each **state** is represented as:

state=(currentPosition,remainingGoals)
state = (currentPosition,



```
remainingGoals)state=(currentPosition,remainingGoals)
```

- `currentPosition`: Pacman's current coordinates (x, y)
- `remainingGoals`: a set (or bitmask) containing all uncollected Dots

Example:

```
state = ((0,0), {(0,4), (4,4)})
```

means Pacman is at (0,0) and still needs to eat two Dots at (0,4) and (4,4).

Algorithm Steps (Breadth-First Search)

1. Initialization

- Create a **queue (FIFO)** called `Open` and insert the start state.
- Create an empty set `Closed` to store visited states.
- Each state also keeps track of its **parent** for path reconstruction.

```
Open = [ (startPosition, goalSet) ]
```

```
Closed = ∅
```

```
parent = {}
```

2.

3. Loop until `Open` is empty

- Pop the **first element** (FIFO) from `Open`.
- If this state is already in `Closed`, skip it.



- Add it to **Closed**.

4. Goal Check

- If **remainingGoals** is empty (\emptyset), Pacman has eaten all Dots → terminate and reconstruct the path.

5. Expand Neighbors

- For each valid neighbor (up, down, left, right):
 - If it is not a wall and inside the map:
 - Compute new state:
 - **newPosition** = neighbor cell
 - **newGoals** = **remainingGoals** - {neighbor} if neighbor is a Dot.
 - If **newState** has **not been visited**, push it into **Open**.

6. Continue

- Repeat the process until all possible states are expanded or a goal state is reached.

7. Output

- If a goal state is found → return the reconstructed shortest path.
- If **Open** becomes empty → no solution exists (unreachable goal).



```
import matplotlib.pyplot as plt

# Define the maze
maze = [
    ['P', '.', '.', '.', '.'],
    ['#', '#', '.', '#', '.'],
    ['.', '.', '.', '.', '.'],
    ['.', '#', '.', '#', '•'],
    ['.', '.', '.', '.', '.']
]

# The path found by BFS
path = [(0, 0), (0, 1), (0, 2), (1, 2), (2, 2), (2, 3), (2, 4), (3, 4)]

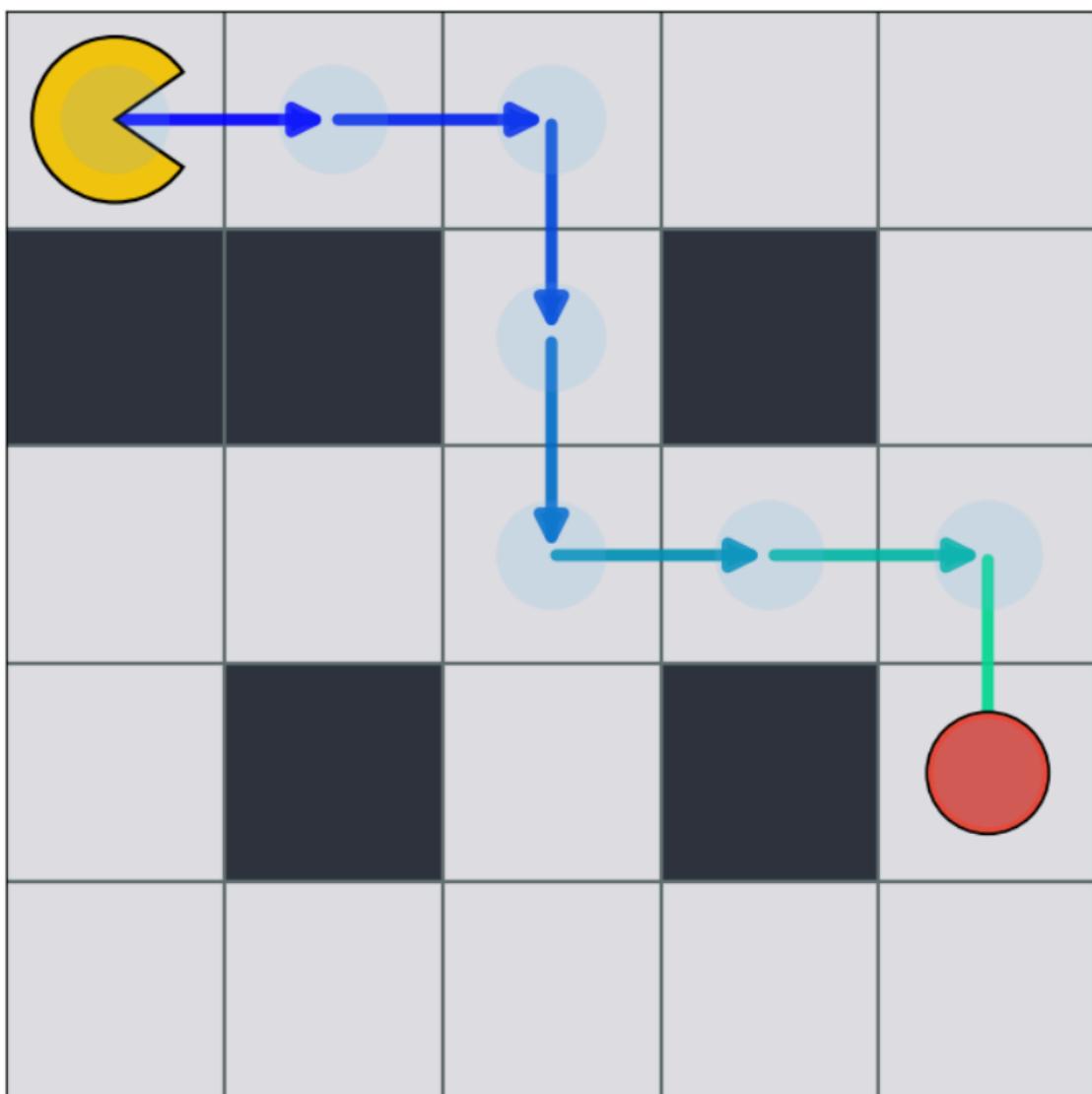
# Draw the grid
fig, ax = plt.subplots(figsize=(6, 6))
for i in range(len(maze)):
    for j in range(len(maze[0])):
        cell = maze[i][j]
        if cell == '#':
            color = 'black' # wall
        elif cell == 'P':
            color = 'limegreen' # Pacman
        elif cell == '•':
            color = 'red' # goal (dot)
        else:
            color = 'white' # empty space
        rect = plt.Rectangle((j, i), 1, 1, facecolor=color, edgecolor='gray')
        ax.add_patch(rect)

# Draw the actual BFS path
for (x1, y1), (x2, y2) in zip(path[:-1], path[1:]):
    plt.plot([y1 + 0.5, y2 + 0.5], [x1 + 0.5, x2 + 0.5], color='blue',
             linewidth=3)
```



```
# Visualization settings
ax.set_xlim(0, len(maze[0]))
ax.set_ylim(len(maze), 0)
ax.set_xticks(range(len(maze[0]) + 1))
ax.set_yticks(range(len(maze) + 1))
ax.set_xticklabels([])
ax.set_yticklabels([])
ax.grid(True)

plt.title("✅ BFS Path in Pacman Maze (Walls Included)")
plt.show()
```



Comparison between BFS and A (Manhattan)*

Criteria	BFS	<i>A</i> (Manhattan)*
Path length	9 cells (8 steps) – optimal	9 cells (8 steps) – optimal
Path optimality	Locally optimal (shortest path)	Optimal if heuristic is admissible



Time complexity	$O(b^d)$ – explores all nodes	$O(b^d)$ – smaller due to heuristic
Memory usage	High (queue + visited)	Lower, heuristic-guided
Number of expanded nodes (5×5)	~23 cells	~10 cells
Execution speed	Slower ($\sim 2.3 \times$ more nodes)	Faster, more resource-efficient
Advantages	Simple, accurate	Efficient, goal-directed
Disadvantages	Time and memory consuming	Requires suitable heuristic selection

Algorithm	Search Space	Heuristic Function	Time Complexity	Space Complexity	Main Characteristics	General Remarks
BFS – Multi Goal	$O(N \times M \times 2^k) O(N \times M \times 2^k) O(N \times M \times 2^k)$	None	$O(N \times M \times 2^k) O(N \times M \times 2^k) O(N \times M \times 2^k)$	$O(N \times M \times 2^k) O(N \times M \times 2^k) O(N \times M \times 2^k)$	Each state is represented as (current position, remaining set of Dots). BFS must explore all possible state combinations.	Extremely time- and memory-consuming; impractical for large maps or many Dots.
<i>A – Multi Goal (weak heuristic)*</i>	$O(N \times M \times 2^k) O(N \times M \times 2^k) O(N \times M \times 2^k)$	Yes, but weak (e.g., only distance to the nearest	$\approx O(N \times M \times 2^k) O(N \times M \times 2^k) O(N \times M \times 2^k)$ (almost	$\approx O(N \times M \times 2^k) O(N \times M \times 2^k) O(N \times M \times 2^k)$	The heuristic provides poor guidance, so few	Inefficient — performs nearly like BFS.



	$M \times 2k$)	Dot)	same as BFS)	$\times M \times 2k$) (almost same as BFS)	nodes are pruned.	
<i>A – Multi Goal (strong heuristic)*</i>	$O(N \times M \times 2k)O(N \times M \times 2^k)O(N \times M \times 2^k)$	Yes (e.g., total Manhattan distance, MST, Tuấn Anh's heuristic)	$\ll O(N \times M \times 2k)O(N \times M \times 2^k)O(N \times M \times 2^k)$ (significantly reduced in practice)	$\ll O(N \times M \times 2k)O(N \times M \times 2^k)O(N \times M \times 2^k)$	The heuristic effectively directs the search, expanding only promising areas → much faster solution.	Optimal choice for large maps or many Dots.

3 Discuss

3.1 Limitations of the Current Model

Small map size (5×5):

- Limits the ability to evaluate the algorithm's performance in more complex environments.
- Few Dots and walls → does not fully reflect the challenges of real-world problems.

Simple heuristic (total Manhattan distance):

- Not always admissible (may overestimate the true cost).
- Does not account for relationships between Dots (e.g., optimal paths visiting nearby Dots).

No dynamic elements (Ghosts / other agents):

- Static environment with no risks or changes during movement.
- Cannot evaluate the algorithm's ability to react or avoid opponents.



3.2 Future Development Directions

Expand map scale:

- Increase size (e.g., 10×10 , 20×20) to test scalability.
- Add more walls and obstacles to raise complexity.

Integrate multiple agents:

- Introduce Ghosts (chasers) or multiple cooperating Pac-Men.
- Explore cooperative or avoidance strategies (multi-agent pathfinding).

Improve heuristic:

- Use advanced heuristics (e.g., Minimum Spanning Tree among Dots, or Nearest Neighbor Heuristic) for more accurate estimates.
- Combine machine learning–based heuristics for complex environments.

4 Conclusion

The application of the A* algorithm in the Pac-Man game demonstrates its effectiveness in finding optimal paths and enabling intelligent movement control. A* maintains a balance between processing speed and accuracy, allowing characters to find the shortest route in real time. This proves that A* is a suitable choice for AI systems in games, contributing to enhanced strategy, difficulty, and overall player experience.